Nucleon resonances in Compton scattering

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Bonn, Germany

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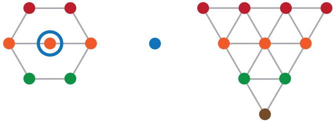
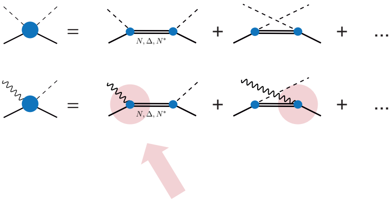
Nucleon resonances

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
$\Delta(1910)$	$\Delta(1620)$ $\Delta(1900)$	$\Delta(1232)$ $\Delta(1600)$ $\Delta(1920)$	$\Delta(1700)$ $\Delta(1940)$	$\Delta(1905)$ $\Delta(2000)$	$\Delta(1930)$	$\Delta(1950)$
$\Lambda(1116)$ $\Lambda(1600)$ $\Lambda(1810)$	$\Lambda(1405)$ $\Lambda(1670)$ $\Lambda(1800)$	$\Lambda(1890)$	$\Lambda(1520)$ $\Lambda(1690)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Sigma(1189)$ $\Sigma(1660)$ $\Sigma(1880)$	$\Sigma(1750)$	$\Sigma(1385)$	$\Sigma(1670)$ $\Sigma(1940)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				



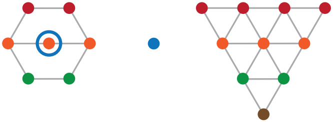
Extraction of resonances?



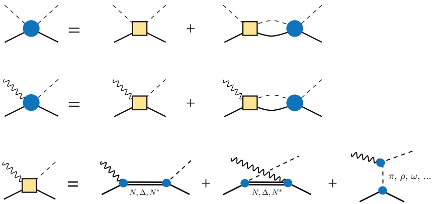
Electromagnetic transition form factors

Nucleon resonances

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
N(939) N(1440) N(1710) N(1880)	N(1535) N(1650) N(1895)	N(1720) N(1900)	N(1520) N(1700) N(1875)	N(1680) N(1860) N(2000)	N(1675)	N(1990)
Δ(1910)	Δ(1620) Δ(1900)	Δ(1232) Δ(1600) Δ(1920)	Δ(1700) Δ(1940)	Δ(1905) Δ(2000)	Δ(1930)	Δ(1950)
Λ(1116) Λ(1600) Λ(1810)	Λ(1405) Λ(1670) Λ(1800)	Λ(1890)	Λ(1520) Λ(1690)	Λ(1820)	Λ(1830)	
Σ(1189) Σ(1660) Σ(1880)	Σ(1750)	Σ(1385)	Σ(1670) Σ(1940)	Σ(1915)	Σ(1775)	
Ξ(1315)		Ξ(1530)	Ξ(1820)			
		Ω(1672)				



Extraction of resonances?



Sato-Lee/EBAC/ANL-Osaka,
Dubna-Mainz-Taiwan,
Valencia,
Jülich-Bonn,
GSI,

JLab,
MAID,
SAID,
KSU,
Giessen,
Bonn-Gatchina,
JPAC, . . .

Nucleon resonances

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$
$N(939)$ $N(1440)$ $N(1710)$ $N(1880)$	$N(1535)$ $N(1650)$ $N(1895)$	$N(1720)$ $N(1900)$	$N(1520)$ $N(1700)$ $N(1875)$	$N(1680)$ $N(1860)$ $N(2000)$	$N(1675)$	$N(1990)$
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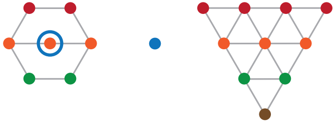
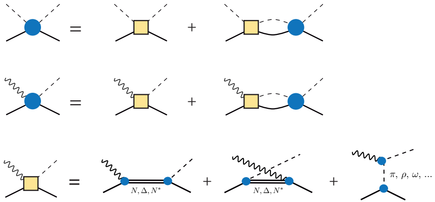
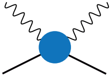


Extraction of resonances?

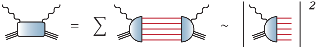


- **Lorentz invariance?**
- **Em. gauge invariance?**
- **EFT $\leftrightarrow$ QCD?**
- What is an “offshell hadron”?

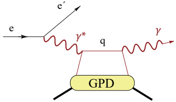
Compton scattering



Structure functions
& PDFs in forward limit

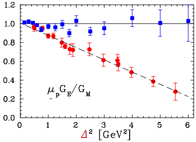


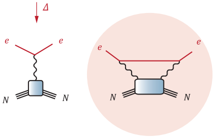
Handbag dominance
& GPDs in DVCS



TPE corrections to form factors

Guichon, Vanderhaeghen, PRL 91 (2003)



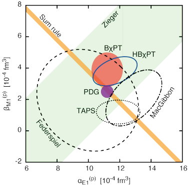


Proton radius puzzle?

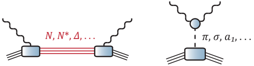
Antonigni et al., 2013, Pohl et al. 2013,
Birse, McGovern 2012, Carlson 2015

Nucleon polarizabilities

Hagelstein, Miskimen, Pascalutsa,
Prog. Part. Nucl. Phys. 88 (2016)

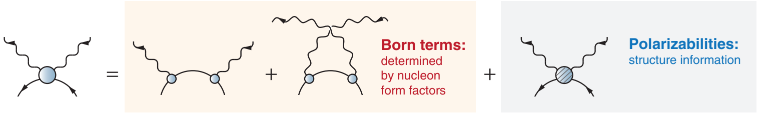


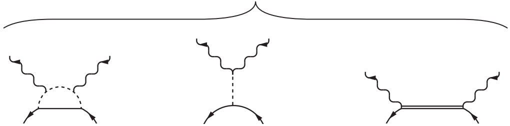
Resonances!



Compton scattering

Compton amplitude = sum of **Born terms + 1PI structure part:**

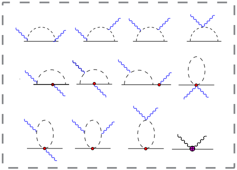




„Pion cloud“
(ChPT)

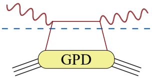
t-channel
meson exchange
$(\pi, \sigma, a_1, \ldots)$

s/u-channel
nucleon resonances
$(\Delta, N^*, \ldots)$



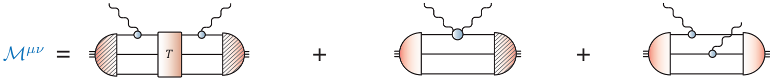
Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

but also:



⇒ is there a common underlying
quark-level description?

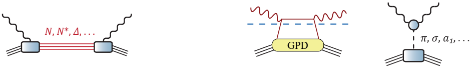
Compton scattering



Nucleon resonances

quark **Compton vertex:** reproduces perturbative handbag & t-channel meson poles

cat's ears diagrams

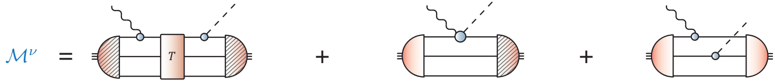


GE, Fischer, PRD 85 (2012), PRD 87 (2013)

- **Lorentz invariance**
- **Crossing symmetry**
- **Em. gauge invariance**
- **Chiral symmetry**
- **Perturbative QCD** included
- **s, t, u channel poles** dynamically generated, no offshell hadrons inside - only quarks & gluons

But: consistency important, simple approximations can be dangerous $\longrightarrow$ **challenge**

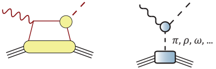
Meson electroproduction



Nucleon resonances

quark **meson-photon vertex:** reproduces perturbative diagrams & t-channel meson poles

cat's ears diagrams



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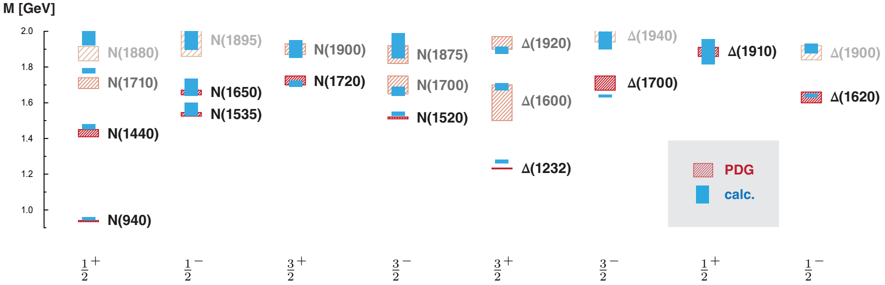
But: consistency important, simple approximations can be dangerous $\longrightarrow$ **challenge**

Baryons with functional methods



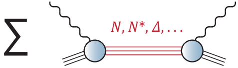
Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

→ see **Christian Fischer's** plenary Thursday morning

→ talks by **Craig Roberts, José Rodriguez-Quintero, Chen Chen**

Nucleon resonances

GE, Ramalho, PRD 98 (2018)



$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940)	**N(1720)**	**N(1535)**	**N(1520)**
N(1440)	$N(1900)$	**N(1650)**	$N(1700)$
$N(1710)$		$N(1895)$	$N(1875)$
$N(1880)$			
Δ(1910)	**Δ(1232)**	**Δ(1620)**	**Δ(1700)**
	Δ(1600)	Δ(1900)	Δ(1940)
	Δ(1920)		

Need **em. transition FFs**

But vertices are half offshell:
need 'consistent couplings'
Pascalutsa, Timmermans, PRC 60 (1999)

- **em gauge invariance:** $Q^\mu \, \Gamma^{\alpha\mu} = 0$
- **spin-3/2 gauge invariance:** $k^\alpha \, \Gamma^{\alpha\mu} = 0$
- invariance under **point transformations:** $\gamma^\alpha \, \Gamma^{\alpha\mu} = 0$
- no kinematic dependencies, **"minimal" basis**

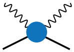
E.g. Jones-Scadron current **cannot be used offshell:**

$$\Gamma^{\alpha\mu} \sim \bar{u}^\alpha(k) \Big[m^2 \lambda_- (G_M^* - G_E^*) \, \varepsilon_{kQ}^{\alpha\mu} - G_E^* \, \varepsilon_{kQ}^{\alpha\beta} \, \varepsilon_{kQ}^{\beta\mu} - \tfrac{1}{2} \, G_C^* \, Q^\alpha k^\beta t_{QQ}^{\beta\mu} \Big] \, u(k')$$

$$t_{AB}^{\alpha\beta} = A \cdot B \, \delta^{\alpha\beta} - B^\alpha A^\beta$$

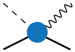
$$\varepsilon_{AB}^{\alpha\beta} = \gamma_5 \, \varepsilon^{\alpha\beta\gamma\delta} A^\gamma B^\delta$$

Minimal tensor bases



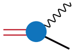
Compton scattering

Tarrach, Nuovo Cim. A28 (1975)
GE, Ramalho, PRD 98 (2018)



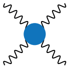
Meson electroproduction

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)



Nucleon-to-resonance transition currents

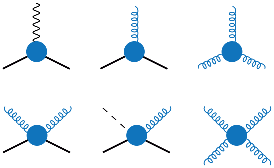
GE, Ramalho, PRD 98 (2018)



Light-by-light scattering?

GE, Fischer, Heupel, PRD 92 (2015)

... also **n-point functions with quarks & gluons**

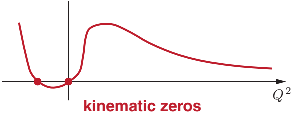


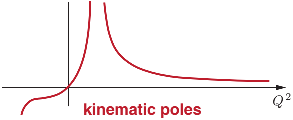
Minimal tensor bases

$$\Gamma^{\mu\nu} = \sum_i c_i\, K_i^{\mu\nu} = \underbrace{\sum_i g_i\, G_i^{\mu\nu}}_{\textbf{G}} + \underbrace{\sum_j f_j\, X_j^{\mu\nu}}_{\textbf{T}}$$

Minimal basis: neither g_i, f_j nor G_i, X_j become singular

Without minimal basis:





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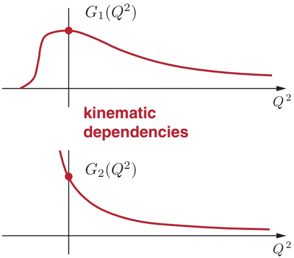

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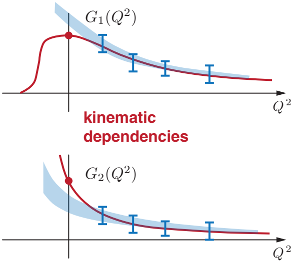

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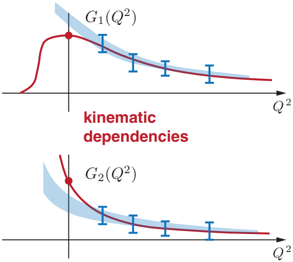
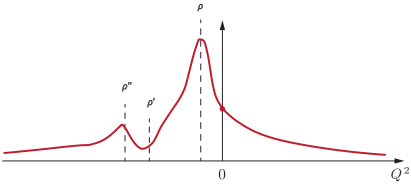


With minimal basis:
no kinematic dependencies,
only ‘physical’ poles and cuts!



Minimal tensor bases

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Transversality constraints:

$$\begin{aligned} Q'^{\mu}\,\Gamma^{\mu\nu} &= 0 \\ Q^{\nu}\,\Gamma^{\mu\nu} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ \\ c_n \end{bmatrix} = 0$$

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Row-reduced echelon form:

$$\left[\begin{array}{ccc|cccccc} \multicolumn{3}{c|}{\text{dim G}} & \multicolumn{6}{c}{\text{dim T}} \\ 1 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 1 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} = 0$$

Minimal tensor bases

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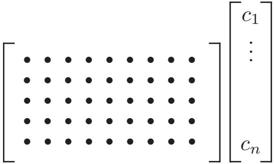
Minimal basis: neither g_i, f_j nor G_i, X_j become singular

Transversality constraints:

$$Q'^{\mu}\,\Gamma^{\mu\nu} = 0$$
$$Q^{\nu}\,\Gamma^{\mu\nu} = 0 \quad \Rightarrow$$

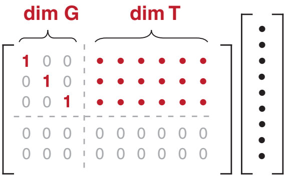


$= 0$

A **minimal basis** exists, if

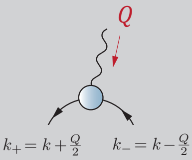
- by swapping columns (= renaming basis tensors)
- adding / subtracting rows, multiplying rows with scalars (Gauss-Jordan elimination)

one can find a **row-reduced echelon form** where  is nonsingular in any kinematic limit



$= 0$

An example



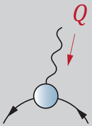
3 variables:

$$k^2,\ Q^2,\ w = k\cdot Q$$

$$\Gamma^\mu(k,Q) = c_1\, k^\mu + c_2\, Q^\mu$$

$$\overline{\Gamma}^\mu(k,Q) := \Gamma^\mu(-k,-Q) \stackrel{!}{=} -\Gamma^\mu(k,-Q) \qquad \text{(charge conjugation)}$$

An example



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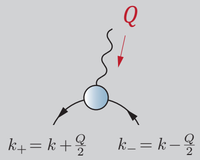

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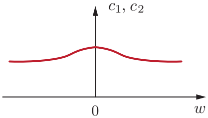

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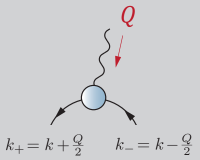

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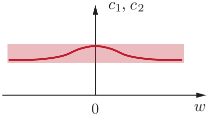

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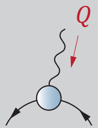

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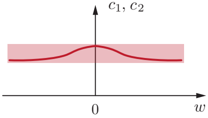
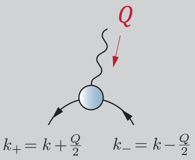


Transversality:

$$Q^\mu\,\Gamma^\mu = c_1\, w + c_2\, w\, Q^2 = 0$$

An example



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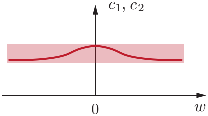

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$$\Rightarrow \begin{bmatrix} w & w\,Q^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$

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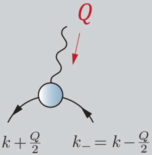

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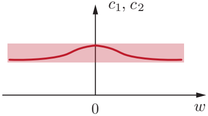
$$\Rightarrow \begin{bmatrix} 1 & Q^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$



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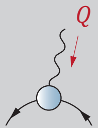

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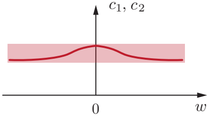
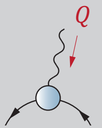


but **not**

$$\begin{bmatrix} 1 & \frac{1}{Q^2} \end{bmatrix} \begin{bmatrix} c_2 \\ c_1 \end{bmatrix} = 0 \quad \textbf{!!}$$

An example



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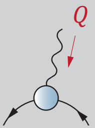
$$c_1 = -c_2\, Q^2 \quad \Rightarrow \quad \Gamma^\mu = -c_2\, (Q^2\, k^\mu - w\, Q^\mu) = -c_2\, (Q^2\, \delta^{\mu\nu} - Q^\mu Q^\nu)\, k^\nu = -c_2\, t^{\mu\nu}_{QQ}\, k^\nu$$

$$\Rightarrow \quad \Gamma^\mu(k,Q) = \underbrace{g_1\, k^\mu}_{\textbf{G}} + \underbrace{f_1\, t^{\mu\nu}_{QQ}\, k^\nu}_{\textbf{T}}$$

An example



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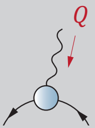
Ward-Takahashi identity only affects **G:**

$$Q^\mu\, \Gamma^\mu = D(k_+)^{-1} - D(k_-)^{-1} = g_1\, w$$

$$\Rightarrow \quad g_1 = 2\,\frac{D(k_+)^{-1} - D(k_-)^{-1}}{k_+^2 - k_-^2} = 2\Delta$$

$$\Rightarrow \quad \boldsymbol{\Gamma^\mu(k,Q) = \underbrace{g_1\, k^\mu}_{\textbf{G}} + \underbrace{f_1\, t^{\mu\nu}_{QQ}\, k^\nu}_{\textbf{T}}}$$

An example



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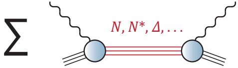
Transverse-longitudinal separation?

$$\Gamma^\mu(k,Q) = \tilde{g}_1\, w\, Q^\mu + \tilde{f}_1\, t^{\mu\nu}_{QQ}\, k^\nu \qquad \Rightarrow \qquad \Gamma^\mu(k,Q) = \underbrace{g_1\, k^\mu}_{\textbf{G}} + \underbrace{f_1\, t^{\mu\nu}_{QQ}\, k^\nu}_{\textbf{T}}$$

$$Q^\mu\, \Gamma^\mu = D(k_+)^{-1} - D(k_-)^{-1} = \tilde{g}_1 w\, Q^2$$

$\Rightarrow \quad \tilde{g}_1 = \frac{2\Delta}{Q^2} \quad \Rightarrow \quad \tilde{f}_1 = f_1 + \frac{2\Delta}{Q^2} \quad \Rightarrow$ both **kinematically dependent** and **singular!**

Nucleon resonances



$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940)	**N(1720)**	**N(1535)**	**N(1520)**
N(1440)	$N(1900)$	**N(1650)**	$N(1700)$
$N(1710)$		$N(1895)$	$N(1875)$
$N(1880)$			
Δ(1910)	**Δ(1232)**	**Δ(1620)**	**Δ(1700)**
$\Delta(1600)$		$\Delta(1900)$	$\Delta(1940)$
	$\Delta(1920)$		

Need **em. transition FFs**

But vertices are half offshell:
need 'consistent couplings'
Pascalutsa, Timmermans, PRC 60 (1999)

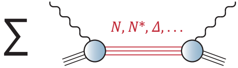
- **em gauge invariance:** $Q^\mu \, \Gamma^{\alpha\mu} = 0$
- **spin-3/2 gauge invariance:** $k^\alpha \, \Gamma^{\alpha\mu} = 0$
- invariance under **point transformations:** $\gamma^\alpha \, \Gamma^{\alpha\mu} = 0$
- no kinematic dependencies, **"minimal" basis**

Most general **offshell vertices** satisfying these constraints:
GE, Ramalho, 1806.04579

$$\tfrac{1}{2}^+ \to \tfrac{1}{2}^\pm: \quad \Gamma^\mu = \begin{bmatrix} \mathbf{1} \\ \gamma_5 \end{bmatrix} \sum_{i=1}^{8} F_i \, T_i^\mu \qquad \begin{cases} t^{\mu\nu}_{QQ}\gamma^\nu \\ [\gamma^\mu, \not{Q}] \\ \dots \end{cases}$$

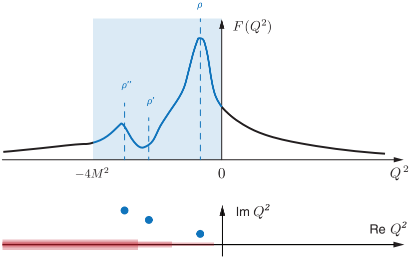
$$\tfrac{1}{2}^+ \to \tfrac{3}{2}^\pm: \quad \Gamma^{\alpha\mu} = \begin{bmatrix} \gamma_5 \\ \mathbf{1} \end{bmatrix} \sum_{i=1}^{12} F_i \, T_i^{\alpha\mu} \qquad \begin{cases} \varepsilon^{\alpha\mu}_{kQ} \\ t^{\alpha\mu}_{kQ} \\ it^{\alpha\beta}_{k\gamma} t^{\beta\mu}_{QQ} \\ \dots \end{cases}$$

Nucleon resonances

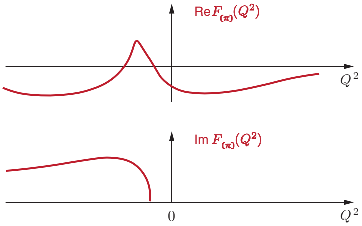


Constraint-free transition FFs:
only physical poles and cuts

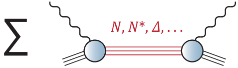
- ρ **poles** $\rightsquigarrow$ monotonous behavior (+ zero crossings for excited states)



- Non-monotonicity at low Q2 $\rightsquigarrow$ signature for cuts ($\rho \to \pi\pi$, etc.): **meson cloud**



Nucleon resonances



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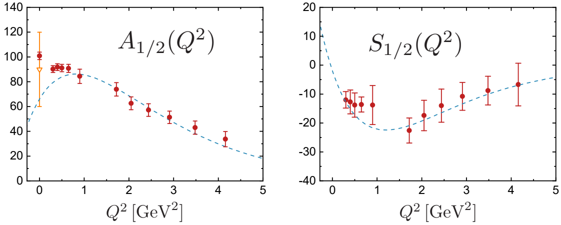


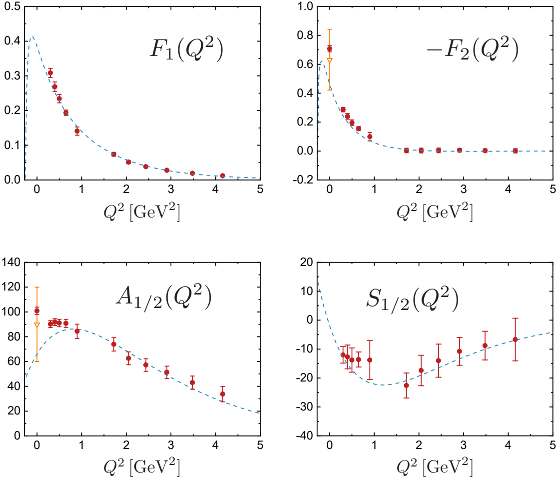
Example:
N(1535) helicity amplitudes

PDG

CLAS data
userweb.jlab.org/~mokeev/resonance_electrocouplings

MAID
Tiator, Drechsel, Kamalov, Vanderhaeghen, EPJ 198 (2011)

Nucleon resonances



N(1535) transition FFs:
no kinematic constraints

Example:
N(1535) helicity amplitudes

PDG

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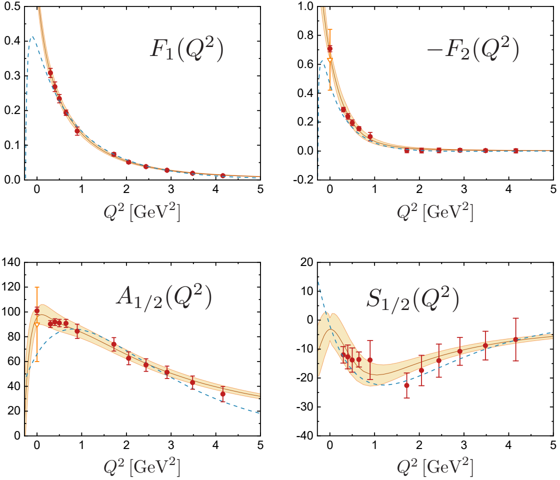

N(1535) transition FFs:
no kinematic constraints

Fit

Example:
N(1535) helicity amplitudes

PDG

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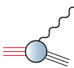

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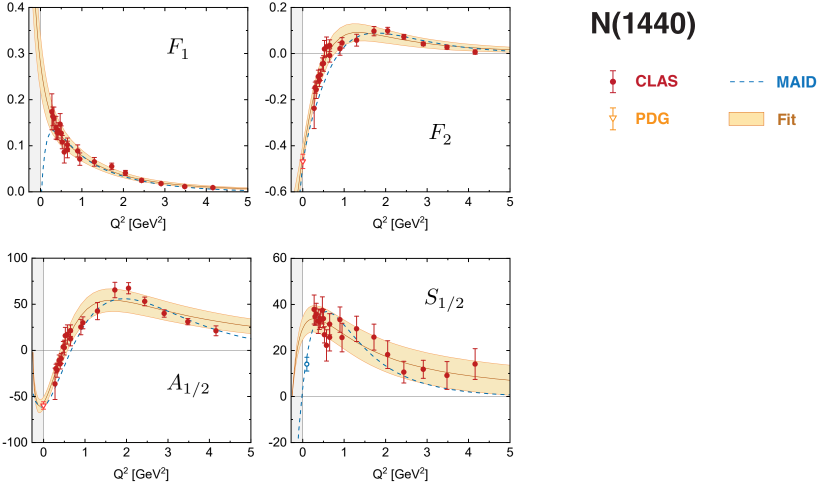
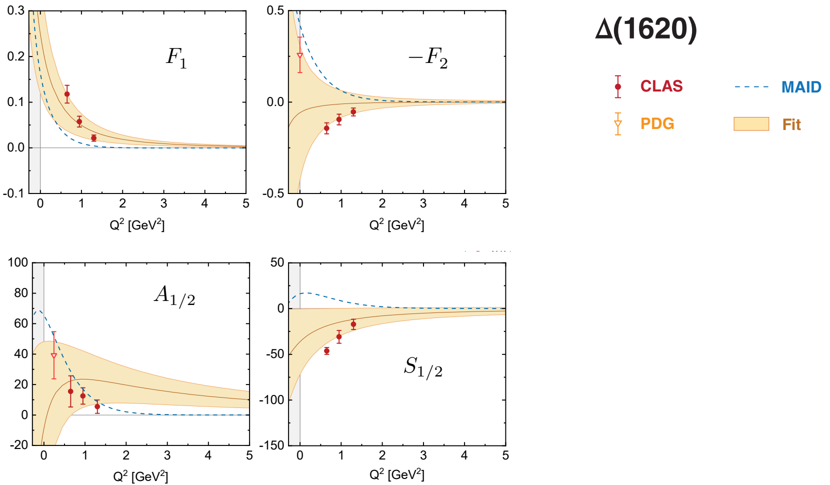
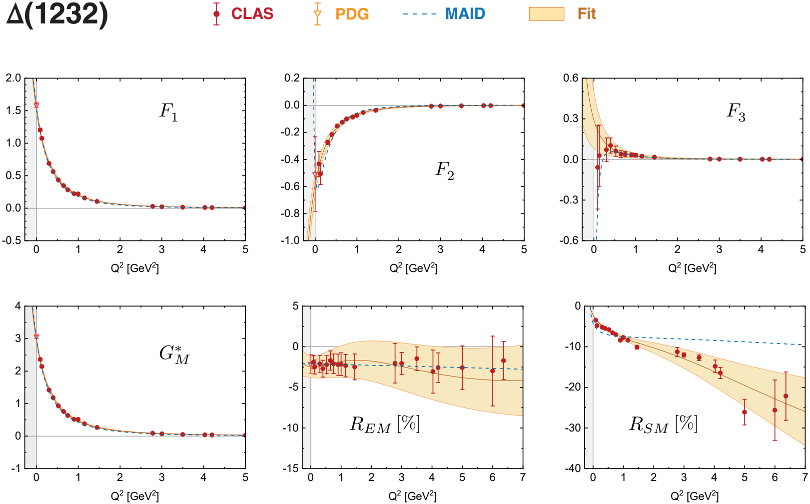


Nucleon resonances

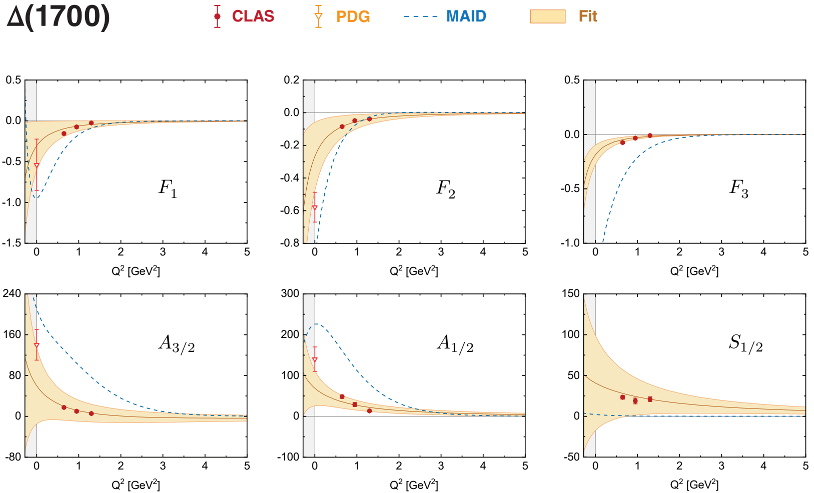


Nucleon resonances

$\Delta(1232)$

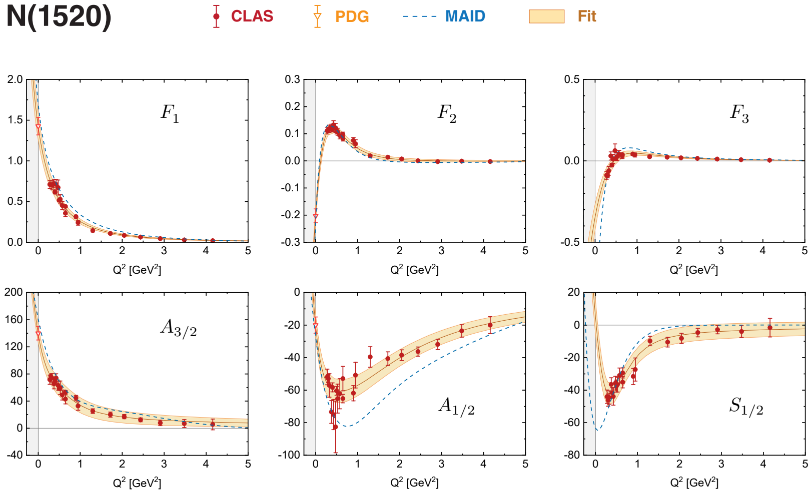


Nucleon resonances

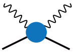


Nucleon resonances

N(1520)



Kinematics

 $$= \sum_{i=1}^{18} c_i(\eta_+,\eta_-,\omega,\lambda)\,\bar{u}(p_f)\,X_i^{\mu\nu}(p,Q,Q')\,u(p_i)$$

18 CFFs

4 kinematic variables:

$$\eta_+ = \frac{Q^2+Q'^2}{2m^2}$$
$$\eta_- = \frac{Q\cdot Q'}{m^2}$$
$$\omega = \frac{Q^2-Q'^2}{2m^2}$$
$$\lambda = -\frac{p\cdot Q}{m^2}$$

18 Compton tensors,
form minimal basis

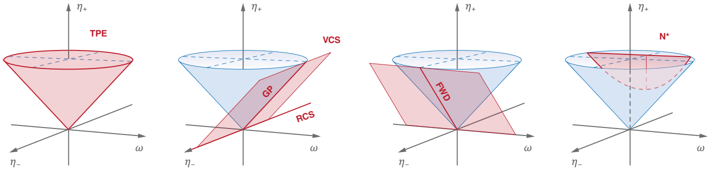
- systematic derivation
- similar to Tarrach basis
  Tarrach, Nuovo Cim. A28 (1975)

$$X_i' = U_{ij}\,X_j\,, \qquad \det U = const.$$

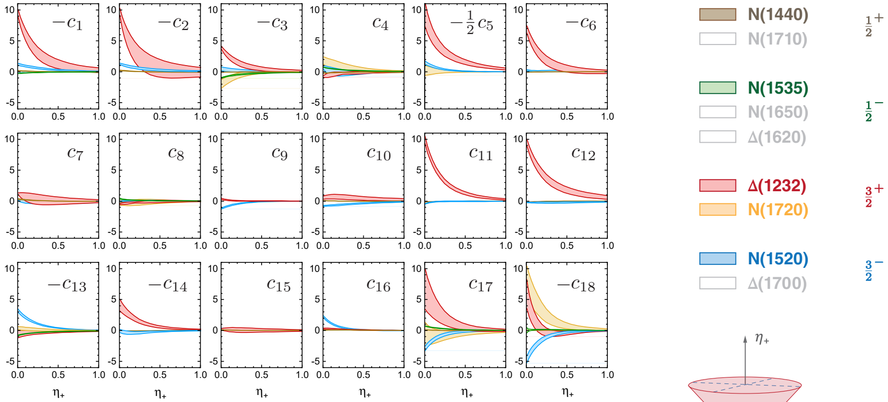
- CFFs free of kinematics

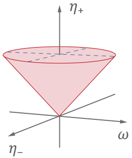
$$X_1^{\mu\nu} = \frac{1}{m^4}\,t^{\mu\alpha}_{Q'p}\,t^{\alpha\nu}_{pQ}\,,$$
$$X_2^{\mu\nu} = \frac{1}{m^2}\,t^{\mu\nu}_{Q'Q}\,,$$
$$X_3^{\mu\nu} = \frac{1}{m^4}\,t^{\mu\alpha}_{Q'Q'}\,t^{\alpha\nu}_{QQ}\,,$$
$$X_4^{\mu\nu} = \frac{1}{m^6}\,t^{\mu\alpha}_{Q'Q'}\,p^\alpha p^\beta\,t^{\beta\nu}_{QQ}\,,$$
$$X_5^{\mu\nu} = \frac{\lambda}{m^4}\left(t^{\mu\alpha}_{Q'Q'}\,t^{\alpha\nu}_{pQ} + t^{\mu\alpha}_{Q'p}\,t^{\alpha\nu}_{QQ}\right),$$
$$X_6^{\mu\nu} = \frac{1}{m^2}\,\varepsilon^{\mu\nu}_{Q'Q}\,,$$
$$X_7^{\mu\nu} = \frac{1}{im^3}\left(t^{\mu\alpha}_{Q'Q'}\,\varepsilon^{\alpha\nu}_{\gamma Q} - \varepsilon^{\mu\alpha}_{Q'\gamma}\,t^{\alpha\nu}_{QQ}\right),$$
$$X_8^{\mu\nu} = \frac{\omega}{im^3}\left(t^{\mu\alpha}_{Q'Q'}\,\varepsilon^{\alpha\nu}_{\gamma Q} + \varepsilon^{\mu\alpha}_{Q'\gamma}\,t^{\alpha\nu}_{QQ}\right),$$
$$\vdots$$

GE, Ramalho, PRD 98 (2018)



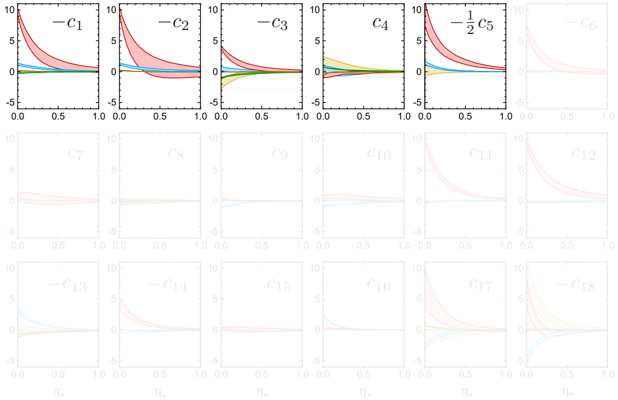
Compton form factors





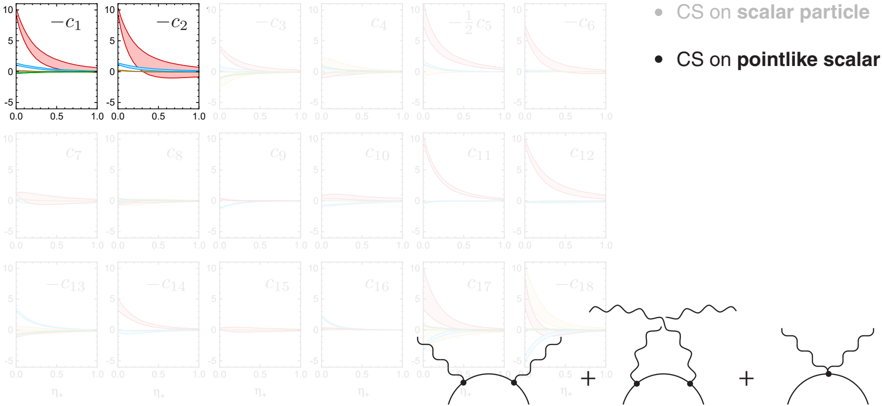
To be multiplied with $\dfrac{(m_R^2-m^2)^2}{(s-m_R^2)(u-m_R^2)} = \dfrac{\delta^2}{(\eta_-+\delta)^2-4\lambda^2}$

Compton form factors



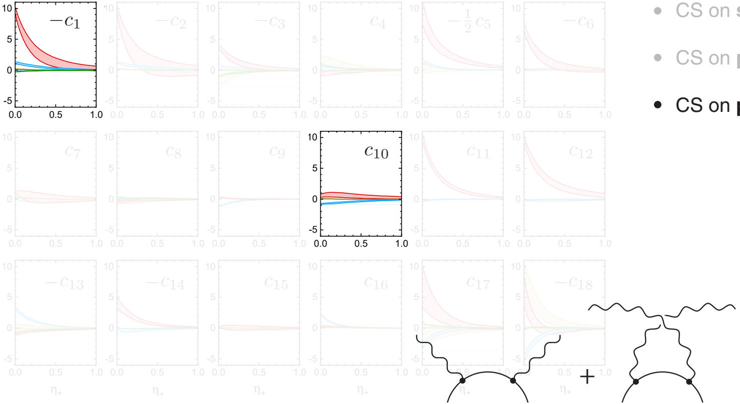
- CS on **scalar particle**

Compton form factors



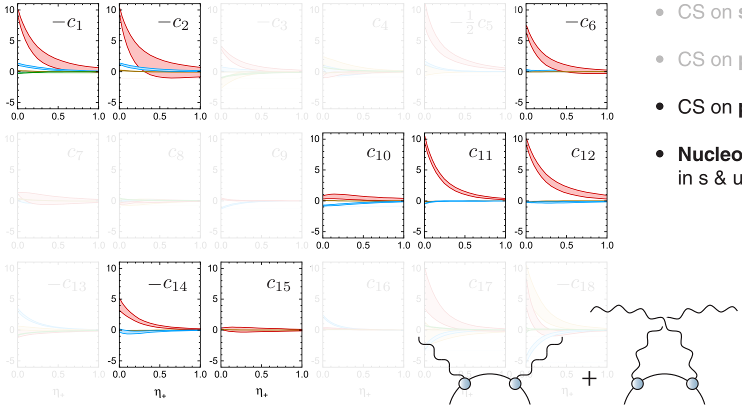
- CS on **scalar particle**
- CS on **pointlike scalar**

Compton form factors



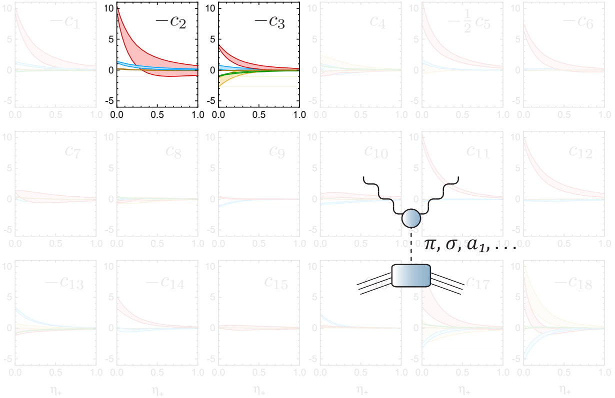
- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**

Compton form factors



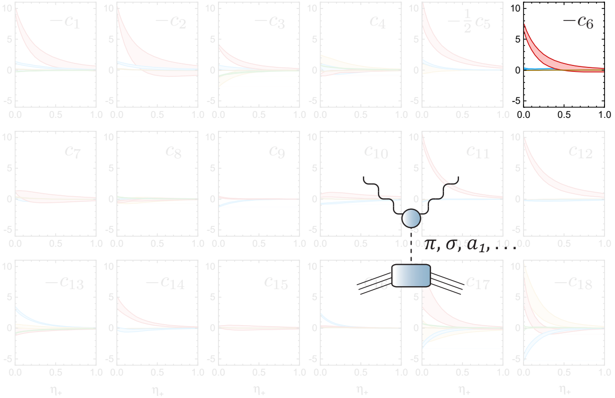
- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel

Compton form factors

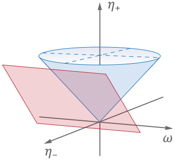


- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel
- **Scalar pole** in t channel

Compton form factors

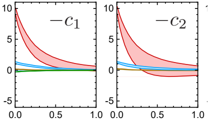


- CS on **scalar particle**
- CS on **pointlike scalar**
- CS on **pointlike fermion**
- **Nucleon Born poles** in s & u channel
- **Scalar pole** in t channel
- **Pion pole** in t channel ($\pi^0 \to \gamma^*\gamma^*$)



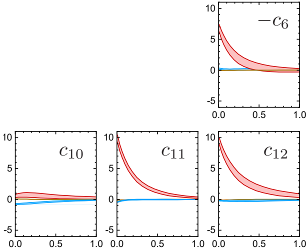
GE, Fischer, Weil, Williams, PLB 774 (2017)

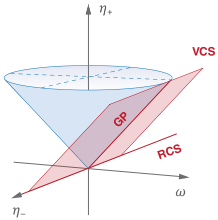
Polarizabilities



Scalar polarizabilities:

$$\begin{bmatrix} \alpha+\beta \\ \beta \end{bmatrix} = -\frac{\alpha_{\text{em}}}{m^3} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$



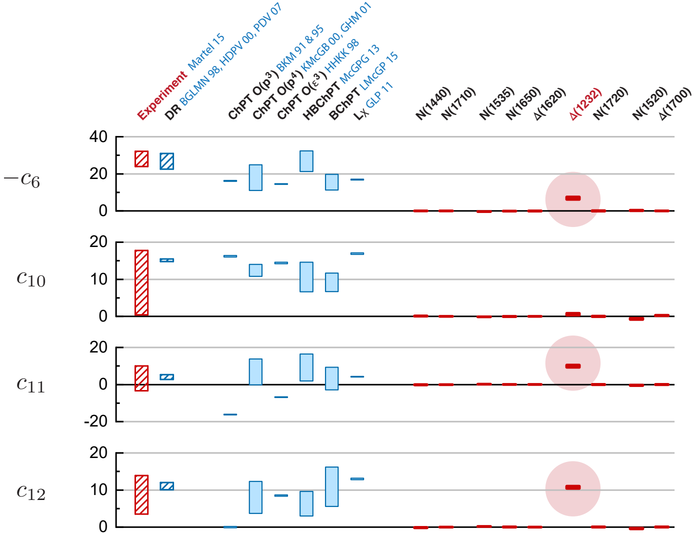


Spin polarizabilities:

$$\begin{bmatrix} \gamma_{E1E1} \\ \gamma_{M1M1} \\ \gamma_{E1M2} \\ \gamma_{M1E2} \end{bmatrix} = \frac{\alpha_{\text{em}}}{2m^4} \begin{bmatrix} c_6 + 4c_{11} - 4c_{12} \\ -c_6 - 2c_{10} + 4c_{12} \\ c_6 + 2c_{10} \\ -c_6 \end{bmatrix}$$

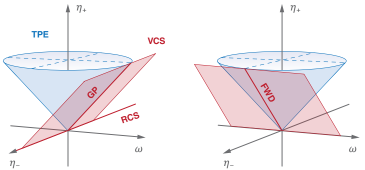
$$\begin{bmatrix} \gamma_0 \\ \gamma_\pi \end{bmatrix} = -\frac{2\alpha_{\text{em}}}{m^4} \begin{bmatrix} c_{11} \\ c_6 + c_{10} + c_{11} - 2c_{12} \end{bmatrix}$$

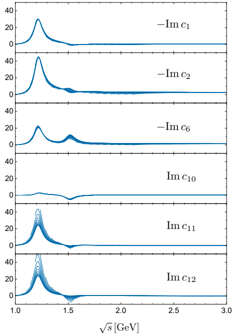
Spin polarizabilities



Only $\Delta(1232)$ important

Summary





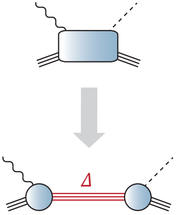
Resonance contributions in **RCS**

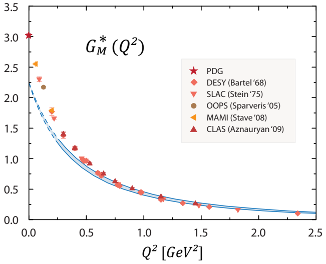
- **Compton scattering:** minimal basis, constraint-free CFFs
- **Electromagnetic transition currents:** minimal bases, constraint-free TFFs
- **Fits** of experimental TFFs
- **Nucleon resonances** in Compton scattering: derived in general kinematics
- Only **Δ(1232)** and **N(1520)** relevant for **polarizabilities**

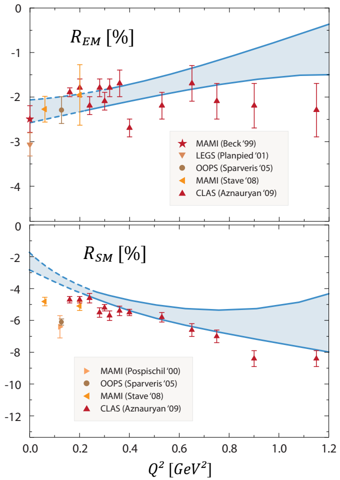
GE, Ramalho, PRD 98 (2018)

Backup slides

Nucleon-Δ-γ transition







- **Magnetic dipole transition (G_M^*) dominant:** quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole ratios** small & negative, encode deformation. Reproduced without pion cloud: **OAM** from **p waves!**

GE, Nicmorus, PRD 85 (2012)