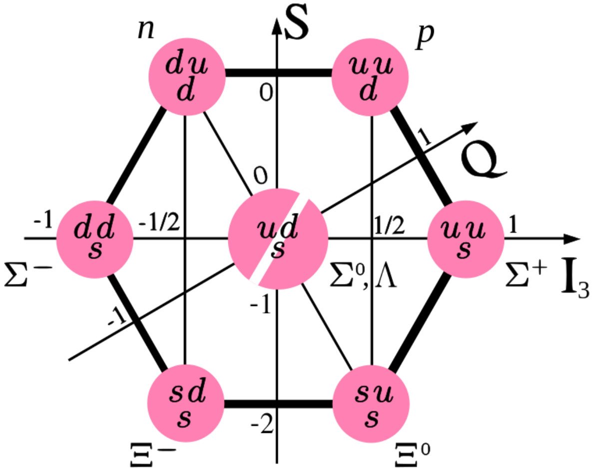
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Spectrum and structure of octet and decuplet baryons and their positive-parity excitations

Chen Chen

University of Giessen



NSTAR2019

The 12th International Workshop on the Physics of Excited Nucleons

June 11th, 2019
Bonn University

UNIVERSITÄT BONN

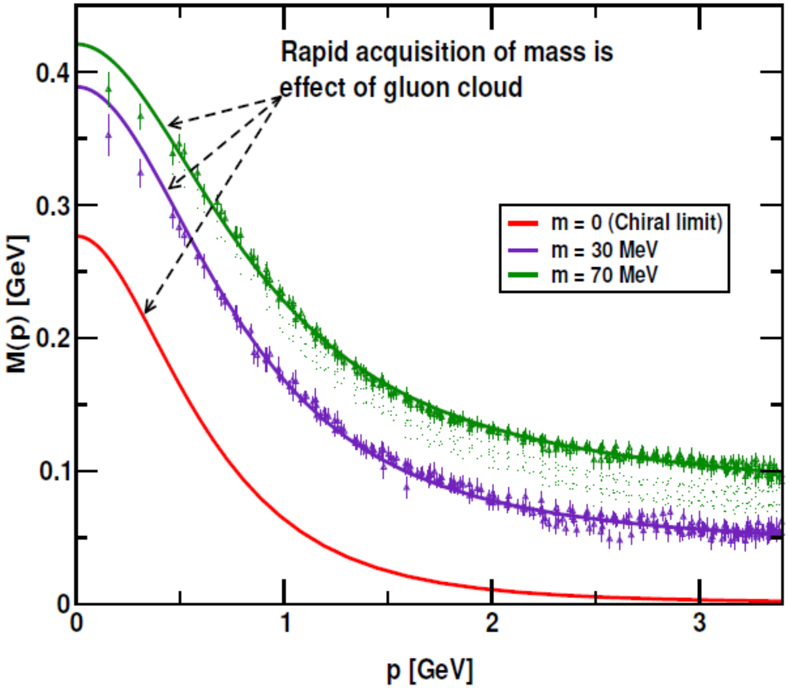
Non-Perturbative QCD:

- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics –** *Two emergent phenomena*
  - **Confinement**: Colored particles have never been seen isolated
  - Explain how quarks and gluons bind together
  - **DCSB**: Hadrons do not follow the chiral symmetry pattern
  - Explain the most important mass generating mechanism for visible matter in the Universe
  - Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

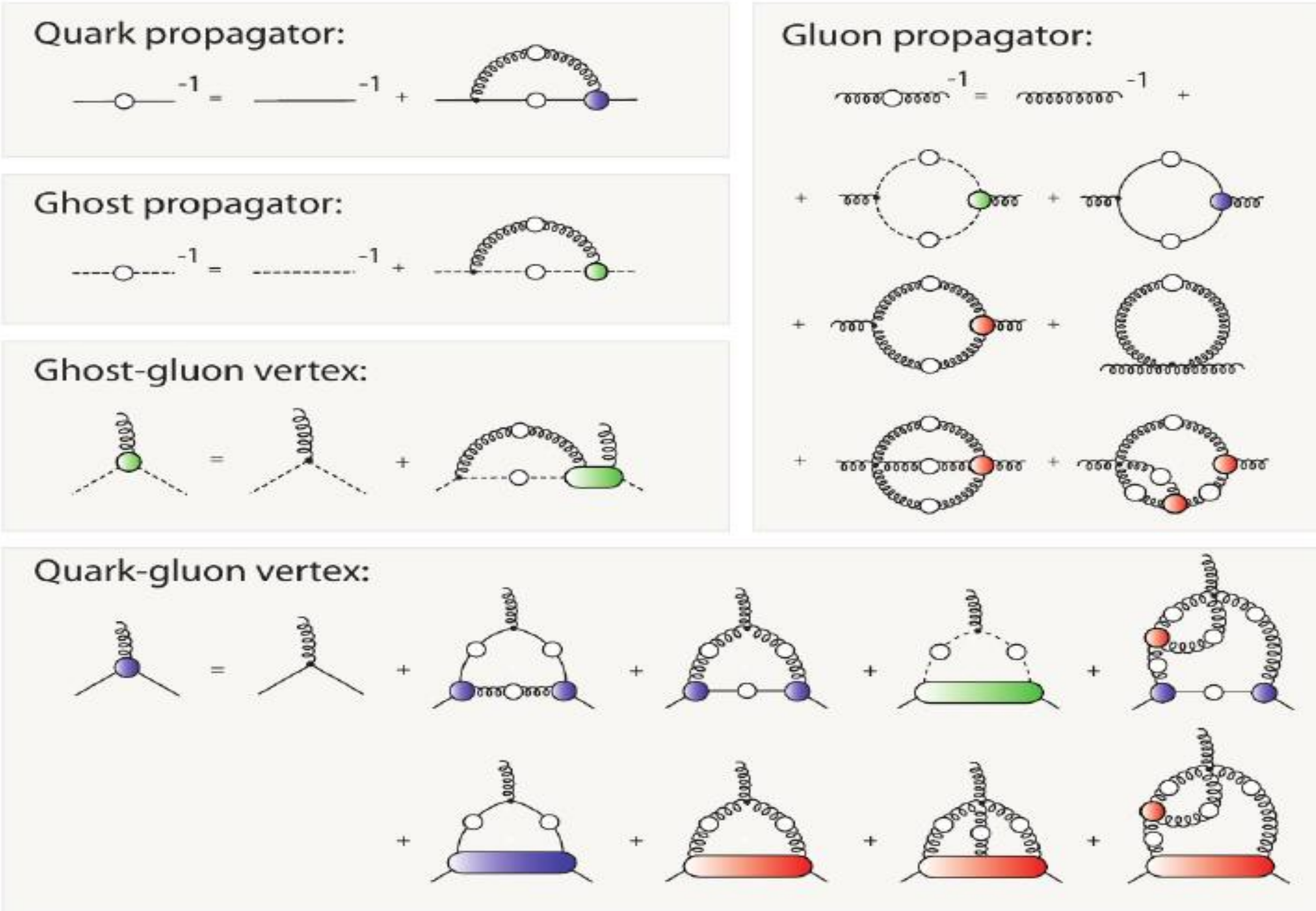
Non-Perturbative QCD:

- **From a quantum field theoretical point of view**, these emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD 's Schwinger functions (propagators and vertices). The Schwinger functions are solutions of the quantum equations of motion (**Dyson-Schwinger equations**).
- **Dressed-quark propagator:**

  - Mass generated from the interaction of quarks with the gluon.
  - Light quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton and the large splitting between parity partners.



Dyson-Schwinger equations (DSEs)



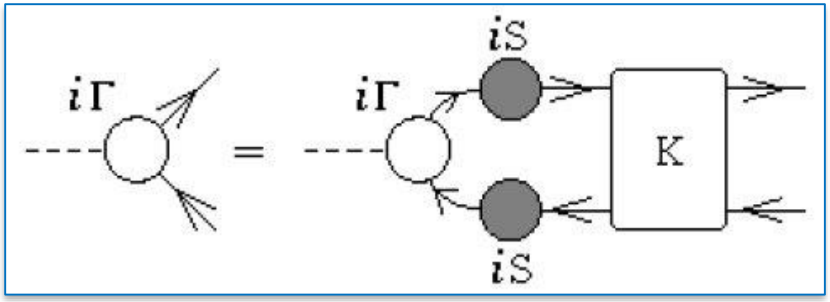
Dyson-Schwinger equations (DSEs)

Dyson-Schwinger equations

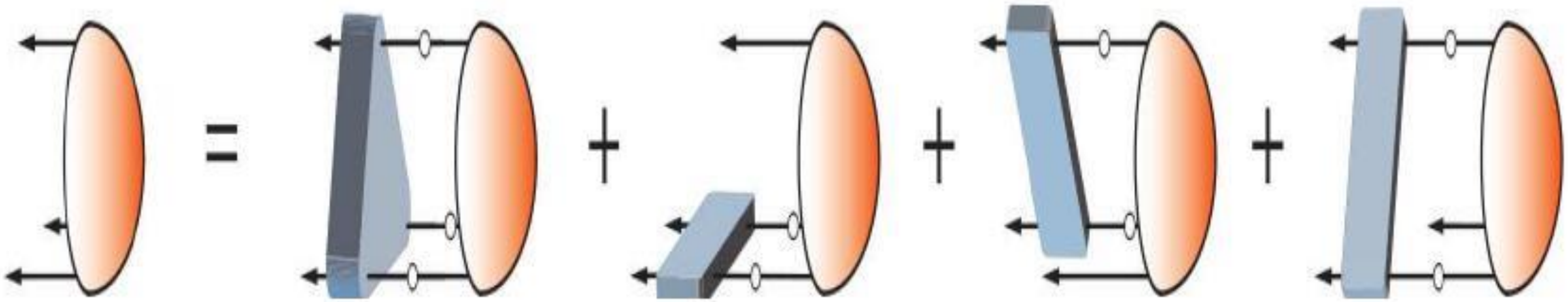
- ✓ A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
- ✓ Well suited to Relativistic Quantum Field Theory
- ✓ A method connects observables with long-range behaviour of the running coupling
- ✓ Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

Hadrons: Bound-states in QFT

- **Mesons: a 2-body bound state problem in QFT**
  - ***Bethe-Salpeter Equation***
  - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**

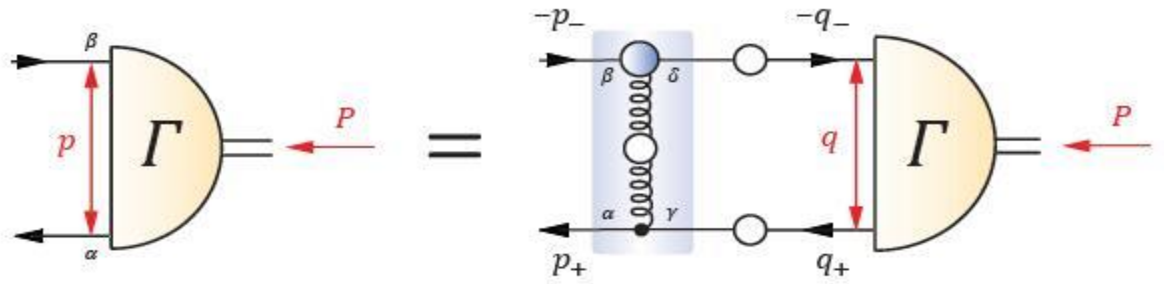


- **Baryons: a 3-body bound state problem in QFT.**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**



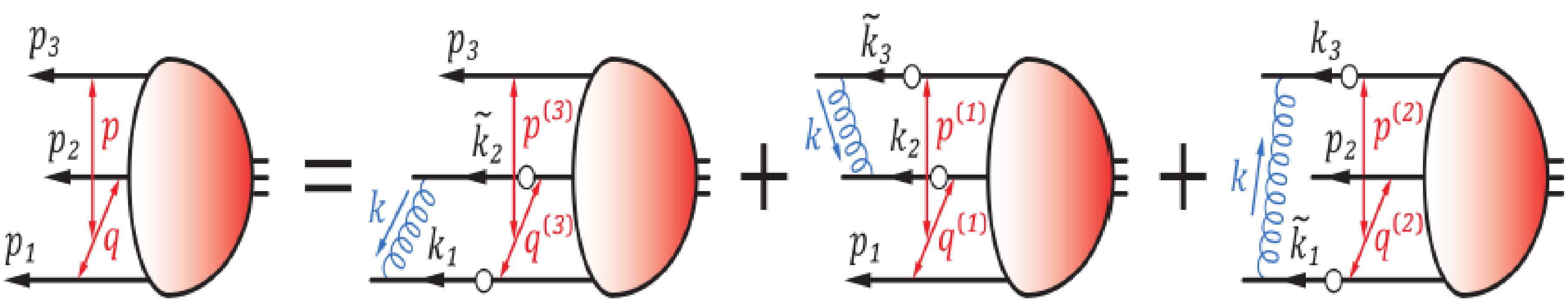
Hadrons: Bound-states in QFT

- **Mesons: a 2-body bound state problem in QFT**
  - ***Bethe-Salpeter Equation***
  - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons: a 3-body bound state problem in QFT.**
- ***Faddeev equation*: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**

Faddeev equation in rainbow-ladder truncation



2-body correlations

- **Mesons: quark-antiquark correlations -- color-singlet**
- **Diquarks: quark-quark correlations within a color-singlet baryon.**

- **Diquark correlations:**
  - In our approach: non-pointlike color-antitriplet and fully interacting.
  - Diquark correlations are soft, they possess an electromagnetic size.
  - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p;P) = - \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

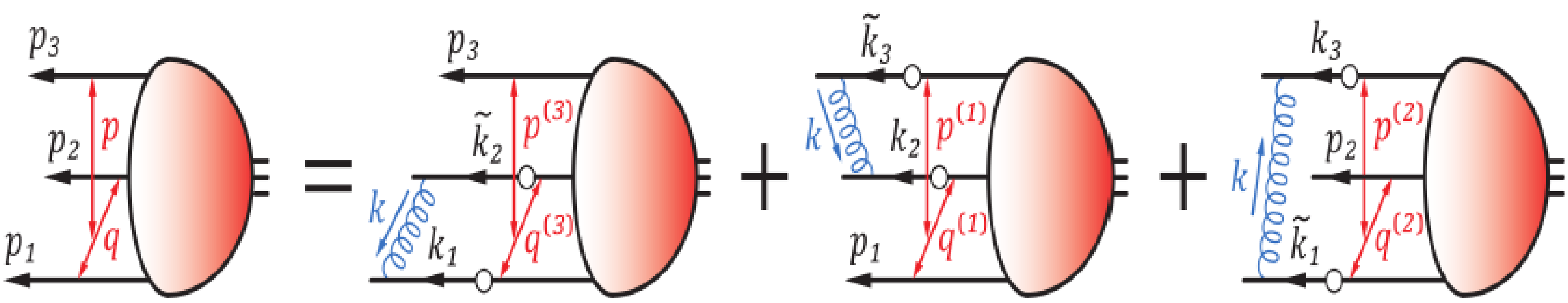
$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

2-body correlations

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
  - (I=0, J^P=1^-): isoscalar-vector diquark
  - (I=1, J^P=1^-): isovector-vector diquark
  - Tensor diquarks
- Three-body bound states ➡

  Quark-Diquark two-body bound states

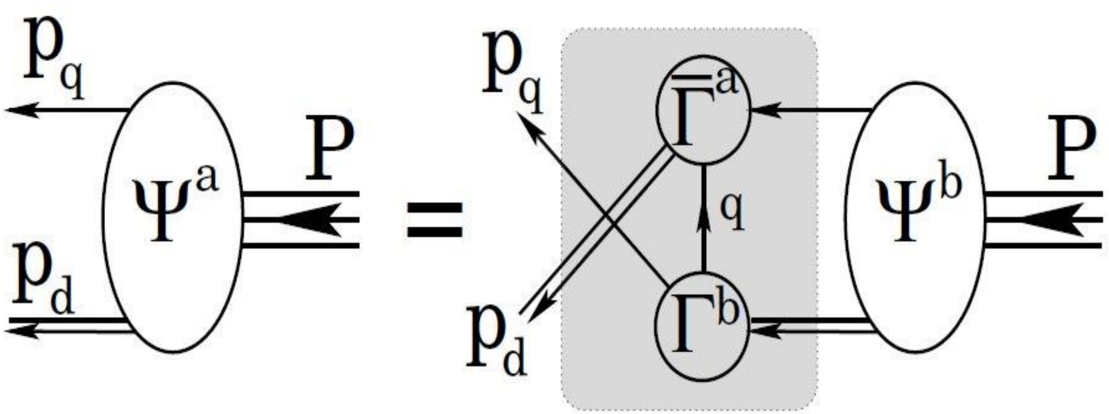
Faddeev equation in rainbow-ladder truncation



2-body correlations

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
  - (I=0, J^P=1^-): isoscalar-vector diquark
  - (I=1, J^P=1^-): isovector-vector diquark
  - Tensor diquarks
- Three-body bound states ➡

  Quark-Diquark two-body bound states



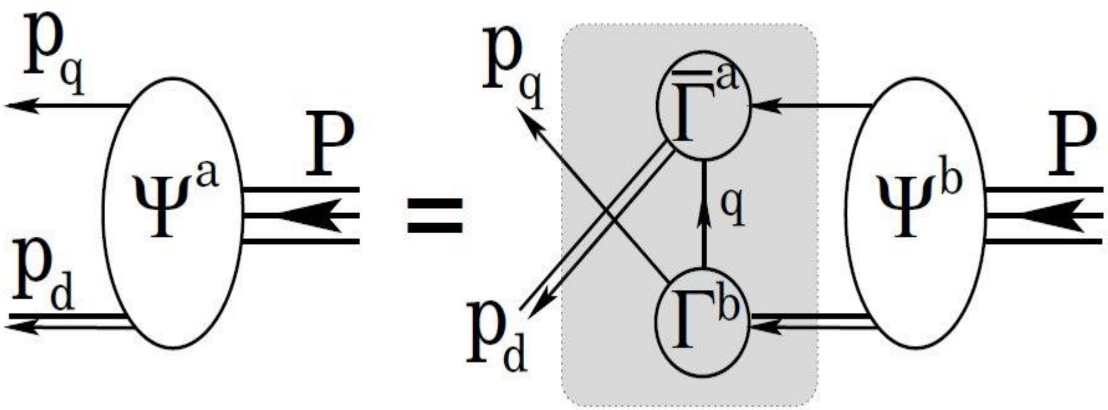
2-body correlations

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector diquark
  - (I=0, J^P=0^-): isoscalar-pesudoscalar diquark
  - (I=0, J^P=1^-): isoscalar-vector diquark
  - (I=1, J^P=1^-): isovector-vector diquark
  - Tensor diquarks
- Three-body bound states ➡

  Quark-Diquark two-body bound states

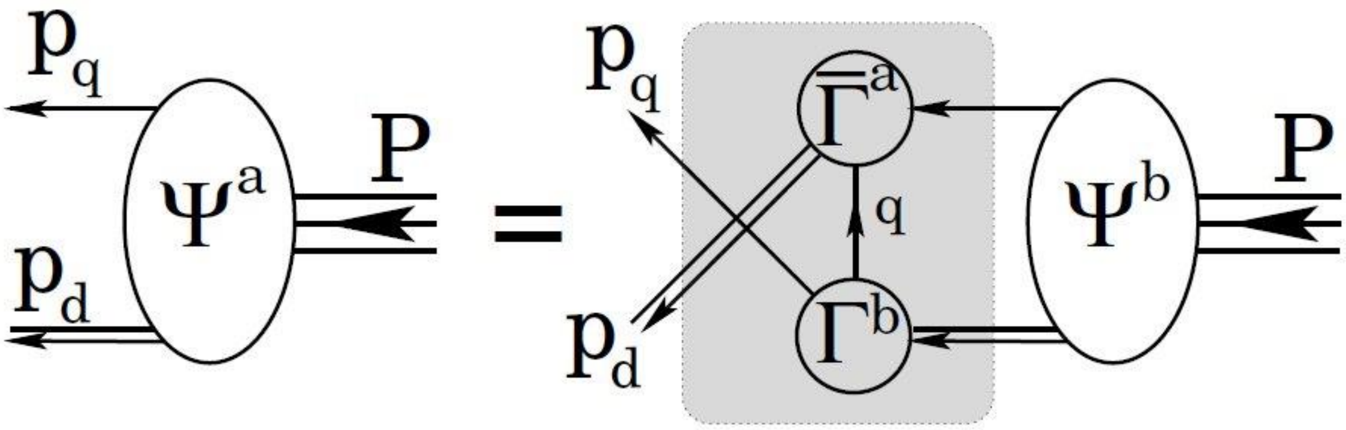
✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog.Part.Nucl.Phys. 91 (2016) 1-100

✓ CC, B. El-Bennich, C. D. Roberts, S. M. Schmidt, J. Segovia, S-L. Wan, Phys.Rev. D97 (2018) no.3, 034016



QCD-kindred model

- The dressed-quark propagator
- Diquark amplitudes
- Diquark propagators
- Faddeev amplitudes



QCD-kindred model

- **Spin-flavor structure:**

$$u_p = \begin{bmatrix} u[ud]_{0^+} \\ d\{uu\}_{1^+} \\ u\{ud\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_p^1 \\ a_p^4 \\ a_p^5 \end{bmatrix},$$

$$u_\Lambda = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2}\, s[ud]_{0^+} \\ d[us]_{0^+} - u[ds]_{0^+} \\ d\{us\}_{1^+} - u\{ds\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Lambda^1 \\ s_\Lambda^{[2,3]} \\ a_\Lambda^{[6,8]} \end{bmatrix},$$

$$u_\Sigma = \begin{bmatrix} u[us]_{0^+} \\ s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^4 \\ a_\Sigma^6 \end{bmatrix},$$

$$u_\Xi = \begin{bmatrix} s[us]_{0^+} \\ s\{us\}_{1^+} \\ u\{ss\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^6 \\ a_\Sigma^9 \end{bmatrix},$$

$$u_\Delta = [u\{uu\}_{1^+}] \leftrightarrow [a_\Delta^4],$$

$$u_{\Sigma^*} = \begin{bmatrix} s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} a_{\Sigma^*}^4 \\ a_{\Sigma^*}^6 \end{bmatrix},$$

$$u_{\Xi^*} = \begin{bmatrix} s\{us\}_{1^+} \\ u\{ss\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} a_{\Xi^*}^6 \\ a_{\Xi^*}^9 \end{bmatrix},$$

$$u_\Omega = [s\{ss\}_{1^+}] \leftrightarrow [a_\Omega^9].$$

- **Diquark masses (in GeV):**

$$m_{[ud]_{0^+}} = 0.80,$$
$$m_{[us]_{0^+}} = m_{[ds]_{0^+}} = 0.95,$$
$$m_{\{uu\}_{1^+}} = m_{\{ud\}_{1^+}} = m_{\{dd\}_{1^+}} = 0.90,$$
$$m_{\{us\}_{1^+}} = m_{\{ds\}_{1^+}} = 1.05,$$
$$m_{\{ss\}_{1^+}} = 1.20.$$

- **The values of m_[ud] & m_{uu/ud/dd} provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**

- **Other masses are derived therefrom via an equal-spacing rule: *viz*. replacing by a *s-quark* bring an extra 0.15 GeV (~ *Ms - Mu*).**

QCD-kindred model

- **Solution to the 50 year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is *50%* greater and it is unstable…**

QCD-kindred model

- **Solution to the 50 year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is *50%* greater and it is unstable…**

PRL 115, 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5]
Shu-Sheng Xu,[6] and Hong-Shi Zong[6]

[1]*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain*
[2]*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*
[3]*Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil*
[4]*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*
[5]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[6]*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801 PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

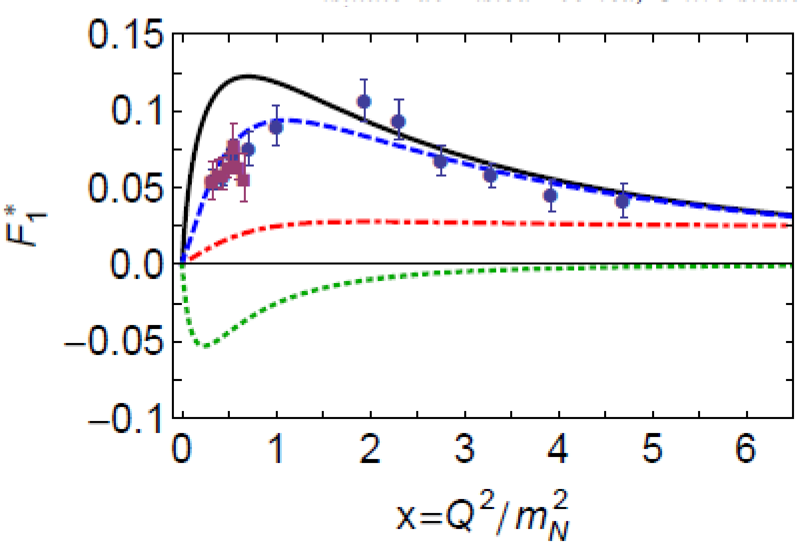
QCD-kindred model

- **Solution to the 50 year puzzle -- Roper resonance: Discovered in 1963, the Roper resonance appears to be an exact copy of the proton except that its mass is *50%* greater and it is unstable…**

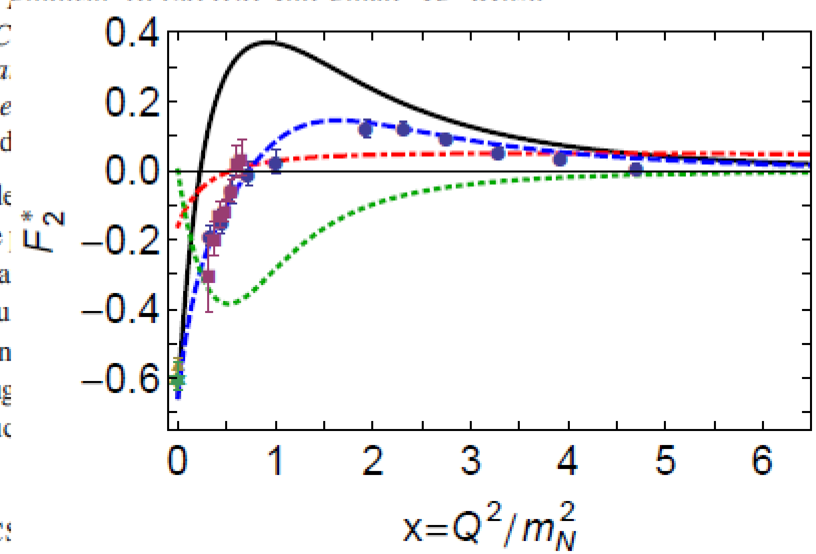
PRL 115, 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5] Shu-Sheng Xu,[6] and Hong-Shi Zong[6]
[1]*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain*
[2]*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*
[3]*Instituto de Física Teórica, Universidade Estadual*



ntioquia, C
nal Labora
njing Unive
ipt received
e three vale
erties of the
alysis indica
dressed qu
ne proton an
ass by roug
lectroproduc
PACS



SOLUTIONS & THEIR PROPERTIES

arXiv.org > nucl-th > arXiv:1901.04305

Spectrum and structure of octet and decuplet baryons and their positive-parity excitations

Chen Chen,[1,*] Gastão Krein,[1] Craig D. Roberts,[2,†] Sebastian M. Schmidt,[3] and Jorge Segovia[4]

[1] Instituto de Física Teórica, Universidade Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271, 01140-070 São Paulo, SP, Brazil

[2] Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA

[3] Institute for Advanced Simulation, Forschungszentrum Jülich and JARA, D-52425 Jülich, Germany

[4] Departamento de Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, E-41013 Sevilla, Spain

(Dated: 10 January 2019)

SOLUTIONS & THEIR PROPERTIES

- Spectrum and structure of *octet & decuplet baryons* and their positive-parity excitations
- **Masses**
- **Rest-frame orbital angular momentum**
- **Diquark content**
- **Pointwise structure**

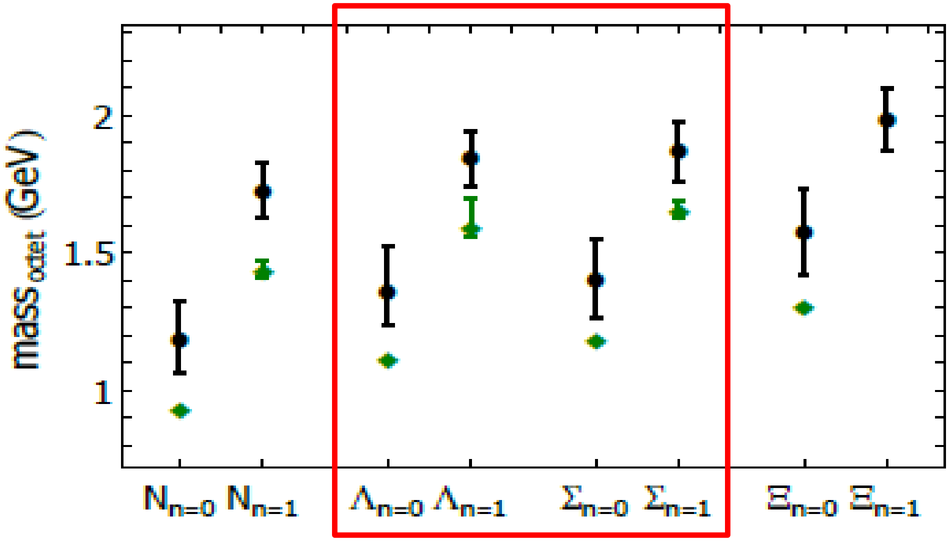
Masses

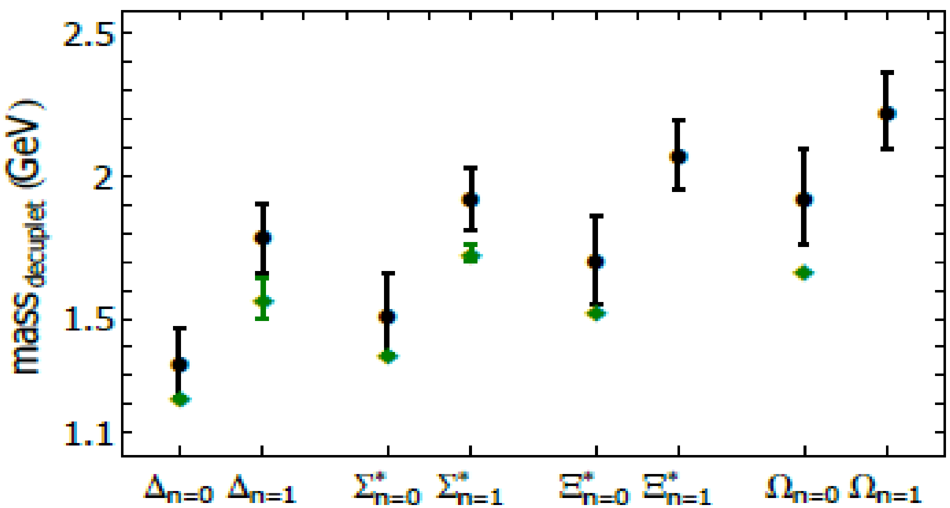
Row			N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
n=0	1	DSE	1.19(13)	1.37(14)	1.41(14)	1.58(15)	1.35(12)	1.52(14)	1.71(15)	1.93(17)
	3	expt.	0.94	1.12	1.19	1.31	1.23	1.38	1.53	1.67
n=1	5	DSE	1.73(10)	1.85(09)	1.88(11)	1.99(11)	1.79(12)	1.93(11)	2.08(12)	2.23(13)
	7	expt.	1.44(03)	$1.51^{+0.10}_{-0.04}$	1.66(03)	-	1.57(07)	1.73(03)	-	-

- Σ-Λ mass splitting
- While the Σ and Λ are associated with the same combination of valence-quarks, their spin-flavor wave functions are different: the Λ contains more of the (lighter) scalar diquark correlations than the Σ

$$u_\Sigma = \begin{bmatrix} u[us]_{0^+} \\ s\{uu\}_{1^+} \\ u\{us\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Sigma^2 \\ a_\Sigma^4 \\ a_\Sigma^6 \end{bmatrix} \qquad u_\Lambda = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2}\, s[ud]_{0^+} \\ d[us]_{0^+} - u[ds]_{0^+} \\ d\{us\}_{1^+} - u\{ds\}_{1^+} \end{bmatrix} \leftrightarrow \begin{bmatrix} s_\Lambda^1 \\ s_\Lambda^{[2,3]} \\ a_\Lambda^{[6,8]} \end{bmatrix}$$

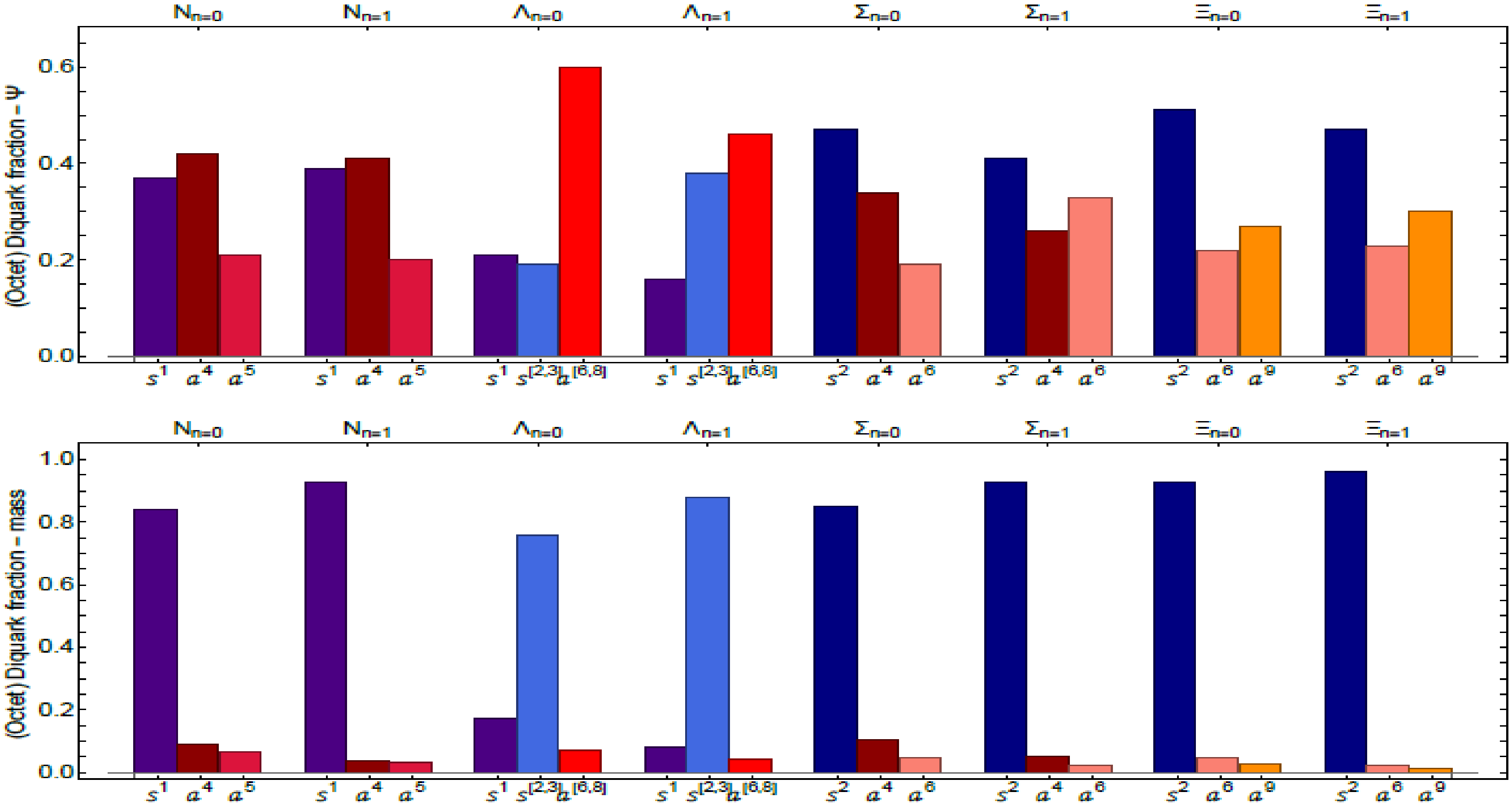
Masses





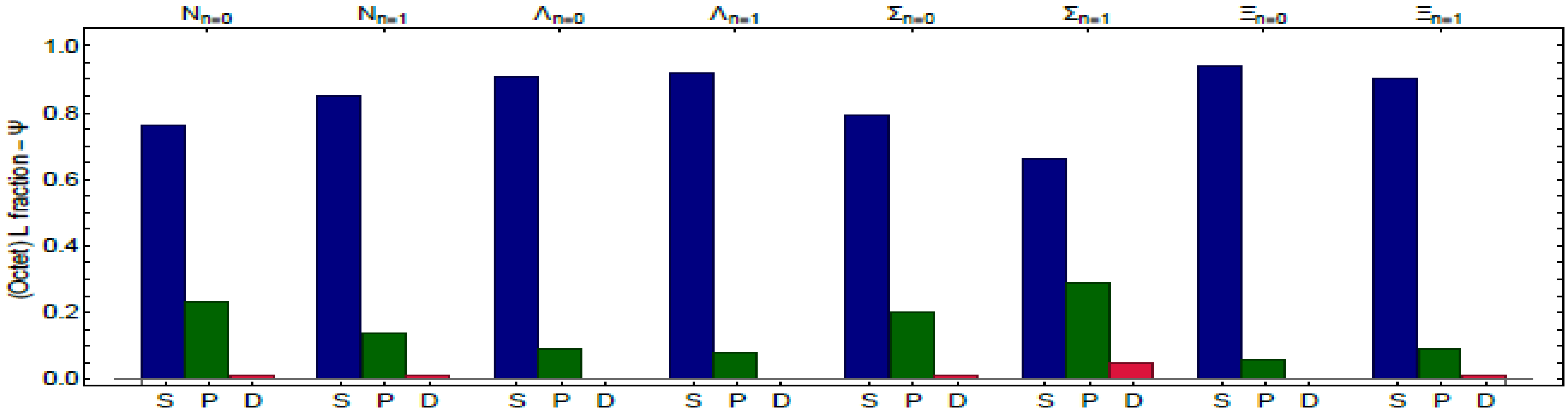
- The computed masses are uniformly larger than the corresponding empirical values.
- The quark-diquark kernel omits all those resonant contributions associated with meson-baryon final-state interactions, which typically generate a measurable reduction.
- The Faddeev equations analyzed to produce the results should be understood as producing the dressed-quark core of the bound state, NOT the completely dressed and hence observable object.

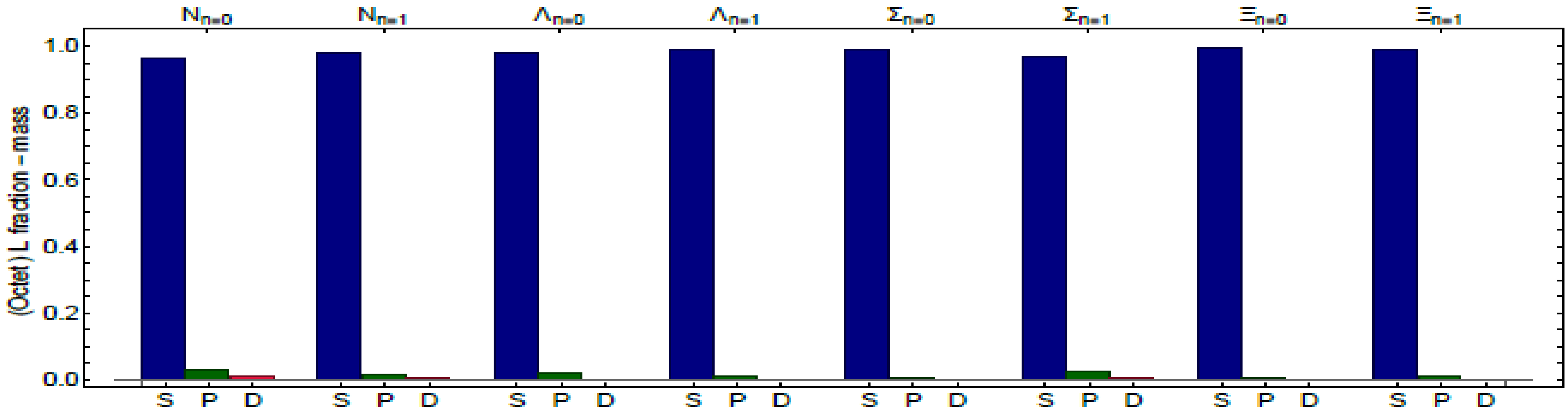
Diquark content (Octet)



- Upper: Computed from the amplitudes directly.
  Lower: Computed from the relative contributions to the masses.
- Lower : In each, there is a single dominant diquark component: scalar diquark
- Difference -> the lack of interference between diquark components

Rest-frame orbital angular momentum (Octet)



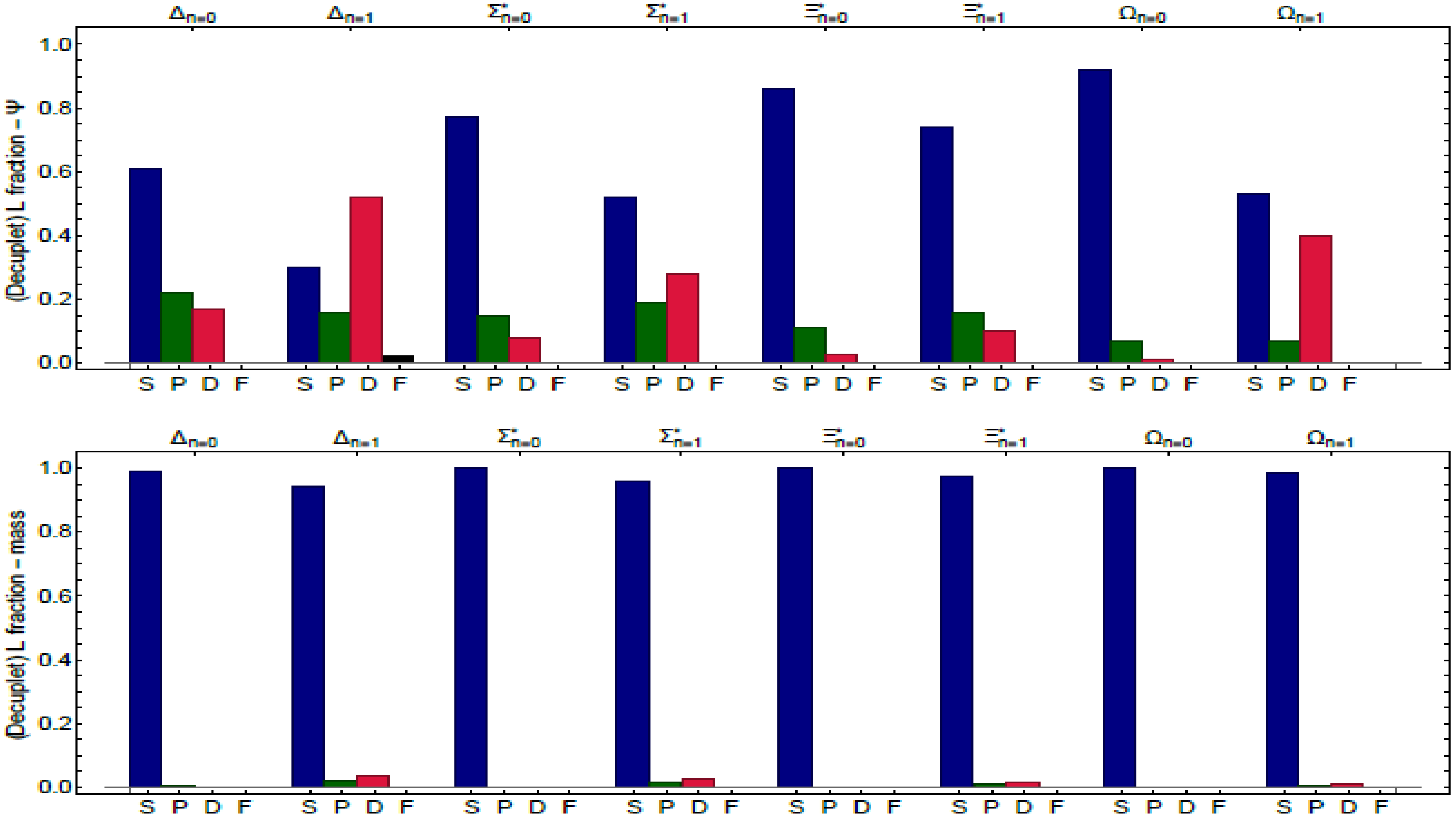


- Upper: Computed from the wave functions directly.

  Lower: Computed from the relative contributions to the masses.

- Both measures deliver the same qualitative picture of each baryon's internal structure. So there is little mixing between partial waves in the computation of a baryon's mass.

Rest-frame orbital angular momentum (Decuplet)



- Upper: Computed from the wave functions directly.

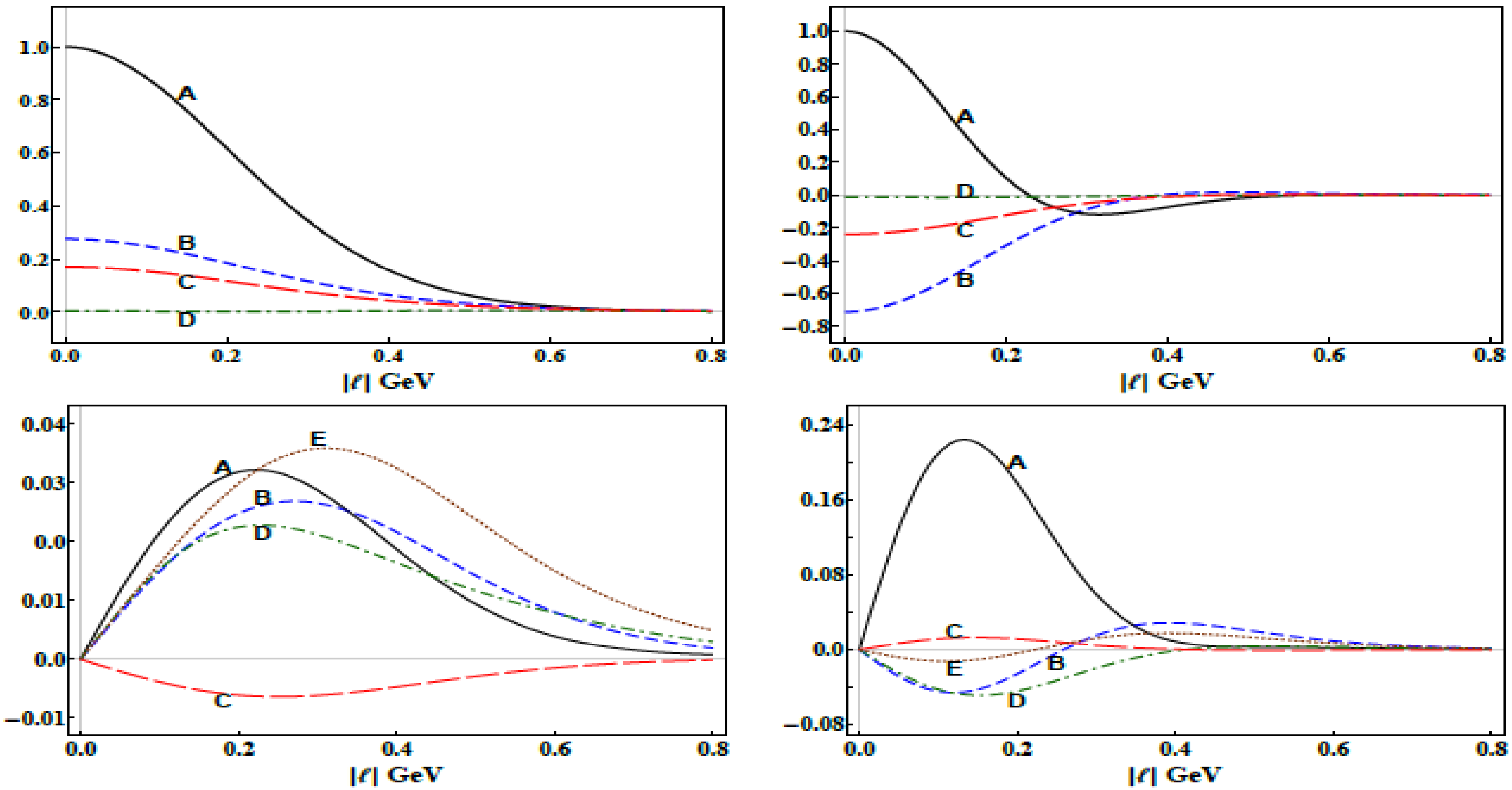
  Lower: Computed from the relative contributions to the masses.
- In both panels that *S-wave* strength is shifted into *D-wave* contributions within decuplet positive-parity excitations.

Rest-frame orbital angular momentum

L content	$N_{n=0}$	$N_{n=1}$	$\Lambda_{n=0}$	$\Lambda_{n=1}$	$\Sigma_{n=0}$	$\Sigma_{n=1}$	$\Xi_{n=0}$	$\Xi_{n=1}$
S, P, D	1.19	1.73	1.37	1.85	1.41	1.88	1.58	1.99
$-, P, D$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S, -, D$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97
$S, P, -$	1.20	1.74	1.37	1.85	1.41	1.89	1.58	1.99
$S, -, -$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97

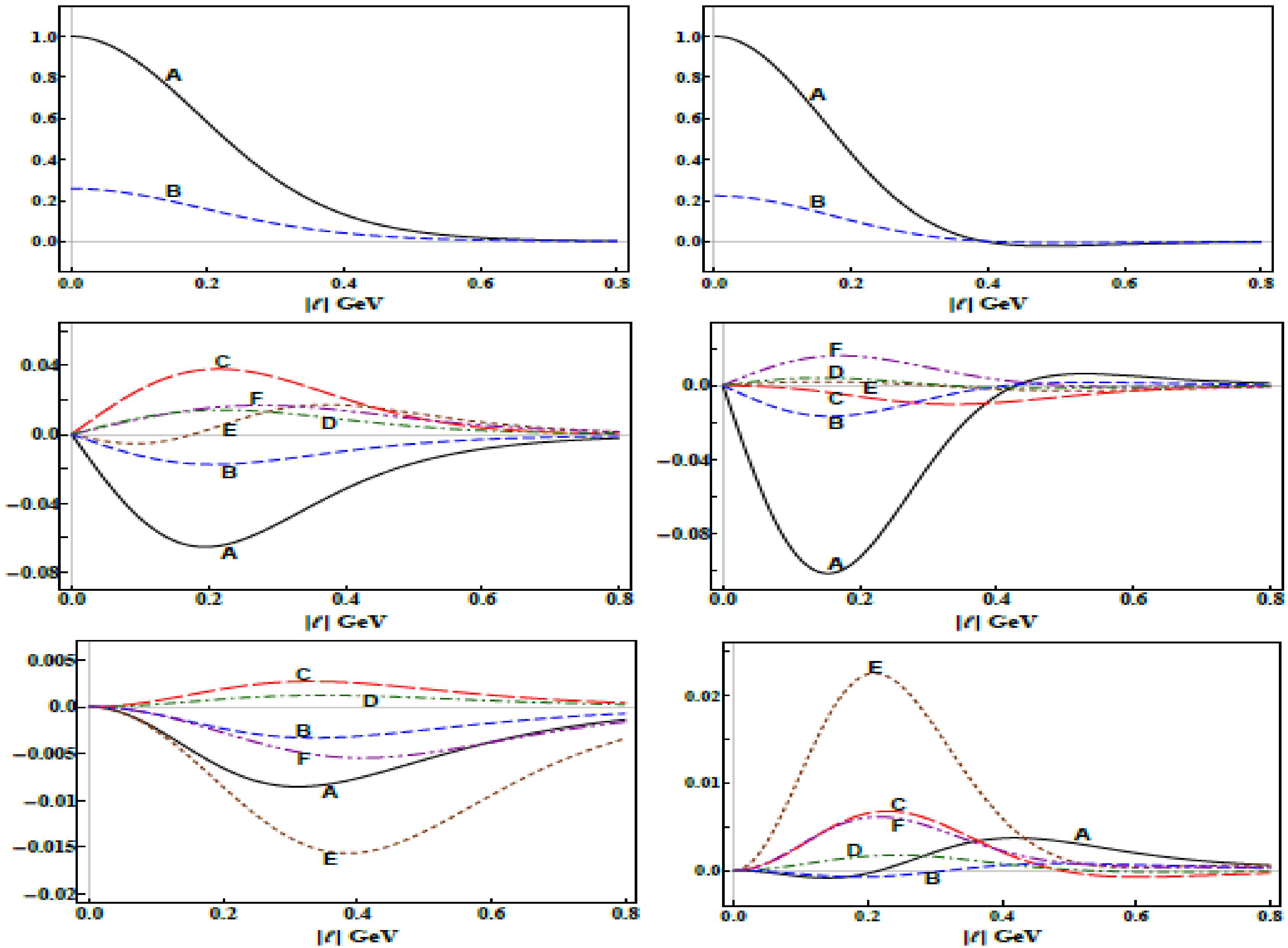
L content	$\Delta_{n=0}$	$\Delta_{n=1}$	$\Sigma^*_{n=0}$	$\Sigma^*_{n=1}$	$\Xi^*_{n=0}$	$\Xi^*_{n=1}$	$\Omega_{n=0}$	$\Omega_{n=1}$
S, P, D, F	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$-, P, D, F$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$S, -, D, F$	1.36	1.75	1.52	1.90	1.71	2.06	1.93	2.22
$S, P, -, F$	1.35	1.82	1.52	1.95	1.71	2.09	1.93	2.24
$S, P, D, -$	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$S, -, -, -$	1.35	1.80	1.52	1.93	1.71	2.08	1.93	2.23

Pointwise structure (Octet – Λ baryon)



- The zeroth Chebyshev moment of all *S-* and *P-wave* components in a given baryon's Faddeev wave function.
- Each projection for the ground-state is of a single sign (+ or -).
- First excitation: all *S-* and *P-wave* components exhibit a single zero at some point. It may be interpreted as the simplest *radial excitation* of its ground-state partner.

Pointwise structure (Decuptet – Ξ* baryon)



Summary & Outlook

- We computed the spectrum and Poincare-covariant wave functions for all favor-SU(3) octet and decuplet baryons and their first positive-parity excitations.
- Negative-parity diquarks are **negligible** in these positive-parity baryons.
- In its rest-frame, every system considered may be judged as primarily *S-wave* in character; and the first positive-parity excitation of each octet or decuplet baryon exhibits the characteristics of a radial excitation of the ground-state.
- Next: Negative-parity partners; Form factors & axial couplings; PDFs, PDAs, GPDs, TMDs...

Thank you!

Rest-frame orbital angular momentum

L content	$N_{n=0}$	$N_{n=1}$	$\Lambda_{n=0}$	$\Lambda_{n=1}$	$\Sigma_{n=0}$	$\Sigma_{n=1}$	$\Xi_{n=0}$	$\Xi_{n=1}$
S, P, D	1.19	1.73	1.37	1.85	1.41	1.88	1.58	1.99
$-, P, D$	–	–	–	–	–	–	–	–
$S, -, D$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97
$S, P, -$	1.20	1.74	1.37	1.85	1.41	1.89	1.58	1.99
$S, -, -$	1.24	1.71	1.40	1.83	1.42	1.84	1.59	1.97

L content	$\Delta_{n=0}$	$\Delta_{n=1}$	$\Sigma^*_{n=0}$	$\Sigma^*_{n=1}$	$\Xi^*_{n=0}$	$\Xi^*_{n=1}$	$\Omega_{n=0}$	$\Omega_{n=1}$
S, P, D, F	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$-, P, D, F$	–	–	–	–	–	–	–	–
$S, -, D, F$	1.36	1.75	1.52	1.90	1.71	2.06	1.93	2.22
$S, P, -, F$	1.35	1.82	1.52	1.95	1.71	2.09	1.93	2.24
$S, P, D, -$	1.35	1.79	1.52	1.93	1.71	2.08	1.93	2.23
$S, -, -, -$	1.35	1.80	1.52	1.93	1.71	2.08	1.93	2.23

- Upper: octet baryons
  Lower: decuplet baryons
- Every one of the systems considered is primarily *S-wave* in nature.
- *P-wave* components play a measurable role in octet ground-states & first positive-parity excitations: they are attractive in ground-states & repulsive in the excitations
- Decuplet systems: the ground-state masses are almost completely insensitive to non-S-wave components; and in the first positive parity excitations, *P-wave* components generate a little repulsion, some attraction is provided by *D-waves*.

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- **algebraic form:**

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}(2(x+\bar{m}^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)], \quad \text{(A3a)}$$

$$\bar{\sigma}_V(x) = \frac{1}{x+\bar{m}^2}[1-\mathcal{F}(2(x+\bar{m}^2))], \quad \text{(A3b)}$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}, \quad \text{(A4)}$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$\bar{m}$	b_0	b_1	b_2	b_3
0.00897	0.131	2.90	0.603	0.185

(A5)

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate-p^2 domains.]

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

  - Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
  - Mass function has a real-world value at p^2 = 0, NOT the highly inflated value typical of ~~RL~~ truncation.
  - Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from ~~RL~~ truncation.
  - Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
  - Compare with that computed using the DCSB-improved gap equation kernel (DB). The parametrization is a sound representation numerical results, although simple and introdu long beforehand.

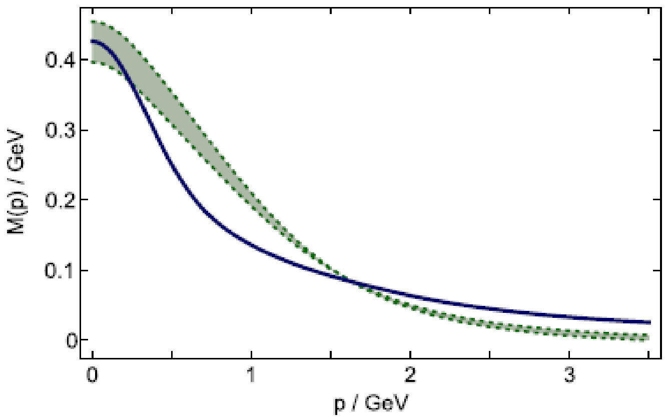


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes: five types of correlation are possible in a *J=1/2* bound state: isoscalar scalar(I=0,J^P=0^+), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.**
- **The LEADING structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),**

$$\Gamma^{0^+}(k;K) = g_{0^+}\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\vec{\Gamma}_\mu^{1^+}(k;K) = i g_{1^+}\gamma_\mu C\vec{t}\,\vec{H}\,\mathcal{F}(k^2/\omega_{1^+}^2),$$

$$\Gamma^{0^-}(k;K) = i g_{0^-} C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^-}^2),$$

$$\Gamma_\mu^{1^-}(k;K) = g_{1^-}\gamma_\mu\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{1^-}^2),$$

$$\vec{\Gamma}_\mu^{\bar{1}^-}(k;K) = i g_{\bar{1}^-}[\gamma_\mu, \gamma\cdot K]\gamma_5 C\vec{t}\,\vec{H}\,\mathcal{F}(k^2/\omega_{\bar{1}^-}^2),$$

- **Simple form. Just one parameter: diquark masses.**
- **Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.**

QCD-kindred model

- **The diquark propagators**

$$\Delta^{0^\pm}(K) = \frac{1}{m_{0^\pm}^2}\mathcal{F}(k^2/\omega_{0^\pm}^2),$$

$$\Delta_{\mu\nu}^{1^\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^\pm}^2}\right]\frac{1}{m_{1^\pm}^2}\mathcal{F}(k^2/\omega_{1^\pm}^2).$$

- **The *$\mathcal{F}$-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and 1/q^2 evolution (UV) of meson propagators.**

- **Diquarks are confined.**
  - **free-particle-like at spacelike momenta**
  - **pole-free on the timelike axis**
  - **This is NOT true of ~~RL~~ studies. It enables us to reach arbitrarily high values of momentum transfer.**

QCD-kindred model

- **The Faddeev amplitudes:**

$$\begin{aligned}\psi^{\pm}(p_i,\alpha_i,\sigma_i) = {} & [\Gamma^{0^+}(k;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\Delta^{0^+}(K)[\varphi^{\pm}_{0^+}(\ell;P)u(P)]^{\alpha_3}_{\sigma_3} \\ & + [\Gamma^{1^+j}_{\mu}]\Delta^{1^+}_{\mu\nu}[\varphi^{j\pm}_{1^+\nu}(\ell;P)u(P)] \\ & + [\Gamma^{0^-}]\Delta^{0^-}[\varphi^{\pm}_{0^-}(\ell;P)u(P)] \\ & + [\Gamma^{1^-}_{\mu}]\Delta^{1^-}_{\mu\nu}[\varphi^{\pm}_{1^-\nu}(\ell;P)u(P)], \qquad (9)\end{aligned}$$

- **Quark-diquark vertices:**

$$\varphi^{\pm}_{0^+}(\ell;P) = \sum_{i=1}^{2} \mathscr{s}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{j\pm}_{1^+\nu}(\ell;P) = \sum_{i=1}^{6} \mathscr{a}^{j\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_{\nu}(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{\pm}_{0^-}(\ell;P) = \sum_{i=1}^{2} \mathscr{p}^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\mp},$$

$$\varphi^{\pm}_{1^-\nu}(\ell;P) = \sum_{i=1}^{6} \mathscr{v}^{\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_{\nu}(\ell;P)\mathcal{G}^{\mp},$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_{\rm D}(\gamma_5)$ and

$$\mathcal{S}^1 = \mathbf{I}_{\rm D}, \qquad \mathcal{S}^2 = i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\mathbf{I}_{\rm D}$$

$$\mathcal{A}^1_{\nu} = \gamma\cdot\ell^{\perp}\hat{P}_{\nu}, \qquad \mathcal{A}^2_{\nu} = -i\hat{P}_{\nu}\mathbf{I}_{\rm D}, \qquad \mathcal{A}^3_{\nu} = \gamma\cdot\hat{\ell}^{\perp}\hat{\ell}^{\perp}_{\nu}$$

$$\mathcal{A}^4_{\nu} = i\hat{\ell}^{\perp}_{\nu}\mathbf{I}_{\rm D}, \quad \mathcal{A}^5_{\nu} = \gamma^{\perp}_{\nu} - \mathcal{A}^3_{\nu}, \quad \mathcal{A}^6_{\nu} = i\gamma^{\perp}_{\nu}\gamma\cdot\hat{\ell}^{\perp} - \mathcal{A}^4_{\nu},$$

QCD-kindred model

- **Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.**
- **Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.**
- **In consequence, the manner by which the dressed quarks' spin, *S*, and orbital angular momentum, *L*, add to form the total momentum *J*, is frame dependent: *L*, *S* are not independently Poincare invariant.**
- **The set of baryon rest-frame quark-diquark angular momentum identifications:**

$$^2S:\ \mathcal{S}^1, \mathcal{A}_\nu^2, (\mathcal{A}_\nu^3 + \mathcal{A}_\nu^5),$$

$$^2P:\ \mathcal{S}^2, \mathcal{A}_\nu^1, (\mathcal{A}_\nu^4 + \mathcal{A}_\nu^6),$$

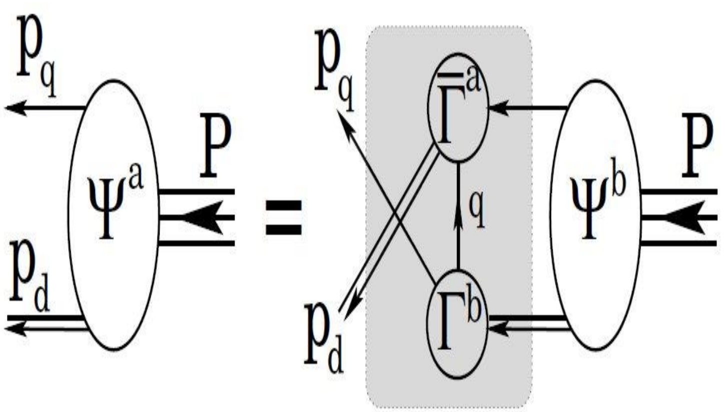
$$^4P:\ (2\mathcal{A}_\nu^4 - \mathcal{A}_\nu^6)/3,$$

$$^4D:\ (2\mathcal{A}_\nu^3 - \mathcal{A}_\nu^5)/3,$$

- **The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.**

Quark-diquark picture

- A baryon can be viewed as a Borromean bound-state, the binding within which has two contributions:
  - ✓ Formation of tight diquark correlations.
  - ✓ Quark exchange depicted in the shaded area.



- The exchange ensures that diquark correlations within the baryon are fully dynamical: no quark holds a special place.
- The rearrangement of the quarks guarantees that the baryon's wave function complies with Pauli statistics.
- Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.
- The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.