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LARGE *N* QCD AT STRONG COUPLING

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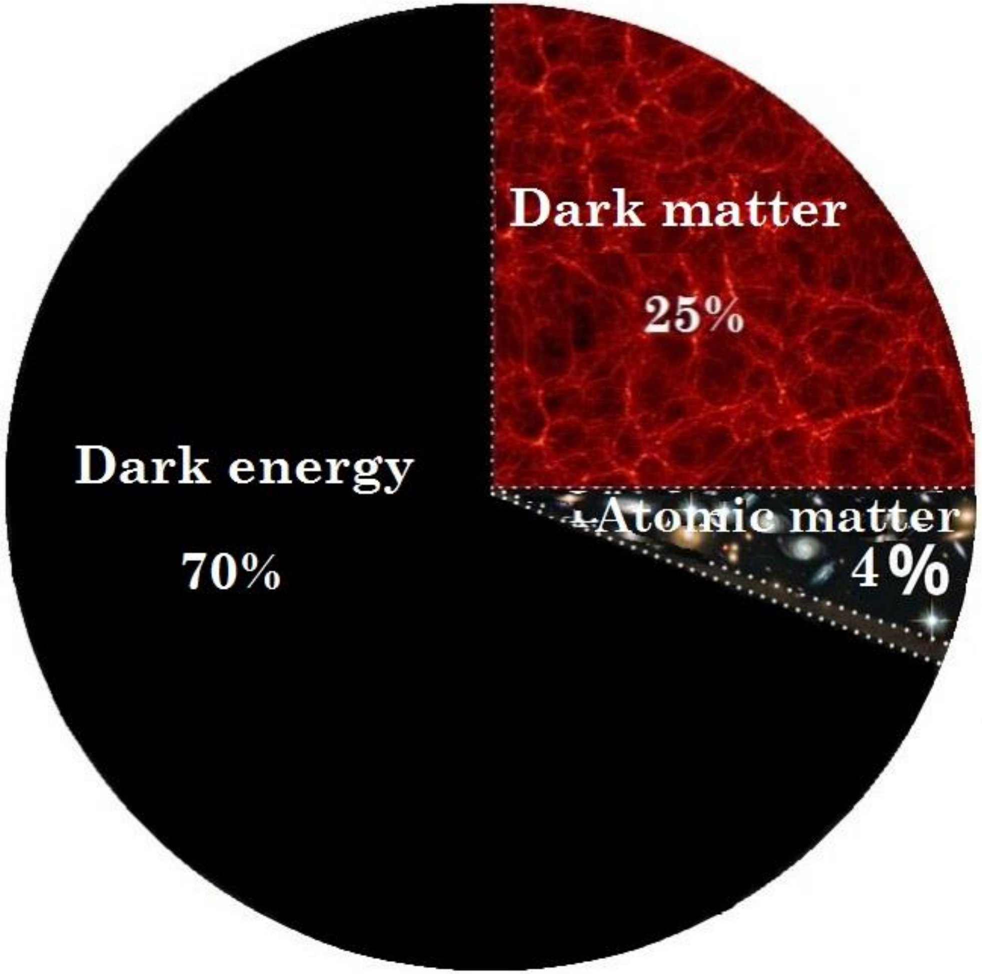
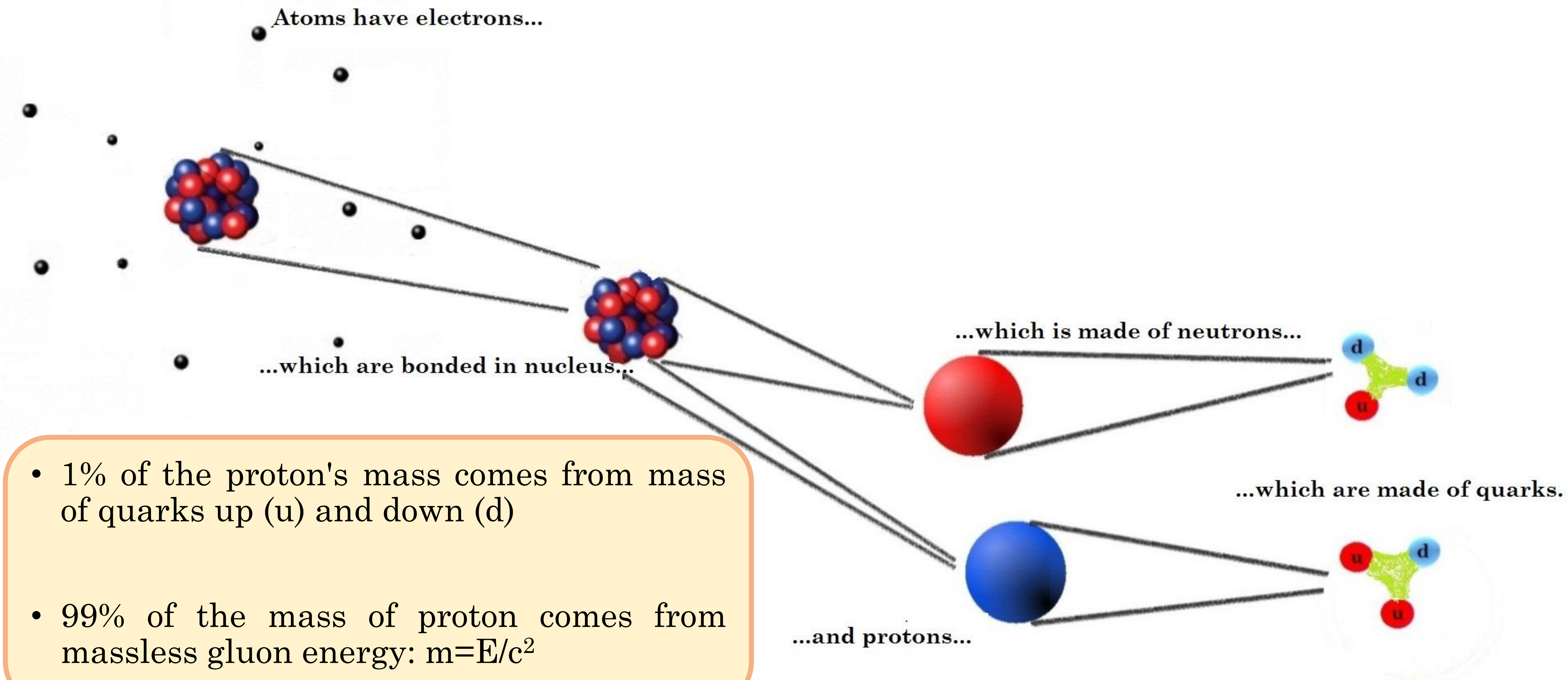
Outline

- Introduction
- Energy Distribution at Universe
  - Atomic matter
- Isospin symmetry
- Quantum Chromodynamics (QCD)
  - QCD at low energies
  - Large N QCD
- Large N solution at strong coupling
- Conclusions

Energy Distribution at Universe…



Why we focus on 4% ?



- 1% of the proton's mass comes from mass of quarks up (u) and down (d)
- 99% of the mass of proton comes from massless gluon energy: $m=E/c^2$

Isospin Symmetry

- *Protons* and *neutrons* are very close in mass

$$m_{proton} \simeq m_{neutron}$$

  - *Protons and neutrons are up/down states of isospin* 1/2 *system*

- *Pi-mesons* also have approximate masses

$$m_{\pi^+} \simeq m_{\pi^0} \simeq m_{\pi^-}$$

  - *Pions are* +1, 0, -1 *states of isospin* 1 *system*

Mesons on Quark Model

- One of the main meson is pi-meson:

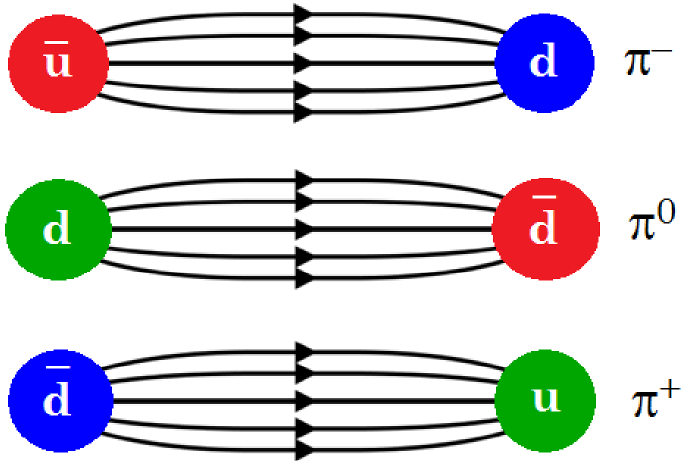
$$\pi^{+} \qquad \pi^{0} \qquad \pi^{-}$$

- It consist of one *quark* and one *antiquark*:

$$\pi^{-} \sim \bar{u}d$$

$$\pi^{0} \sim \bar{u}u \quad or \quad \bar{d}d$$

$$\pi^{+} \sim \bar{d}u$$



Yukawa Theory

- Yukawa used the idea of forces being mediated by particles to explain the nuclear force

- A new particle was introduced whose exchange between nucleons causes the nuclear force
  - It was called a *meson* (*pion*)

- Pions are the associated quantum-field particles that transmit the nuclear force between hadrons that pull those together into a nucleus.

- Our key point of interest is to go into the interior of nucleons and pi-mesons

Quantum Chromodynamics (QCD)

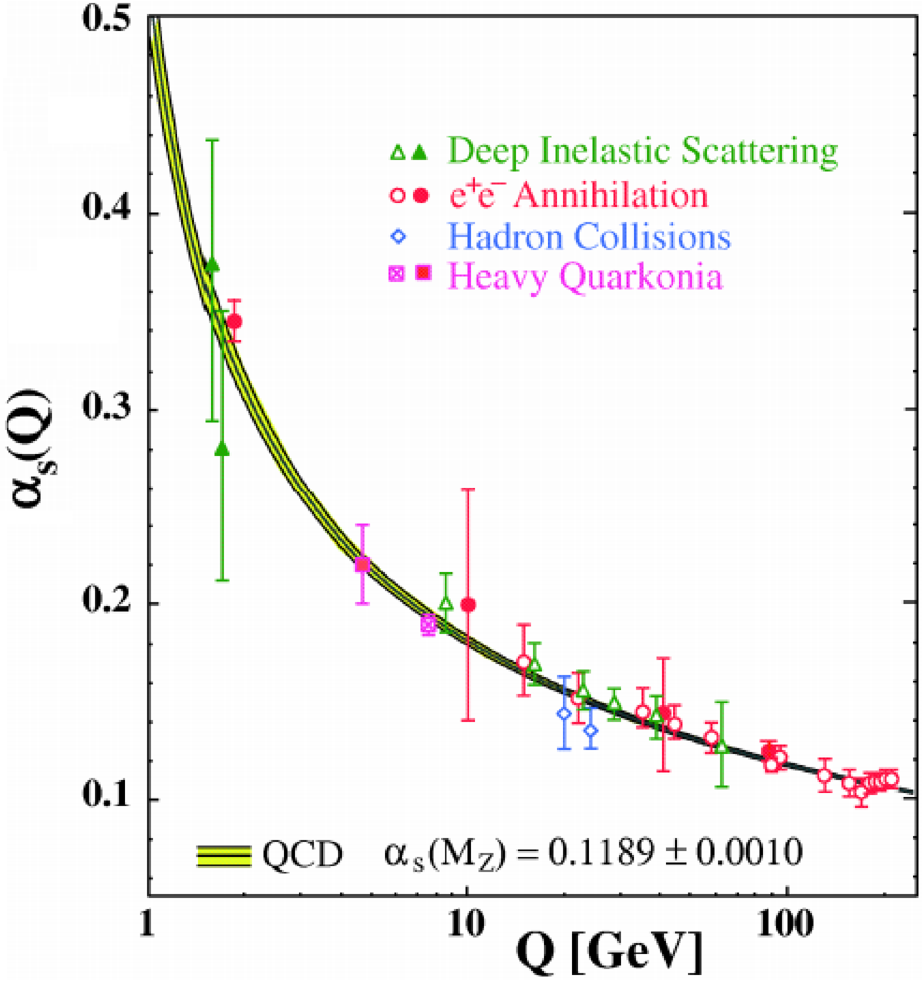
- QCD is the theory of strong interaction between quarks and gluons

  - Quarks have another type of charge called "*color charge*"
    - There are three different charges ("colors"): "*red*", "*blue*" and "*green*"
      - Note: *in QED there is only one charge, electric*
  - Gluons are massless particles
    - There are eight different gluons with color charge

- The Lagrange density of QCD schematically is written as

$$\mathcal{L}_{QCD} = \mathcal{L}_{Quark-Gluon} + \mathcal{L}_{Gluon-Gluon}$$

Quantum Chromodynamics (QCD)

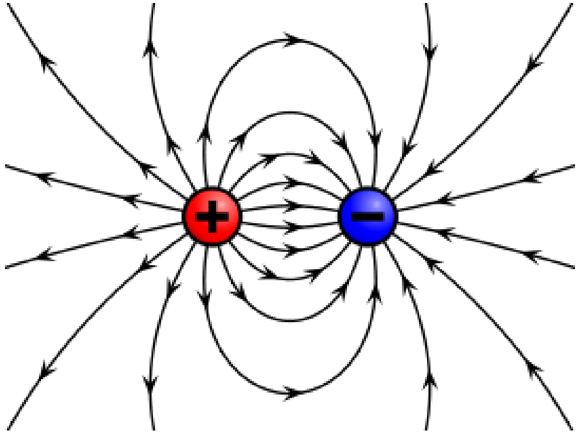


S. Bethke, Prog.Part.Nucl.Phys. 58 (2007) 351-386

- Running coupling constant:
  - ***In high energy***
    - *Asymptotic freedom*
      - Inside hadrons, quarks can be considered to be "free"
    - Small coupling constant
  - ***At low energies***
    - *Quark confinement*
      - It is impossible to detach individual quarks from hadron
    - Large coupling constant

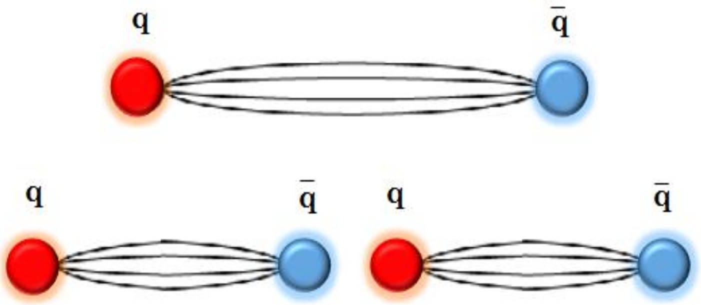
Interaction Between Two Charged Particles

- QED



$$V_{QED}(r) \sim -\frac{1}{r}$$

- QCD

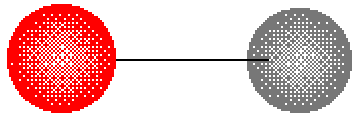


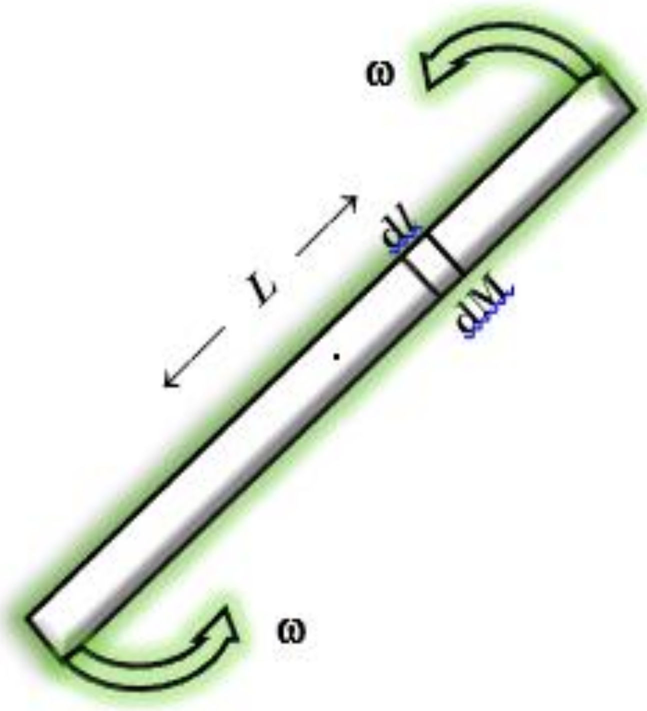
$$V_{QCD}(r) \sim Kr$$

- Full potential energy of quarks:

$$V(r) = V_0 + \frac{\alpha_s}{r} + Kr$$

String Model of Mesons



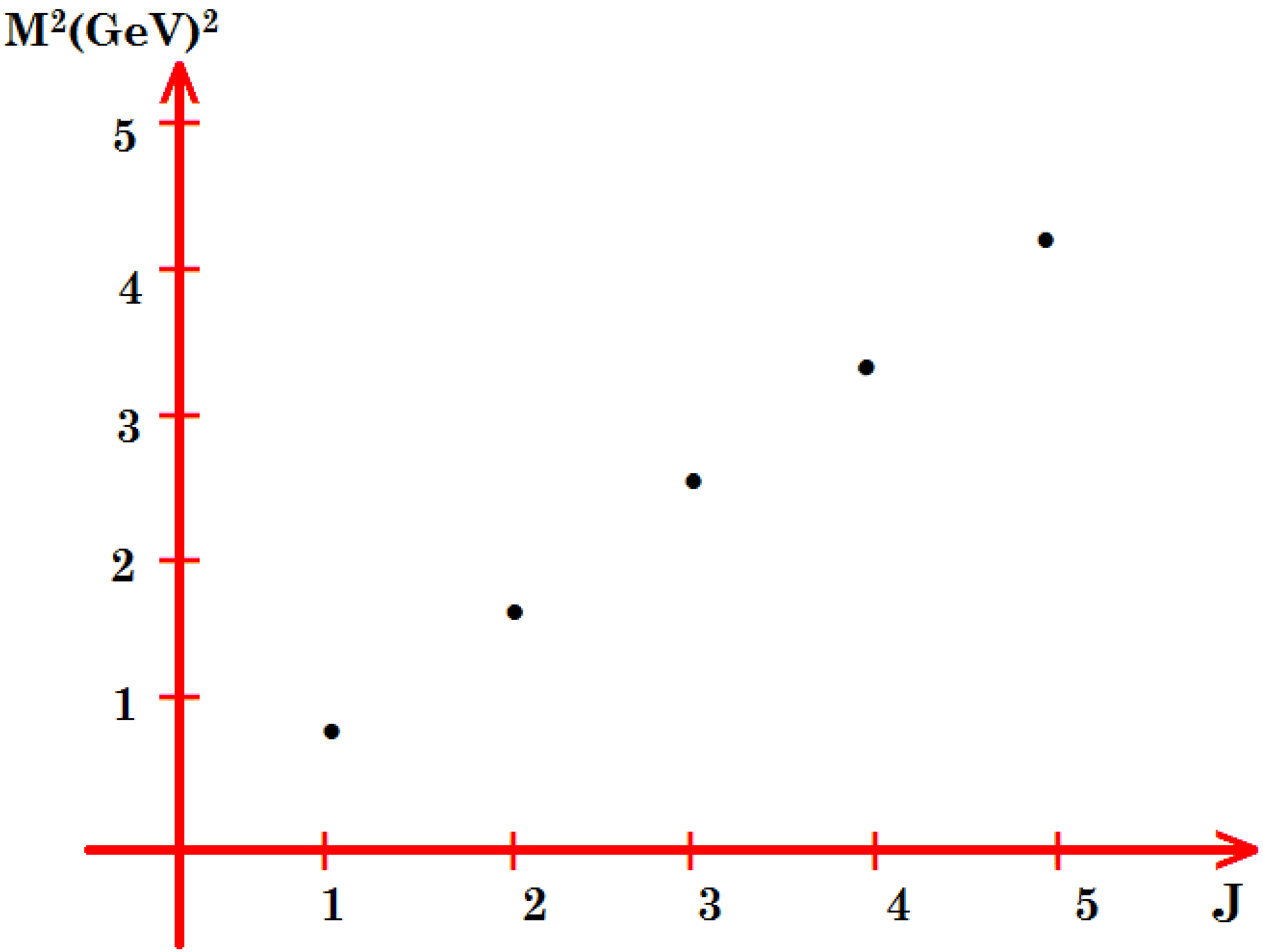


Schematic representation of the connection string, of length *L*, between a quark and an antiquark

- Hadrons behave like relativistic string with gluon material, as a bosonic string

$$\left.\begin{aligned} E &\propto K\cdot L \\ J &\propto K\cdot L^2 \end{aligned}\right\} \Rightarrow \frac{E^2}{J} \propto K$$

Regge Trajectory

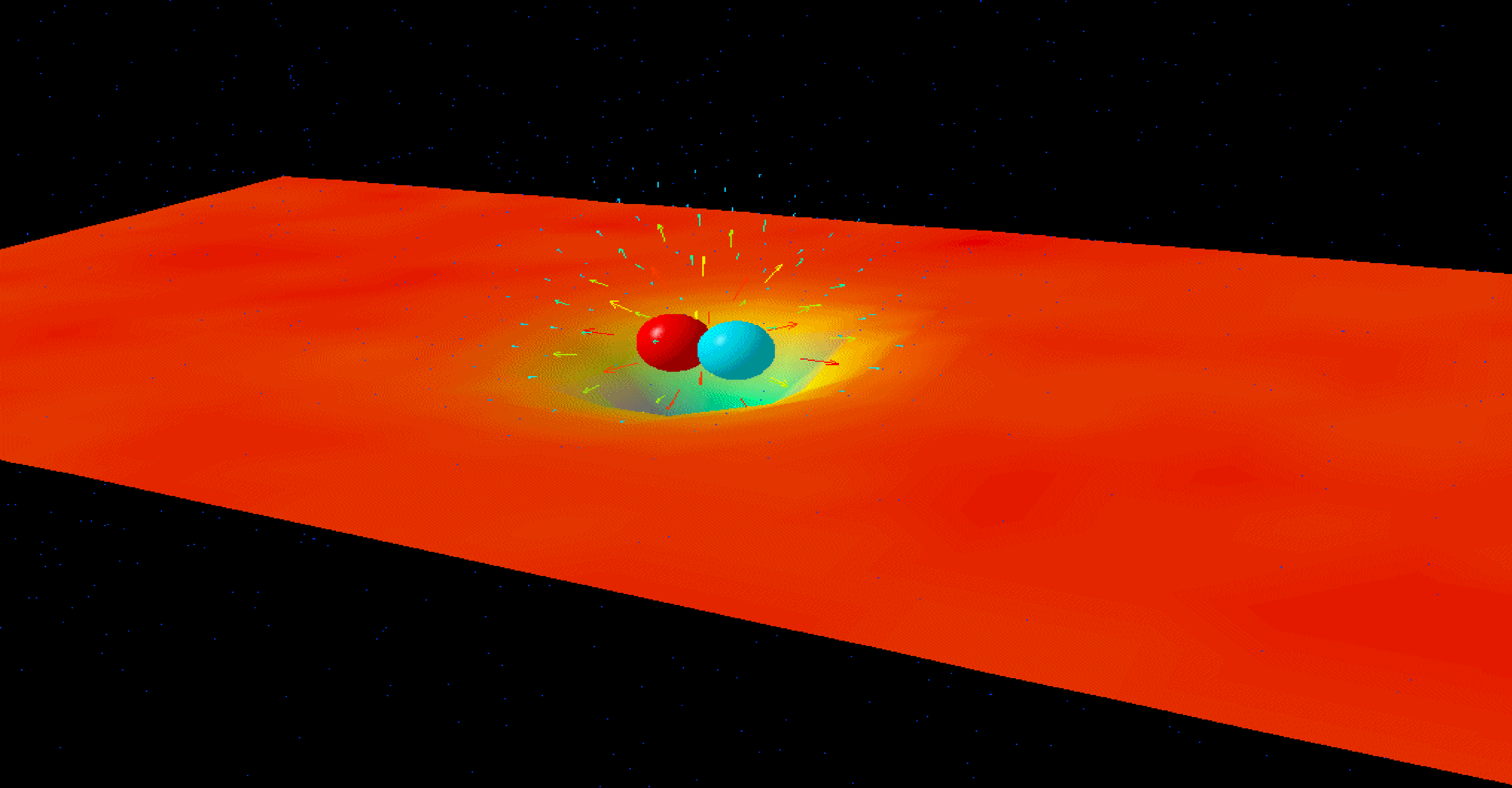


- The binding energy of hadron as a function of spin

- Linearization of Regge trajectories gives the K constant

$$K = \frac{1}{2\pi\alpha'} \approx 200 MeV^2$$

where $\alpha' \simeq 1 GeV^{-2}$



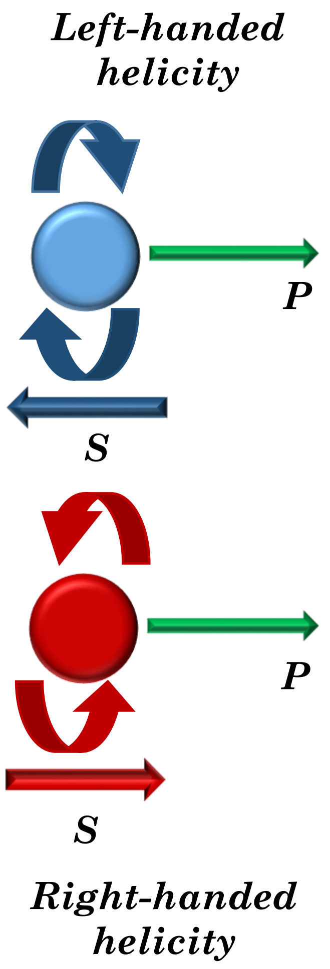
Quarks Flux Tube

Chiral Symmetry

- In strong interaction theory, Lagrangian has a chiral symmetry in the limit that the quark masses vanish, $m \to 0$.

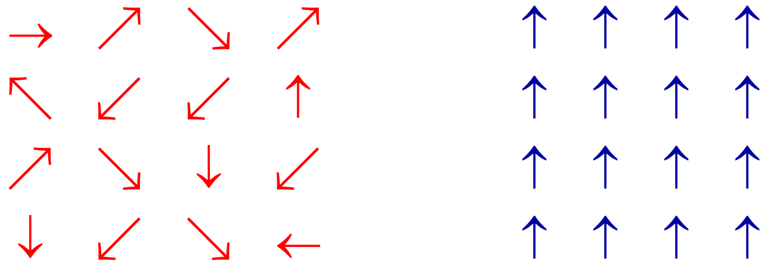
This symmetry is *broken* in two ways:

- *Explicitly* broken by the quark *masses*
- *Spontaneously* broken by *dynamical* chiral symmetry breaking of QCD



Spontaneous Rotational Symmetry Breaking

- Spontaneous symmetry breaking occurs in systems that under certain conditions are symmetric, but whose lowest energy state is not.
- Spontaneous breaking of symmetry can be illustrated through solid-state physics
- $|\vec{B}| = 0$



$|\vec{M}| = 0$ $\qquad$ $|\vec{M}| \neq 0$

- The same situation can be described for chiral symmetry by means of a *nonzero chiral condensate* in the ground state

Goldstone Theorem

- Spontaneous symmetry breaking is associated with existence of *massless particles* which appear in the energetic spectrum of system

- The number of particles is equal to the number of residual symmetry group generators

$$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \rightarrow \mathrm{SU}(2)$$

- Number of SU(2) group generators is 3

- Experimentally, these three particles are identified by:

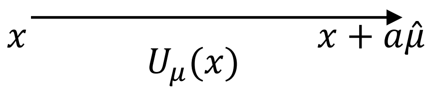
$$\pi^+ \qquad \pi^0 \qquad \pi^-$$

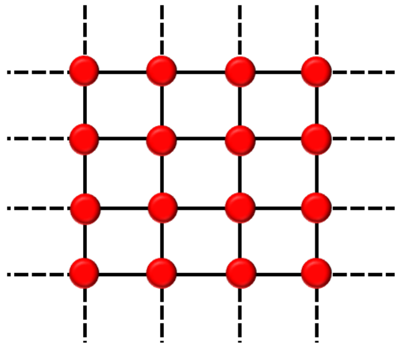
Large N QCD at Strong Coupling

- The theory is non–perturbatively solvable in the large N approximation

- The formulation of a gauge theory on a lattice offers a possibility for studying spontaneous symmetry breaking
  - This makes it necessary to introduce fermions in the lattice theory, Kogut-Susskind fermions

> **Discretize space-time**
>
> - ***Fermions on sites*** $\psi(x)$
> - ***Gauge fields on links*** $U_\mu(x)$





- We consider this theory in the strong coupling limit, $g \rightarrow \infty$

$$S_{QCD} = S_F + S_{Gauge}, \qquad S_{Gauge} = \frac{1}{2g^2} \sum_{plaquette} Tr(U_\mu U_\nu U_\mu^* U_\nu^*) \xrightarrow[g\to\infty]{} 0$$

The Dual Theory Model

- The action [Kogut, Susskind 1975]:

$$S_F = \sum_{x,f} m_f \bar{\psi}(x,f)\psi(x,f) + \frac{1}{2}\sum_{x,f,\mu} \eta_\mu(x)[\bar{\psi}(x,f)U_\mu(x)\psi(x+\hat{\mu},f) - \bar{\psi}(x+\hat{\mu},f)U_\mu^*(x)\psi(x,f)], \quad f = 1,2,\ldots,N_f$$

where

$\psi(x,f), \quad \bar{\psi}(x,f)$	*N-component Grassmann fermion field*
m_f	*flavor mass*
$x = (x_1, x_2, \ldots, x_d)$	*lattice sites*
$f = 1, 2, \ldots, N_f$	*flavor index*
$U_\mu(x)$	*SU(N) matrix at each directed link (x, x+$\hat{\mu}$)*

$$\eta_1(x) = 1, \quad \eta_\mu(x) = (-1)^{x_1 + \cdots x_{\mu-1}}, \mu = 2, \ldots, d$$

- Why dual?...

Integration of Gauge Fields

- The partition function of the theory:

$$Z = \int \prod_{x,\mu}^{x_d,d} dU_\mu(x) \prod_{x,f,\mu}^{x_d,N_f,d} d\overline{\psi}(x,f)_\mu d\psi(x,f)_\mu e^{S_F} = \int DUD\overline{\psi}D\psi e^{S_F}$$

where $d\overline{\psi}(x,f),\quad d\psi(x,f)$ *Grassmann integration measure*

$dU_\mu(x)$ *Haar measure of SU(N) group integration*

- The action of effective fermion theory:

$$S = \sum_{x,f} m_f \overline{\psi}(x,f)\psi(x,f) + \frac{1}{4N} \sum_{x,\mu,f,f'} \left[\overline{\psi}(x,f)\psi(x,f')\overline{\psi}(x+\mu,f')\psi(x+\mu,f)\right]$$

- The key point is *bosonization* of fermions…

Analytical Techniques for Fermions Bosonization

- Bosonization on Kawamoto-Smit paper:

$$S = Nm_f \sum_x tr\mathcal{M}(x) - Ntr\sum_x \ln \mathcal{M}(x) + \frac{N}{4}\sum_{x,\mu}\mathcal{M}(x)\mathcal{M}(x+\mu)$$

$$J^N = \int \prod_i d\overline{\psi}_i d\psi_i e^{\overline{\psi}_i J \psi_i}$$

$$\frac{dz}{2\pi iz} z^{-N} = \frac{dz}{2\pi iz} z^{-N\ln z}, \quad z = \overline{\psi}\psi$$

- Bosonization on Kluberg-Stern *et al*. paper:

$$\int \prod_c (d\eta_c d\overline{\eta}_c) f(\eta_a \overline{\eta}_b) = \det\left(\frac{\partial}{\partial \sigma_{ab}}\right) f(\sigma)\Big|_{\sigma=0}$$

$$S = Ntr\left[\sum_r im\sigma_r + \sum_{link} W(\overline{\Lambda})\right], \quad \sigma_r = \overline{\psi}_r \psi_r / N$$

where $\overline{\Lambda} = -\frac{1}{4N^2}\overline{\psi}_r \psi_r \overline{\psi}_{r+\mu}\psi_{r+\mu} = -\frac{1}{4}\sigma_r \sigma_{r+\mu}$

Bosonization of Fermions

- Our bosonization method relies on the leading order approximation:

$$F(\lambda) = 1-(1-\lambda)^{1/2} + \ln\frac{1+(1-\lambda)^{1/2}}{2} = \frac{1}{4}\lambda + O(\lambda^2), \qquad \lambda = \frac{1}{N^2}\overline{\psi}(x,f)\psi(x,f')\overline{\psi}(x+\mu,f')\psi(x+\mu,f)$$

- *n-dimensional* Gaussian integral over Grassmann variables:

$$\int d^n\overline{\psi} d^n\psi \exp\left(-\frac{1}{2}\overline{\psi}^T A\psi + J_1^T\psi + J_2^T\psi\right) = \det A\cdot\exp\left[\frac{1}{2}\left(J_2^T A^{-1} J_1^T\right)\right]$$

- The *effective action*, after integrating the Gaussian integral with respect to the flavor-symmetric bosonic field $\Sigma(x,f,f')$, $f,f'=1,2,\ldots,N_f$.

$$S_\Sigma = N\sum_x tr\ln\left[m_f + \Sigma(x,f)\right] - 2N\sum_{x,y} tr\Sigma(x,f,f')A^{-1}(x,y)\Sigma(y,f',f), \qquad A(x,y) = \sum_\mu\left(\delta_{x+\mu,y} + \delta_{x-\mu,y}\right)$$

Large N Solution

- In order to find the large N solution we first find the field that makes the action stationary.
  - ➢ The saddle point solution is *constant lattice field* $G_0(f,f')$ with *small fluctuation field* $g(x,f,f')$.
- The *effective action* in the large N approximation is:

$$S_g/N = -N_f V(\ln G_0 + \frac{d}{4}G_0^2) + \frac{1}{8}(\frac{4}{G_0^2} - 2d)\sum_x trg(x)^2 + \frac{1}{8}\sum_{x,\mu} tr\left[g(x+\mu) - g(x)\right]^2 + O\left[G_0^{-3}g(x)^3\right]$$

- The *free energy* of the theory is:

$$\mathcal{F} = NN_f V(\ln G_0 + \frac{d}{4}G_0^2)$$

- *Our interest is the fluctuation which can be written in form:*

$$S_g / N \propto -M^2 \sum_x tre^{ig(x)} - \sum_{x,\mu} tre^{ig(x+\mu)}e^{-ig(x)} + O\left[G_0^{-3}g(x)^3\right], \qquad M^2 = \frac{4}{G_0^2} - 2d$$

Spontaneous Symmetry Breaking

- In stationary state, the Goldstone bosons which are described as $g(x)$ matrices of $N_f \times N_f$ dimensions, are invariant under the global transformations $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$

$$e^{ig(x)} \rightarrow U e^{ig(x)} U^*$$

- For vanishing quark mass the theory has an exact global symmetry:

$$e^{ig(x+\mu)} \rightarrow U e^{ig(x+\mu)} V^*, \qquad e^{-ig(x)} \rightarrow V e^{ig(x)} U^*$$

  - The effective action is invariant with respect to global transformations

- For nonzero quark mass the symmetry is explicitly broken.

Fermion-Antifermion Condensate Pi-Meson Mass

- The chiral condensate of the theory in the large N solution is defined by expression:

$$\zeta = \lim_{m_f \to 0} \lim_{V \to \infty} \frac{1}{V} \sum_{x,f,\mu} \left\langle \overline{\psi}(x,f)\psi(x,f') \right\rangle = \lim_{m_f \to 0} \lim_{V \to \infty} \frac{1}{V} \frac{\partial \ln Z}{\partial m_f} = - \lim_{m_f \to 0} \lim_{V \to \infty} \frac{1}{V} \frac{\partial \ln \mathcal{F}}{\partial m_f} \simeq N N_f \sqrt{\frac{2}{d}}$$

  - *Our approximation differ by a $O(1)$ order from two mentioned papers*

- Chiral symmetry is spontaneously broken to:

$$\mathrm{U}(N_f)_L \otimes \mathrm{U}(N_f)_R \to \mathrm{U}(N_f)$$

  - According to Goldstone theorem → N_f^2 Goldstone bosons

- The mass of observed particles, quark-antiquark pair, on the spectrum is:

$$M^2 = 2 m_f \sqrt{2d}$$

  - For 2 flavor, SU(2) group → 4 massless bosons

Conclusions

The following conclusions have been taken with a new analytical method:

- The main result of this work is that the color confinement of the strong interactions may be solved non-perturbatively in the large N approximation.
- Besides the Kawamoto-Smit and Kluberg-Stern et al. papers there is *another analytical method* to solve the QCD problem at strong coupling.
- The analytical method which *is computed based on Boriçi 2018 technique*, is formulated over the fermion lattice model. Its purpose is a *new fermion's bosonization*.

Conclusions

- Calculation of the hadron spectrum is essential in the Theory of Strong Couplings but also of particle spectrum in any other Theory that has in focus Elementary Particles. The number of observed particles in the spectrum is 4.
- The action is in quadratic form with the exact symmetry for the zero quark mass. Because the chiral condensate of the theory in the large N approximation does not vanish, therefore *the chiral symmetry is spontaneously broken*. As a result, *the spectrum* of particles produced by this breaking of symmetry *that corresponds to the Goldstone bosons* can be studied.
- Another important result of this calculation is that in the strong interaction at low energy *the quadratic mass of the light meson is proportional to the mass of the quark*.

Thanks a lot to my Professor Artan Boriçi for his help and very helpful comments on this work.

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THANK YOU FOR LISTENING