Different partial-wave analysis tools and recent results of the Jülich-Bonn model

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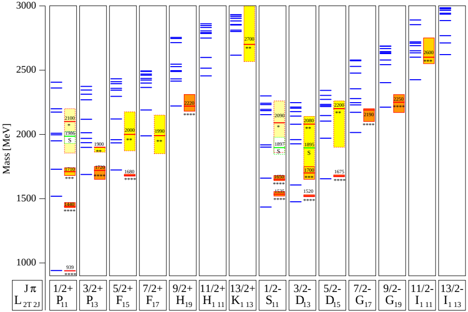
N^* and Δ^* spectrum in the past

- Most information from elastic or charge exchange πN scattering,
  e.g. Karlsruhe-Helsinki (KH), Carnegie-Mellon-Berkeley (CMB), George-Washington U (GWU)

- Theoretical predictions, e.g., from lattice calculations and quark models
  $\rightarrow$ "Missing resonance problem": above 1.8 GeV much more states are predicted than observed

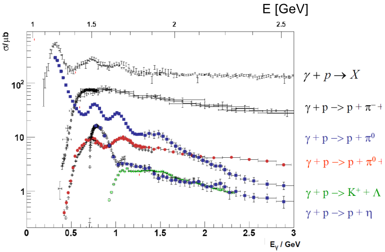
Relativistic quark model:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Experimental studies of hadronic reactions: major progress in recent years

Photoproduction: e.g. from JLab, ELSA, MAMI, GRAAL, SPring-8, …



source: ELSA; data: ELSA, JLab, MAMI

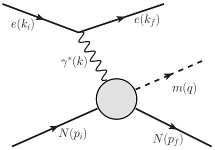
- enlarged data base with high quality for different final states
- (double) polarization observables

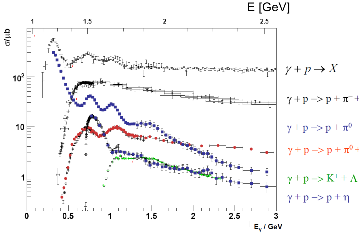
  $\to$ alternative source of information besides $\pi N \to X$

  $\to$ towards a complete experiment: unambiguous determination of the amplitude (up to an overall phase)

Electroproduction:

- electroproduction of πN, ηN, KY, $\pi\pi N$
- access the Q^2 dependence of the amplitude, information on the internal structure of resonances





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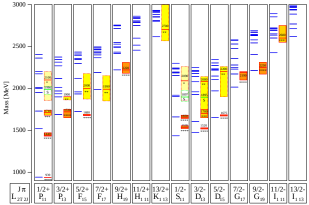
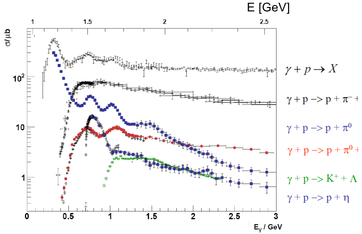


Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Different modern analyses frameworks:

- **unitary isobar models:** unitary amplitudes + Breit-Wigner resonances
  MAID, Yerevan/JLab, KSU
- **(multi-channel) K-matrix:** GWU/SAID, BnGa (phenomenological),
  Gießen (microscopic Bgd)
- **dynamical coupled-channel (DCC):** 3d scattering eq., off-shell intermediate states
  ANL-Osaka (EBAC), Dubna-Mainz-Taipeh, Jülich-Bonn
- **other groups:** JPAC (high energies), Mainz-Tuzla-Zagreb PWA (MAID + fixed-t dispersion relations, L+P), Gent, truncated PWA
- ...

From experimental data to the resonance spectrum



source: ELSA; data: ELSA, JLab, MAMI

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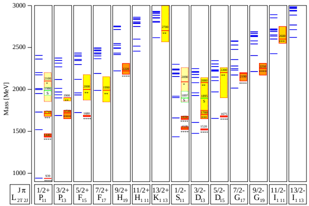

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The SAID, MAID, BnGa and JüBo approaches

SAID PWA

based on Chew-Mandelstam K-matrix

- K-matrix elements parameterized as energy-dependent polynomials
- resonance poles are dynamically generated (except for the $\Delta(1232)$)
- masses, width and hadronic couplings from fits to pion-induced πN and ηN production
- photocouplings from photoproduction

Bonn-Gatchina (BnGa) PWA

Multi-channel PWA based on K-matrix (N/D)

- mostly phenomenological model
- resonances added by hand
- resonance parameters determined from large experimental data base:
  pion-, photon-induced reactions, 3-body final states

MAID PWA

unitary isobar model

- resonances as multi-channel Breit-Wigner amplitudes
- background: Born terms + Regge exchanges
- so far: masses, widths taken from PDG (upcoming: pole positions with L+P method)
- photo- and electroproduction of pions, etas and kaons

Jülich-Bonn (JüBo) DCC model

Lippmann-Schwinger eq. formulated in TOPT

- hadronic potential from effective Lagrangians
- photoproduction parameterized by energy-dependent polynomials
- resonances as s-channel states ("by hand"), dynamical generation possible
- resonance parameters determined from pion- and photon-induced data

The SAID, MAID, BnGa and JüBo approaches

All four approaches:

- (some) coupled channel effects
- unitarity (2 body)
- amplitudes are analytic functions of the invariant mass

Differences:

(besides the details in the construction of the amplitude)

- underlying (chiral) Lagrangian or not
- channel space, degree of channel coupling
- data base: simultaneous fits, which properties are determined from which data
- energy range
- resonances dynamically generated/added "by hand" (model selection)
- fitting techniques, computational effort

Detailed comparison + joint analysis of new polarization data: EPJ A 52, 284 (2016)

- new polarization data for pion photoproduction included in SAID, BnGa, JüBo
  $\Rightarrow$ agreement between multipoles considerably improved

- similar analysis planned for eta photoproduction (Afternoon session on Tuesday)

Recent results

Recent results:

- elastic πN PWA used as input by many other groups
  - covariance matrices for WI08 solution available (PRC 93 (2016) 065205)
  - XP15 solution: including new $\pi^{\pm}p \rightarrow \pi^{\pm}p$ data (EPECUR, PRC 91 (2015) 025205, see also PRC 93 (2016) 062201(R))
- NN elastic PWA (PRC 94 065203 (2016))
- Pion photoproduction:
  - MA27 multipole analysis of $\gamma n \rightarrow \pi^- p$
    $\rightarrow$ first determination of photon decay amplitudes $N^* \rightarrow \gamma n$ at the pole for $N(1440)1/2^+$, $N(1535)1/2^-$, $N(1650)1/2^-$, $N(1720)3/2^+$ (PRC 96 (2017) 035204)
  - Multipole analysis of E for $\gamma n \rightarrow \pi^- p$ (PRL 118 (2017) 242002)
    $\rightarrow$ revised $\gamma n N^*$ couplings

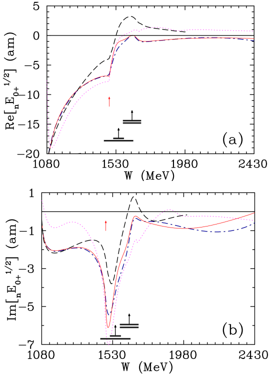


figure: PRC 96 (2017) 035204. Red line: MA27

Future plans:

- analysis of new (neutron-target) data
- new and more modern web interface

Talk by W. Briscoe on Wednesday

MAID collaboration has widened: **Mainz-Tuzla-Zagreb** collaboration

Recent results:

- η, η' photoproduction: "EtaMAID2018" (EPJ A54 (2018) 210, talk by V. Kashevarov on Tuesday)
  4 coupled channels: ηp, ηn, $\eta' p$, $\eta' n$ with up to 20 N^* and Regge phenomenology
- Role of angle-dependent phase rotations of reaction amplitudes in $\gamma p \to \eta p$ (PR C98 (2018) 045206)
- Fixed-t analyticity as a constraint in SE PWA of meson photoproduction reactions (PR C97 (2018) 015207, talk by A. Svarc on Tuesday)

$\gamma p \to \eta p$

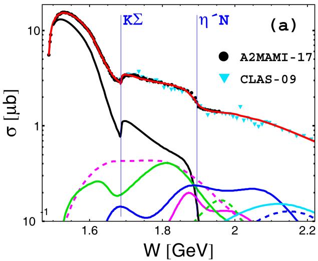


figure: EPJ A 54, 210. Red: EtaMAID2018. Black: S_{11}

Future plans:

- SE PWA for γ,π^0 on the proton with fixed-t analyticity (publication in preparation)
- Update of PionMAID for πN photoproduction in full isospin
- Dispersion relations for γ,η and γ,π^0
- Resonance pole analysis and inelastic residues for γ,η

See also talk by M. Gorshteyn on Wednesday

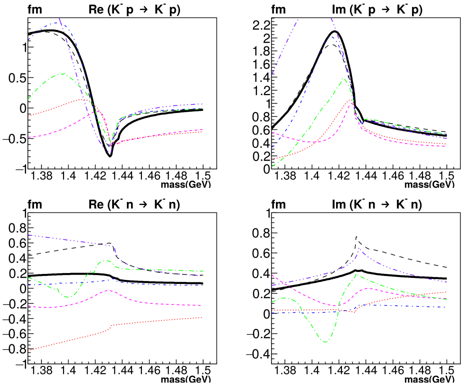
Recent results:

- photoproduction: $\gamma p \to \eta p, \eta' p$ all polarization observables included (PLB 785, 626)
  $\to$ existence of $N(1895)1/2^-$ (talk by A. Sarantsev on Tuesday)

  $\gamma n \to K^0\Lambda^0$, $K^0\Sigma^0$ (CLAS), $\gamma p \to \pi^0 p\eta$ (MAMI)

- pionproduction: $\pi^- p \to \pi^+\pi^- n$, $\pi^0\pi^- p$ (HADES)

- PWA of $\Sigma\pi$ and $K^- p$ interactions (arXiv:1905.05456, talk by E. Klempt on Thursday)



Future plans:

- analysis of new data
- extension to electroproduction

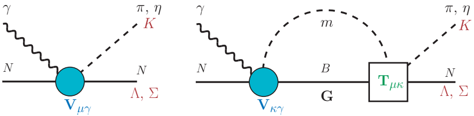
figure: arXiv:1905.05456, black line: S-wave, one pole $\Lambda(1405)$ BnGa

- DCC analysis including $\gamma p \to K^+\Lambda$ (EPJA 54, 110 (2018)) and $\gamma p \to K^+\Sigma^0$, $K^0\Sigma^+$ (preliminary)

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_\kappa T^{IJ}_{\mu\kappa} G_\kappa V^{IJ}_{\kappa\gamma}$$

$$\gamma \sim \gamma N \; ; \; \mu, \kappa \sim mB$$

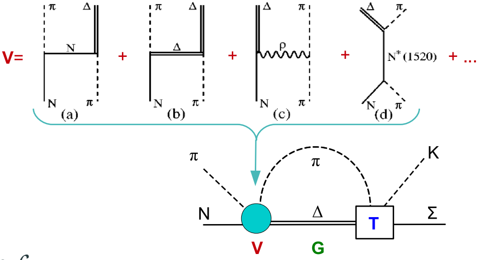


- $V_{\mu\gamma} \sim$ energy-dependent polynomials

Hadronic amplitude

$$T^{IJ}_{\mu\nu} = V^{IJ}_{\mu\nu} + \sum_\kappa V^{IJ}_{\mu\kappa} G_\kappa T^{IJ}_{\kappa\nu}$$

$$\mu, \nu, \kappa \sim mB$$



- potentials $V_{\mu\nu}$ constructed from effective $\mathcal{L}$
- t- and u-channel: "Background" (dynamical generation of poles possible)
- s-channel: genuine resonances (poles on the 2nd Riemann sheet)

Data Base and fit

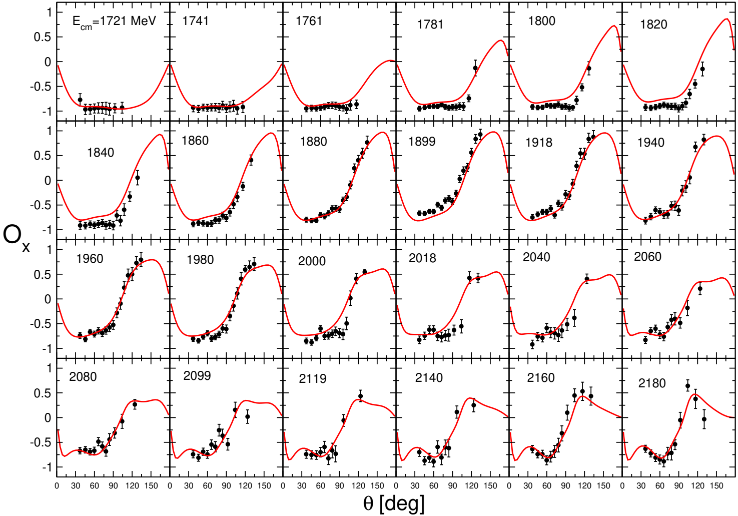
- $\pi N \to X$: $>$ **7,000** data points ($\pi N \to \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \to X$:

Reaction	Observables (# data points)	p./channel
$\gamma p \to \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140),	
	G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	25,542
$\gamma p \to \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231),	
	G (86), H (128), E (903)	9,748
$\gamma p \to \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \to K^+\Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383),	
	$C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	**57,027**

- Fit paramters: – m_{bare}, f_{mBN^*} (s-chan.), contact terms, couplings of polynomials $V_{\mu\gamma}$
  – more than 900, calculations on JURECA supercomputer [JSC, *JURECA: General-purpose supercomputer at Jülich Supercomputing Centre*, Journal of large-scale research facilities, 2, A62 (2016)]

Selected fit results: O_x and O_z in $\gamma p \to K^+\Lambda$, $K^+\Sigma^0$

- $\gamma p \to K^+\Lambda$:



data: Paterson (CLAS) PRC 93, 065201 (2016), red line: fit JüBo2019

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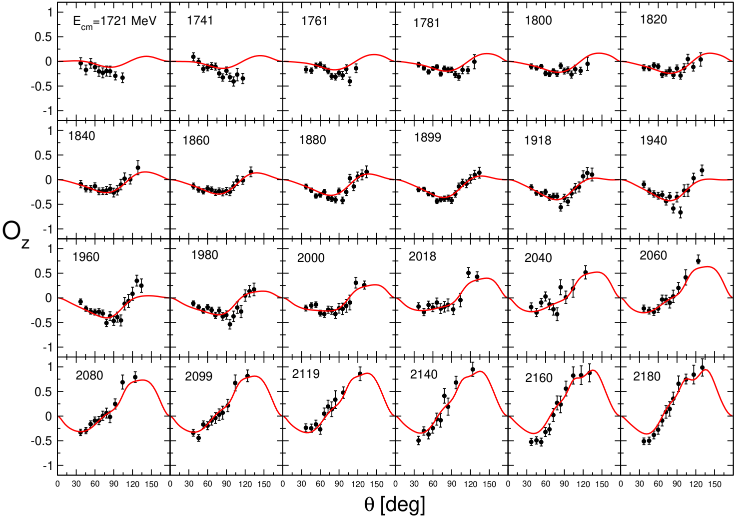

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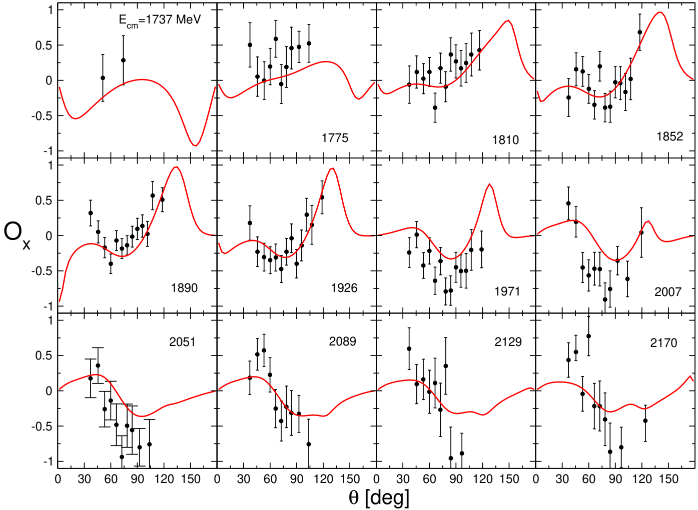
- $\gamma p \to K^+\Sigma^0$:



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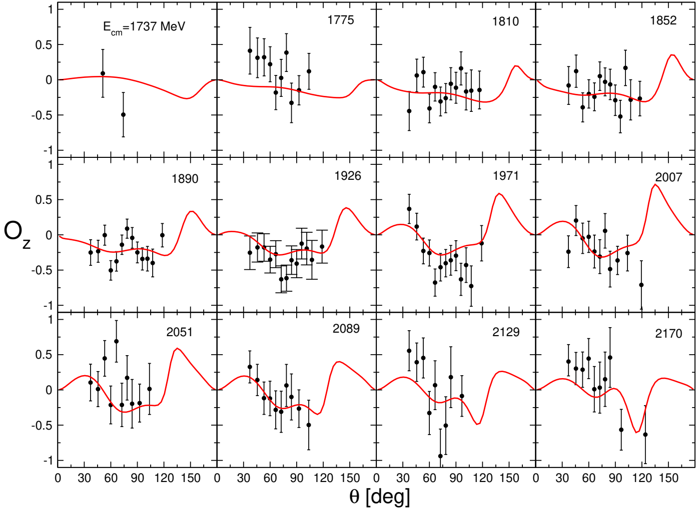
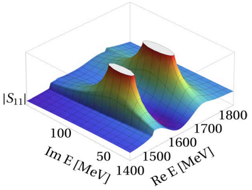


data: Paterson (CLAS) PRC 93, 065201 (2016), red line: fit JüBo2019

Resonance spectrum

Resonance states: **Poles in the T-matrix** on the 2^{nd} Riemann sheet



- $\text{Re}(E_0)$ = "mass", $-2\text{Im}(E_0)$ = "width"
- elastic πN residue $(|r_{\pi N}|, \theta_{\pi N \to \pi N})$, normalized residues for inelastic channels $(\sqrt{\Gamma_{\pi N}\Gamma_\mu}/\Gamma_{\text{tot}}, \theta_{\pi N \to \mu})$
- photocouplings at the pole: $\tilde{A}^h_{pole} = A^h_{pole} e^{i\vartheta^h}$, $h = 1/2,\ 3/2$

$$\tilde{A}^h_{pole} = I_F \sqrt{\frac{q_p}{k_p} \frac{2\pi\,(2J+1)\mathbf{E_0}}{m_N\ \mathbf{r}_{\pi\mathbf{N}}}}\ \text{Res}\, A^h_{L\pm}$$

I_F: isospin factor
q_p (k_p): meson (photon) momentum at the pole
$J = L \pm 1/2$ total angular momentum
E_0: pole position
$r_{\pi N}$: elastic πN residue
$A^h_{L\pm}$: helicity multipole

In the present analysis ("JüBo2019", preliminary):

- all 4-star N and Δ states up to $J = 9/2$ are seen (exception: $N(1895)1/2^-$)
  + some states rated with less than 4 stars
- one additional s-channel diagram included: $N(2000)5/2^+$
- **more information on Δ states** than in JüBo2017: $\gamma p \to K^+\Sigma^0$, $K^0\Sigma^+$ is mixed isospin

Resonance spectrum: selected results (JüBo2019, preliminary)

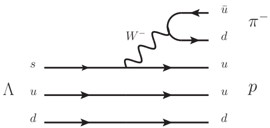
$\Delta(1910)\ 1/2^+$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\lvert r_{\pi N}\rvert$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]	$\frac{\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
JüBo2019	1873	346	0.6	178	5.4	12
JüBo2017	1798(5)	621(35)	81(68)	$-87(18)$	5.1(2.2)	$-96(58)$
PDG 2019	1860 ± 30	300 ± 100	25 ± 5	130 ± 50	7 ± 0.2	-110 ± 30

$\Delta(1700)\ 3/2^-$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\lvert r_{\pi N}\rvert$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]	$\frac{\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
JüBo2019	1601	248	13	-2.4	0.83	171
JüBo2017	1667(28)	305(45)	22(6)	$-8.6(32.1)$	0.7(1.8)	176(152)
PDG 2019	1665 ± 25	250 ± 50	25 ± 15	-20 ± 20	—	—

Resonance spectrum: impact of the Λ decay parameter α_-

Advantage in KY photoproduction: self-analysing decay of the hyperons
$\rightarrow$ measurement of recoil polarization easier

- Λ decays weakly to $\pi^- p$ with decay parameter α_- (PDG average: $\alpha_- = 0.642 \pm 0.013$)

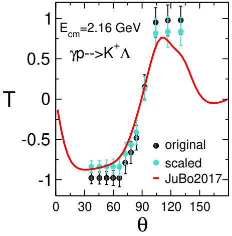


- recent BESIII measurement ($e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$): $\alpha_- = 0.750 \pm 0.009 \pm 0.004$ (Ablikim, Nature (2019))

  $\rightarrow$ polarizations affected by α_- are $\sim 17\%$ too large!

- independent estimation of α_- from $\gamma p \rightarrow K^+\Lambda$ CLAS data using Fierz identities
  $\Rightarrow$ $\alpha_- = 0.721 \pm 0.006 \pm 0.005$ (Ireland et al. arXiv:1904.0761)

($\rightarrow$ Talk by D. Ireland on Thursday!)



data: Paterson (CLAS) PRC 93, 065201 (2016)

Has impact on

- observables P, T, C_x, C_z, O_x, O_z
- reactions $\gamma p \rightarrow K^+\Lambda$, $K^+\Sigma^0$ ($\rightarrow K^+\gamma\Lambda$), $\pi^- p \rightarrow K^0\Lambda$, $K^0\Sigma^0$
- resonance spectrum? $\Rightarrow$ **JüBo re-fit to data scaled by new α_-**

Resonance spectrum: impact of the Λ decay parameter α_- (preliminary)

Most resonances very stable, example:

$N(1720)\ 3/2^+$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\lvert r_{\pi N}\rvert$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
JüBo2019	1731	233	12	-85	4.1	50
Re-fit $\alpha_- = 0.721$	1730	233	12	-85	4.1	50

Some exceptions, also among the well established states:

$\Delta(1910)\ 1/2^+$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\lvert r_{\pi N}\rvert$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
JüBo2019	1873	346	0.6	178	5.4	12
Re-fit $\alpha_- = 0.721$	1859	303	4.4	129	14	-165

$\Delta(1700)\ 3/2^-$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\lvert r_{\pi N}\rvert$ [MeV]	$\theta_{\pi N\to\pi N}$ [deg]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
JüBo2019	1601	248	13	-2.4	0.83	171
Re-fit $\alpha_- = 0.721$	1647	265	7.1	-24	0.43	175

Resonance spectrum: impact of the Λ decay parameter α_- (preliminary)

Photocouplings at the pole less stable:

$N(1720)\ 3/2^+$	$A^{1/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$	$\vartheta^{1/2}$ [deg]	$A^{3/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$	$\vartheta^{3/2}$ [deg]
JüBo2019	66	58	14	-71
Re-fit $\alpha_- = 0.72$	59	65	20	-65

$\Delta(1910)\ 1/2^+$	$A^{1/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$	$\vartheta^{1/2}$ [deg]	$A^{3/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$	$\vartheta^{3/2}$ [deg]
JüBo2019	99	-77	-	-
Re-fit $\alpha_- = 0.72$	64	159	-	-

$\Delta(1700)\ 3/2^-$	$A^{1/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$	$\vartheta^{1/2}$ [deg]	$A^{3/2}_{\mathrm{pole}}$ $[10^{-3}\ \mathrm{GeV}^{-\frac{1}{2}}]$	$\vartheta^{3/2}$ [deg]
JüBo2019	96	-16	101	-44
Re-fit $\alpha_- = 0.72$	82	-14	87	-17

Summary

Extraction of the N^* and Δ spectrum from experimental data:

- new information from photoproduction data
- also electroproduction
- recent results from different PW analysis groups

Jülich-Bonn model:

- extension of the coupled-channel approach to kaon photoproduction
- $\gamma p \to K\Sigma$ especially interesting for $I = 3/2$ states
- impact of a new value of the Λ decay parameter α_-:
  - many resonances more or less stable
  - some exceptions with major changes in the resonance parameters
  - photo couplings at the pole more sensitive than other parameter

Future plans JüBo:

- electroproduction (already in progress)
- inclusion of the further channels, e.g. photoproduction on the neutron

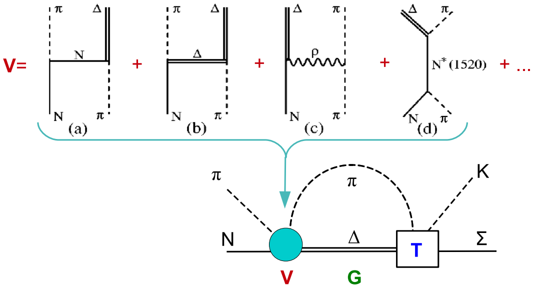
Thank you for your attention!

Appendix

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

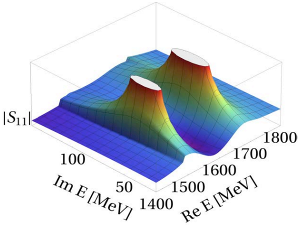
The scattering equation **in partial-wave basis**

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$



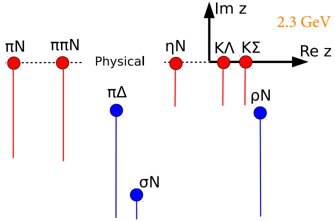
- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P
  genuine resonance states
- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated
- contact terms

Resonance states: **Poles in the T-matrix** on the 2^{nd} Riemann sheet



- pole position E_0 is the same in all channels
- residues→ branching ratios

Re(E_0) = "mass", -2Im(E_0) = "width"

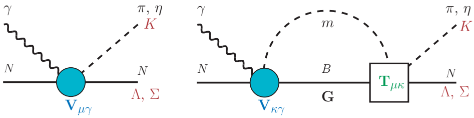


- (2-body) unitarity and analyticity respected
- 3-body $\pi\pi N$ channel:
  - parameterized effectively as $\pi\Delta$, σN, ρN
  - $\pi N/\pi\pi$ subsystems fit the respective phase shifts
  - ↳ branch points move into complex plane

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_\kappa V^{IJ}_{\kappa\gamma}$$

(partial wave basis)

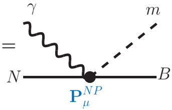


$m = \pi, \eta, K$, $B = N, \Delta, \Lambda$

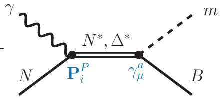
$T_{\mu\kappa}$: Jülich hadronic T-matrix → Watson's theorem fulfilled by construction
→ **analyticity of T**: extraction of resonance parameters

Photoproduction potential: approximated by energy-dependent polynomials

$\mathbf{V}_{\mu\gamma}(E,q) =$



$+$



$$= \frac{\tilde\gamma^a_\mu(q)}{m_N} P^{NP}_{\mu}(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^P_i(E)}{E - m^b_i}$$

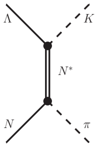
$\tilde\gamma^a_\mu$, $\gamma^a_{\mu;i}$: hadronic vertices → correct threshold behaviour, cancellation of singularity at $E = m^b_i$
→ $\gamma^a_{\mu;i}$ affects pion- and photon-induced production of final state mB

i: resonance number per multipole; μ: channels πN, ηN, $\pi\Delta$, KY

Polynomials

Fit parameters:

s-channel: resonances (T^P)



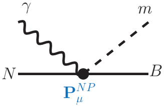
- $\pi N \to \pi N$
  $\pi^- p \to \eta n,\ K^0\Lambda,\ K^0\Sigma^0,\ K^+\Sigma^-$
  $\pi^+ p \to K^+\Sigma^+$

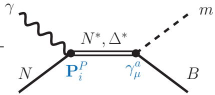
$m_{bare} + f_{mBN*}$

$\Rightarrow$ 134 free parameters
  11 N^* resonances $\times$ (1 m_{bare}+ couplings to πN, ρN, ηN, $\pi\Delta$, $K\Lambda$, $K\Sigma$)
  + 10 Δ resonances $\times$ (1 m_{bare}+ couplings to πN, ρN, $\pi\Delta$, $K\Sigma$)

- contact terms: one per partial wave, couplings to πN, ηN, ($\pi\Delta$,) $K\Lambda$, $K\Sigma$
  $\Rightarrow$ 61 free parameters

- $\gamma p \to \pi^0 p$, $\pi^+ n$, ηp, $K^+\Lambda$:
  couplings of the polynomials
  $\Rightarrow$ 566 free parameters





$\Rightarrow$ 761 in total, calculations on the JURECA supercomputer [Jülich Supercomputing Centre, *JURECA: General-purpose supercomputer at Jülich Supercomputing Centre,* Journal of large-scale research facilities, 2, A**62** (2016)]

- *t*- & *u*-channel parameters: fixed to values of hadronic DCC analysis (JüBo 2013)

Uncertainties of extracted resonance parameters

Challenges in determining resonance uncertainties, e.g.:

- **elastic πN channel**: not data but GWU SAID PWA

  $\rightarrow$ correlated χ^2 fit including the covariance matrix $\hat{\Sigma}$ (available on SAID webpage!)

  PRC 93, 065205 (2016)

  $$\chi^2(A) = \chi^2(\hat{A}) + (A - \hat{A})^T \hat{\Sigma}^{-1} (A - \hat{A})$$

  $A \sim$ vector of fitted PWs, $\hat{A} \sim$ vector of SAID SE PWs

  $\rightarrow$ same χ^2 as fitting to data up to nonlinear and normalization corrections

- **error propagation** data $\rightarrow$ fit parameters $\rightarrow$ derived quantities:
  bootstrap method: generate pseudo data around actual data, repeat fit

- **model selection**, significance of resonance signals:
  determine minimal resonance content using Bayesian evidence [PRL 108, 182002; PRC 86, 015212 (2012)]
  or the LASSO method [PRC 95, 015203 (2017); J. R. Stat. Soc. B 58, 267 (1996)]:

  $$\chi^2_T = \chi^2 + \lambda \sum_{i=1}^{i_{max}} |a_i|$$

  $\lambda \sim$ penalty factor, $a_i \sim$ fit parameter

Uncertainties of extracted resonance parameters

In JüBo framework: such methods are nummerically challenging, but planned for the (near) future

Estimation of uncertainties of extracted resonance parameters in the present study:

- from 9 re-fits to re-weighted data sets
- individually increase the weight in each reaction channel
- extract resonance parameters from refits
- maximal deviation of resonance parameters of the refits = ”error”
- only a qualitative estimation of relative uncertainties, absolute size not well determined

Details of the formalism

Polynomials:

$$P_i^{\mathrm{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathrm{P}} \left(\frac{E-E_0}{m_N}\right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathrm{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathrm{NP}} \left(\frac{E-E_0}{m_N}\right)^j e^{-g_{\mu,n+1}^{\mathrm{NP}}(E-E_0)}$$

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathrm{P}}$, $g_{\mu,j}^{\mathrm{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

back

The scattering potential: s-channel resonances

$$V^{\mathrm{P}} = \sum_{i=0}^{n} \frac{\gamma^a_{\mu;i}\,\gamma^c_{\nu;i}}{z - m_i^b}$$

- i: resonance number per PW
- $\gamma^c_{\nu;i}$ ($\gamma^a_{\mu;i}$): creation (annihilation) vertex function with **bare coupling** f (free parameter)
- z: center-of-mass energy
- m_i^b: **bare mass** (free parameter)

- $J \leq 3/2$:

  $\gamma^c_{\nu;i}$ ($\gamma^a_{\mu;i}$) from effective $\mathcal{L}$

Vertex	$\mathcal{L}_{int}$
$N^*(S_{11})N\pi$	$\frac{f}{m_\pi}\bar{\Psi}_{N*}\gamma^\mu\vec{\tau}\,\partial_\mu\vec{\pi}\,\Psi \;+\;$ h.c.
$N^*(S_{11})N\eta$	$\frac{f}{m_\pi}\bar{\Psi}_{N*}\gamma^\mu\partial_\mu\eta\,\Psi \;+\;$ h.c.
$N^*(S_{11})N\rho$	$f\,\bar{\Psi}_{N*}\gamma^5\gamma^\mu\vec{\tau}\,\vec{\rho}_\mu\,\Psi \;+\;$ h.c.
$N^*(S_{11})\Delta\pi$	$\frac{f}{m_\pi}\bar{\Psi}_{N*}\gamma^5\vec{S}\partial_\mu\vec{\pi}\,\Delta^\mu \;+\;$ h.c.

- $5/2 \leq J \leq 9/2$:

  correct dependence on L (centrifugal barrier)

$$(\gamma^{a,c})_{\frac{5}{2}-} = \frac{k}{M}(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{5}{2}+} = \frac{k}{M}(\gamma^{a,c})_{\frac{3}{2}-}$$

$$(\gamma^{a,c})_{\frac{7}{2}-} = \frac{k^2}{M^2}(\gamma^{a,c})_{\frac{3}{2}-} \qquad (\gamma^{a,c})_{\frac{7}{2}+} = \frac{k^2}{M^2}(\gamma^{a,c})_{\frac{3}{2}+}$$

$$(\gamma^{a,c})_{\frac{9}{2}-} = \frac{k^3}{M^3}(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{9}{2}+} = \frac{k^3}{M^3}(\gamma^{a,c})_{\frac{3}{2}-}$$

The scattering potential: t- and u-channel exchanges

	πN	ρN	ηN	πΔ	σN	KΛ	KΣ
πN	N,Δ,$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$	N, Δ, Ct., π, ω, a_1	N, a_0	N, Δ, ρ	N, π	Σ, Σ*, K*	Λ, Σ, Σ*, K*
ρN		N, Δ, Ct., ρ	-	N, π	-	-	-
ηN			N, f_0	-	-	K*, Λ	Σ, Σ*, K*
πΔ				N, Δ, ρ	π	-	-
σN					N, σ	-	-
KΛ						Ξ, Ξ*, f_0, ω, ϕ	Ξ, Ξ*, ρ
KΣ							Ξ, Ξ*, f_0, ω, ϕ, ρ

Free parameters: cutoffs Λ in the form factors: $F(q) = \left(\frac{\Lambda^2 - m_x^2}{\Lambda^2 + \vec{q}^{\,2}}\right)^n$, $n = 1, 2$

Interaction potential from effective Lagrangian

J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); U.-G. Meißner, Phys. Rept. **161**, 213 (1988); B. Borasoy and U.-G. Meißner, Int. J. Mod. Phys. A **11**, 5183 (1996).

- consistent with the approximate (broken) chiral $SU(2) \times SU(2)$ symmetry of QCD

Vertex	$\mathcal{L}_{int}$	Vertex	$\mathcal{L}_{int}$
$NN\pi$	$-\frac{g_{NN\pi}}{m_\pi}\Psi\gamma^5\gamma^\mu\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi$	$NN\omega$	$-g_{NN\omega}\bar{\Psi}[\gamma^\mu - \frac{\kappa_\omega}{2m_N}\sigma^{\mu\nu}\partial_\nu]\omega_\mu\Psi$
$N\Delta\pi$	$\frac{g_{N\Delta\pi}}{m_\pi}\bar{\Delta}^\mu\vec{S}^\dagger\cdot\partial_\mu\vec{\pi}\Psi + \text{h.c.}$	$\omega\pi\rho$	$\frac{g_{\omega\pi\rho}}{m_\omega}\epsilon_{\alpha\beta\mu\nu}\partial^\alpha\vec{\rho}^\beta\cdot\partial^\mu\vec{\pi}\omega^\nu$
$\rho\pi\pi$	$-g_{\rho\pi\pi}(\vec{\pi}\times\partial_\mu\vec{\pi})\cdot\vec{\rho}^\mu$	$N\Delta\rho$	$-i\frac{g_{N\Delta\rho}}{m_\rho}\bar{\Delta}^\mu\gamma^5\gamma^\mu\vec{S}^\dagger\cdot\vec{\rho}_{\mu\nu}\Psi + \text{h.c.}$
$NN\rho$	$-g_{NN\rho}\Psi[\gamma^\mu - \frac{\kappa_\rho}{2m_N}\sigma^{\mu\nu}\partial_\nu]\vec{\tau}\cdot\vec{\rho}_\mu\Psi$	$\rho\rho\rho$	$g_{NN\rho}(\vec{\rho}_\mu\times\vec{\rho}_\nu)\cdot\vec{\rho}^{\mu\nu}$
$NN\sigma$	$-g_{NN\sigma}\bar{\Psi}\Psi\sigma$	$NN\rho\rho$	$\frac{\kappa_\rho g^2_{NN\rho}}{2m_N}\bar{\Psi}\sigma^{\mu\nu}\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\rho}_\nu)$
$\sigma\pi\pi$	$\frac{g_{\sigma\pi\pi}}{2m_\pi}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\sigma$	$\Delta\Delta\pi$	$\frac{g_{\Delta\Delta\pi}}{m_\pi}\bar{\Delta}_\mu\gamma^5\gamma^\nu\vec{T}\Delta^\mu\partial_\nu\vec{\pi}$
$\sigma\sigma\sigma$	$-g_{\sigma\sigma\sigma}m_\sigma\sigma\sigma\sigma$	$\Delta\Delta\rho$	$-g_{\Delta\Delta\rho}\bar{\Delta}_\tau(\gamma^\mu - i\frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta}\sigma^{\mu\nu}\partial_\nu)$ $\cdot\vec{\rho}_\mu\cdot\vec{T}\Delta^\tau$
$NN\rho\pi$	$\frac{g_{NN\pi}}{m_\pi}2g_{NN\rho}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\pi})$	$NN\eta$	$-\frac{g_{NN\eta}}{m_\pi}\bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\eta\Psi$
NNa_1	$-\frac{g_{NN\pi}}{m_\pi}m_{a_1}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi\vec{a}_\mu$	NNa_0	$g_{NNa_0}m_\pi\bar{\Psi}\vec{\tau}\Psi\vec{a}_0$
$a_1\pi\rho$	$-\frac{2g_{\pi a_1\rho}}{m_{a_1}}[\partial_\mu\vec{\pi}\times\vec{a}_\nu - \partial_\nu\vec{\pi}\times\vec{a}_\mu]\cdot[\partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu]$ $+\frac{2g_{\pi a_1\rho}}{2m_{a_1}}[\vec{\pi}\times(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)]\cdot[\partial^\mu\vec{a}^\nu - \partial^\nu\vec{a}^\mu]$	$\pi\eta a_0$	$g_{\pi\eta a_0}m_\pi\eta\vec{\pi}\cdot\vec{a}_0$

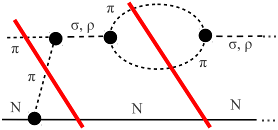
Theoretical constraints of the S-matrix

Unitarity: probability conservation

- 2-body unitarity
- 3-body unitarity:

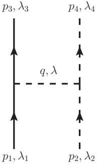
  discontinuities from t-channel exchanges

  → Meson exchange from requirements of the S-matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]

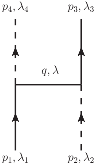


Analyticity: from unitarity and causality

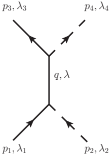
- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut → Baryon u-channel exchange



$\vec{q} = \vec{p}_1 - \vec{p}_3$



$\vec{q} = \vec{q}_1 - \vec{p}_4$



$\vec{q} = \vec{p}_1 + \vec{p}_2 = 0$

→ Resonances