Theory of Baryon Resonances

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• Lesson 2: Well separated resonances
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Introduction
QCD LAGRANGIAN

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_f \bar{q}_f (i \mathbb{D} - \mathcal{M}) q_f + \ldots \]

\[ D_{\mu} = \partial_{\mu} - ig A_{\mu}^a \lambda^a / 2 \]
\[ G_{\mu\nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a - g [A_{\mu}^b, A_{\nu}^c] \]
\[ f = (u, d, s, c, b, t) \]

- running of \( \alpha_s = \frac{g^2}{4\pi} \) \( \Rightarrow \) \( \Lambda_{\text{QCD}} = 210 \pm 14 \text{ MeV} \) \( (N_f = 5, \overline{M}\bar{S}, \mu = 2 \text{ GeV}) \)

- light (u,d,s) and heavy (c,b,t) quark flavors:

<table>
<thead>
<tr>
<th>Light Quark</th>
<th>Heavy Quark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_u = 2.2^{+0.6}_{-0.4} \text{ MeV} )</td>
<td>( m_c = 1.28 \pm 0.03 \text{ GeV} )</td>
</tr>
<tr>
<td>( m_d = 4.7^{+0.5}_{-0.4} \text{ MeV} )</td>
<td>( m_b = 4.18^{+0.04}_{-0.03} \text{ GeV} )</td>
</tr>
<tr>
<td>( m_s = 96^{+8}_{-4} \text{ MeV} )</td>
<td>( m_t = 173.1 \pm 0.6 \text{ GeV} )</td>
</tr>
</tbody>
</table>
**LIMITS of QCD**

**light quarks:** \[ \mathcal{L}_{QCD} = \bar{q}_L i \gamma \cdot D q_L + \bar{q}_R i \gamma \cdot D q_R + \mathcal{O}(m_f/\Lambda_{QCD}) \]

- L and R quarks decouple \( \Rightarrow \) chiral symmetry
- spontaneous chiral symmetry breaking \( \Rightarrow \) pseudo-Goldstone bosons
- pertinent EFT \( \Rightarrow \) chiral perturbation theory (CHPT)

**heavy quarks:** \[ \mathcal{L}_{QCD} = \bar{Q}_f iv \cdot D Q_f + \mathcal{O}(\Lambda_{QCD}/m_f) \]

- independent of quark spin and flavor
  \( \Rightarrow \) SU(2) spin and SU(2) flavor symmetries (HQSS and HQFS)
- pertinent EFT \( \Rightarrow \) heavy quark effective field theory (HQEFT)

**heavy-light systems:**
- heavy quarks act as matter fields coupled to light pions
- combine CHPT and HQEFT
WHY EXCITED STATES?

• The spectrum of QCD is its **least** understood feature
  → why only $qqq$ and $\bar{q}q$ states? XYZ states? “exotics”? glueballs?
  → important players: hadronic molecules ↔ nuclear physics
  → the quark model is much too simple . . .
  → need insight from EFTs ↔ symmetries!

• Many recent high-precision data (utilizing e.g. double polarization exp’s)
  → ELSA at Bonn, CEBAF at Jefferson Lab, LHCb at CERN,
    BESIII at BEPCII, . . ., PANDA at FAIR, GlueX at JLab12, . . .

• Lattice QCD can get ground-states at almost physical pion masses
  → most distinctive feature of excited states: *decays*
  → only captured for very few states in lattice QCD
  → must explore this (almost complete) *terra incognita*
Lesson 1
What is a resonance?
WHAT is a RESONANCE?

- “Not every bump is a resonance and not every resonance is a bump”
  Moorhouse 1960ties

- Resonances have **complex** properties (mass & width, photo-couplings, ...)
  $\leftrightarrow$ these intrinsic properties do not depend on the experiment or theory (model)

- Resonances correspond to S-matrix poles on unphysical Riemann sheets
  $\leftrightarrow$ only model-independent definition!
  $\leftrightarrow$ matrix-elements from analytic cont.
  to the resonance pole $p_R$
  $\leftrightarrow$ pics next slide

- That’s all nice in the continuum, but . . .
PICTURES of RESONANCES

- Resonances as complex poles on unphysical sheets:

- A view of the two close-by baryon resonances:
  the two lowest nucleon excitations in the
  $S_{11}$ partial wave of $\pi N \rightarrow \pi N$
  JüBo approach, D. Rönchen et al.

→ talk by D. Rönchen

Abs|T(J^P=1/2-)| for $\pi N \rightarrow \pi N$
• Resonances in a box: not eigenstates of the Hamiltonian
  ⇒ volume dependence of the energy spectrum

• consider a narrow resonance → avoided level crossing
Lesson 2
Well separated resonances
ISOLATED RESONANCES in a BOX

- Two identical particles of mass $m$ in a box, no interaction:
  \[ E = 2\sqrt{m^2 + |\vec{p}|^2}, \quad p_i = \frac{2\pi}{L} n_i, \quad n_i \in \mathbb{Z} \]

- turn on interaction → scattering phase → Lüscher formula:  
  \[
  \delta(p) = -\phi(q) \mod \pi, \quad q = \frac{pL}{2\pi} \\
  \phi(q) = -\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1; q^2)}, \quad \mathcal{Z}_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2}
  \]

- assume resonance with mass $m_R > 2m$ → effective range expansion (Breit-Wigner shape):
  \[
  \tan \left( \delta - \frac{\pi}{2} \right) = \frac{E^2 - m_R^2}{m_R \Gamma_R} \quad \text{[not general!]} 
  \]

⇒ measure the phase shift in the resonance region and fit $m_R, \Gamma_R$ & extension to moving frames 
  Rummukainen, Gottlieb (1995) + ...
RESULTS for the $\rho(770)$-MESON

- The $\rho(770)$ is a well separated meson resonance in the $\pi\pi$ system
- P-wave $\pi\pi$ scattering, $M_\pi = 280 - 500$ MeV, three different $a$, three different $L$, boosts $\vec{d} = 0, 1, 2, 3, 4$, all irreps

Phase shift

Mass

Width

consistent with other collaborations world-wide

pioneered in: Feng, Jansen, Renner (2011)
RESULTS for the $\Delta(1232)$

- The $\Delta(1232)$ is a well separated baryon resonance in the $\pi N$ system
- $l = 1, I = 3/2$ $\pi N$ phase shift
- $M_\pi = 160 - 390$ MeV, large volumes
- consistent with the experimental width
- precision determination of $g_{\pi N\Delta}$ requires more precise data around $\delta = \pi/2$

→ for new quantitative results, see the talk by Marcus Petschlies on Wednesday, 14:30

overview: Colin Morningstar
Lesson 3: Coupled channels / thresholds
EXTENSION to COUPLED CHANNELS

- Isolated (well-separated) resonances are the exception

- Coupled channel effects, close-by thresholds: $f_0(980)$, $a_0(980)$, $\Lambda(1405)$, . . .

- various extensions of Lüscher’s approach:
  - purely quantum mechanical treatment
    - Feng, He, Liu, Li, . . .
  - non-relativistic EFT (NREFT)
    - Beane, Savage, Bernard, Lage, UGM, Rusetsky, Briceno, Davoudi, Luu, . . .
  - finite-volume unitarized CHPT
    - Döring, UGM, Rusetsky, Oset, . . .

- Mostly done in the meson sector, very little for baryons
  - talk by Colin Morningstar

- Be aware of methods that can mislead you (K-matrix and alike)
- $D\pi$, $D\eta$, $D_s\bar{K}$ scattering with $I = 1/2$:

- 3 volumes, one $a_s$, one $a_t$, $M_\pi \simeq 390$ MeV, various K-matrix type extrapolations

- S-wave pole at $(2275.9 \pm 0.9)$ MeV

- close to the $D\pi$ threshold

- consistent w/ $D_0^*(2400)$ of PDG

- BUT: chiral symmetry ignored... :-(

Theory of baryon resonances – Ulf-G. Meißner – NSTAR’19, Bonn, June 11, 2019
COUPLED CHANNEL DYNAMICS


• $D\phi$ bound states: Poles of the T-matrix (potential from CHPT and unitarization)

\[
T = V + V G V + V G G V + \ldots
\]

• Unitarized CHPT as a non-perturbative tool:

\[
T^{-1}(s) = V^{-1}(s) - G(s)
\]

• $V(s)$: derived from the SU(3) chiral Lagrangian, 6 LECs up to NLO → next slide

• $G(s)$: 2-point scalar loop function, regularized w/ a subtraction constant $a(\mu)$

• $T, V, G$: all these are matrices, channel indices suppressed
COUPLED CHANNEL DYNAMICS cont’d

• NLO effective chiral Lagrangian for coupled channel dynamics

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} \]

\[ \mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - M_D^2 D D^\dagger, \quad D = (D^0, D^+, D_s^+) \]

\[ \mathcal{L}^{(2)} = D \left[ -h_0 \langle \chi_+ \rangle - h_1 \chi_+ + h_2 \langle u_\mu u^\mu \rangle - h_3 u_\mu u^\mu \right] D \]

\[ + \mathcal{D}_\mu D \left[ h_4 \langle u^\mu u^\nu \rangle - h_5 \{u^\mu, u^\nu\} \right] D \nu D \]

with \[ u_\mu \sim \partial_\mu \phi, \quad \chi_+ \sim M_{\text{quark}}, \ldots \]

• LECs:

\[ \leftrightarrow h_0 \text{ absorbed in masses} \]

\[ \leftrightarrow h_1 = 0.42 \text{ from the } D_s^-D \text{ splitting} \]

\[ \leftrightarrow h_{2,3,4,5} \text{ from a fit to lattice data } (D\pi \rightarrow D\pi, D\bar{K} \rightarrow D\bar{K}, ...) \]

Liu, Orginos, Guo, Hanhart, UGM, Phys. Rev. D 87 (2013) 014508
FIT to LATTICE DATA

Liu, Orginos, Guo, Hanhart, UGM, PRD 87 (2013) 014508

• Fit to lattice data in 5 “simple” channels: no disconnected diagrams

Prediction: Pole in the \((S, I) = (1, 0)\) channel: \(2315^{+18}_{-28}\) MeV

Experiment: \(M_{D_s^*(2317)} = (2317.7 \pm 0.6)\) MeV

\([M_\pi = 600\ MeV \text{ not fitted}]\)
FINITE VOLUME FORMALISM

- Goal: postdict the finite volume (FV) energy levels for $I = 1/2$ and compare with the recent LQCD results from Moir et al. using the already fixed LECs → parameter-free insights into the $D_0^*(2400)$

- In a FV, momenta are quantized: $\mathbf{q} = \frac{2\pi}{L} \mathbf{n}$, $\mathbf{n} \in \mathbb{Z}^3$

$\Rightarrow$ Loop function $G(s)$ gets modified:

$$\int d^3 \mathbf{q} \to \frac{1}{L^3} \sum_{\mathbf{q}}$$

$$\tilde{G}(s, L) = G(s) = \lim_{\Lambda \to \infty} \left[ \frac{1}{L^3} \sum_{\mathbf{n}} I(\mathbf{q}) - \int_0^{\Lambda} \frac{q^2 dq}{2\pi^2} I(\mathbf{q}) \right]$$

- FV energy levels from the poles of $\tilde{T}(s, L)$:

$$\tilde{T}^{-1}(s, L) = V^{-1}(s) - \tilde{G}(s, L)$$

WHAT ABOUT the $D_0^*(2400)$?

- Results for $I = 1/2 D\phi$ scattering
  Albaladejo, Fernandez-Soler, Guo, Nieves (2017)

- this is NOT a fit!
- all LECs taken from the earlier study of Liu et al. (discussed before)
WHAT ABOUT the $D_0^*(2400)$?

- reveals a two-pole scenario! [cf. $\Lambda(1405)$]
- understood from group theory
  \[
  \text{3} \otimes \text{8} = \text{3} \oplus \text{6} \oplus \text{15} \]
  attractive
- this was seen earlier in various calc's
- Again: important role of chiral symmetry
- Easy lattice QCD test:
  sextet pole becomes a bound state
  for $M_\phi > 575$ MeV in the SU(3) limit
  Du et al. (2018)
TWO-POLE SCENARIO in the HEAVY-LIGHT SECTOR

- Two states in various $I = 1/2$ states in the heavy meson sector ($M, \Gamma/2$)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$D_0^*$</td>
<td>$2105_{-8}^{+6}, 102_{-11}^{+10}$</td>
<td>$2451_{-26}^{+36}, 134_{-8}^{+7}$</td>
<td>$(2318 \pm 29, 134 \pm 20)$</td>
</tr>
<tr>
<td>$D_1$</td>
<td>$2247_{-6}^{+5}, 107_{-10}^{+11}$</td>
<td>$2555_{-30}^{+47}, 203_{-9}^{+8}$</td>
<td>$(2427 \pm 40, 192_{-55}^{+65})$</td>
</tr>
<tr>
<td>$B_0^*$</td>
<td>$5535_{-11}^{+9}, 113_{-17}^{+15}$</td>
<td>$5852_{-19}^{+16}, 36 \pm 5$</td>
<td>—</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$5584_{-11}^{+9}, 119_{-17}^{+14}$</td>
<td>$5912_{-18}^{+15}, 42_{-4}^{+5}$</td>
<td>—</td>
</tr>
</tbody>
</table>

→ but is their experimental support for this? YES, but this is another talk... ($B \rightarrow D\pi\pi$ from LHCb)
Lesson 4: Hadronic molecules
What are HADRONIC MOLECULES?

- QCD offers yet another set of bound states, first seen in nuclear physics
  \( \rightarrow \) hadronic molecules (made of 2 or 3 hadrons)

- Bound states of two hadrons in an S-wave very close a 2-particle threshold or between two close-by thresholds \( \Rightarrow \) particular decay patterns

- Weak binding entails a large spatial extension

- The classical example:
  - The deuteron
    \[ m_p + m_n = 938.27 + 939.57 \text{ MeV}, \]
    \[ m_d = m_p + m_n - E_B \rightarrow E_B = 2.22 \text{ MeV} \]
    \[ r_d = 2.14 \text{ fm} \quad [r_p = 0.85 \text{ fm}] \]

- Other examples: \( \Lambda(1405), f_0(980), X(3872), \ldots \)

⇒ how to distinguish these from compact multi-quark states?
COMPOSITENESS CRITERION


- Wave fct. of a bound state with a compact & a two-hadron component in S-wave:

\[
|\Psi\rangle = \left(\sqrt{Z}|\psi_0\rangle, \chi(\vec{k})|h_1 h_2\rangle\right)
\]

compact comp. w/ probability \(\sqrt{Z}\)

two-hadron comp. w/ relative w.f. \(\chi(\vec{k})\)

- consider the scattering amplitude and compare with the ERE:

\[
a = -2 \frac{1 - Z}{2 - Z} \left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right), \quad r = -\frac{Z}{1 - Z} \left(\frac{1}{\gamma}\right) + \mathcal{O}\left(\frac{1}{\beta}\right) \quad \gamma = \sqrt{2\mu E_b}
\]

\(a\) = scattering length, \(\gamma/E_B = \) binding momentum/energy (shallow b.s.)

\(\mu = \) reduced mass of the two-particle system, \(\beta = \) range of forces

\(\Rightarrow\) pure molecule \((Z = 0)\): maximal scattering length \(a = -1/\gamma\)

natural effective range \(r = \mathcal{O}(1/\beta)\)

\(\Rightarrow\) compact state \((Z = 1)\): the scattering length is \(a = -\mathcal{O}(1/\beta)\)

effective range diverges, \(r \to -\infty\)
The DEUTERON
Weinberg, Phys. Rev. **137** (1965) B672

- The deuteron: shallow neutron-proton bound state ($E_B \ll m_d$):
  \[
  E_B = 2.22 \text{ MeV} \rightarrow \gamma = 45.7 \text{ MeV} = 0.23 \text{ fm}^{-1}
  \]

- range of forces set by the one-pion-exchange:
  \[
  \frac{1}{\beta} \sim \frac{1}{M_\pi} \simeq 1.4 \text{ fm}
  \]

- set $Z = 0$ in the Weinberg formula:
  \[
  a_{\text{mol}} = -(4.3 \pm 1.4) \text{ fm}
  \]

- this is consistent with the data:
  \[
  a = -5.419(7) \text{ fm}, \ r = 1.764(8) \text{ fm}
  \]

One begins to suspect that Nature is doing her best to keep us from learning whether the "elementary" particles deserve that title. (Weinberg, 1965)
EXTENSION to RESONANCES


● Still assume closeness to a two-particle threshold:

\[
T(E) = \frac{g^2/2}{E - E_r + (g^2/2)(ik + \gamma) + i\Gamma_0/2}
\]

with \( E = k^2/(2\mu) \), \( \Gamma_0 \) accounts for the inelasticities of other channels

● leads to very different line shapes for compact and molecular states:

\[ M = m_1 + m_2 + E \]

\( k^2 \) term dominates \( \rightarrow \) symmetric  \( g^2 \) term dominates \( \rightarrow \) asymmetric/cusp

● extension to instable particles/additional poles have also been worked out
SOME CANDIDATES

- Prominent examples in the light quark sector:
  \( f_0(980), a_0(980), \text{the two } \Lambda(1405), \ldots \)
  ↦ see talks by Mai and Oset

- Prominent examples in the \( c\bar{c} \) spectrum:
  \( X(3872), Z_c(3900), Y(4260), Y(4660), \ldots \)

- Prominent examples of heavy-light mesons:
  \( D^*_s(2317), D_s(2460), D^*_s(2860), \ldots \)

- Prominent examples in the \( b\bar{b} \) spectrum:
  \( Z_b(10610), Z_b(10650) \)

- and some examples of heavy baryons:
  \( \Lambda_c(2595), \Lambda_c(2940), P_c(4312), P_c(4557), \ldots \)

- suitable EFTs: UCHPT, NREFT\(_1\), NREFT\(_2\), CMS, \ldots

MISCONCEPTIONS on HADROPRODUCTION

Albaladejo, Guo, Hanhart, UGM, Nieves, Nogga, Yang, Chin.Phys. C 41 (2017) 121001

- It is often claimed that molecules due to their large spatial extent can not be produced in high-energy collisions, say at the LHC → this is wrong!


\[ \sigma(\bar{p}p \rightarrow X) \sim \left| \int d^3k \langle X | D^0 \bar{D}^* (k) \rangle \langle D^0 \bar{D}^* (k) | \bar{p}p \rangle \right|^2 \]

\[ \simeq \left| \int_{\mathcal{R}} d^3k \langle X | D^0 \bar{D}^* (k) \rangle \langle D^0 \bar{D}^* (k) | \bar{p}p \rangle \right|^2 \]

\[ \leq \int_{\mathcal{R}} d^3k |\Psi(k)|^2 \int_{\mathcal{R}} d^3k \left| \langle D^0 \bar{D}^* (k) | \bar{p}p \rangle \right|^2 \]

\[ \leq \int_{\mathcal{R}} d^3k \left| \langle D^0 \bar{D}^* (k) | \bar{p}p \rangle \right|^2 \]

- The result depends crucially on the value of \( \mathcal{R} \) which specifies the region where the bound state wave function “\( \Psi(k) \) is significantly different from zero”

- assumption by Bignamini et al: \( \mathcal{R} \simeq 35 \text{ MeV} \) of the order of \( \gamma \)

\[ \leftrightarrow \sigma(\bar{p}p \rightarrow X) \simeq 0.07 \text{ nb} \text{ way smaller than experiment} \]

\[ \leftrightarrow \text{the } X(3872) \text{ can not be a molecule} \]

\[ \leftrightarrow \text{so what goes wrong?} \]
• Consider the relevant integral for the deuteron:  
\[ \bar{\Psi}_\lambda(\mathcal{R}) \equiv \int_{\mathcal{R}} d^3k \, \Psi_\lambda(k) \]

• the binding momentum is \( \gamma \simeq 45 \text{ MeV} \), use that for the support \( \mathcal{R} \):

\( \mathcal{R} \) is by far not saturated for \( \mathcal{R} = \gamma \), need \( \mathcal{R} \simeq 2M_\pi \simeq 300 \text{ MeV} \)

• Similar misconception: Molecules can not be produced at large \( p_T \)

\( \downarrow \) true for nuclei but not quarkonia and alike (\( q \) versus \( \bar{q} \))
HADROPRODUCTION of the X(3872)

- Nice example of a process involving short-distance physics
  → still, factorization is at work, best seen using EFT


  consider production at the Tevatron and at LHC

\[
\begin{align*}
\sigma[X] &= \frac{1}{4m_H m_{H'}} g^2 |G|^2 \left( \frac{d\sigma[HH'(k)]}{dk} \right)_{\text{MC}} \frac{4\pi^2 \mu}{k^2} \\
G(E, \Lambda) &= -\frac{\mu}{\pi^2} \left[ \sqrt{2\pi} \frac{\Lambda}{4} + \sqrt{\pi} \gamma D \left( \frac{\sqrt{2}\gamma}{\Lambda} \right) - \frac{\pi}{2} \gamma e^{2\gamma^2/\Lambda^2} \right]
\end{align*}
\]

- typical results (using PYTHIA/HERWIG):


<table>
<thead>
<tr>
<th>[pp/\bar{p} \rightarrow X(3872)]</th>
<th>[\Lambda = 0.5 - 1.0 \text{ GeV}]</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron</td>
<td>5 - 29 [nb]</td>
<td>37 - 115 [nb]</td>
</tr>
<tr>
<td>LHC7</td>
<td>4 - 55 [nb]</td>
<td>13 - 39 [nb]</td>
</tr>
</tbody>
</table>

⇒ not very precise, but perfectly consistent with the data!
⇒ also predictions for the charm-strange mesons

Lesson 5:
The width of baryon resonances from EFT
EFT including the $\Delta$-RESONANCE

● Task: calculate the width of the $\Delta$ at two-loop order [one-loop too simple]

Gegelia, UGM, Siemens, Yao, Phys. Lett. B763 (2016) 1

● Consider the effective chiral Lagrangian of pions, nucleons and deltas:

\[
\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \{ i \not{D} - m + \frac{1}{2} g \not{\gamma} \gamma^5 \} \Psi_N
\]

\[
\mathcal{L}_{\pi \Delta}^{(1)} = -\bar{\Psi}_\mu \xi_{ij}^{\frac{3}{2}} \left\{ \left( i \not{D} j^k - m_\Delta \delta^{jk} \right) g^{\mu \nu} - i \left( \gamma^\mu D^\nu, j^k + \gamma^\nu D^\mu, j^k \right) + i \gamma^\mu \not{D} j^k \gamma^\nu \\
+ m_\Delta \delta^{jk} \gamma^\mu \gamma^\nu + g_1 \frac{1}{2} \gamma^j k^5 g^{\mu \nu} + g_2 \frac{1}{2} (\gamma^\mu u^\nu, j^k + u^\nu, j^k \gamma^\mu) \gamma^5 \\
+ g_3 \frac{1}{2} \gamma^\mu \gamma^j k^5 \gamma^\nu \right\} \xi_{kl} \Psi^l_\nu
\]

\[
\mathcal{L}_{\pi N \Delta}^{(1)} = h \bar{\Psi}_\mu \xi_{ij}^{\frac{3}{2}} \Theta^{\mu \alpha}(z_1) \omega^j_\alpha \Psi_N + \text{h.c.}
\]

\[
\mathcal{L}_{\pi N \Delta}^{(2)} = \bar{\Psi}_\mu \xi_{ij}^{\frac{3}{2}} \Theta^{\mu \alpha}(z_2) \left[ i b_3 \omega^j_\alpha \beta \gamma^\beta + i b_8 \frac{1}{m} \omega^j_\alpha \beta i D^\beta \right] \Psi_N + \text{h.c.} + \ldots
\]

\[
\mathcal{L}_{\pi N \Delta}^{(3)} = \bar{\Psi}_\mu \xi_{ij}^{\frac{3}{2}} \Theta^{\mu \nu}(z_3) \left[ f_1 \frac{1}{m} [D^\nu, \omega^j_\alpha \beta] \gamma^\alpha i D^\beta - f_2 \frac{1}{2 m^2} [D^\nu, \omega^j_\alpha \beta] \{D^\alpha, D^\beta\} + f_4 \omega^j_\nu \langle \chi_+ \rangle \\
+ f_5 [D^\nu, i \chi^j_-] \right] \Psi_N + \text{h.c.} + \ldots
\]

● Power counting rests on $m_\Delta - m_N$ being a small quantity

● So many LECs, how can one possibly make a prediction?
COMPLEX-MASS RENORMALIZATION

- Method originally introduced for $W, Z$-physics, later transported to chiral EFT

  Stuart (1990), Denner, Dittmaier et al. (1999), Actis, Passarino (2007)

- Evaluate the $\Delta$ self-energy on the complex pole:

  $$z - m_\Delta^0 - \Sigma_1(z^2) - z \Sigma_6(z^2) \equiv z - m_\Delta^0 - \Sigma(z) = 0 \text{ with } z = m_\Delta - i \frac{\Gamma_\Delta}{2}$$

- Self-energy diagrams:

  $\rightarrow$ one-loop easy

  $\rightarrow$ two-loops:

    use Cutkovsky rules for instable particles

  $\rightarrow$ width $\sim |A(\Delta \rightarrow N \pi)|^2$

  Veltman, Physica 29 (1963) 186
CALCULATION of the WIDTH

- Remarkable reduction of parameters:
  \[ \Delta_{23} = m_N - m_{\Delta}, \Delta_{123} = (M_\pi^2 + m_N^2 - m_{\Delta}^2)/(2m_N) \]
  \[ h_A = h - (b_3 \Delta_{23} + b_8 \Delta_{123}) - (f_1 \Delta_{23} + f_2 \Delta_{123}) \Delta_{123} + 2(2f_4 - f_5)M_\pi^2 \]

- Very simple formula for the decay width \( \Delta \to N\pi \):
  \[ \Gamma(\Delta \to N\pi) = (53.91 h_A^2 + 0.87 g_1^2 h_A^2 - 3.31 g_1 h_A^2 - 0.99 h_A^4) \text{ MeV} \]

- Correlation:

Siemens et al.,
EFT including the ROPER-RESONANCE

- Task: calculate the width of the Roper $N^*(1440)$ at two-loop order
  
  \[ \text{Gegelia, UGM, Yao, Phys. Lett. B760 (2016) 736} \]

- Remarkable feature: $\Gamma(R \rightarrow N\pi) \simeq \Gamma(R \rightarrow N\pi\pi)$

- Consider the effective chiral Lagrangian of pions, nucleons and deltas:
  
  
  \[ \text{Long, van Kolck, Nucl. Phys. A870-871 (2011) 72} \]

$$ L_{\text{eff}} = L_{\pi\pi} + L_{\pi N} + L_{\pi \Delta} + L_{\pi R} + L_{\pi N \Delta} + L_{\pi N R} + L_{\pi \Delta R} $$

$$ L_{\pi R}^{(1)} = \overline{\Psi}_R \left\{ i \slashed{D} - m_R + \frac{1}{2} g_R \slashed{u} \gamma^5 \right\} \Psi_R $$

$$ L_{\pi R}^{(2)} = \overline{\Psi}_R \left\{ c_R^1 \langle \chi^+ \rangle \right\} \Psi_R + \ldots $$

$$ L_{\pi N R}^{(1)} = \overline{\Psi}_R \left\{ \frac{1}{2} g_{\pi NR} \gamma^\mu \gamma_5 u_\mu \right\} \Psi_N + \text{h.c.} $$

$$ L_{\pi \Delta R}^{(1)} = h_R \overline{\Psi}_\mu \xi_{ij}^{3/2} \Theta^{\mu \alpha} (\tilde{z}) \omega_\alpha^j \Psi_R + \text{h.c.} $$
The power counting is complicated, but can be set up around the complex pole:

\[ m_R - m_N \sim \varepsilon, \quad m_R - m_\Delta \sim \varepsilon^2, \quad m_\Delta - m_N \sim \varepsilon^2, \quad M_\pi \sim \varepsilon^2 \]

Calculate the two-loop self-energy and the corresponding decay amplitudes.
CALCULATION of the WIDTH

A lengthy calculation leads to:

\[ \Gamma(R \to N\pi) = 550(58) \, g_{\pi NR}^2 \, \text{MeV} \]

\[ \Gamma(R \to N\pi\pi) = \left( 1.49(0.58) \, g_A^2 \, g_{\pi NR}^2 - 2.76(1.07) \, g_A \, g_{\pi NR}^2 \, g_R \right. \\
\left. + \ 1.48(0.58) \, g_{\pi NR}^2 \, g_R^2 + 2.96(0.94) \, g_A \, g_{\pi NR} \, h h_R \right. \\
\left. - \ 3.79(1.37) \, g_{\pi NR} \, g_R \, h h_R + 9.93(5.45) \, h^2 h_R^2 \right) \, \text{MeV} \]

Fix \( g_{\pi NR} \) from the PDG value:

\[ g_{\pi NR} = \pm (0.47 \pm 0.05) \]  

Maximal mixing assumption:

\[ g_R = g_A, \ h_R = h \]


\[ \Gamma(R \to N\pi\pi) = (41 \pm 22_{\text{LECs}} \pm 17_{\text{h.o.}}) \, \text{MeV} \]

consistent with the PDG value of \((67 \pm 10)\) MeV

need an improved determination of the LECs \( g_R \) and \( h_R \)
Summary & Outlook
SUMMARY & OUTLOOK

- Lessons learned / take home:
  
  - The QCD spectrum is more than a collection of quark model states
  
  - Structure formation in QCD ties nuclear and hadron physics together
  
  - Lattice QCD is making progress in addressing complex resonance properties (must respect chiral symmetry)
  
  - EFTs are of utmost importance in pushing this program forward
SPARES
LATTICE QCD

- In principle ab initio calcs of non-pert. QCD on a discretized space–time
  \[ \leftrightarrow \text{already some successes but only now entering the chiral regime} \]

- Extrapolations necessary:
  - finite volume \( V = L^3 \times L_t \to \infty \)
  - finite lattice spacing \( a \to 0 \)
  - chiral extrapolation \( m_q \to m_q^{\text{phys}} \)

- All these effects can be treated in suitably tailored EFTs

- how are resonances defined in such a finite space-time?
  \[ \Rightarrow \text{consider finite volume effects for low-lying hadron resonances} \]
Amplitude Analysis of

\[ B \rightarrow D\pi\pi \]
DATA for $B \rightarrow D\pi\pi$

- Recent high precision results for $B \rightarrow D\pi\pi$ from LHCb
  Aaji et al. [LHCb], Phys. Rev. D 94 (2016) 072001
- Spectroscopic information in the angular moments ($D\pi$ FSI):

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{data_b_to_dpipi}
\end{figure}
**CHIRAL LAGRANGIAN for $B \rightarrow D$ TRANSITIONS**


- Consider $\bar{B} \rightarrow D$ transition with the emission of two light pseudoscalars (pions)
  - chiral symmetry puts constraints on one of the two pions
  - the other pion moves fast and does not participate in the final-state interactions

- Chiral effective Lagrangian:

$$L_{\text{eff}} = \bar{B} \left[ c_1 (u_\mu t M + M t u_\mu) + c_2 (u_\mu M + M u_\mu) t 
+ c_3 t (u_\mu M + M u_\mu) + c_4 (u_\mu \langle M t \rangle + M \langle u_\mu t \rangle) 
+ c_5 t \langle M u_\mu \rangle + c_6 \langle (M u_\mu + u_\mu M) t \rangle \right] \partial^\mu D^\dagger$$

with

$$\bar{B} = (B^-, \bar{B}^0, \bar{B}_s^0), \quad D = (D^0, D^+, D_s^+)$$

$M$ is the matter field for the fast-moving pion

$t = u H u$ is a spurion field for Cabbibo-allowed decays

$\Rightarrow$ only some combinations of the LECs $c_i$ appear

$H = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
THEORY of $B \to D\pi\pi$

- $B^- \to D^+\pi^-\pi^-$ contains coupled-channel $D\pi$ FSI

- Consider $S, P, D$ waves: $A(B^- \to D^+\pi^-\pi^-) = A_0(s) + A_1(s) + A_2(s)$
  
  \[ \to \text{P-wave: } D^*, D^*(2860); \text{D-wave: } D_2(2460) \text{ as by LHCb} \]
  
  \[ \to \text{S-wave: use coupled channel } (D\pi, D\eta, D_s\bar{K}) \text{ amplitudes} \]
  
  with all parameters fixed before
  
  \[ \to \text{only two parameters in the S-wave} \]
  
  (one combination of the LECs $c_i$ and
  
  one subtraction constant in the $G_{ij}$)

\[ A_0(s) \propto E_\pi \left[ 2 + G_{D\pi}(s) \left( \frac{5}{3} T_{11}^{1/2}(s) + \frac{1}{3} T_{11}^{3/2}(s) \right) \right] + \frac{1}{3} E_\eta G_{D\eta}(s) T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}} E_{\bar{K}} G_{D_s\bar{K}}(s) T_{31}^{1/2}(s) + C E_\eta G_{D\eta}(s) T_{21}^{1/2}(s) \]
THEORY of $B \rightarrow D\pi\pi$ continued


- More appropriate combinations of the angular moments:

$$
\langle P_0 \rangle \propto |A_0|^2 + |A_1|^2 + |A_2|^2
$$

$$
\langle P_2 \rangle \propto \frac{2}{5}|A_1|^2 + \frac{2}{7}|A_2|^2 + \frac{2}{\sqrt{5}}|A_0||A_2|\cos(\delta_2 - \delta_0)
$$

$$
\langle P_{13} \rangle = \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|A_0||A_1|\cos(\delta_1 - \delta_0)
$$

- The S-wave $D\pi$ can be very well described using pre-fixed amplitudes

- Fast variation in [2.4,2.5] GeV in $\langle P_{13} \rangle$: cusps at the $D\eta$ and $D_s\bar{K}$ thresholds

$\leftrightarrow$ should be tested experimentally
A CLOSER LOOK at the S–WAVE

- LHCb provides anchor points, where the strength and the phase of the S-wave were extracted from the data and connected by cubic spline

- Higher mass pole at 2.46 GeV clearly amplifies the cusps predicted in our amplitude
THEORY of $B^0_s \rightarrow \bar{D}^0 K^- \pi^+$

LHCb has also data on $B^0_s \rightarrow \bar{D}^0 K^- \pi^+$, but less precise

Same formalism as before, one different combination of the LECs $c_i$

same resonances in the P- and D-wave as LHCb $\rightarrow$ one parameter fit!

⇒ these data are also well described
⇒ better data for $\langle P_{13} \rangle$ would be welcome
⇒ even more channels, see Du, Guo, UGM, Phys. Rev. D 99 (2019) 114002