Quark model calculations of transition form factors at high photon virtualities

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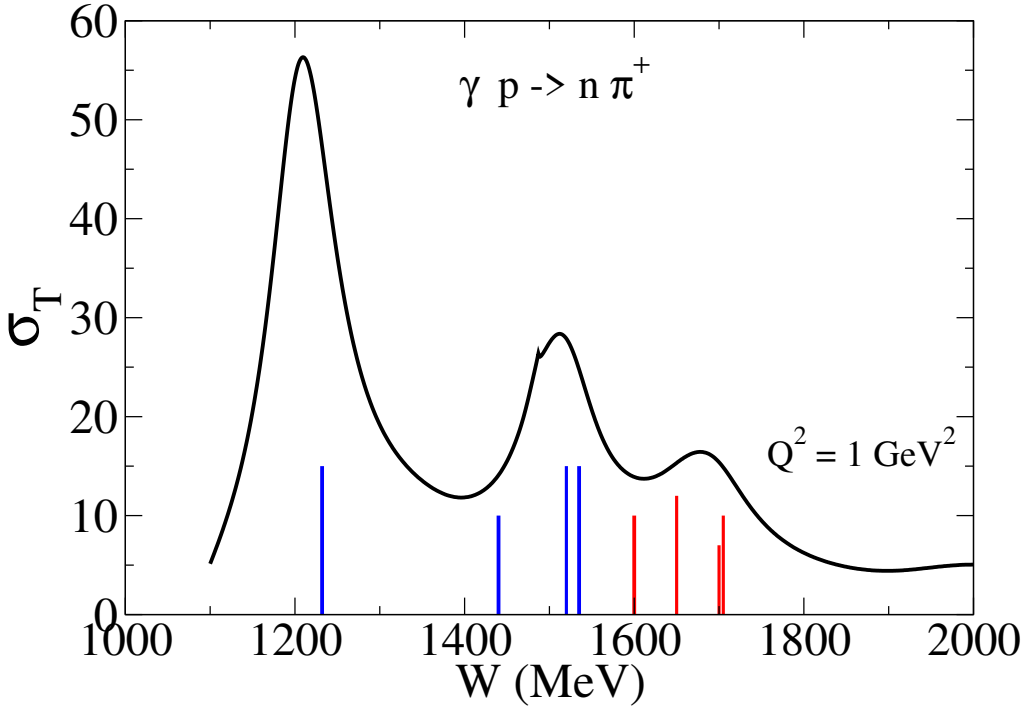
Motivation

- New accurate data from modern accelerators (Jlab, MAMI, ELSA,...) associated with N^* states with increasing W (1.4–1.8 GeV) and large Q^2 (2–6 GeV^2)
- $\Rightarrow$ **New challenges**:
  - Interpret the data $\leftrightarrow$ Theory $\leftrightarrow$ relativistic models
  - Make predictions: higher W, higher Q^2 – **Jlab-12 GeV–upgrade**

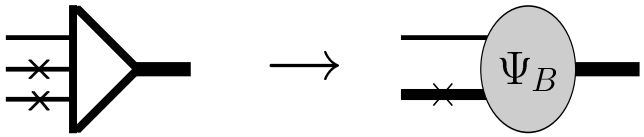


Plan of the talk

- **Theoretical framework**
  Covariant Spectator Quark Model
- **Calculations of N^* transition form factors at large Q^2**
  $\Delta(1232)\frac{3}{2}^+$, $N(1440)\frac{1}{2}^+$, $N(1535)\frac{1}{2}^-$, $N(1520)\frac{3}{2}^-$, $\Delta(1600)\frac{3}{2}^+$
  ... include results from
  Single Quark Transition model
  $N(1650)\frac{1}{2}^-$, $N(1700)\frac{3}{2}^-$, $\Delta(1620)\frac{1}{2}^-$, $\Delta(1700)\frac{3}{2}^-$
- ... some results at low-Q^2
- **Summary and conclusions**

Covariant Spectator Quark Model – Introduction (1)

- Baryon: 3 constituent quark system
- Covariant Spectator Theory: wave function Ψ_B defined in terms of a 3-quark vertex Γ with 2 on-mass-shell quarks – integrate into quark-pair degrees of freedom



$$\int_{k_1}\int_{k_2} = \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s-4m_q^2}{s}} \int \frac{d^3\mathbf{k}}{2\sqrt{s+\mathbf{k}^2}}$$

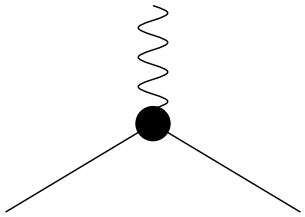
Mean value theorem: $s = (k_1+k_2)^2 \to m_D^2$; effective diquark mass m_D

Gross and Agbakpe PRC 73, 015203 (2006); Gross, GR and Peña PRC 77, 015202 (2008)

- $\Rightarrow$ reduction to a quark-diquark structure: $\Psi_B(P_B,k)$
  Baryon wave function $\Psi_B(P_B,k)$ free of singularities

  Stadler, Gross and Frank PRC 56, 2396 (1998); Savkli and Gross PRC 63, 035208 (2001)
- Spin-flavor structure $\approx$ relativistic $SU_F(3)\times SU_S(2)$ structure
- Radial wave function $\psi_B(P_B,k)$ determined phenomenologically
  Not a solution of a dynamical wave equation – mass $M_B \equiv M_B^{\rm exp}$
  Shape determined by momentum scale parameters using experimental data or lattice data of some ground state systems
- $\Psi_B(P_B,k)$ defined at rest frame; generalized **covariantly** to an arbitrary frame using Lorentz transformations

Covariant Spectator Quark Model – Introduction (2)

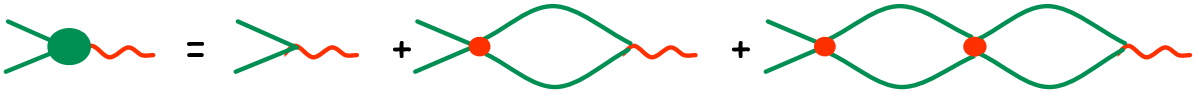


- Quarks with electromagnetic structure (**impulse approximation**)

$$j_q^\mu = \left(\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-}\tau_3\right)\gamma^\mu + \left(\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-}\tau_3\right)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}$$

form factors $f_{i\pm}$ parametrize dressing of quarks (gluons and $q\bar{q}$) $\kappa_q = f_{2q}(0) \approx 2$

- Vector meson dominance parameterization at quark level:



Quark current parametrized in terms of vector meson poles (m_v, M_h)

F Gross, GR, MT Peña, PRC 77, 015202 (2008); GR, MT Peña, JPG 36, 115011 (2009); PRD 80, 013008 (2009); GR, K Tsushima, AW Thomas, JPG 40, 015102 (2013); GR, K Tsushima, F Gross, PRD 80, 033004 (2009)

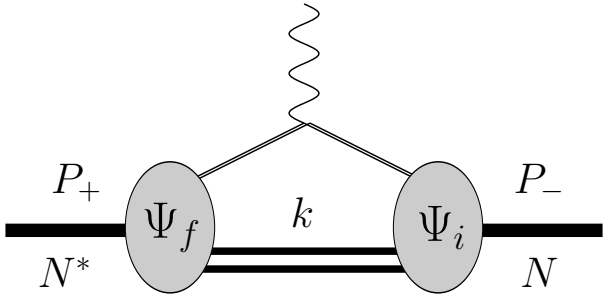
- 4 parameters determined by the fit to the **nucleon data**

F Gross, GR, MT Peña, PRC 77, 015202 (2008)

Covariant Spectator Quark Model – Introduction (3)

- Transition current – relativistic impulse approximation

$$J^\mu = 3 \sum_\lambda \int_k \bar{\Psi}_f(P_+, k) j_q^\mu \Psi_i(P_-, k)$$



diquark on-shell

- Quark current j_q^μ and nucleon radial wave function $\psi_N(P_N, k)$ callibrated by **nucleon elastic form factor data**

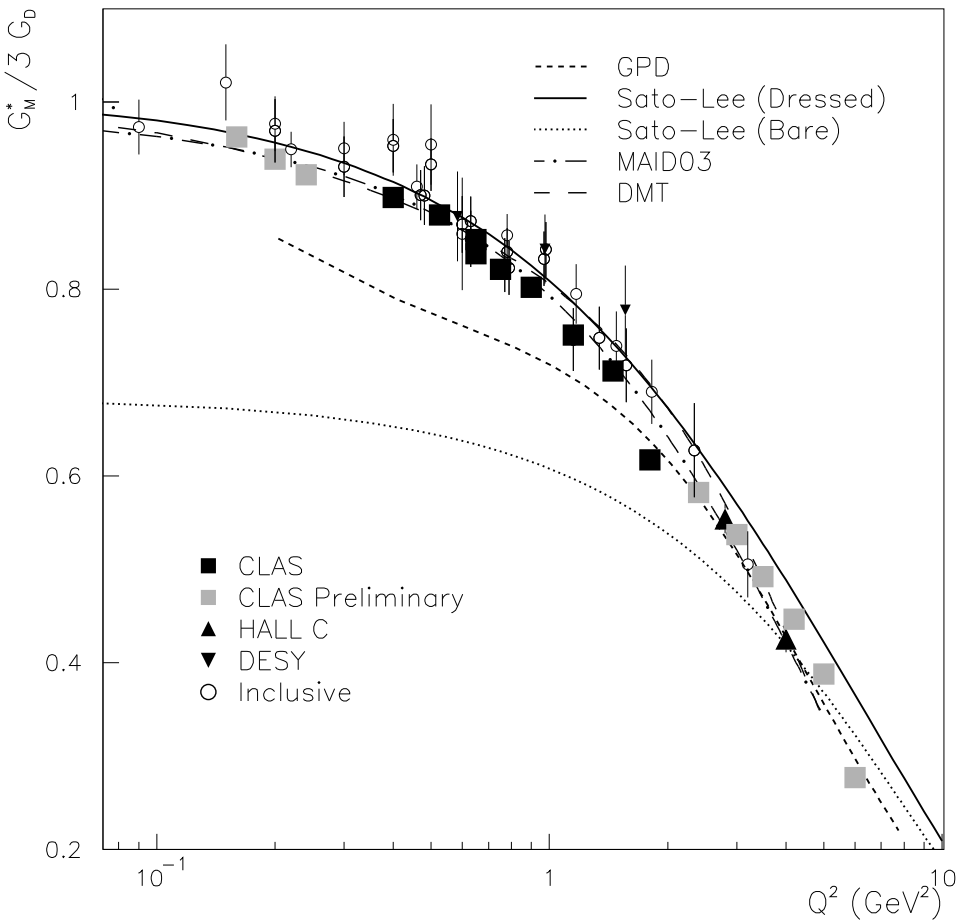
  F Gross, GR, MT Peña, PRC 77, 015202 (2008)

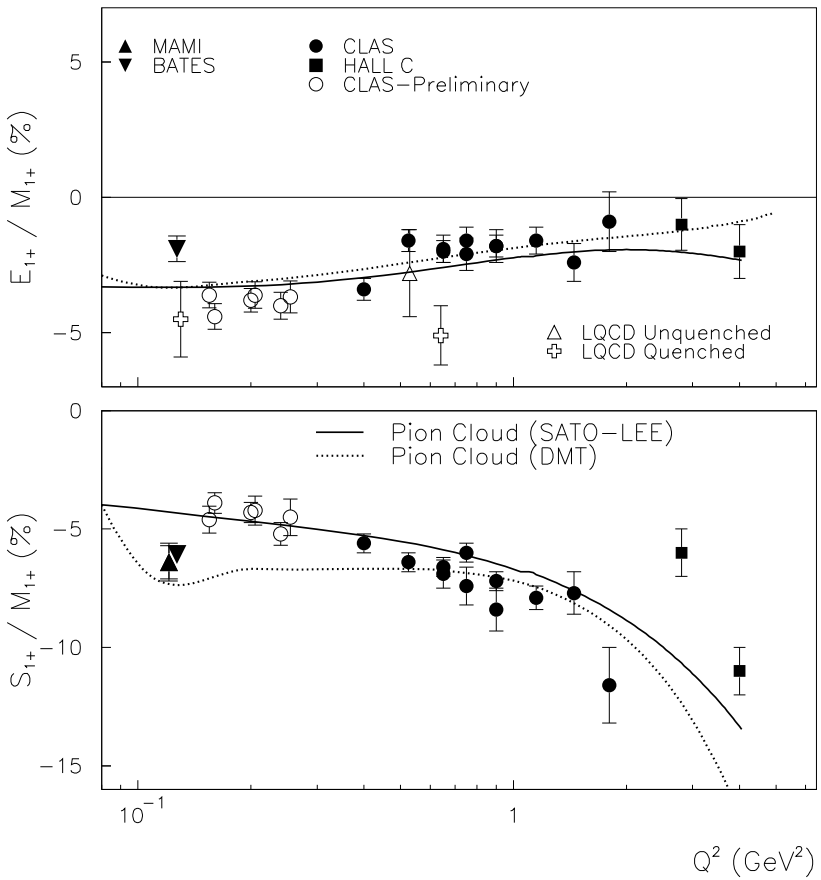
- **Generalization to lattice QCD:**
  - $f_{i\pm}(Q^2; m_\rho, M_N) \to f_{i\pm}(Q^2; m_\rho^{\rm latt}, M_N^{\rm latt})$ – VMD
  - $\psi_B(M_B) \to \psi_B(M_B^{\rm latt})$

  GR, MT Peña, JPG 36, 115011 (2009); PRD 80, 013008 (2009); GR, K Tsushima, F Gross, PRD 80, 033004 (2009); GR, K Tsushima, AW Thomas, JPG 40, 015102 (2013)

Calculations of N^* transition form factors at large Q^2

$\gamma^* N \to \Delta(1232)$ – Introduction (Review: Burkert and H Lee)





- Transition dominated by G_M^*: $N(\uparrow\uparrow\downarrow) \longrightarrow \Delta(\uparrow\uparrow\uparrow)$ (spin-flip)
- Small contributions from G_E^* and $\frac{|\mathbf{q}|}{2M_\Delta} G_C^*$; indication of (small) Δ deformation
  GR, MT Peña, A Stadler, PRD 86, 093022 (2012)
- G_M^* usually underestimated by quark models at small Q^2

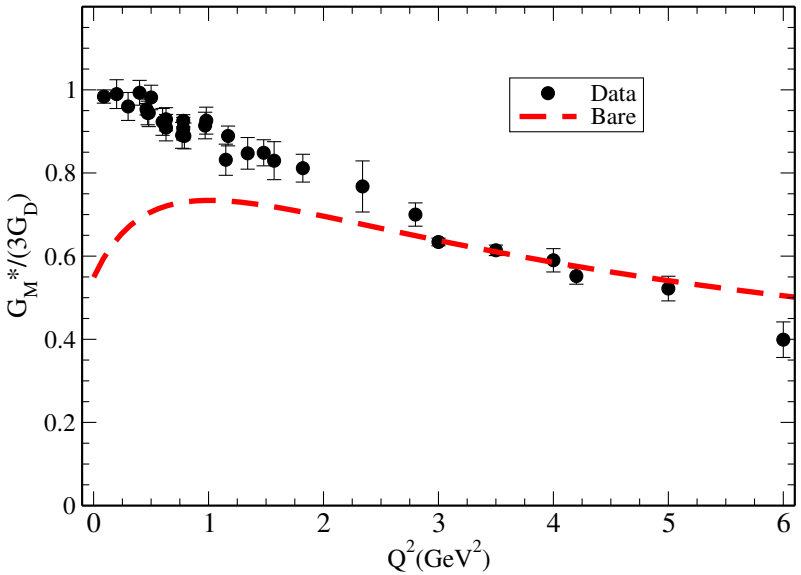
$\Delta(1232)$ - S-wave model - covariant

$$\Psi_N = \frac{1}{\sqrt{2}} \overbrace{\left(\phi_I^0 \phi_S^0 + \phi_I^1 \phi_S^1\right)}^{SU(6)} \psi_N(P,k) \qquad \Psi_\Delta = -\overbrace{\tilde{\phi}_1^I (\varepsilon_P^*)_\alpha u^\alpha(P)}^{SU(6)} \psi_\Delta(P,k)$$

Radial wave functions: $\psi_N(P,k)$; $\psi_\Delta(P,k)$

$\psi_N(P,k)$ **determined by the nucleon data; How to determine** $\psi_\Delta(P,k)$?

ψ_Δ can be determined by lattice data and/or estimates from Dynamical Mod. (core) EBAC/Argonne-Osaka



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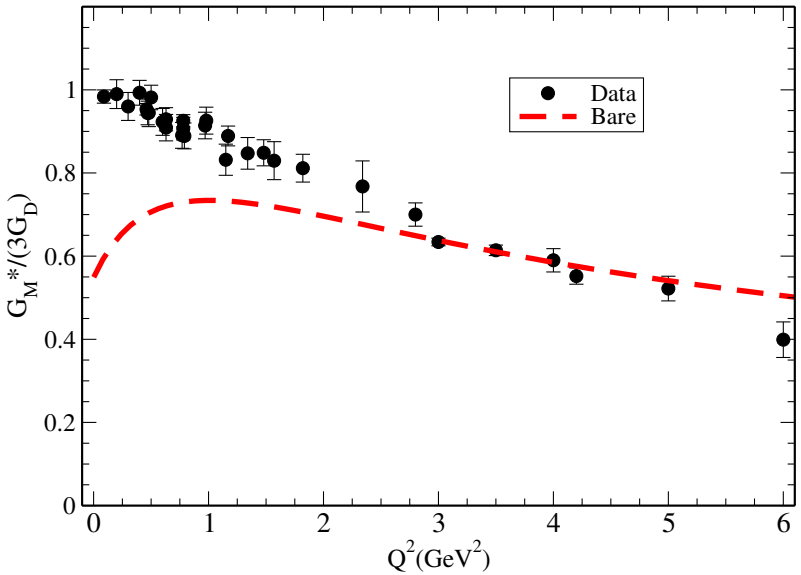
EBAC/Argonne-Osaka **Note that** $Q^2 \approx 0$: $G_M^B(Q^2) \approx 2 \lesssim 3$



$\Delta(1232)$ - S-wave model – underestimation of G_M^*

Understanding the underestimation from Quark Models:

GR, MT Peña, F Gross EPJ A36, 329 (2008)

CSQM – S-wave : $G_E^* = G_C^* = 0$

$$G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} \left(f_{1-} + \frac{M_\Delta + M}{2M} f_{2-} \right) \int_k \psi_\Delta \psi_N$$

Then $G_M^*(0) = 2.07 \int_k \psi_\Delta \psi_N$

Normalization conditions $\oplus$ Cauchy-Schwarz inequality: $\int_k \psi_\Delta \psi_N \leq 1$:

$G_M^*(0) \leq 2.07 \lesssim 3$ (experimental result)

Conclusion:

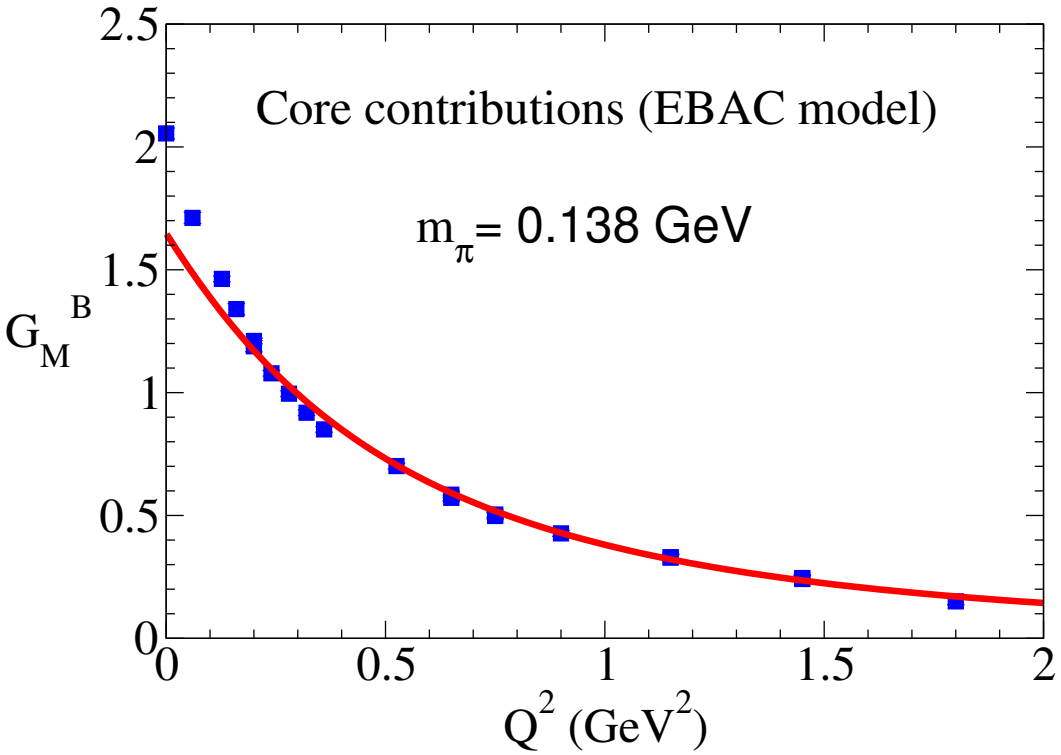
CSQM: **natural explanation** for the underestimation of G_M^*

Missing contributions: pion cloud mechanism

Kamalov, Yang, PRL 83, 4494 (1999); Sato, Lee, PRC 63, 055201 (2001); Diaz et al, PRC 80, 025207 (2009)

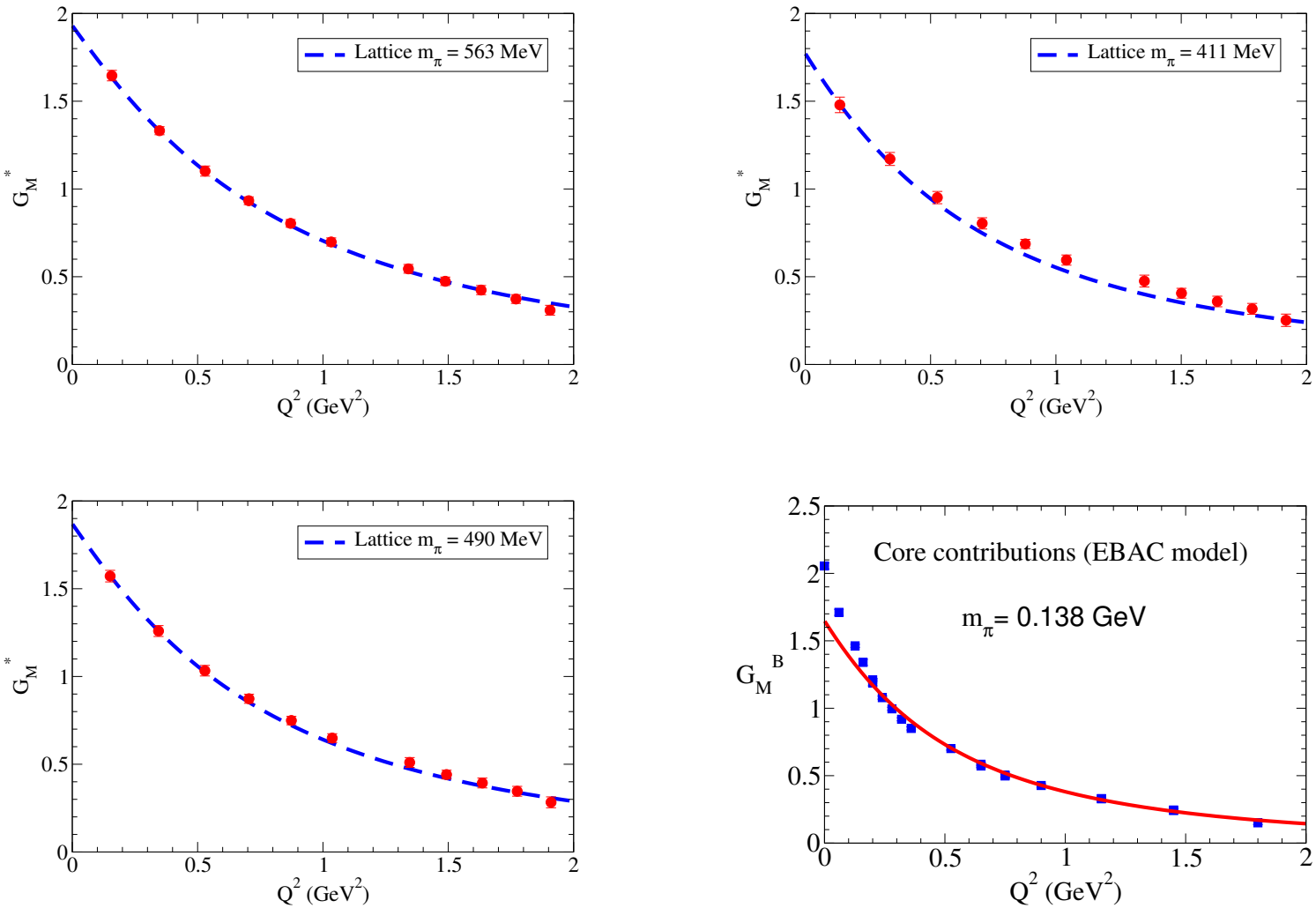
$\Delta(1232)$ - S-wave model – adjusting $\psi_\Delta(P,k)$

GR, F Gross, MT Peña, EJPA 36, 329 (2008); PRD 78, 114017 (2008);
GR, MT Peña, PRD 80, 013008 (2009)



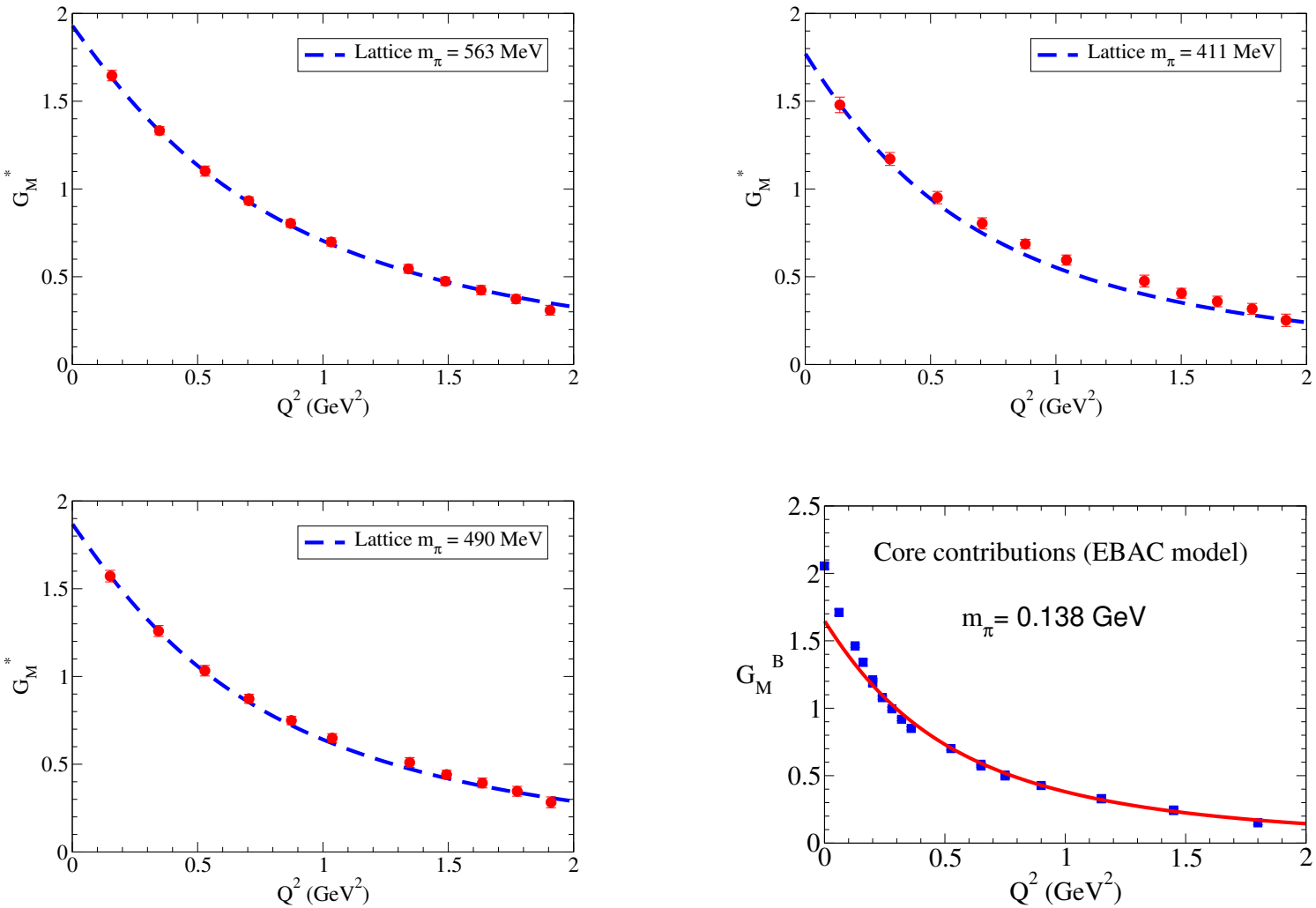
- CSQM fitted to the EBAC *bare* data – adjust $\psi_\Delta(P,k)$
- CSQM describe well the EBAC *bare* results J Diaz et al, PRC 80, 025207 (2009)

$\gamma^* N \to \Delta$: G_M^* in lattice [PRD 80, 013008 (2009)]



- - - - CSQM extended to lattice QCD ($m_\pi = 411$, 490, 563 MeV)
- **Good** description of the lattice data based on EBAC parameters

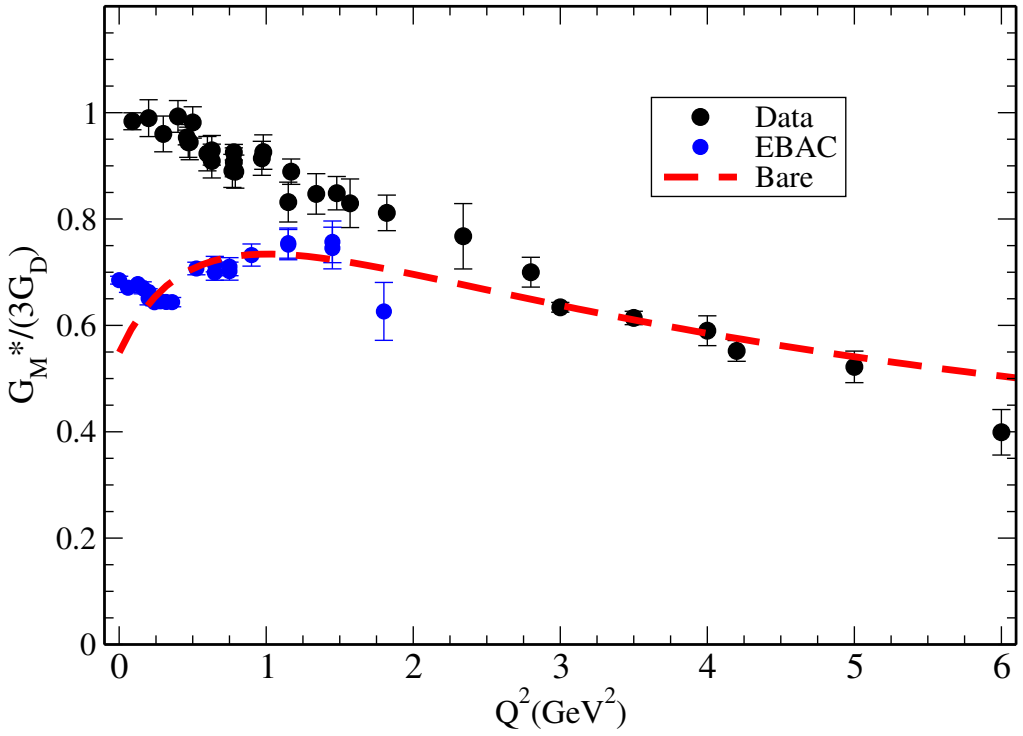
$\gamma^* N \to \Delta$: G_M^* in lattice [PRD 80, 013008 (2009)]



- - - - CSQM extended to lattice QCD Alexandrou, PRD 77, 085012 (2008)
- **Good** description of the lattice data based on EBAC parameters

$\gamma^* N \to \Delta$: G_M^* (valence), physical case

GR and MT Peña PRD 80, 013008 (2009) $G_D = \left(1 + \frac{Q^2}{0.71}\right)^{-2}$



- CSQM explains well lattice data and EBAC estimate (core)
- **How to simulate the effect of the pion cloud ?**

$\gamma^* N \to \Delta$: G_M^* (valence), physical case

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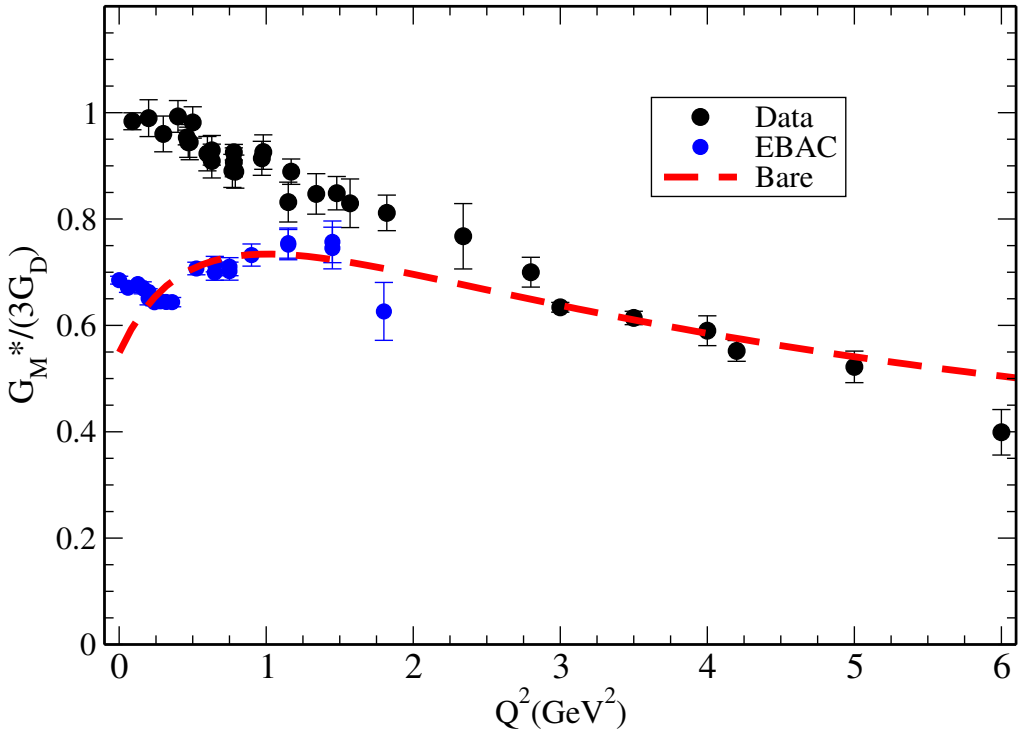


- Good description of the valence quark effects
- **How to simulate the effect of the pion cloud ?**

$\gamma^* N \to \Delta$: How to simulate the pion cloud? [phenomenology]

Recent application – two pion cloud contributions

Motivated by study of the **octet to decuplet transitions**: 50% – 50%

GR, K Tsushima, PRD 88, 053002 (2013)

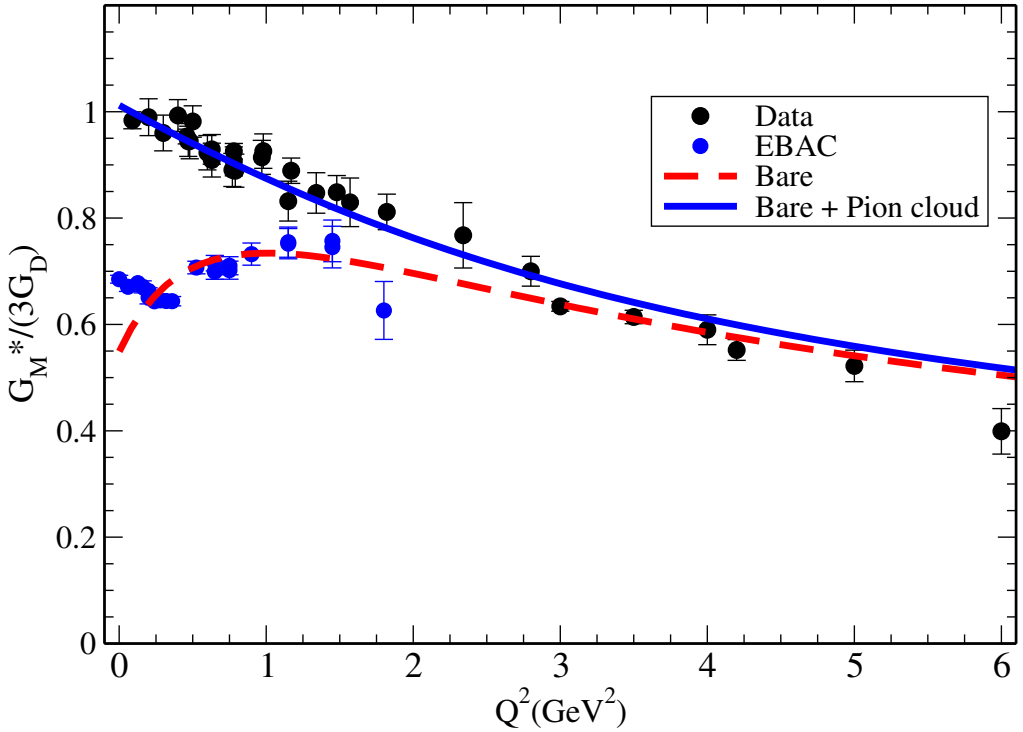
$$G_M^\pi = 3\frac{\lambda_\pi}{2}\underbrace{F_\pi(q^2)\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - q^2}\right)^2}_{\pi-\text{coupling}} + 3\frac{\lambda_\pi}{2}\underbrace{\left(\frac{\Lambda_D^2}{(\Lambda_D^2 - q^2)^2 + \Lambda_D^2(\Gamma_D(q^2))^2}\right)^2}_{B'-\text{coupling}}$$

GR, MT Peña J Weil, H van Hees and U Mosel, PRD 93, 033004 (2016)

$F_\pi(q^2)$: phenomenologic electromagnetic pion form factor;

$\Lambda_D^2 = 0.9$ GeV^2; $\Gamma_D(q^2)$ phenomenological width; $\lambda_\pi = 0.448$

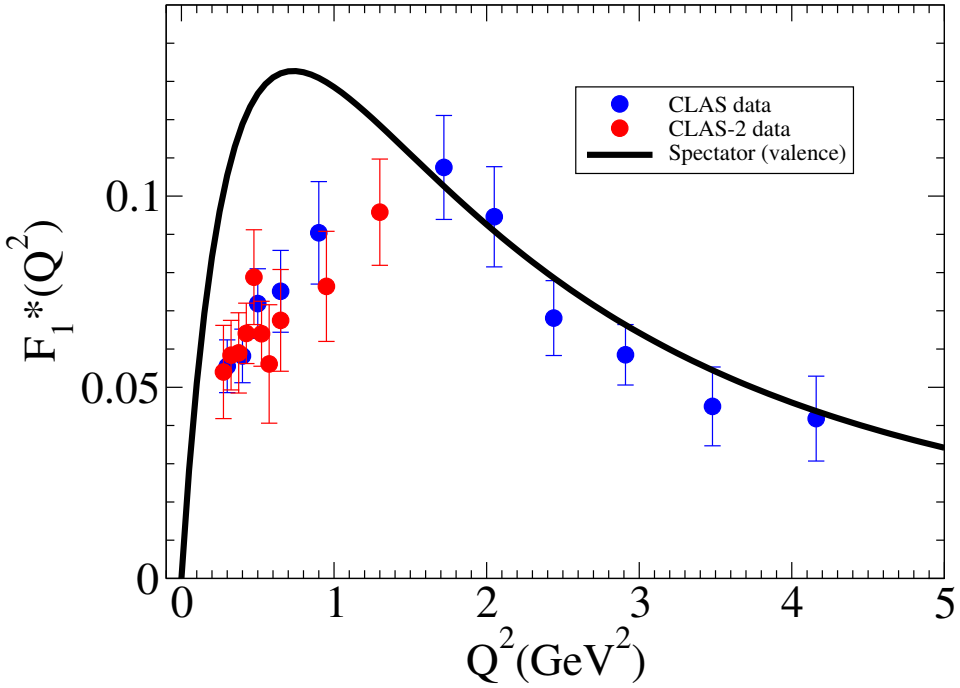
$\gamma^* N \to \Delta$: G_M^* (valence + pion cloud) [phenomenological]

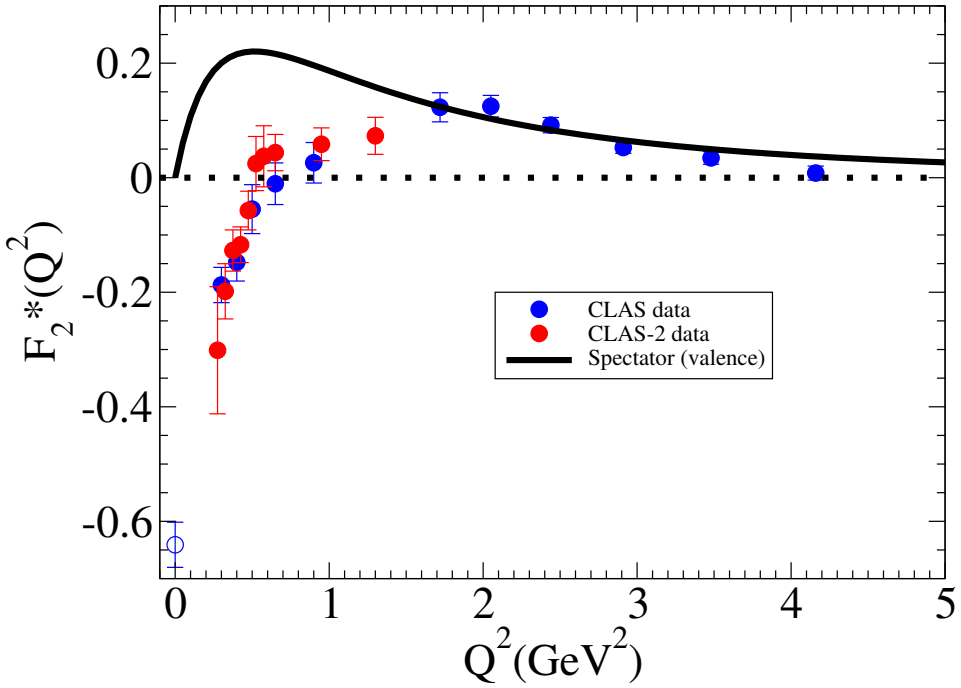


- **Good** description of the **physical data** including the large Q^2 data
- Extension to **timelike** transition – HADES – PRC 95, 065205 (2017)

  GR, MT Peña J Weil, H van Hees and U Mosel, PRD 93, 033004 (2016)

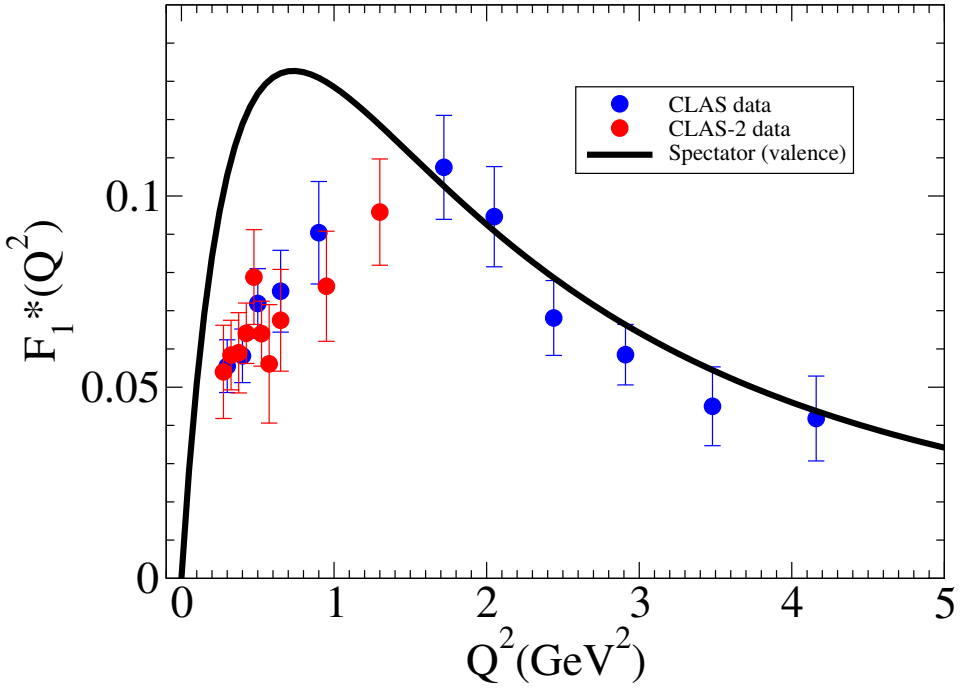
$\gamma^* N \to N(1440)$ – Introduction

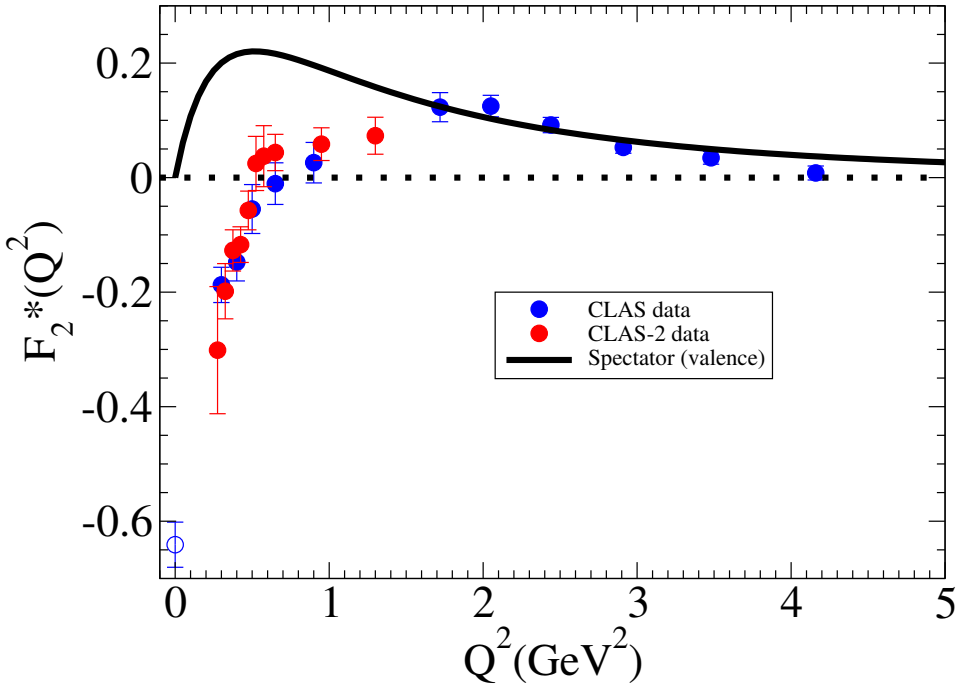




- **CSQM:** Roper defined as the **1st radial** excitation of the nucleon
  Same spin/flavor structure as the nucleon
  Radial wave function defined by the orthogonality with nucleon state
  GR and K Tsushima, PRD 81, 074020 (2010); PRD 89, 073010 (2014)
- No adjustable parameters; **No** meson cloud components included
- **CLAS data:** IG Aznauryan et al., PRC 80, 055203 (2009);
  VI Mokeev et al., PRC 86, 035203 (2012); PRC 93, 025206 (2016)

$\gamma^* N \to N(1440)$ – Results

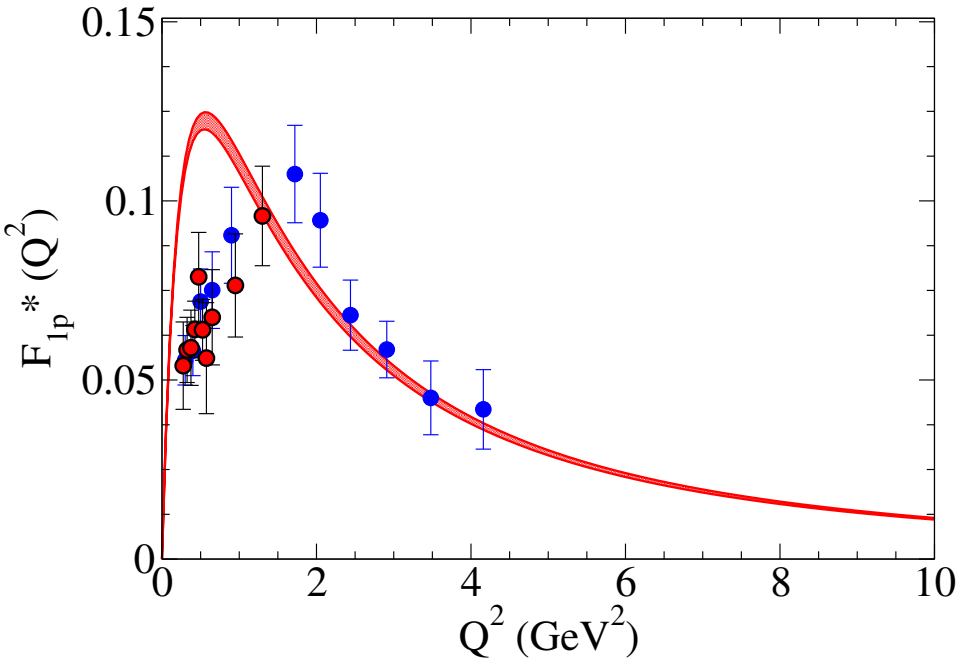


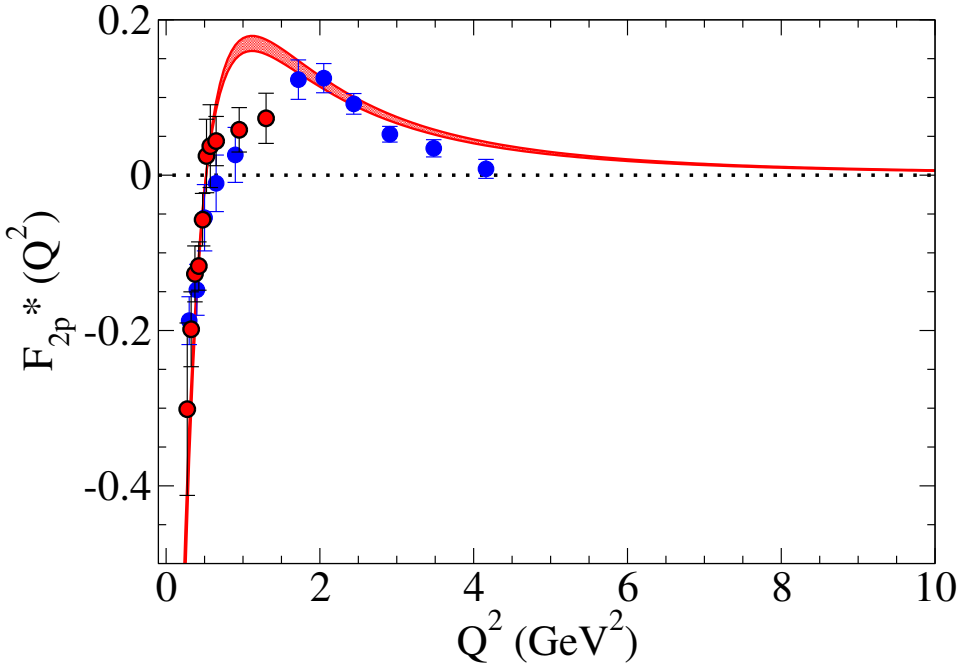


- **Good results** for $Q^2 > 1.5$ GeV^2 – valence quark dominance
  Support **Roper** as **1st radial** excitation of the nucleon
- Failure for $Q^2 < 1.5$ GeV^2 – meson cloud ?
  Used to estimate meson cloud from CLAS data – infered MC
  GR and K Tsushima, AIP Conf.Proc. 1374, 353 (2011)

$\gamma^* N \to N(1440)$ – Holographic estimate

Recent progress from AdS/QCD – Roper form factors





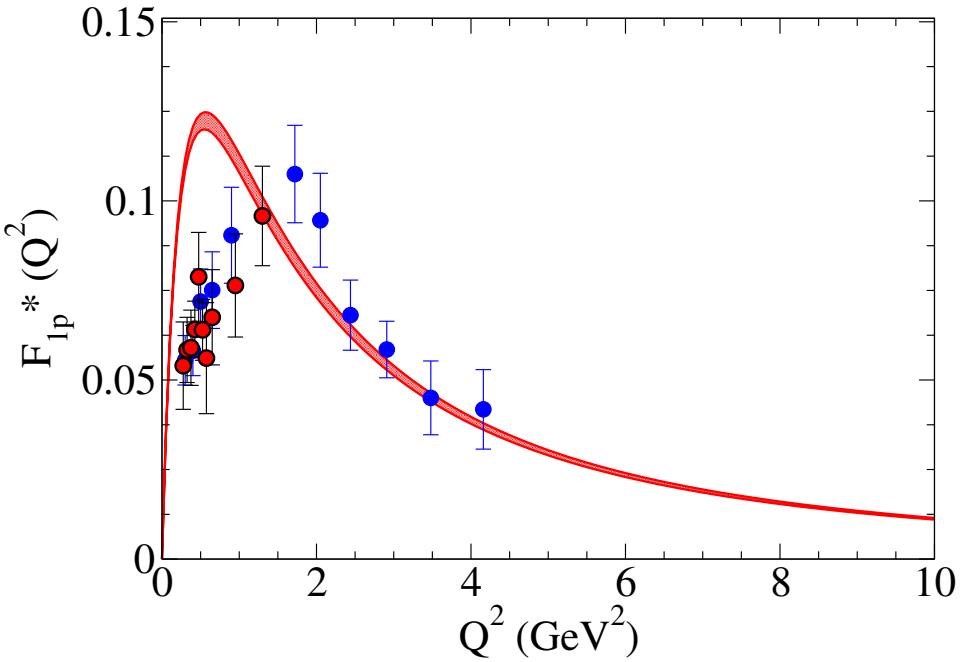
GR, D Melnikov, PRD 97, 034037 (2018); GR, PRD 96, 054021 (2017)

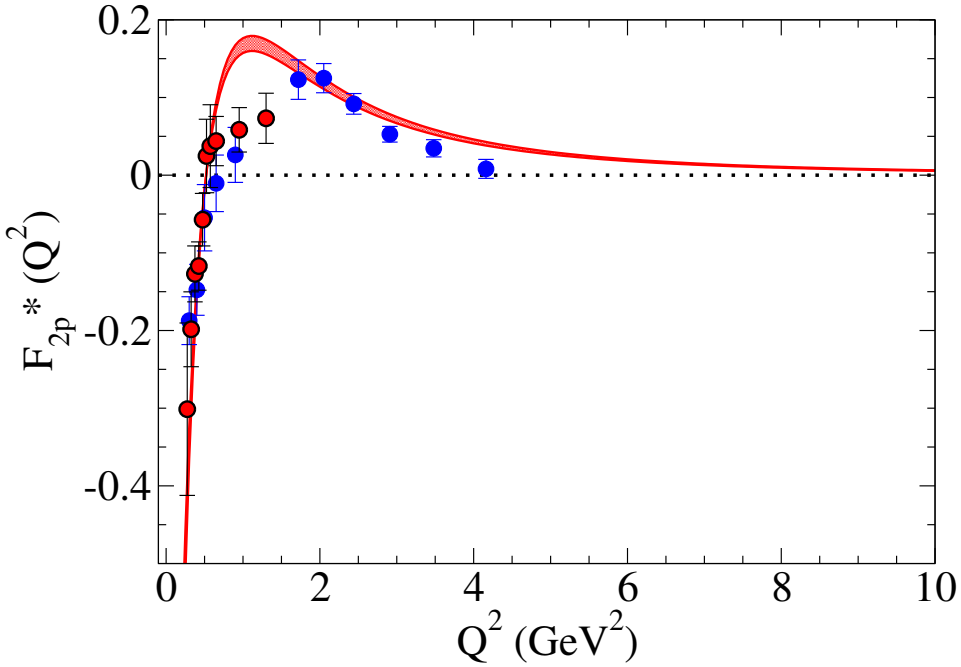
Valence quark approximation – leading twist: 3 adjustable couplings

Parameters adjusted to **nucleon data**: Roper – red band

$\gamma^* N \to N(1440)$ – Holographic estimate

Recent progress from AdS/QCD – Roper form factors



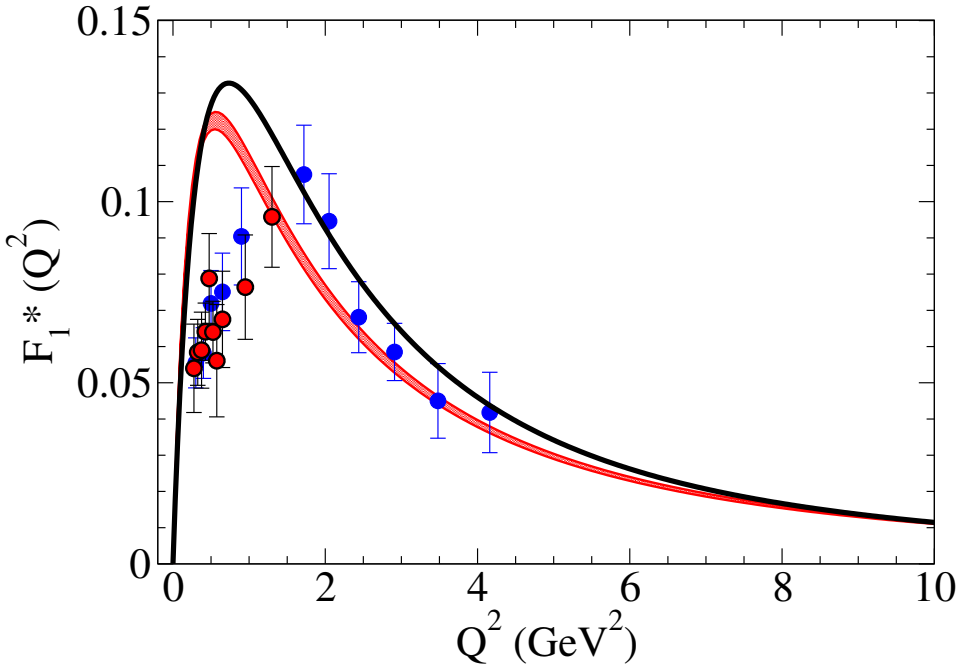


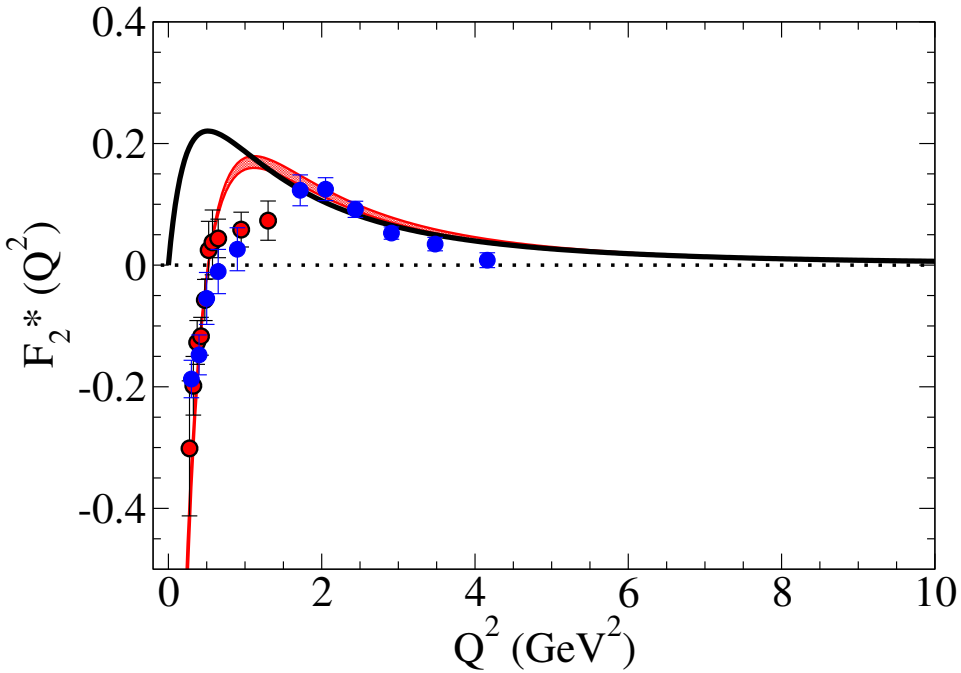
GR, D Melnikov, PRD 97, 034037 (2018); GR, PRD 96, 054021 (2017)

Very good result for F_2^* – **suggest** small meson cloud effects

Very promising method to estimate **valence quark effects** at low Q^2

$\gamma^* N \to N(1440)$ – Comparing results

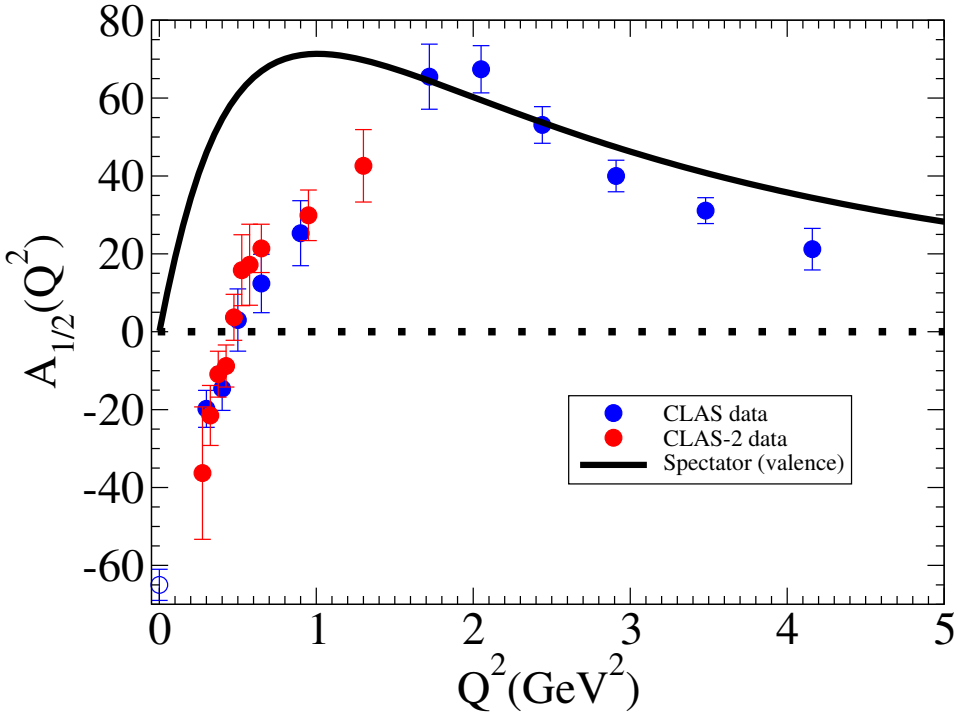


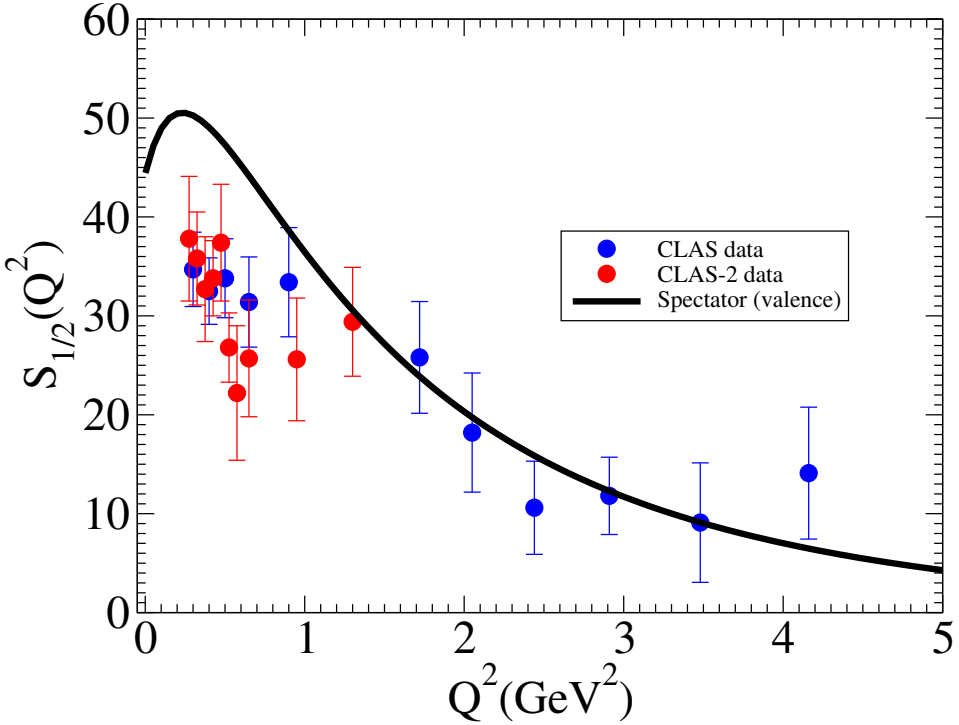


—- CSQM ===== Holography

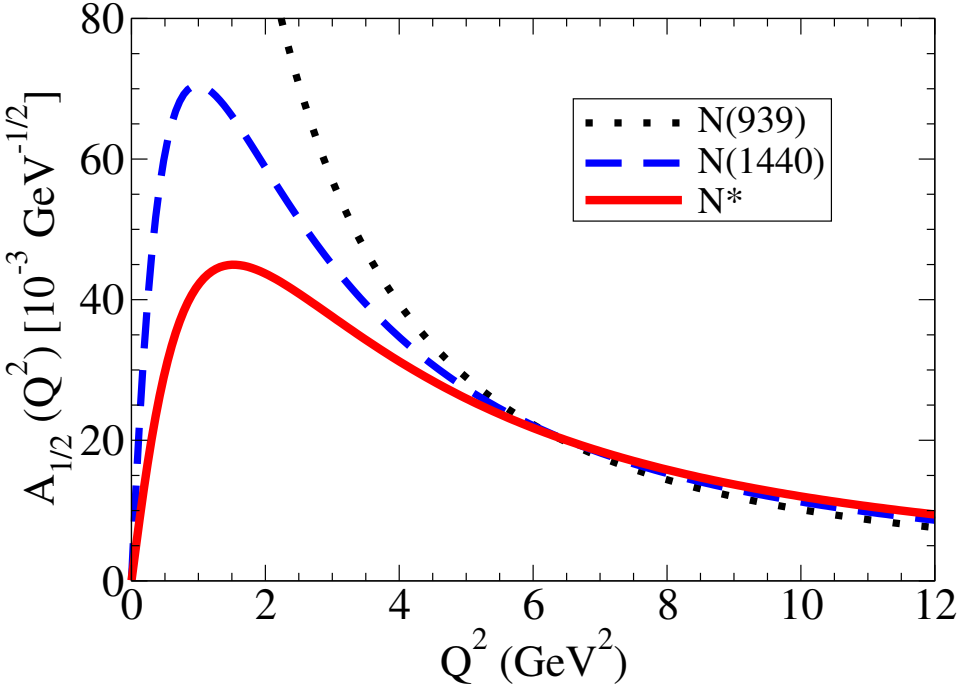
Very similar results at large Q^2, based on very different approximations

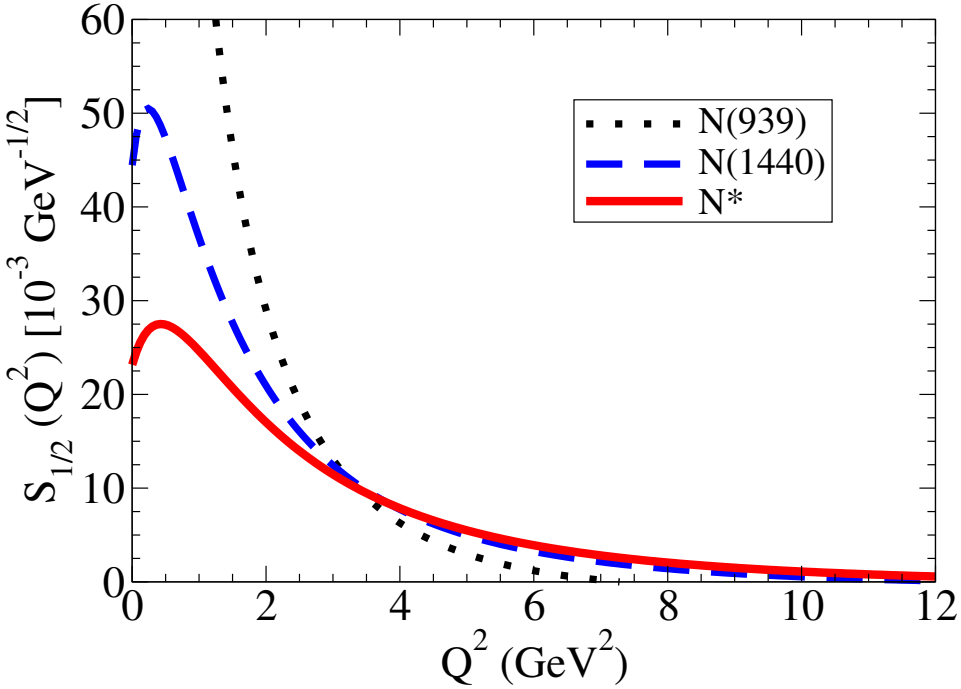
$\gamma N \to N(1440)$ – Helicity amplitudes ††





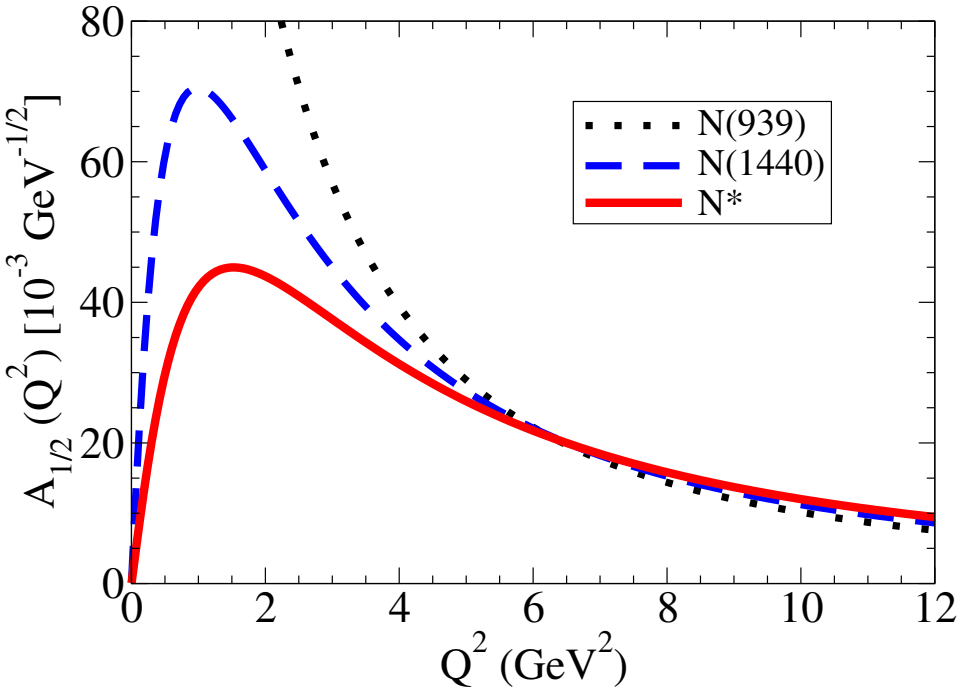
$\gamma^* N \to N^*$ – 2nd radial excitation of the nucleon

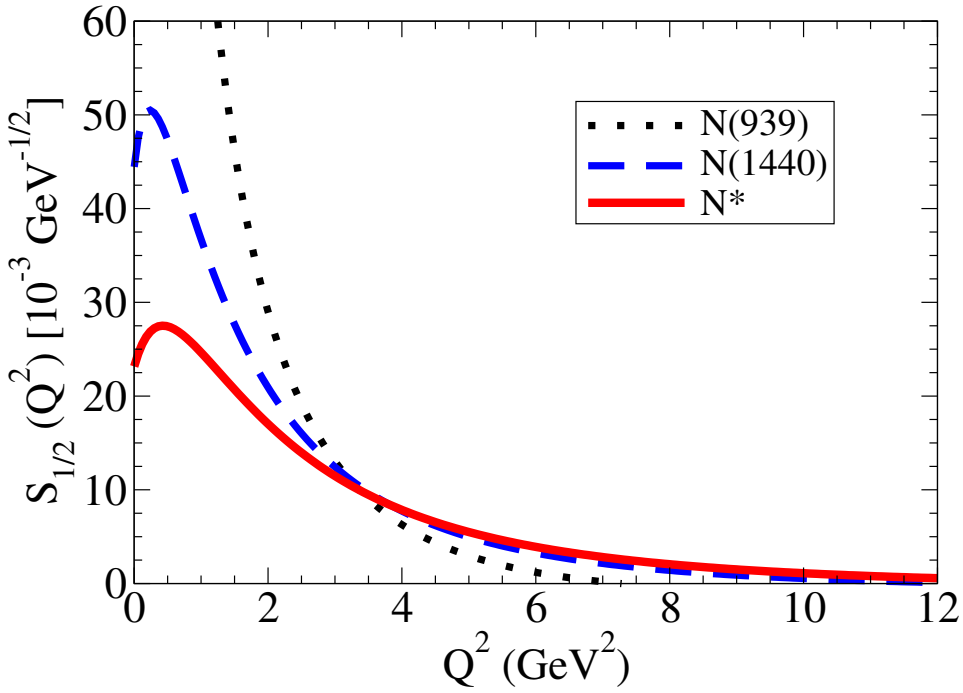




1st assumption $N^* = N(1710)$ not confirmed ($\neq$ radial structure);
Next candidate $N(1880)$; Comparison with Roper and Nucleon data

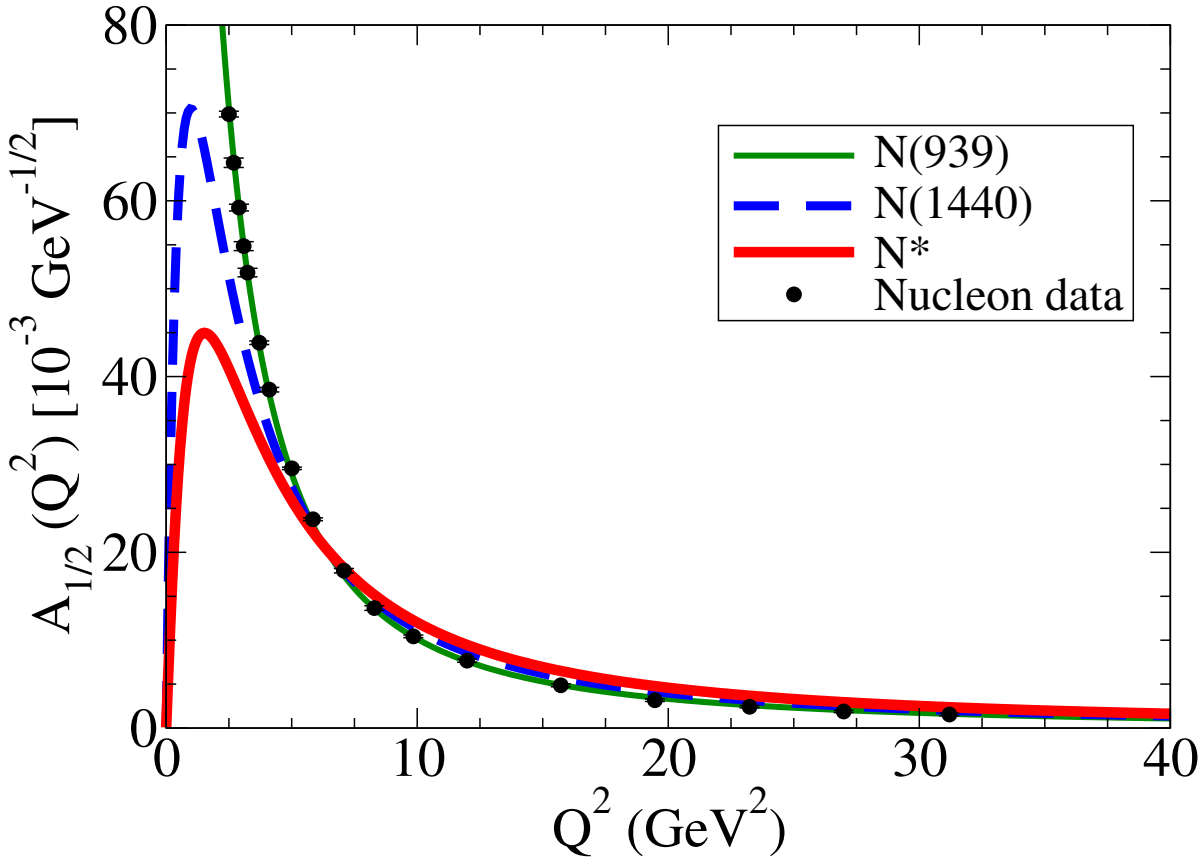
$\gamma^* N \to N^*$: Helicity amplitudes [GR and K Tsushima, PRD 89 073010 (2014)]





- Results compared with Roper and Nucleon ($A_{1/2} \propto G_M$; $S_{1/2} \propto G_E$)
- Large Q^2: similar results for $A_{1/2}$ and $S_{1/2}$ (except nucleon: $G_E(7\text{GeV}^2) \approx 0$); Same short range structure !!

Prediction: Amplitude $A_{1/2} \propto G_M$ (proton): Roper and N^*



Data: J. Arrington, W. Melnitchouk and J. A. Tjon, PRC **76**, 035205 (2007)

$\gamma^* N \to \Delta(1600)$ [GR and K Tsushima, PRD 82, 073007 (2010)]

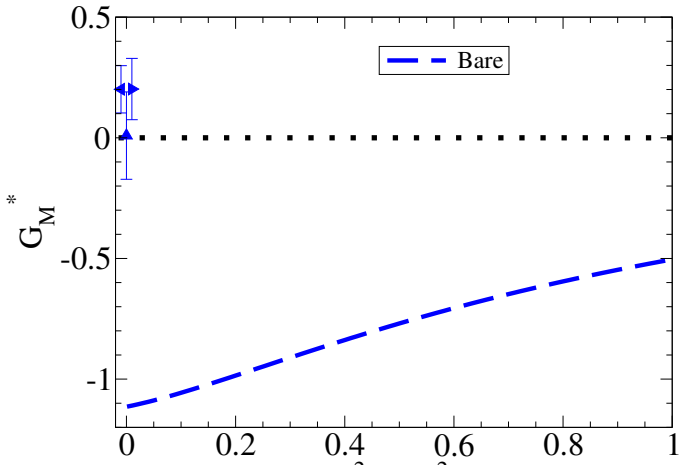
$\Delta(1600)$ as the 1st radial excitation
of $\Delta(1232)$ EPJA, 36, 329 (2008) [S-state]
$G_E^* \equiv 0$, $G_C^* \equiv 0$

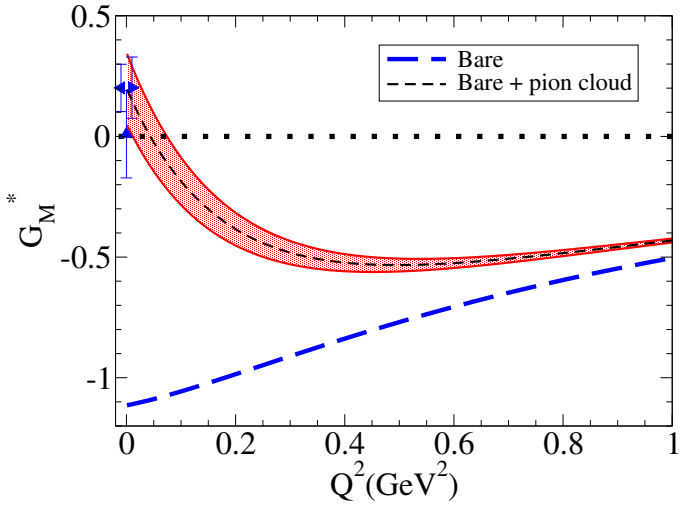
$$\text{Bare}: \quad G_M^B(0) = -1.113$$

Valence quarks **insuficient** to explain data

π cloud effects: estimated w/ $M_{\text{octet}} = M_{\text{decuplet}}$

Decay	BR
$\Delta(1600) \to \pi N$	0.153±0.019
$\Delta(1600) \to \pi \Delta$	0.590±0.100
$\Delta(1600) \to \pi N(1440)$	0.130±0.040





Final result consistent with $Q^2 = 0$ data
Predictions for large Q^2

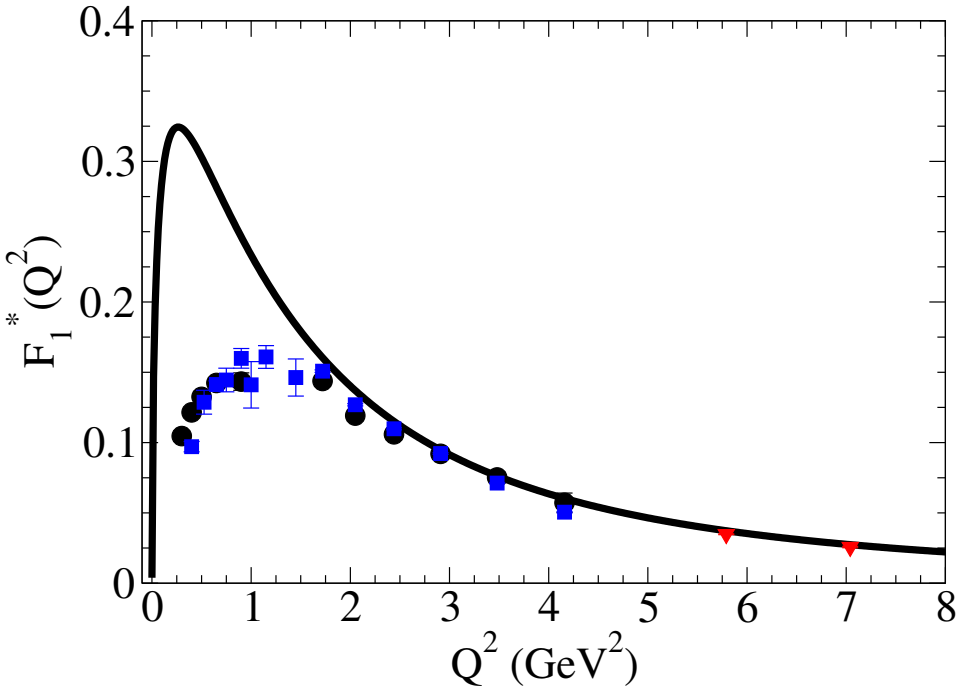
$\gamma^* N \to N(1535)\frac{1}{2}^-$ and $\gamma N^* \to N(1520)\frac{3}{2}^-$

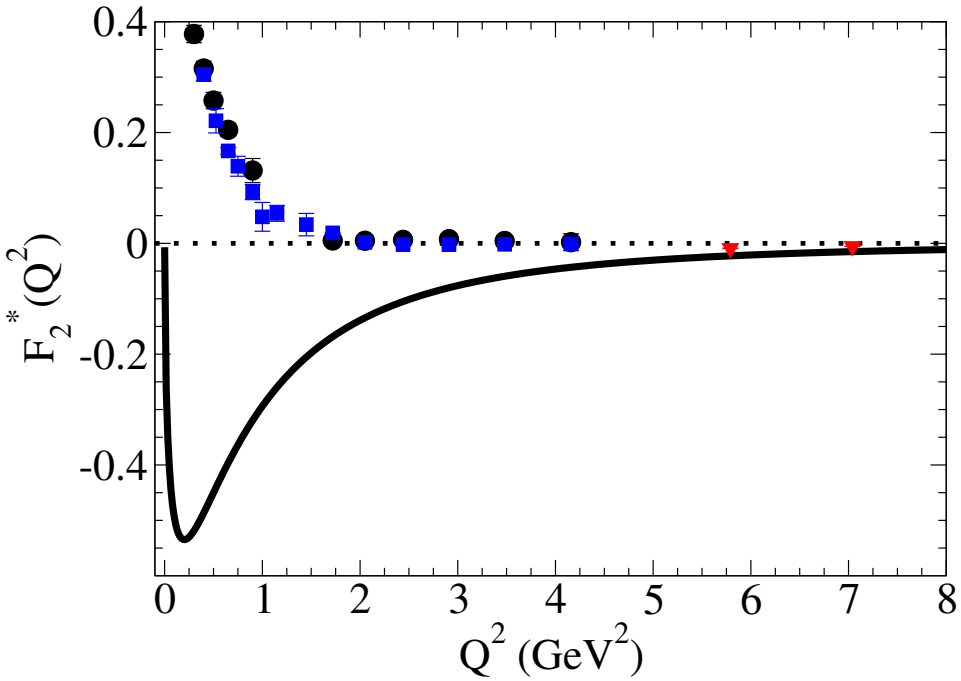
- Negative parity states
- Valence quark contributions estimated within the CSQM framework
  - $N(1525)$ GR and MT Peña, PRD 84, 033007 (2011)
    Estimates valid for large Q^2 ($Q^2 \gg 0.2$ GeV^2)
  - $N(1520)$ GR and MT Peña, PRD 89, 094016 (2014);
    GR and MT Peña, PRD 95, 014003 (2017) – timelike region – Talk B Ramstein
    Radial function ψ_R modified in order to ensure orthogonality

Recent development – **Semirelativistic approximation**
GR, PRD 95, 054008 (2017)

- Mass difference (M_R and M_N) neglected in a first approximation
- Radial wave function determined by ψ_N (nucleon)
  Non-relativistic properties; Covariant expressions
- Orthogonality ensured
- Form factors determined without **any** adjustable parameter
  Model parameters determined by **Nucleon** system

$\gamma^* N \to N(1535)$ – Results





- 2 form factors; Data from **CLAS**, MAID and Jlab/Hall C
- Expected result at $Q^2 = 0$: $F_i^*(0) = 0$
- Good results for F_1^* ($Q^2 > 1.5$ GeV^2); consequence of meson cloud ?
- F_2^* wrong sign; $(F_2^*)_{\rm exp} \approx 0$ for $Q^2 > 1.5$ GeV^2

$\gamma^* N \to N(1535)$: Relation between $A_{1/2}$ and $S_{1/2}$

Implications of $F_2^* = 0$?

$\tau = \frac{Q^2}{(M_R+M_N)^2}$ $Q^2 > 1.5$ GeV^2

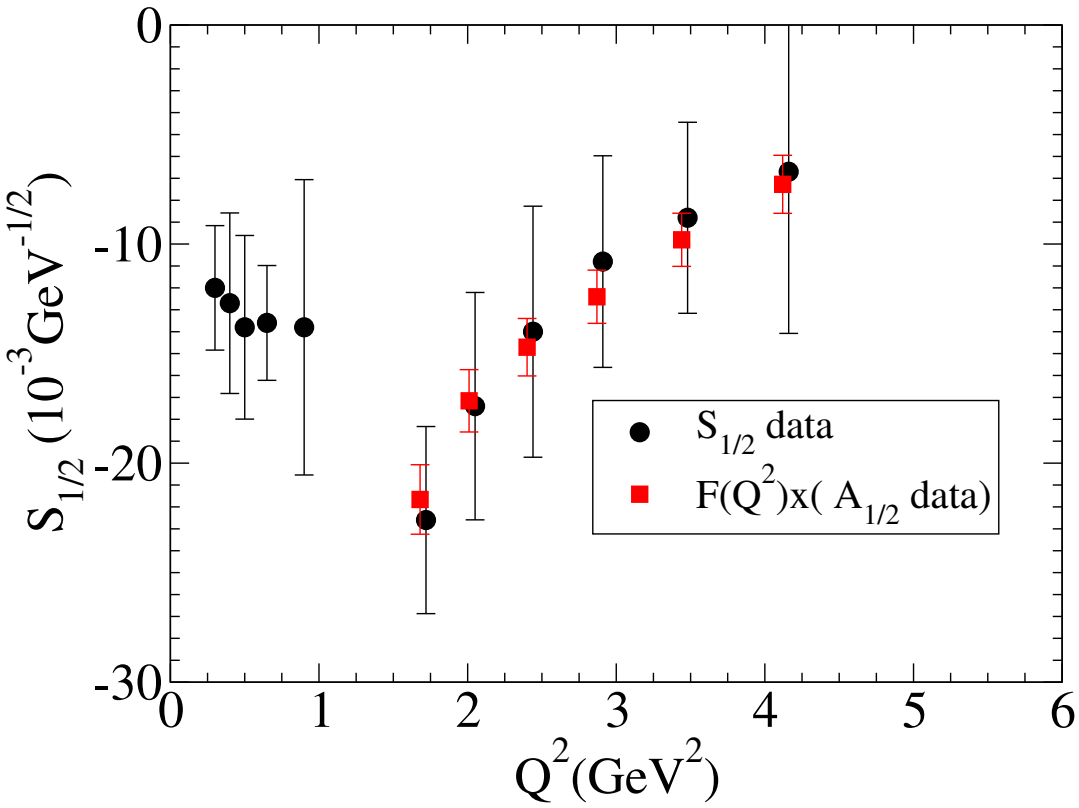
$$S_{1/2} \simeq -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_R^2 - M_N^2}{2M_R Q} A_{1/2}$$

Cancellation between valence and meson cloud

GR, K Tsushima, PRD 84, 051301 (2011)

GR, D Jido, K Tsushima, PRD 85, 093014 (2012)

D Jido, M Doring and E Oset, PRC 77, 065207 (2008)



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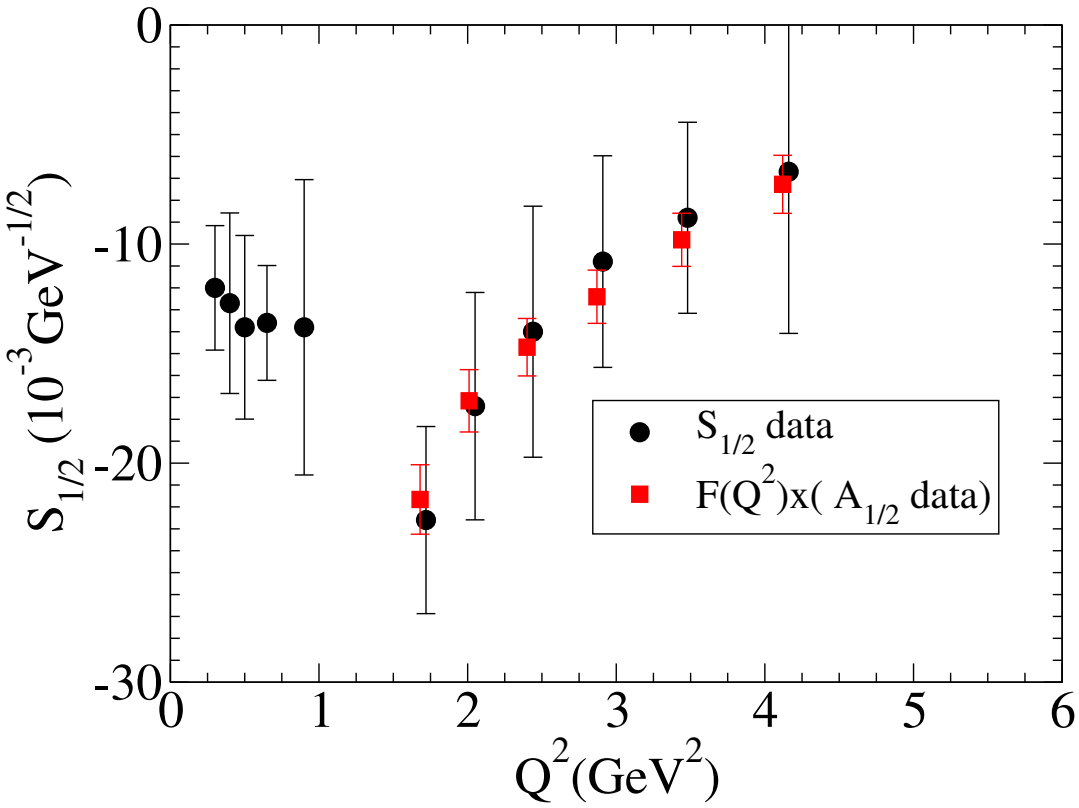
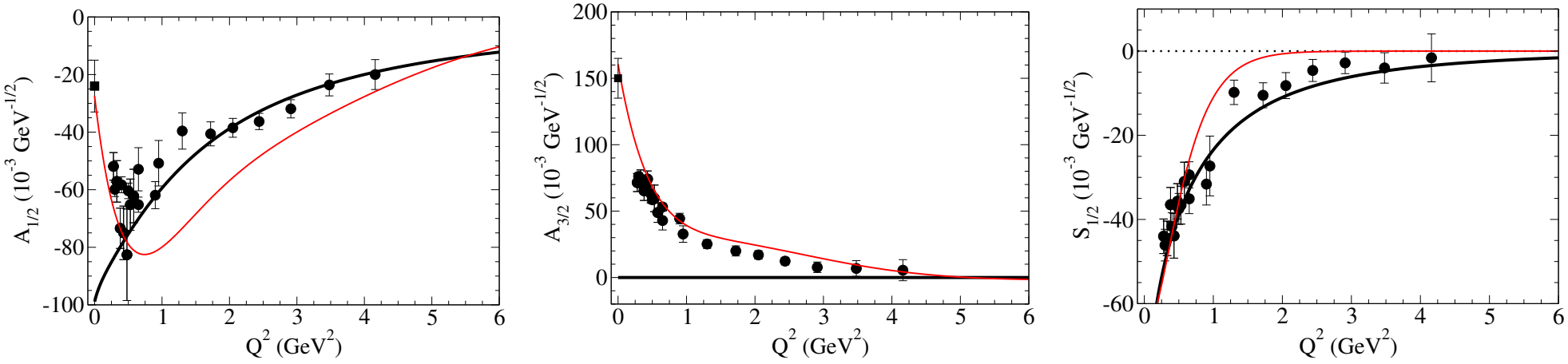


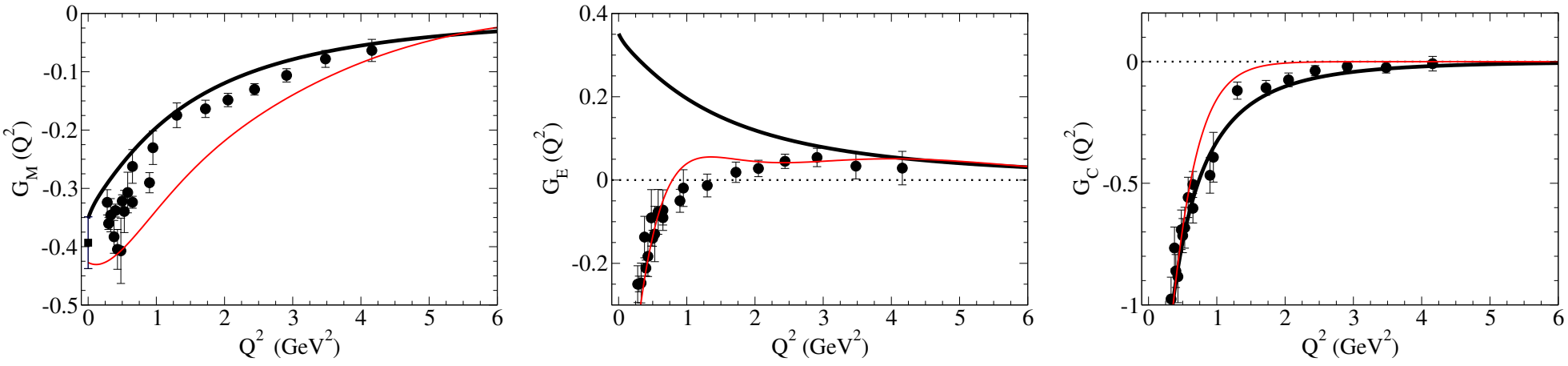
More data are welcome

SR approximation – $N(1520)$ – Results (1)



- 3 independent helicity amplitudes
- —- SemiRelativistic approach (**SR**); data from **CLAS**, include MAID fit
- **SR** very good description of the $Q^2 > 1.5$ GeV^2 data
  Except for $A_{3/2}$ (CSQM: $A_{3/2} \equiv 0$) – $A_{3/2}$ $\leftarrow$ dominated by meson cloud ?
- **Describe well valence quark degress of freedom** ($(A_{3/2})_{\rm bare} \approx 0$)

SR approximation – $N(1520)$ – Results (2)



- 3 independent form factors
- —- SemiRelativistic approach (**SR**); data from **CLAS**, include MAID fit
- **SR** very good description of the $Q^2 > 1.5$ GeV^2 data
  Except for G_E (CSQM: $G_E \equiv -G_M$, $A_{3/2} \equiv 0$)
- **Describe well valence quark degress of freedom** ($(A_{3/2})_{\rm bare} \approx 0$)

Summary

GR, PRD 95, 054008 (2017)

- **No parameters adjusted** ($\psi_R \equiv \psi_N$) – Predictions
- **SR approach** gives a good description of form factor/ helicity amplitude data in the region $Q^2 > 1.5$ GeV^2
- Exceptions (missing meson cloud effects ?):
  - $N(1520)$: $A_{3/2}$ and G_E
  - $N(1535)$: F_2^*
    Dificult to describe based on the valence quark degrees of freedom

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    Dificult to describe based on the valence quark degrees of freedom

In progress:

parametrization of mc effects in the $\gamma^* N \to N(1535)$ transition

Extension to the timelike region (Dalitz decay; HADES)

GR and MT Peña, in preparation

SQTM – Introduction

Single Quark Transition Model $\oplus$ CSQM

- SQTM can be applied to the $[70, 1^-]$ supermultiplet
  Hey and Weyers, PL 48B, 69(1974); Cottingham and Dunbar, ZPC 2, 41 (1979);
  Burkert et al, PRC 67, 035204 (2003) – **Impulse approximation** $SU(6) \otimes O(3)$
  Amplitudes dependent of coefficients A, B, C
- Use amplitudes $N(1535)\frac{1}{2}^-$, $N(1520)\frac{3}{2}^-$ from CSQM
  GR, MT Peña, PRD 84, 033007 (2011); PRD 89, 094016 (2014)
  Valence quark contributions $\longrightarrow$ Calculate A, B, C
- Estimate amplitudes for
  $N(1650)\frac{1}{2}^-$, $N(1650)\frac{3}{2}^-$, $\Delta(1620)\frac{1}{2}^-$, $\Delta(1700)\frac{3}{2}^-$
  GR, PRD 90, 033010 (2014)

Model 2: include parametrization of $A^{\rm MC}_{3/2} \neq 0$ for $N(1520)$

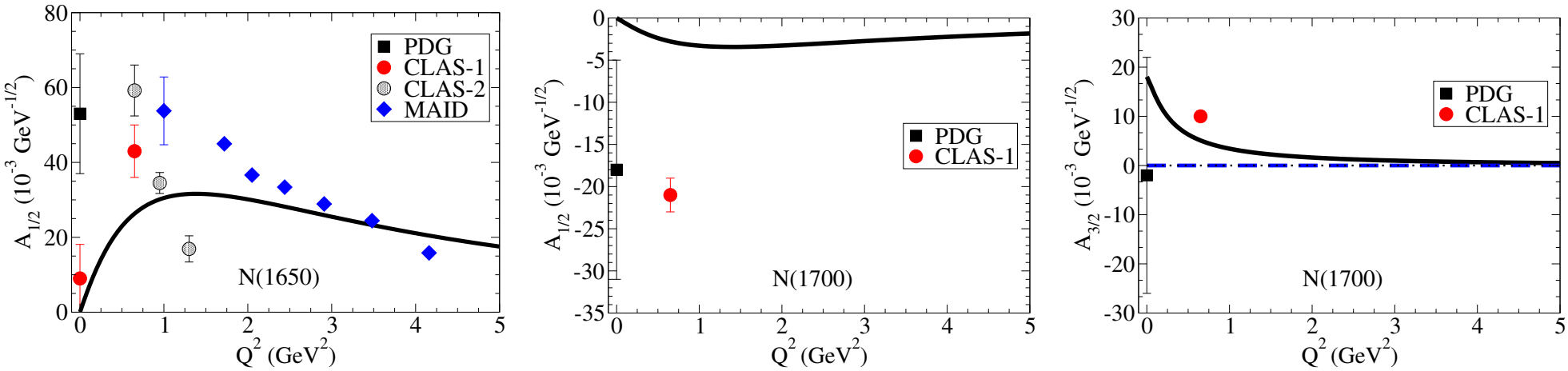
Estimates compared with data from **CLAS, MAID** and **PDG**

SQTM: $[70, 1^-]$ amplitudes – A, B, C; θ_S, θ_D

State	Amplitude	
$N(1535)\frac{1}{2}^-$	$A_{1/2}$	$\frac{1}{6}(A+B-C)\cos\theta_S$
$N(1520)\frac{3}{2}^-$	$A_{1/2}$	$\frac{1}{6\sqrt{2}}(A-2B-C)\cos\theta_D$
	$A_{3/2}$	$\frac{1}{2\sqrt{6}}(A+C)\cos\theta_D$
$N(1650)\frac{1}{2}^-$	$A_{1/2}$	$\frac{1}{6}(A+B-C)\sin\theta_S$
$\Delta(1620)\frac{1}{2}^-$	$A_{1/2}$	$\frac{1}{18}(3A-B+C)$
$N(1700)\frac{3}{2}^-$	$A_{1/2}$	$\frac{1}{6\sqrt{2}}(A-2B-C)\sin\theta_D$
	$A_{3/2}$	$\frac{1}{2\sqrt{6}}(A+C)\sin\theta_D$
$\Delta(1700)\frac{3}{2}^-$	$A_{1/2}$	$\frac{1}{18\sqrt{2}}(3A+2B+C)$
	$A_{3/2}$	$\frac{1}{6\sqrt{6}}(3A-C)$

Results for $N(1650), N(1700)$ PRD 90, 033010 (2014)

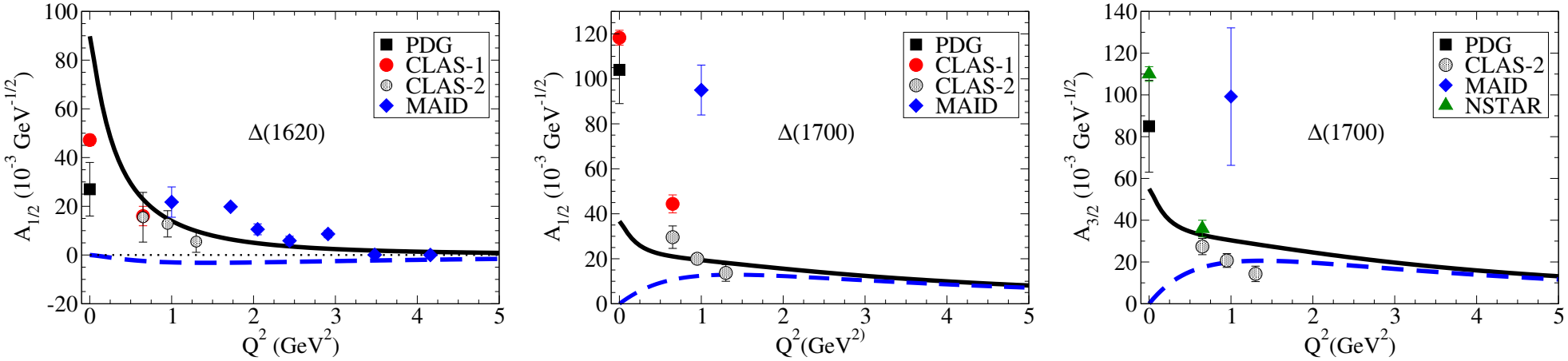
$A_{1/2}$ $A_{1/2}$ $A_{3/2}$



- Almost no data for $Q^2 > 2$ GeV^2
- —- **Model 2** better at large Q^2

Results for $\Delta(1620), \Delta(1700)$ PRD 90, 033010 (2014)

$A_{1/2}$ $A_{1/2}$ $A_{3/2}$



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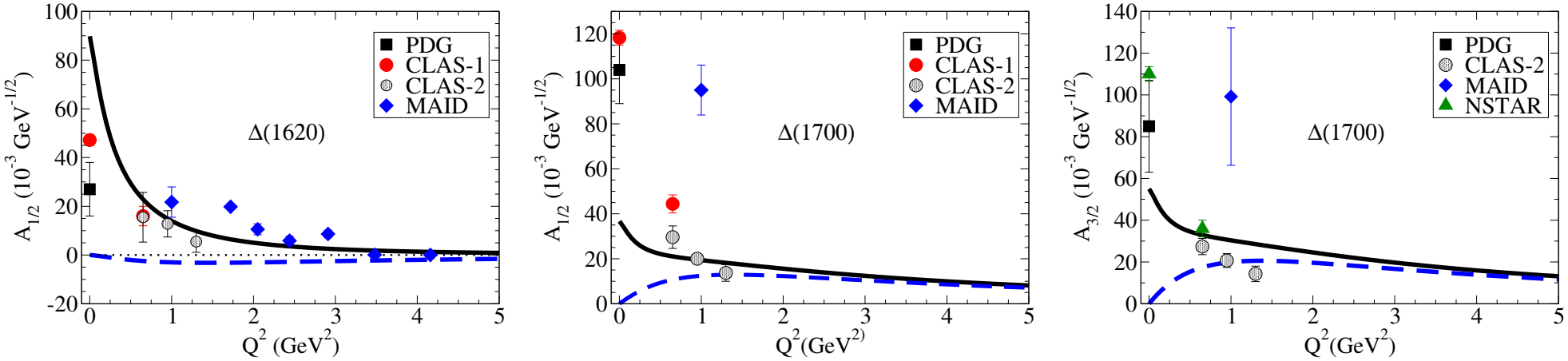


- Almost no data for $Q^2 > 2$ GeV^2 **More large Q^2 data are necessary**
- —- **Model 2** better at large Q^2

Simple parametrization for large Q^2 ††

Facilitate comparison with future data – powers from pQCD

$$A_{1/2}(Q^2) = D\left(\frac{\Lambda^2}{\Lambda^2+Q^2}\right)^{3/2}, \quad A_{3/2}(Q^2) = D\left(\frac{\Lambda^2}{\Lambda^2+Q^2}\right)^{5/2}$$

State	Amplitude	$D(10^{-3}\text{GeV}^{-1/2})$	$\Lambda^2(\text{GeV}^2)$
$S_{11}(1650)$	$A_{1/2}$	68.90	3.35
$S_{31}(1620)$	$A_{1/2}$	...	...
$D_{13}(1700)$	$A_{1/2}$	-8.51	2.82
	$A_{3/2}$	4.36	3.61
$D_{33}(1700)$	$A_{1/2}$	39.22	2.69
	$A_{3/2}$	42.15	8.42

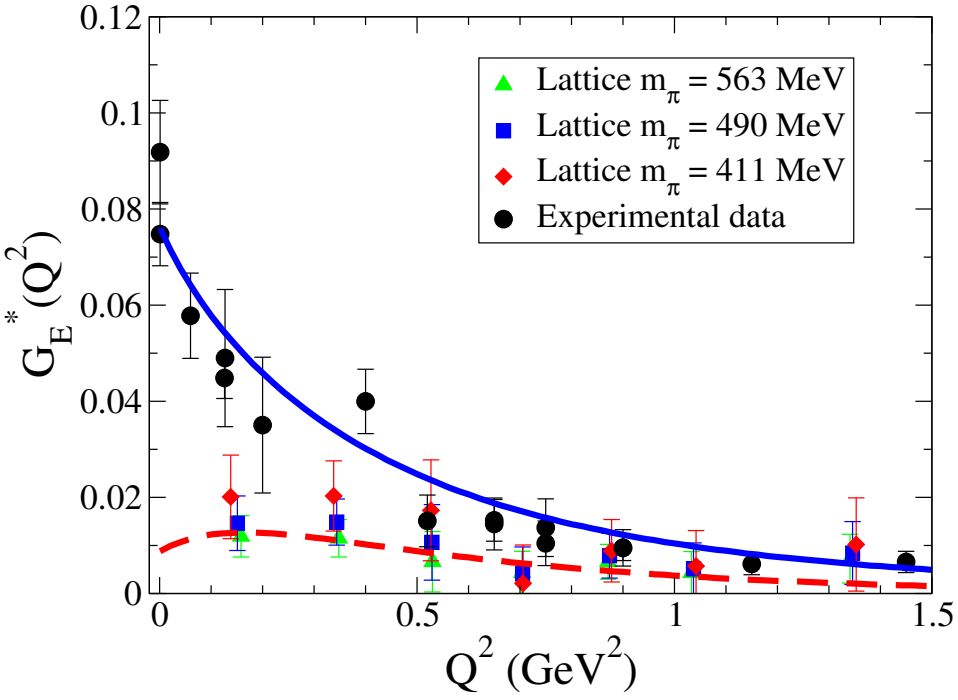
Some calculations of N^* transition form factors at low Q^2

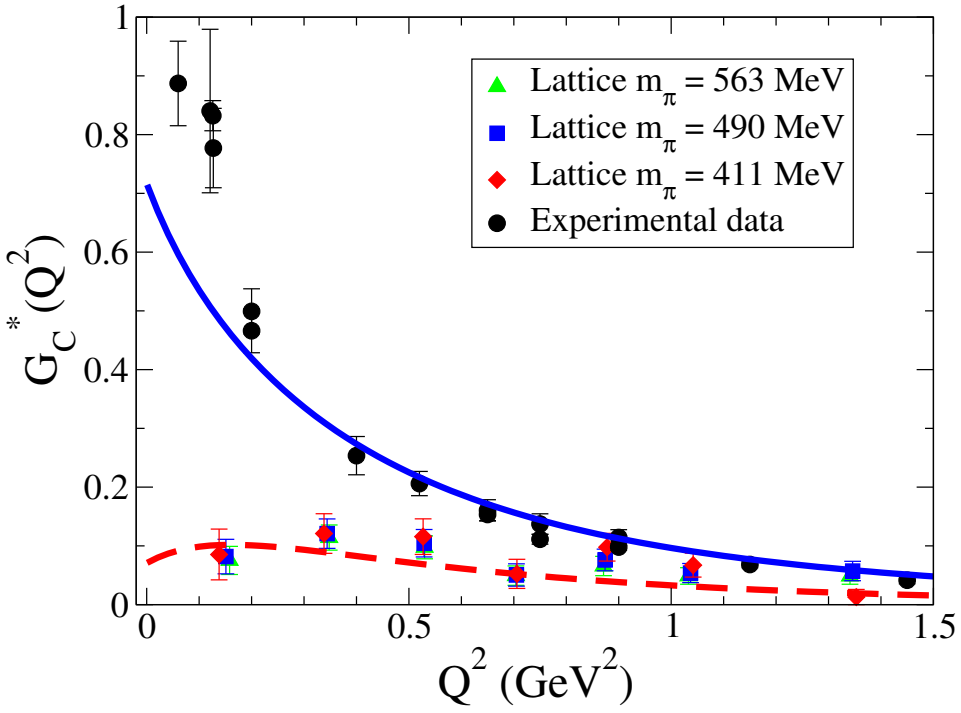
N^* at low Q^2

$\sum$ 

- **Nucleon Compton scattering** G Eichmann and GR, PRD 98, 093007 (2018)
  - Accurate **low-Q^2 data** is necessary to constraint data parametrizations
  - It is important to select the **correct** form factors (no poles and no zeros at $Q^2 = 0$)
  - Take into account the constraints at pseudothreshold (N and N^* at rest; Siegert's theorem)
- **Siegert's theorem** ($S \propto E|\mathbf{q}|$)
  - $\Delta(1232)$, $N(1535)$ and $N(1520)$
    GR, PLB 759, 126, (2016); PRD 93, 113012 (2016); PRD 94, 114001 (2016)
  - Rational parametrizations of the form factors are preferred to parametrizations like **MAID** (avoid exponentials)
    - no fast falloff for large Q^2
    - smoother extensions for timelike $Q^2 < 0$
    - simpler constraints at the pseudothreshold

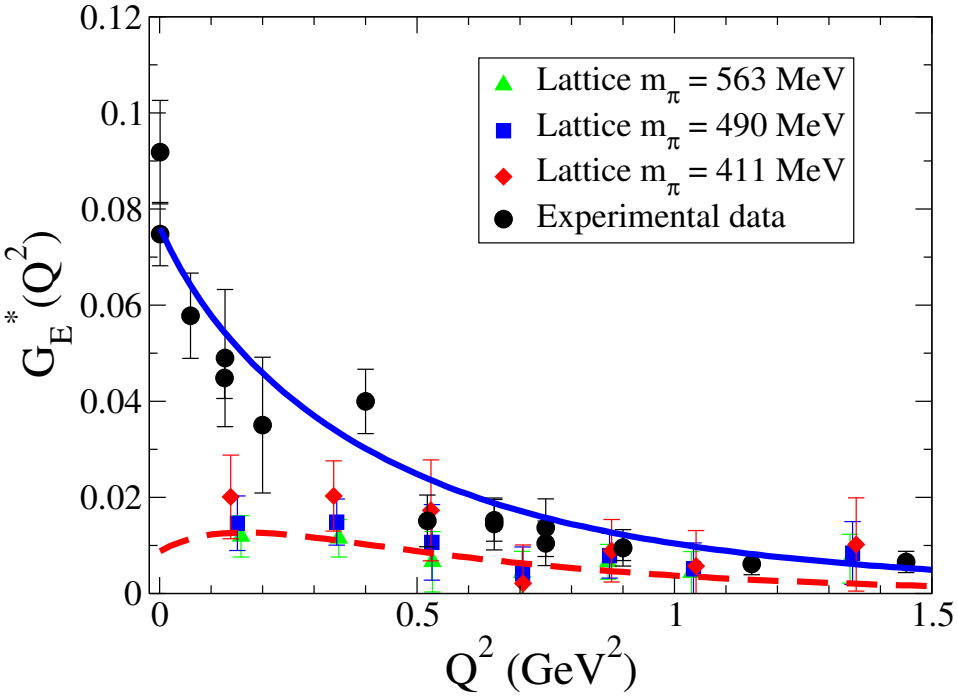
$\gamma^* N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ (bare + pion cloud)

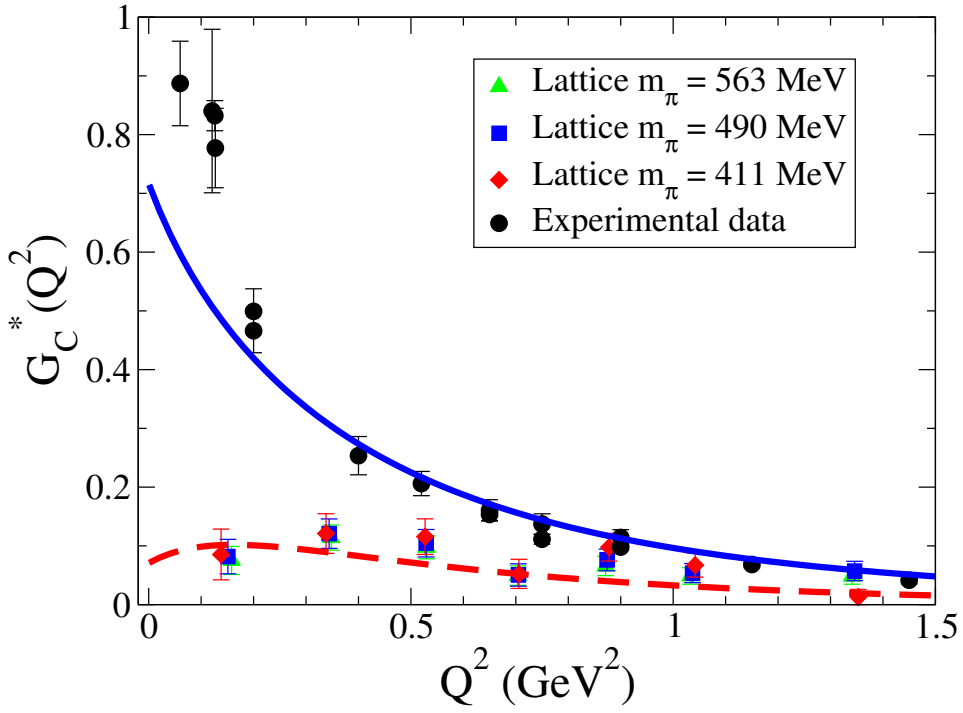




- Bare contributions (QM) determined by fit to lattice Alexandrou, PRD 77, 085012 (2008)
- Pion cloud: large N_c relations $\propto \tilde{G}_{En} = \frac{G_{En}}{Q^2}$;
  Pascalutsa and Vanderhaeghen, PRD 76, 111501 (2007); Buchmann PRD 66, 056002 (2002)
- Bare (QM) $\oplus$ pion cloud (theory) contributions $\approx$ **data**

$\gamma^* N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ (bare + pion cloud)





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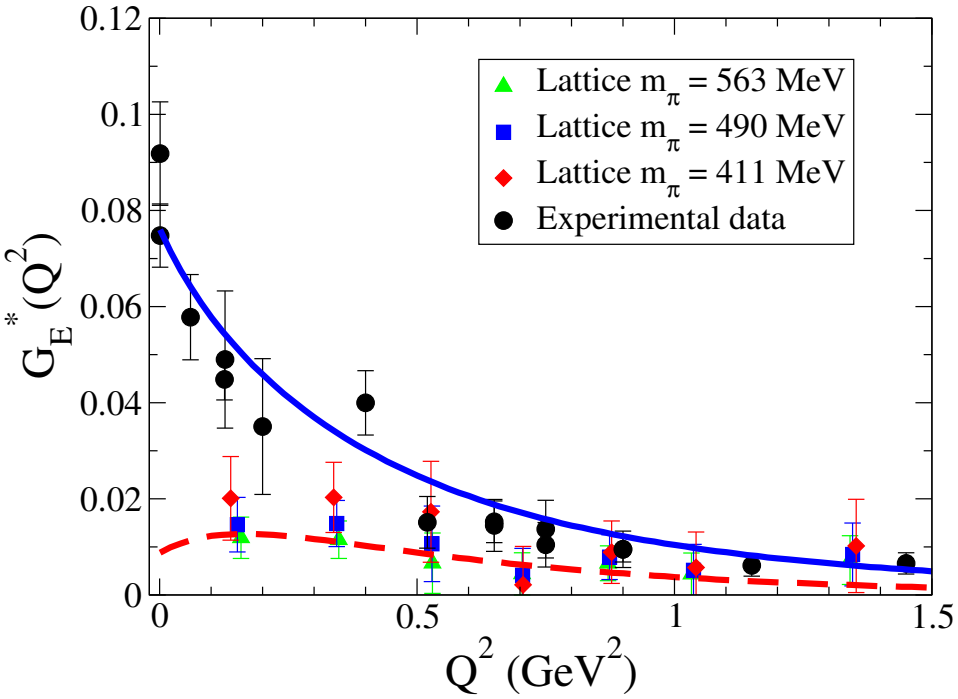
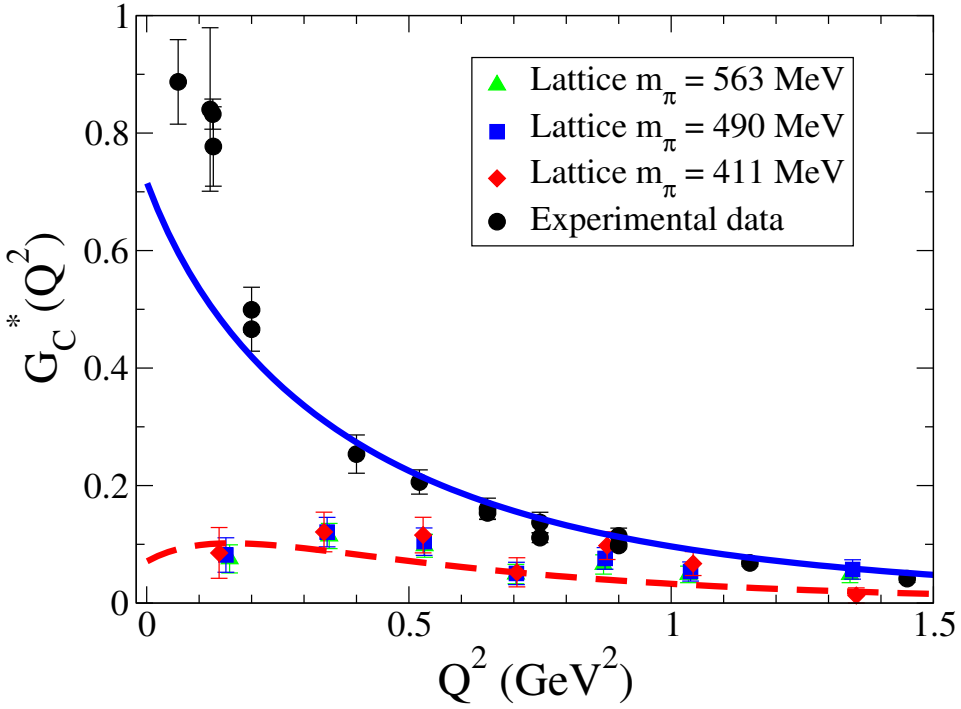
  Pascalutsa and Vanderhaeghen, PRD 76, 111501 (2007); Buchmann PRD 66, 056002 (2002)
- Bare (QM) $\oplus$ pion cloud (theory) contributions $\approx$ **data**

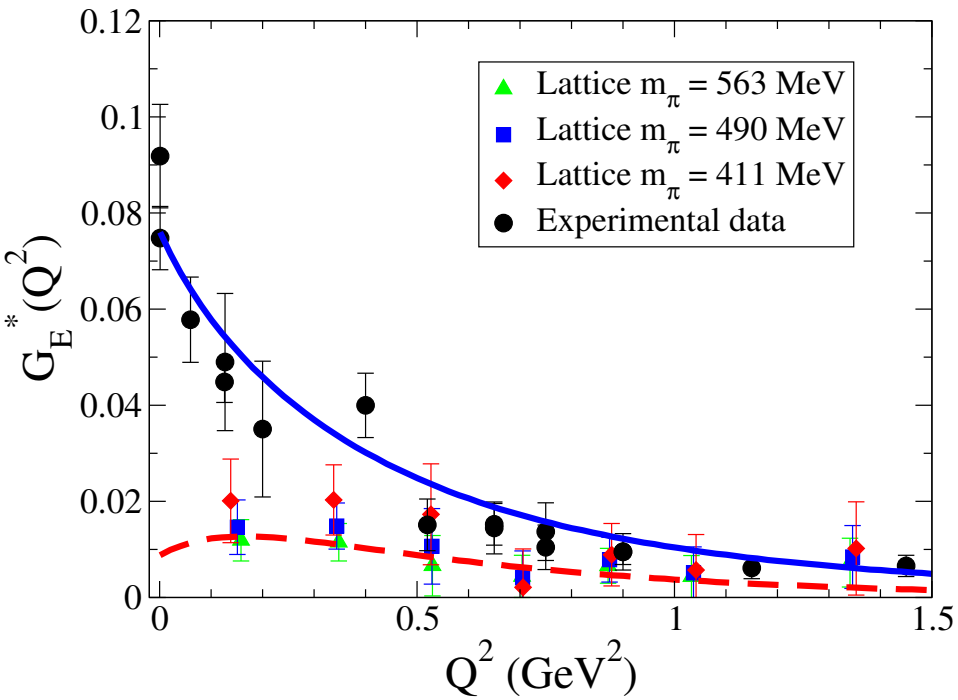
$\gamma^* N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ [GR, MT Peña, PRD 80, 013008 (2009)]

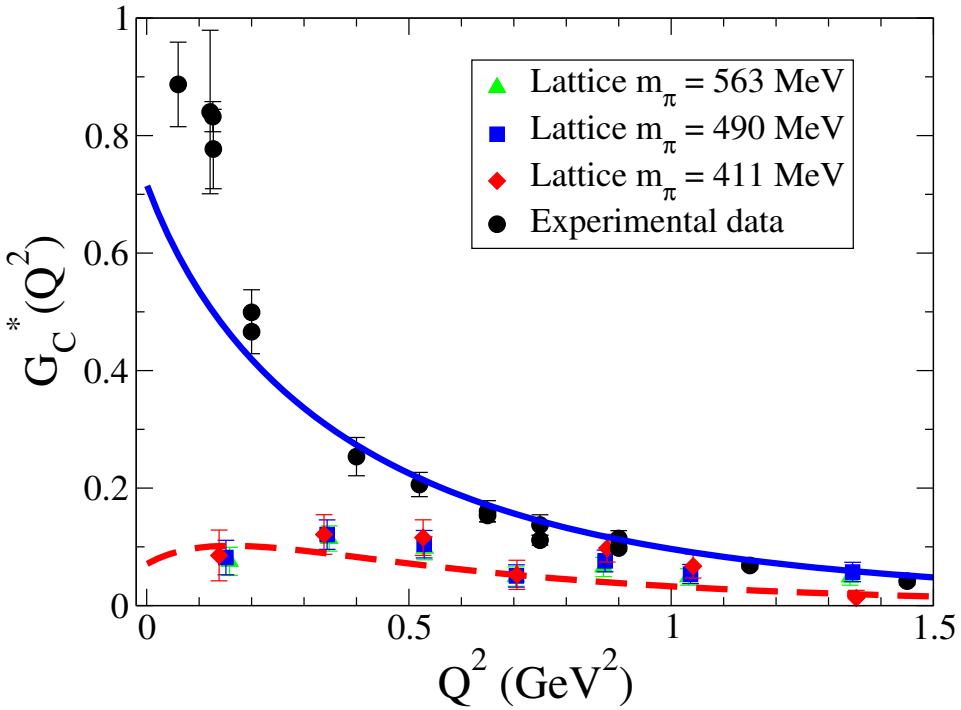




- Data: CLAS, MAMI, MIT-Bates, PDG
- Bare (QM) $\oplus$ pion cloud (theory) contributions $\approx$ **data**

$\gamma^* N \to \Delta$: $G_E^*(Q^2)$, $G_C^*(Q^2)$ [GR, MT Peña, PRD 80, 013008 (2009)]





We can improve the previous picture in two aspects:

- Consider pion cloud parametrization consistent with Siegert's theorem
- Compare the model estimates with the more recent G_C^* data

$\gamma^* N \to \Delta$: quadrupoles – Siegert's theorem – NSTAR 2017 †

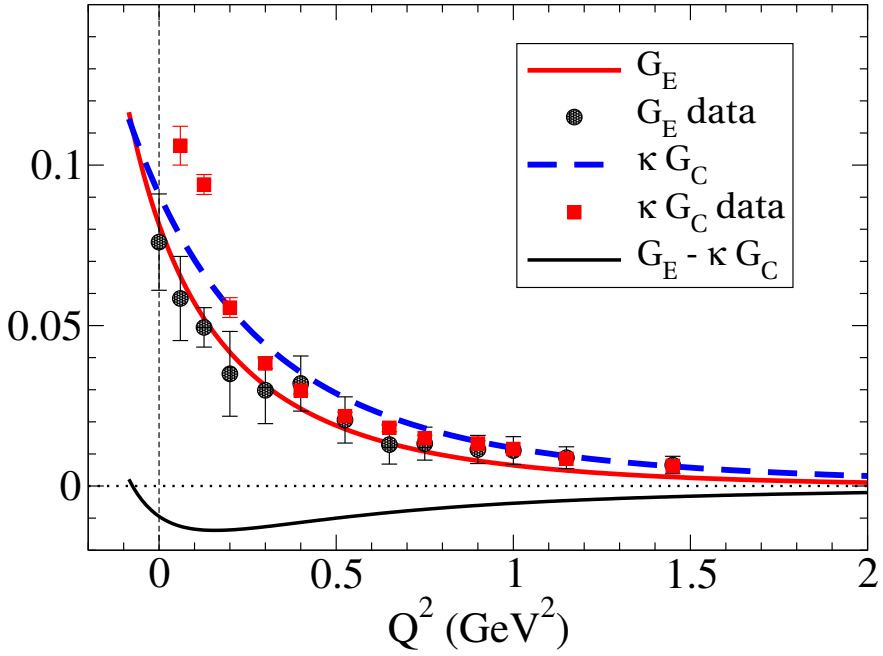
The previous parametrizations **are not** consistent with Siegert's theorem $\kappa = \frac{M_\Delta - M}{2M_\Delta}$

Siegert's theorem: at pseudothreshold $|\mathbf{q}| = 0$; $Q^2 = -(M_\Delta - M)^2$; $G_E^* = \kappa G_C^*$

Large-N_c: GR, PRD 94, 114001 (2016) $\alpha = \frac{Q^2}{M_\Delta^2 - M^2}$, $M_\Delta - M = \mathcal{O}\left(\frac{1}{N_c}\right)$

$$G_E^\pi = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_{En}}{1+\alpha}, \qquad G_C^\pi = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En}$$

Correct G_E^π with a term in $\mathcal{O}(1/N_c^2)$ Error in ST $\approx \mathcal{O}\left(\frac{1}{N_c^4}\right)$



$\gamma^* N \to \Delta$: quadrupoles – Siegert's theorem – NSTAR 2019

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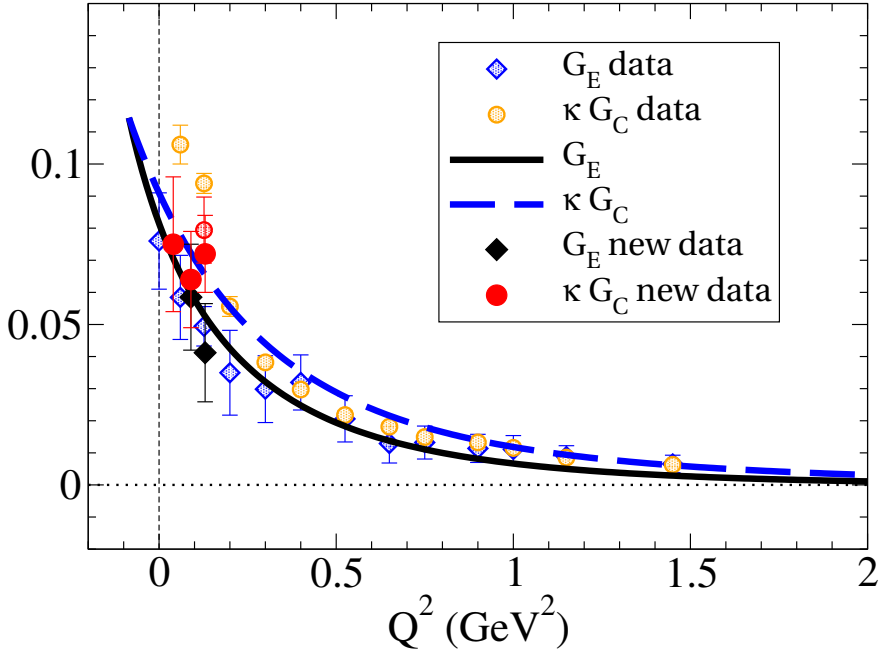
Siegert's theorem: at pseudothreshold $|\mathbf{q}| = 0$; $Q^2 = -(M_\Delta - M)^2$; $G_E^* = \kappa G_C^*$

Improved Large-N_c: GR, EPJA 54, 75 (2018) $\alpha = \frac{Q^2}{2M_\Delta(M_\Delta - M)}$

$$G_E^\pi = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_{En}}{1+\alpha}, \qquad G_C^\pi = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En}$$

Correct G_E^π with a term in $\mathcal{O}(1/N_c^2)$ **NO** breaking of ST



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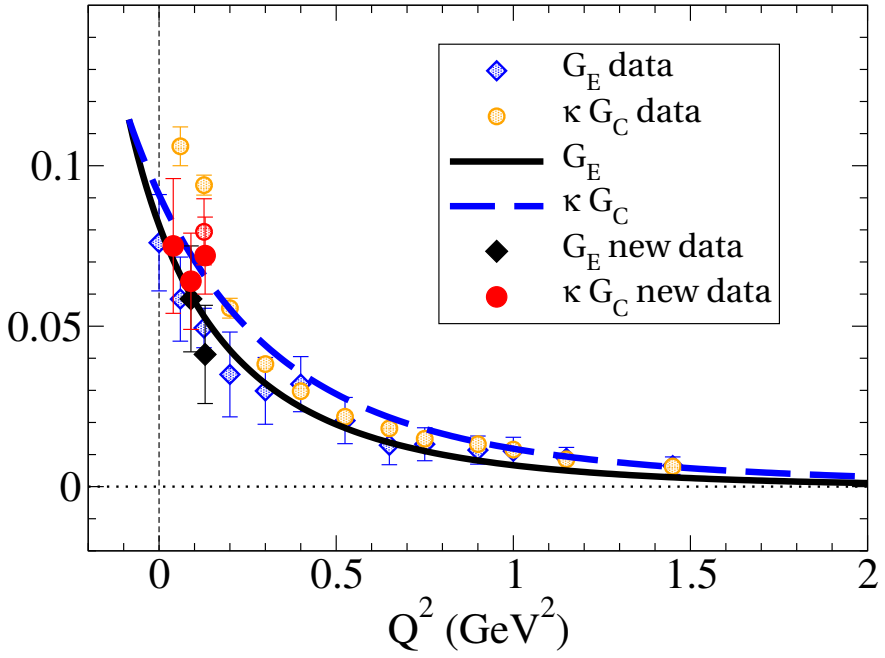
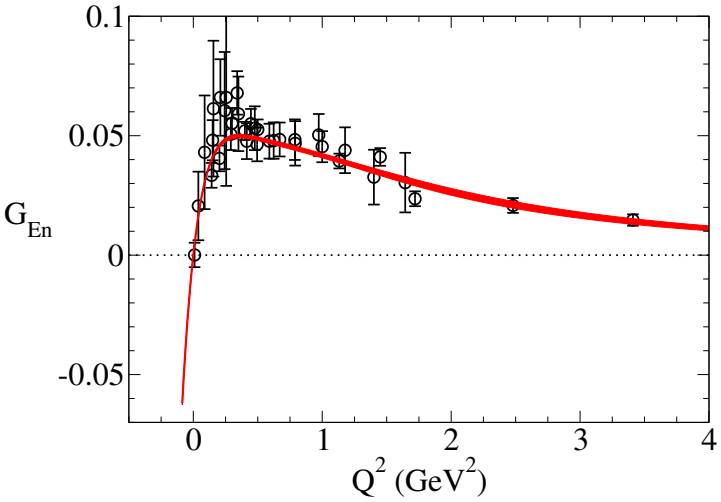
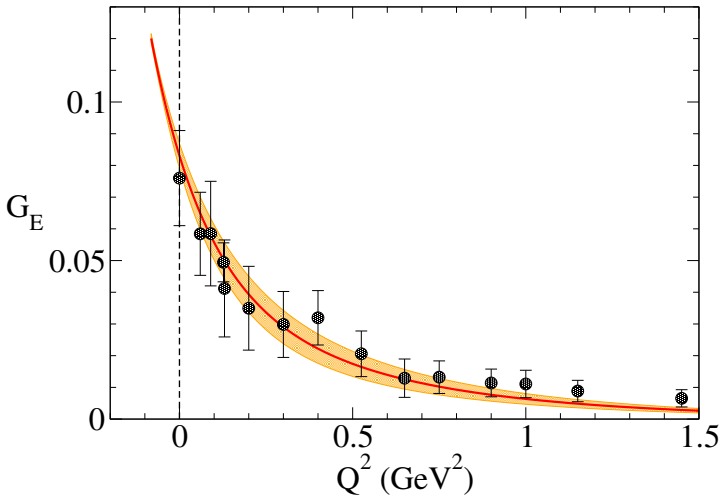
New data JLab/Hall A

A Blomberg, PLB 760, 267 (2016)



$\gamma^* N \to \Delta$: quadrupoles – Global fit





GR, EPJA 55, 32 (2019)

Use quadrupole data to determine best G_{En}

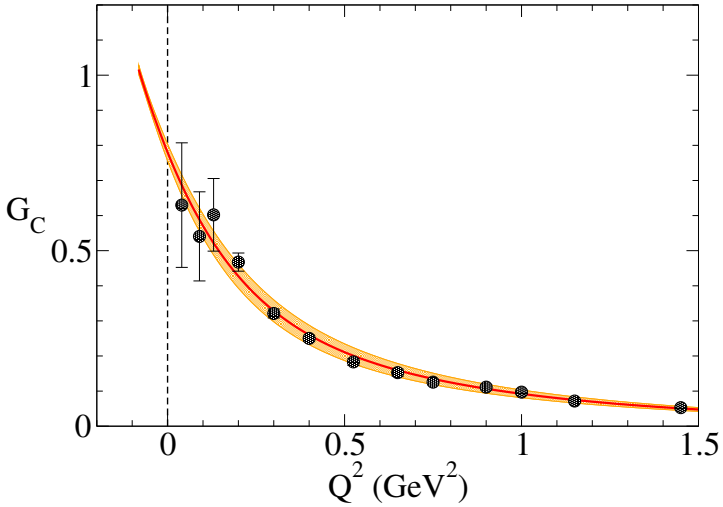
Global fit G_{En}, G_E^*, G_C^*

$$G_{En} = -\frac{r_n^2}{6} Q^2 \frac{1+c_2 Q^2 + c_3 Q^4}{1+c_1 Q^2 + \ldots + \frac{c_1^5}{5!} Q^{10}}$$

Include theoretical errors associated with the Quark Model (fit lattice QCD data)

$r_n^4 \simeq -0.4$ fm^4;

$r_E^2 = 2.33 \pm 0.19$ fm^2; $r_C^2 = 1.75 \pm 0.13$ fm^2



Summary and conclusions

- We present **covariant** estimates of the electromagnetic transition form factors for several N^* states at large Q^2
  $\Delta(1232)$, $N(1440)$, $N(1520)$, $N(1535)$, $\Delta(1600)$, ...
  $N(1650)$, $N(1700)$, $\Delta(1620)$, $\Delta(1700)$
- Estimates based on the **covariant spectator quark model**
  Assume the dominance of valence quark effects for large Q^2 ($Q^2 \gtrsim 2$ GeV^2)
  (... small meson cloud contributions)
- In some cases we conclude that the **infered** meson cloud contributions are significant for $Q^2 = 1$–4 GeV^2 – Valence quarks insuficient
- **Large Q^2:**
  Future data (JLab-12 GeV upgrade) will be useful to test the present predictions; & to refine callibrations of the models [Nucleon and $\Delta(1232)$]
- **Low-Q^2:**
  - Accurate data $Q^2 = 0$–0.3 GeV^2 useful to define parametrizations
  - Lattice QCD data ($N(1440)$, ...); Dynamical Models $\sim$ **bare core**

Summary and conclusions **Thank you** ☺

- We present **covariant** estimates of the electromagnetic transition form factors for several N^* states at large Q^2
  $\Delta(1232)$, $N(1440)$, $N(1520)$, $N(1535)$, $\Delta(1600)$, ...
  $N(1650)$, $N(1700)$, $\Delta(1620)$, $\Delta(1700)$
- Estimates based on the **covariant spectator quark model**
  Assume the dominance of valence quark effects for large Q^2 ($Q^2 \gtrsim 2$ GeV^2)
  (... small meson cloud contributions)
- In some cases we conclude that the **infered** meson cloud contributions are significant for $Q^2 = 1$–4 GeV^2 – Valence quarks insuficient
- **Large Q^2:**
  Future data (JLab-12 GeV upgrade) will be useful to test the present predictions; & to refine callibrations of the models [Nucleon and $\Delta(1232)$]
- **Low-Q^2:**
  - Accurate data $Q^2 = 0$–0.3 GeV^2 useful to define parametrizations
  - Lattice QCD data ($N(1440)$, ...); Dynamical Models $\sim$ **bare core**