Quark model calculations of transition form factors at high photon virtualities

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Motivation

- New accurate data from modern accelerators (Jlab, MAMI, ELSA,...) associated with $N^*$ states with increasing $W$ (1.4–1.8 GeV) and large $Q^2$ (2–6 GeV$^2$)

$\Rightarrow$ **New challenges:**
- Interpret the data $\leftrightarrow$ Theory $\leftrightarrow$ relativistic models
- Make predictions: higher $W$, higher $Q^2$ – Jlab-12 GeV–upgrade
Plan of the talk

- **Theoretical framework**
  Covariant Spectator Quark Model

- **Calculations of $N^*$ transition form factors at large $Q^2$**
  $\Delta(1232)^{3/2}^+, N(1440)^{1/2}^+, N(1535)^{1/2}^-, N(1520)^{3/2}^-, \Delta(1600)^{3/2}^+$
  ... include results from
  **Single Quark Transition model**
  $N(1650)^{1/2}^-, N(1700)^{3/2}^-, \Delta(1620)^{1/2}^-, \Delta(1700)^{3/2}^-$

- ... some results at low-$Q^2$

- **Summary and conclusions**
Baryon: 3 constituent quark system

Covariant Spectator Theory: wave function $\Psi_B$ defined in terms of a 3-quark vertex $\Gamma$ with 2 on-mass-shell quarks – integrate into quark-pair degrees of freedom

\[
\int_{k_1} \int_{k_2} = \int_{4m_q^2}^{+\infty} ds \sqrt{s - 4m_q^2} \int \frac{d^3k}{2\sqrt{s + k^2}}
\]

Mean value theorem: $s = (k_1 + k_2)^2 \rightarrow m_D^2$; effective diquark mass $m_D$

Gross and Agbakpe PRC 73, 015203 (2006); Gross, GR and Peña PRC 77, 015202 (2008)

$\Rightarrow$ reduction to a quark-diquark structure: $\Psi_B(P_B, k)$

Baryon wave function $\Psi_B(P_B, k)$ free of singularities

Stadler, Gross and Frank PRC 56, 2396 (1998); Savkli and Gross PRC 63, 035208 (2001)

Spin-flavor structure $\approx$ relativistic $SU_F(3) \times SU_S(2)$ structure

Radial wave function $\psi_B(P_B, k)$ determined phenomenologically

Not a solution of a dynamical wave equation – mass $M_B \equiv M_B^{\text{exp}}$

Shape determined by momentum scale parameters using experimental data or lattice data of some ground state systems

$\Psi_B(P_B, k)$ defined at rest frame; generalized covariantly to an arbitrary frame using Lorentz transformations
Quarks with electromagnetic structure (impulse approximation)

\[ j^\mu_q = \left( \frac{1}{6} f_1^+ + \frac{1}{2} f_1^- \tau_3 \right) \gamma^\mu + \left( \frac{1}{6} f_2^+ + \frac{1}{2} f_2^- \tau_3 \right) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \]

form factors \( f_i^\pm \) parametrize dressing of quarks (gluons and \( q\bar{q} \)) \( \kappa_q = f_{2q}(0) \approx 2 \)

- Vector meson dominance parameterization at quark level:

Quark current parametrized in terms of vector meson poles \( (m_v, M_h) \)

F Gross, GR, MT Peña, PRC 77, 015202 (2008); GR, MT Peña, JPG 36, 115011 (2009); PRD 80, 013008 (2009);
GR, K Tsushima, AW Thomas, JPG 40, 015102 (2013); GR, K Tsushima, F Gross, PRD 80, 033004 (2009)

- 4 parameters determined by the fit to the nucleon data

F Gross, GR, MT Peña, PRC 77, 015202 (2008)
Covariant Spectator Quark Model – Introduction (3)

- Transition current – *relativistic impulse approximation*

\[ J^\mu = 3 \sum_\lambda \int_k \bar{\Psi}_f (P_+, k) j^\mu_q \Psi_i (P_-, k) \]

- Quark current \( j^\mu_q \) and nucleon radial wave function \( \psi_N (P_N, k) \)
  - calibrated by **nucleon elastic form factor data**

  F Gross, GR, MT Peña, PRC 77, 015202 (2008)

- **Generalization to lattice QCD:**
  - \( f_{i\pm} (Q^2; m_\rho, M_N) \rightarrow f_{i\pm} (Q^2; m^{\text{latt}}_\rho, M^{\text{latt}}_N) \) – VMD
  - \( \psi_B (M_B) \rightarrow \psi_B (M^{\text{latt}}_B) \)

  GR, MT Peña, JPG 36, 115011 (2009); PRD 80, 013008 (2009); GR, K Tsushima, F Gross, PRD 80, 033004 (2009); GR, K Tsushima, AW Thomas, JPG 40, 015102 (2013)
Calculations of $N^*$ transition form factors at large $Q^2$
$\gamma^* N \rightarrow \Delta(1232)$ – Introduction (Review: Burkert and H Lee)

- Transition dominated by $G_M^*$: $N(\uparrow\uparrow\downarrow) \rightarrow \Delta(\uparrow\uparrow\uparrow)$ (spin-flip)
- Small contributions from $G_E^*$ and $\frac{|q|}{2M_\Delta} G_C^*$; indication of (small) $\Delta$ deformation
- $G_M^*$ usually underestimated by quark models at small $Q^2$
$\Delta(1232)$ - S-wave model - covariant

$$\Psi_N = \frac{1}{\sqrt{2}} \left( \phi^0_I \phi^0_S + \phi^1_I \phi^1_S \right) \psi_N(P, k)$$

$$\Psi_\Delta = -SU(6) \left( \tilde{\phi}^I_1 (\varepsilon^*_P) \alpha u^\alpha(P) \right) \psi_\Delta(P, k)$$

Radial wave functions: $\psi_N(P, k); \psi_\Delta(P, k)$

$\psi_N(P, k)$ determined by the nucleon data; How to determine $\psi_\Delta(P, k)$?

$\psi_\Delta$ can be determined by lattice data and/or estimates from Dynamical Mod. (core)

EBAC/Argonne-Osaka
\[ \Psi_N = \frac{1}{\sqrt{2}} \left( \phi_0^0 \phi_S^0 + \phi_1^1 \phi_S^1 \right) \psi_N(P, k) \]

\[ \Psi_\Delta = - \frac{SU(6)}{\hat{\phi}_1^I(\varepsilon_P^*) \alpha u^\alpha(P)} \psi_\Delta(P, k) \]

Radial wave functions: \( \psi_N(P, k) \); \( \psi_\Delta(P, k) \)

\( \psi_N(P, k) \) determined by the nucleon data; **How to determine \( \psi_\Delta(P, k) \) ?**

\( \psi_\Delta \) can be determined by lattice data and/or estimates from Dynamical Mod. (core)

EBAC/Argonne-Osaka

**Note that** \( Q^2 \approx 0: \ G_M^B(Q^2) \approx 2 < 3 \)
\(\Delta(1232)\) - S-wave model – underestimation of \(G_M^*\)

Understanding the underestimation from Quark Models:


CSQM – S-wave : \(G_E^* = G_C^* = 0\)

\[
G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} \left( f_{1-} + \frac{M_\Delta + M}{2M} f_{2-} \right) \int_k \psi_\Delta \psi_N
\]

Then \(G_M^*(0) = 2.07 \int_k \psi_\Delta \psi_N\)

Normalization conditions \(\oplus\) Cauchy-Schwarz inequality: \(\int_k \psi_\Delta \psi_N \leq 1:\)

\(G_M^*(0) \lesssim 2.07 \lesssim 3\) (experimental result)

**Conclusion:**

CSQM: **natural explanation** for the underestimation of \(G_M^*\)

**Missing contributions:** pion cloud mechanism

Kamalov, Yang, PRL 83, 4494 (1999); Sato, Lee, PRC 63, 055201 (2001); Diaz et al, PRC 80, 025207 (2009)
\(\Delta(1232) - S\)-wave model – adjusting \(\psi_{\Delta}(P, k)\)

GR, F Gross, MT Peña, EJPA 36, 329 (2008); PRD 78, 114017 (2008); GR, MT Peña, PRD 80, 013008 (2009)

- **CSQM** fitted to the EBAC bare data – adjust \(\psi_{\Delta}(P, k)\)
- **CSQM** describe well the EBAC bare results  J Diaz et al, PRC 80, 025207 (2009)
\( \gamma^* N \rightarrow \Delta: G_M^* \) in lattice [PRD 80, 013008 (2009)]

- CSQM extended to lattice QCD \((m_\pi = 411, 490, 563 \text{ MeV})\)
- Good description of the lattice data based on EBAC parameters
$\gamma^* N \to \Delta$: $G_M^*$ in lattice [PRD 80, 013008 (2009)]

- - - CSQM extended to lattice QCD  
Alexandrou, PRD 77, 085012 (2008)

Good description of the lattice data based on EBAC parameters
\[ \gamma^* N \rightarrow \Delta: G_M^* \text{ (valence), physical case} \]

GR and MT Peña PRD 80, 013008 (2009)

\[ G_D = \left(1 + \frac{Q^2}{0.71}\right)^{-2} \]

- CSQM explains well lattice data and EBAC estimate (core)
- How to simulate the effect of the pion cloud?
\( \gamma^* N \rightarrow \Delta: G_M^* \) (valence), physical case

GR and MT Peña PRD 80, 013008 (2009)

\[ G_D = \left(1 + \frac{Q^2}{0.71}\right)^{-2} \]

- Good description of the valence quark effects
- How to simulate the effect of the pion cloud?
$\gamma^* N \rightarrow \Delta$: How to simulate the pion cloud? [phenomenology]

Recent application – two pion cloud contributions
Motivated by study of the **octet to decuplet transitions**: 50% – 50%
GR, K Tsushima, PRD 88, 053002 (2013)

\[
G^\pi_M = 3 \frac{\lambda_\pi}{2} F_\pi(q^2) \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - q^2} \right)^2 + 3 \frac{\lambda_\pi}{2} \left( \frac{\Lambda^2_D}{(\Lambda^2_D - q^2)^2 + \Lambda^2_D(\Gamma_D(q^2))^2} \right)^2
\]

GR, MT Peña J Weil, H van Hees and U Mosel, PRD 93, 033004 (2016)

$F_\pi(q^2)$: phenomenologic electromagnetic pion form factor;
$\Lambda^2_D = 0.9$ GeV$^2$; $\Gamma_D(q^2)$ phenomenological width; $\lambda_\pi = 0.448$
\( \gamma^* N \rightarrow \Delta: G_M^* \) (valence + pion cloud) [phenomenological]

- **Good** description of the **physical data** including the large \( Q^2 \) data
- Extension to **timelike** transition – HADES – PRC 95, 065205 (2017)
  
  GR, MT Peña J Weil, H van Hees and U Mosel, PRD 93, 033004 (2016)
\[ \gamma^* N \rightarrow N(1440) \rightarrow \text{Introduction} \]

- **CSQM:** Roper defined as the 1st radial excitation of the nucleon
  - Same spin/flavor structure as the nucleon
  - Radial wave function defined by the orthogonality with nucleon state
  - GR and K Tsushima, PRD 81, 074020 (2010); PRD 89, 073010 (2014)

- No adjustable parameters; No meson cloud components included

- **CLAS data:** IG Aznauryan et al., PRC 80, 055203 (2009); VI Mokeev et al., PRC 86, 035203 (2012); PRC 93, 025206 (2016)
**Results**

- **Good results** for $Q^2 > 1.5$ GeV$^2$ – valence quark dominance
  - Support **Roper** as 1st radial excitation of the nucleon

- **Failure** for $Q^2 < 1.5$ GeV$^2$ – meson cloud?
  - Used to estimate meson cloud from CLAS data – inferred MC

Recent progress from AdS/QCD – Roper form factors

Valence quark approximation – leading twist: 3 adjustable couplings
Parameters adjusted to nucleon data: Roper – red band

GR, D Melnikov, PRD 97, 034037 (2018); GR, PRD 96, 054021 (2017)
\[ \gamma^* N \to N(1440) \] – Holographic estimate

Recent progress from AdS/QCD – Roper form factors

GR, D Melnikov, PRD 97, 034037 (2018); GR, PRD 96, 054021 (2017)

**Very good** result for \( F_2^* \) – suggest small meson cloud effects

**Very promising method** to estimate **valence quark effects** at low \( Q^2 \)
$\gamma^* N \rightarrow N(1440)$ – Comparing results

Very similar results at large $Q^2$, based on very different approximations

--- CSQM === Holography
\[ \gamma N \rightarrow N(1440) \] – Helicity amplitudes ††

**Gilberto Ramalho** (UNICSUL, SP, Brazil)

Quark model calculations ... at high \( Q^2 \)
$\gamma^* N \rightarrow N^* \rightarrow 2$nd radial excitation of the nucleon

1st assumption $N^* = N(1710)$ not confirmed ($\neq$ radial structure);
Next candidate $N(1880)$; Comparison with Roper and Nucleon data
$\gamma^* N \rightarrow N^*$: Helicity amplitudes [GR and K Tsushima, PRD 89 073010 (2014)]

- Results compared with Roper and Nucleon ($A_{1/2} \propto G_M$; $S_{1/2} \propto G_E$)
- Large $Q^2$: similar results for $A_{1/2}$ and $S_{1/2}$
  (except nucleon: $G_E(7\text{GeV}^2) \approx 0$); Same short range structure!!
\( \gamma^* N \rightarrow N^* \): Helicity amplitudes [GR and K Tsushima, PRD 89 073010 (2014)]

**Prediction:** Amplitude \( A_{1/2} \propto G_M \) (proton): Roper and \( N^* \)

\[
\begin{align*}
A_{1/2}(Q^2) &\propto G_M \text{ (proton)}: \text{ Roper and } N^* \\
Q^2 &\geq 0 \text{ (GeV}^2) \\
A_{1/2}(Q^2) &\propto 10^{-3} \text{ GeV}^{-1/2} \\
\text{Nucleon data} &\text{ (black dots)}
\end{align*}
\]

$\gamma^* N \rightarrow \Delta(1600)$ [GR and K Tsushima, PRD 82, 073007 (2010)]

$\Delta(1600)$ as the 1st radial excitation of $\Delta(1232)$ EPJA, 36, 329 (2008) [S-state]

$G_E^* \equiv 0$, $G_C^* \equiv 0$

**Bare**: $G_M^B(0) = -1.113$

Valence quarks **insufficient** to explain data

$\pi$ cloud effects: estimated w/ $M_{\text{octet}} = M_{\text{decuplet}}$

<table>
<thead>
<tr>
<th>Decay</th>
<th>BR</th>
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<tbody>
<tr>
<td>$\Delta(1600) \rightarrow \pi N$</td>
<td>$0.153 \pm 0.019$</td>
</tr>
<tr>
<td>$\Delta(1600) \rightarrow \pi \Delta$</td>
<td>$0.590 \pm 0.100$</td>
</tr>
<tr>
<td>$\Delta(1600) \rightarrow \pi N(1440)$</td>
<td>$0.130 \pm 0.040$</td>
</tr>
</tbody>
</table>

Final result consistent with $Q^2 = 0$ data

**Predictions for large $Q^2$**
\[ \gamma^* N \rightarrow N(1535) \frac{1}{2}^- \text{ and } \gamma N^* \rightarrow N(1520) \frac{3}{2}^- \]

- Negative parity states
- Valence quark contributions estimated within the CSQM framework
  - \( N(1525) \) GR and MT Peña, PRD 84, 033007 (2011)
    Estimates valid for large \( Q^2 \) (\( Q^2 \gg 0.2 \text{ GeV}^2 \))
  - \( N(1520) \) GR and MT Peña, PRD 89, 094016 (2014);
    GR and MT Peña, PRD 95, 014003 (2017) – timelike region – Talk B Ramstein
    Radial function \( \psi_R \) modified in order to ensure orthogonality

Recent development – **Semirelativistic approximation**
GR, PRD 95, 054008 (2017)

- Mass difference (\( M_R \) and \( M_N \)) neglected in a first approximation
- Radial wave function determined by \( \psi_N \) (nucleon)
  - **Non-relativistic properties;** Covariant expressions
- Orthogonality ensured
- Form factors determined without any adjustable parameter
  Model parameters determined by **Nucleon system**
\( \gamma^* N \rightarrow N(1535) \) – Results

2 form factors; Data from CLAS, MAID and Jlab/Hall C

Expected result at \( Q^2 = 0 \): \( F^*_i(0) = 0 \)

Good results for \( F^*_1 \) (\( Q^2 > 1.5 \text{ GeV}^2 \)); consequence of meson cloud?

\( F^*_2 \) wrong sign; \( (F^*_2)_{\text{exp}} \approx 0 \) for \( Q^2 > 1.5 \text{ GeV}^2 \)
$\gamma^* N \rightarrow N(1535)$: Relation between $A_{1/2}$ and $S_{1/2}$

Implications of $F^*_2 = 0$?

$$\tau = \frac{Q^2}{(M_R+M_N)^2}, \quad Q^2 > 1.5 \text{ GeV}^2$$

$$S_{1/2} \simeq -\frac{\sqrt{1 + \tau}}{\sqrt{2}} \frac{M_R^2 - M_N^2}{2M_RQ} A_{1/2}$$

Cancellation between valence and meson cloud

GR, K Tsushima, PRD 84, 051301 (2011)
GR, D Jido, K Tsushima, PRD 85, 093014 (2012)
D Jido, M Doring and E Oset, PRC 77, 065207 (2008)
\( \gamma^* N \rightarrow N(1535) \): Relation between \( A_{1/2} \) and \( S_{1/2} \)

**Implications of \( F_2^* = 0 \)?**

\[
\tau = \frac{Q^2}{(M_R + M_N)^2} \quad Q^2 > 1.5 \text{ GeV}^2
\]

\[
S_{1/2} \approx -\frac{\sqrt{1 + \tau}}{\sqrt{2}} \frac{M_R^2 - M_N^2}{2M_RQ} A_{1/2}
\]

Cancellation between valence and meson cloud

GR, K Tsushima,
PRD 84, 051301 (2011)

GR, D Jido, K Tsushima,
PRD 85, 093014 (2012)

D Jido, M Doring and E Oset,
PRC 77, 065207 (2008)

More data are welcome
3 independent helicity amplitudes

— SemiRelativistic approach (SR); data from CLAS, include MAID fit

SR very good description of the $Q^2 > 1.5 \text{ GeV}^2$ data
Except for $A_{3/2}$ (CSQM: $A_{3/2} \equiv 0$) – $A_{3/2} \leftarrow$ dominated by meson cloud?

Describe well valence quark degrees of freedom ($A_{3/2}^{bare} \approx 0$)
3 independent form factors

—- SemiRelativistic approach (SR); data from CLAS, include MAID fit

SR very good description of the $Q^2 > 1.5$ GeV$^2$ data
Except for $G_E$ (CSQM: $G_E \equiv -G_M$, $A_{3/2} \equiv 0$)

Describe well valence quark degress of freedom ($A_{3/2}\text{bare} \approx 0$)
\[ \gamma^* N \rightarrow N(1535) \frac{1}{2}^-, \quad \gamma^* N \rightarrow N(1520) \frac{3}{2}^- \] – SR approach

**Summary**

GR, PRD 95, 054008 (2017)

- **No parameters adjusted** \((\psi_R \equiv \psi_N)\) – Predictions

- **SR approach** gives a good description of form factor/helicity amplitude data in the region \(Q^2 > 1.5 \text{ GeV}^2\)

- **Exceptions** (missing meson cloud effects?):
  - \(N(1520)\): \(A_{3/2}\) and \(G_E\)
  - \(N(1535)\): \(F_2^*\)
    - Difficult to describe based on the valence quark degrees of freedom
\[ \gamma^*N \rightarrow N(1535) \frac{1}{2}^-, \gamma^*N \rightarrow N(1520) \frac{3}{2}^- \] – SR approach

**Summary**

GR, PRD 95, 054008 (2017)

- **No parameters adjusted** \((\psi_R \equiv \psi_N)\) – Predictions

- **SR approach** gives a good description of form factor/helicity amplitude data in the region \(Q^2 > 1.5 \text{ GeV}^2\)

- Exceptions (missing meson cloud effects?)
  - \(N(1520)\): \(A_{3/2}\) and \(G_E\)
  - \(N(1535)\): \(F_2^*\)
    Dificult to describe based on the valence quark degrees of freedom

**In progress:**

- Parametrization of mc effects in the \(\gamma^*N \rightarrow N(1535)\) transition

Extension to the timelike region (Dalitz decay; HADES)

GR and MT Peña, in preparation
SQT M – Introduction

Single Quark Transition Model ⊕ CSQM

- SQT M can be applied to the $[70, 1^-]$ supermultiplet
  Hey and Weyers, PL 48B, 69(1974); Cottingham and Dunbar, ZPC 2, 41 (1979);
  Burkert et al, PRC 67, 035204 (2003) – Impulse approximation $SU(6) \otimes O(3)$

- Amplitudes dependent of coefficients $A, B, C$

- Use amplitudes $N(1535)\frac{1}{2}^-, N(1520)\frac{3}{2}^-$ from CSQM
  GR, MT Peña, PRD 84, 033007 (2011); PRD 89, 094016 (2014)
  Valence quark contributions $\rightarrow$ Calculate $A, B, C$

- Estimate amplitudes for
  $N(1650)\frac{1}{2}^-, N(1650)\frac{3}{2}^-, \Delta(1620)\frac{1}{2}^-, \Delta(1700)\frac{3}{2}^-$
  GR, PRD 90, 033010 (2014)

**Model 2**: include parametrization of $A_{3/2}^{MC} \neq 0$ for $N(1520)$

Estimates compared with data from CLAS, MAID and PDG
SQTM: $[70, 1^-]$ amplitudes – $A, B, C; \theta_S, \theta_D$

<table>
<thead>
<tr>
<th>State</th>
<th>Amplitude</th>
<th>Amplitude expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(1535)\frac{1}{2}^-$</td>
<td>$A_{1/2}$</td>
<td>$\frac{1}{6}(A + B - C) \cos \theta_S$</td>
</tr>
<tr>
<td></td>
<td>$A_{3/2}$</td>
<td>$\frac{1}{6\sqrt{2}}(A - 2B - C) \cos \theta_D$</td>
</tr>
<tr>
<td>$N(1520)\frac{3}{2}^-$</td>
<td>$A_{1/2}$</td>
<td>$\frac{1}{6\sqrt{2}}(A - 2B - C) \cos \theta_D$</td>
</tr>
<tr>
<td></td>
<td>$A_{3/2}$</td>
<td>$\frac{1}{2\sqrt{6}}(A + C) \cos \theta_D$</td>
</tr>
<tr>
<td>$N(1650)\frac{1}{2}^-$</td>
<td>$A_{1/2}$</td>
<td>$\frac{1}{6}(A + B - C) \sin \theta_S$</td>
</tr>
<tr>
<td>$\Delta(1620)\frac{1}{2}^-$</td>
<td>$A_{1/2}$</td>
<td>$\frac{1}{18}(3A - B + C)$</td>
</tr>
<tr>
<td>$N(1700)\frac{3}{2}^-$</td>
<td>$A_{1/2}$</td>
<td>$\frac{1}{6\sqrt{2}}(A - 2B - C) \sin \theta_D$</td>
</tr>
<tr>
<td></td>
<td>$A_{3/2}$</td>
<td>$\frac{1}{2\sqrt{6}}(A + C) \sin \theta_D$</td>
</tr>
<tr>
<td>$\Delta(1700)\frac{3}{2}^-$</td>
<td>$A_{1/2}$</td>
<td>$\frac{1}{18\sqrt{2}}(3A + 2B + C)$</td>
</tr>
<tr>
<td></td>
<td>$A_{3/2}$</td>
<td>$\frac{1}{6\sqrt{6}}(3A - C)$</td>
</tr>
</tbody>
</table>
Almost no data for $Q^2 > 2$ GeV$^2$

--- Model 2 better at large $Q^2$
Results for $\Delta(1620), \Delta(1700)$  

Almost no data for $Q^2 > 2$ GeV$^2$

Model 2 better at large $Q^2$
Almost no data for $Q^2 > 2$ GeV$^2$  
--- Model 2 better at large $Q^2$  

More large $Q^2$ data are necessary
Simple parametrization for large $Q^2$ ††

Facilitate comparison with future data – powers from pQCD

\[ A_{1/2}(Q^2) = D \left( \frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^{3/2}, \quad A_{3/2}(Q^2) = D \left( \frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^{5/2} \]

<table>
<thead>
<tr>
<th>State</th>
<th>Amplitude</th>
<th>$D(10^{-3}\text{GeV}^{-1/2})$</th>
<th>$\Lambda^2(\text{GeV}^2)$</th>
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</thead>
<tbody>
<tr>
<td>$S_{11}(1650)$</td>
<td>$A_{1/2}$</td>
<td>68.90</td>
<td>3.35</td>
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<tr>
<td>$S_{31}(1620)$</td>
<td>$A_{1/2}$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$D_{13}(1700)$</td>
<td>$A_{1/2}$</td>
<td>$-8.51$</td>
<td>2.82</td>
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<td>4.36</td>
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<tr>
<td>$D_{33}(1700)$</td>
<td>$A_{1/2}$</td>
<td>39.22</td>
<td>2.69</td>
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<tr>
<td></td>
<td>$A_{3/2}$</td>
<td>42.15</td>
<td>8.42</td>
</tr>
</tbody>
</table>
Some calculations of $N^*$ transition form factors at low $Q^2$
N* at low $Q^2$

- **Nucleon Compton scattering**  
  G Eichmann and GR, PRD 98, 093007 (2018)
  - Accurate **low-$Q^2$ data** is necessary to constraint data parametrizations
  - It is important to select the **correct** form factors
    (no poles and no zeros at $Q^2 = 0$)
  - Take into account the constraints at pseudothreshold
    ($N$ and $N^*$ at rest; Siegert’s theorem)

- **Siegert’s theorem** ($S \propto E|q|$)
  - $\Delta(1232)$, $N(1535)$ and $N(1520)$
  - Rational parametrizations of the form factors are preferred to parametrizations like **MAID** (avoid exponentials)
    - no fast falloff for large $Q^2$
    - smoother extensions for timelike $Q^2 < 0$
    - simpler constraints at the pseudothreshold
$\gamma^* N \rightarrow \Delta: \ G_E^*(Q^2), \ G_C^*(Q^2)$ (bare + pion cloud)

- Bare contributions (QM) determined by fit to lattice [Alexandrou, PRD 77, 085012 (2008)]
- Pion cloud: large $N_c$ relations $\propto \tilde{G}_{En} = \frac{G_{En}}{Q^2}$; [Pascalutsa and Vanderhaeghen, PRD 76, 111501 (2007); Buchmann PRD 66, 056002 (2002)]
- Bare (QM) $\oplus$ pion cloud (theory) contributions $\approx$ data
\( \gamma^* N \rightarrow \Delta: G_E^*(Q^2), G_C^*(Q^2) \) (bare + pion cloud)

- Bare contributions (QM) determined by fit to lattice \[ \text{Alexandrou, PRD 77, 085012 (2008)} \]
- Pion cloud: large \( N_c \) relations \( \propto \tilde{G}_{En} = \frac{G_{En}}{Q^2} \); \[ \text{No extra parameters} \]
  \[ \text{Pascalutsa and Vanderhaeghen, PRD 76, 111501 (2007); Buchmann PRD 66, 056002 (2002)} \]
- Bare (QM) \( \oplus \) pion cloud (theory) contributions \( \approx \text{data} \)
\[ \gamma^* N \rightarrow \Delta: \ G_E^*(Q^2), \ G_C^*(Q^2) \ [GR, \ MT \ Peña, \ PRD \ 80, \ 013008 \ (2009)] \]

Data: CLAS, MAMI, MIT-Bates, PDG

Bare (QM) \( \oplus \) pion cloud (theory) contributions \( \approx \) data
$\gamma^* N \rightarrow \Delta$: $G^*_E(Q^2)$, $G^*_C(Q^2)$ [GR, MT Peña, PRD 80, 013008 (2009)]

We can improve the previous picture in two aspects:

- Consider pion cloud parametrization consistent with Siegert’s theorem
- Compare the model estimates with the more recent $G^*_C$ data
\[ \gamma^* N \rightarrow \Delta: \text{quadrupoles} - \text{Siegert's theorem} - \text{NSTAR 2017} \]

The previous parametrizations are not consistent with Siegert's theorem

\[ \kappa = \frac{M\Delta - M}{2M\Delta} \]

Siegert's theorem: at pseudothreshold \(|q| = 0\);

\[ Q^2 = -(M\Delta - M)^2; \quad G^*_E = \kappa G^*_C \]

Large-\( N_c \): GR, PRD 94, 114001 (2016)

\[ G^\pi_E = \left( \frac{M}{M\Delta} \right)^{3/2} \frac{M\Delta^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_E}{1 + \alpha}, \]

Correct \( G^\pi_E \) with a term in \( \mathcal{O}(1/N^2_c) \)

\[ \alpha = \frac{Q^2}{M\Delta^2 - M^2}, \quad M\Delta - M = \mathcal{O} \left( \frac{1}{N_c} \right) \]

\[ G^\pi_C = \left( \frac{M}{M\Delta} \right)^{1/2} \sqrt{2}MM\Delta \tilde{G}_E \]

Error in ST \( \approx \mathcal{O} \left( \frac{1}{N^4_c} \right) \)

![Graph](image-url)
\( \gamma^* N \to \Delta: \) quadrupoles – Siegert’s theorem – NSTAR 2019

The previous parametrizations are not consistent with Siegert’s theorem:

\[ \kappa = \frac{M_\Delta - M}{2M_\Delta} \]

Siegert’s theorem: at pseudothreshold \(|q| = 0\); \( Q^2 = -(M_\Delta - M)^2 \); \( G^*_E = \kappa G^*_C \)

Improved Large-\( N_c \): GR, EPJA 54, 75 (2018)

\[ \kappa G^*_C = \left( \frac{M}{M_\Delta} \right)^{1/2} \sqrt{2M_\Delta M} \tilde{G}_{En} \]

Correct \( G^*_E \) with a term in \( \mathcal{O}(1/N_c^2) \):

\[ \alpha = \frac{Q^2}{2M_\Delta (M_\Delta - M)} \]

\[ G^*_E = \left( \frac{M}{M_\Delta} \right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \tilde{G}_{En}, \]

\[ G^*_C = \left( \frac{M}{M_\Delta} \right)^{1/2} \sqrt{2M_\Delta M} \tilde{G}_{En} \]

\[ NO \text{ breaking of ST} \]
The previous parametrizations are not consistent with Siegert’s theorem

\[ \kappa = \frac{M_\Delta - M}{2M_\Delta} \]

Siegert’s theorem: at pseudothreshold \( |q| = 0; \)

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Improved Large-\( N_c \): GR, EPJA 54, 75 (2018)

\[ \alpha = \frac{Q^2}{2M_\Delta(M_\Delta - M)} \]

\[ G_E^\pi = \left( \frac{M}{M_\Delta} \right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_{En}}{1 + \alpha}, \]

\[ G_C^\pi = \left( \frac{M}{M_\Delta} \right)^{1/2} \sqrt{2}M\frac{M_\Delta \tilde{G}_{En}}{2M_\Delta - M} \]

New data JLab/Hall A

A Blomberg, PLB 760, 267 (2016)
$\gamma^* N \rightarrow \Delta$: quadrupoles – Global fit

GR, EPJA 55, 32 (2019)

Use quadrupole data to determine best $G_{En}$

Global fit $G_{En}$, $G^*_E$, $G^*_C$

$$G_{En} = -\frac{r^2_n}{6} Q^2 \frac{1+c_2 Q^2+c_3 Q^4}{1+c_1 Q^2+...+\frac{c_5}{5!} Q^{10}}$$

Include theoretical errors associated with the Quark Model (fit lattice QCD data)

$r^4_n \simeq -0.4 \text{ fm}^4$;

$r^2_E = 2.33 \pm 0.19 \text{ fm}^2$; $r^2_C = 1.75 \pm 0.13 \text{ fm}^2$
Summary and conclusions

- We present covariant estimates of the electromagnetic transition form factors for several $N^*$ states at large $Q^2$
  $\Delta(1232), N(1440), N(1520), N(1535), \Delta(1600), \ldots$
  $N(1650), N(1700), \Delta(1620), \Delta(1700)$

- Estimates based on the covariant spectator quark model
  Assume the dominance of valence quark effects for large $Q^2$ ($Q^2 \gtrsim 2$ GeV$^2$)
  (... small meson cloud contributions)

- In some cases we conclude that the inferred meson cloud contributions are significant for $Q^2 = 1–4$ GeV$^2$ – Valence quarks insufficient

- **Large $Q^2$:**
  Future data (JLab-12 GeV upgrade) will be useful to test the present predictions; & to refine calibrations of the models [Nucleon and $\Delta(1232)$]

- **Low-$Q^2$:**
  - Accurate data $Q^2 = 0–0.3$ GeV$^2$ useful to define parametrizations
  - Lattice QCD data ($N(1440), \ldots$); Dynamical Models $\sim$ bare core
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