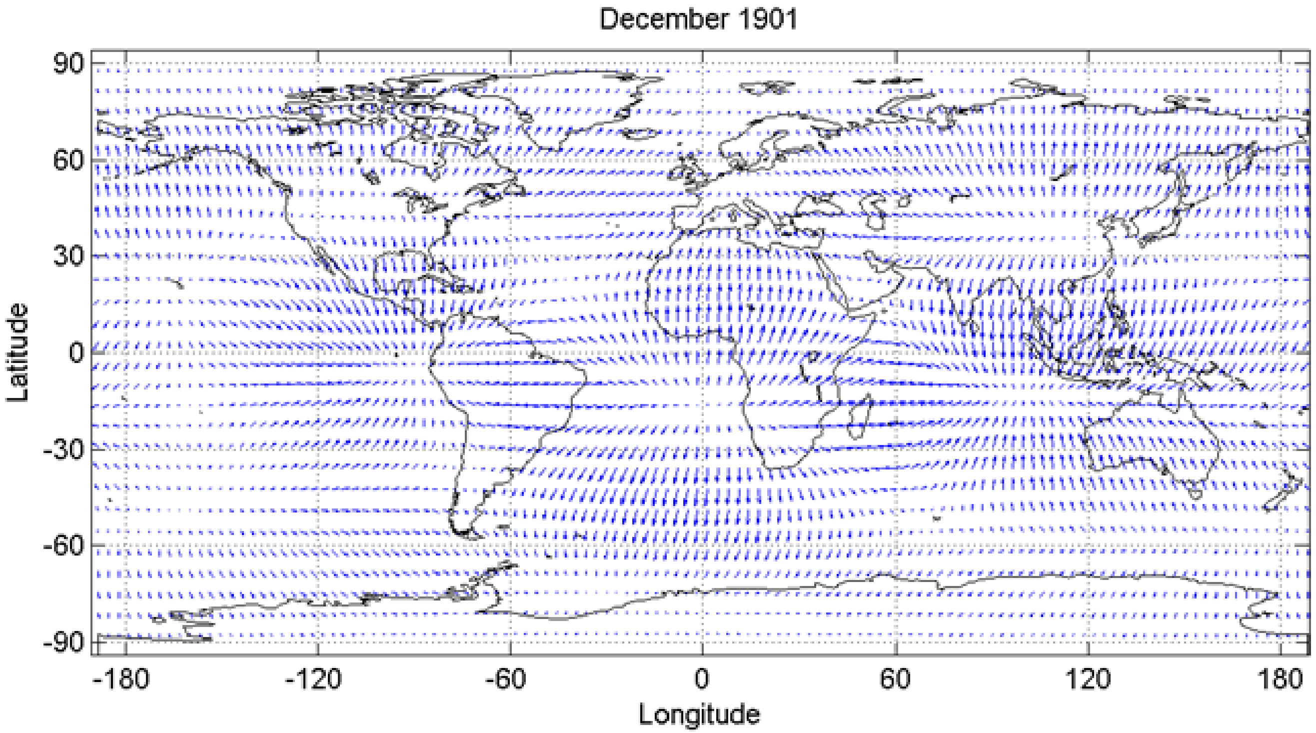


# Flow at the Core-Mantle Boundary and Jerks

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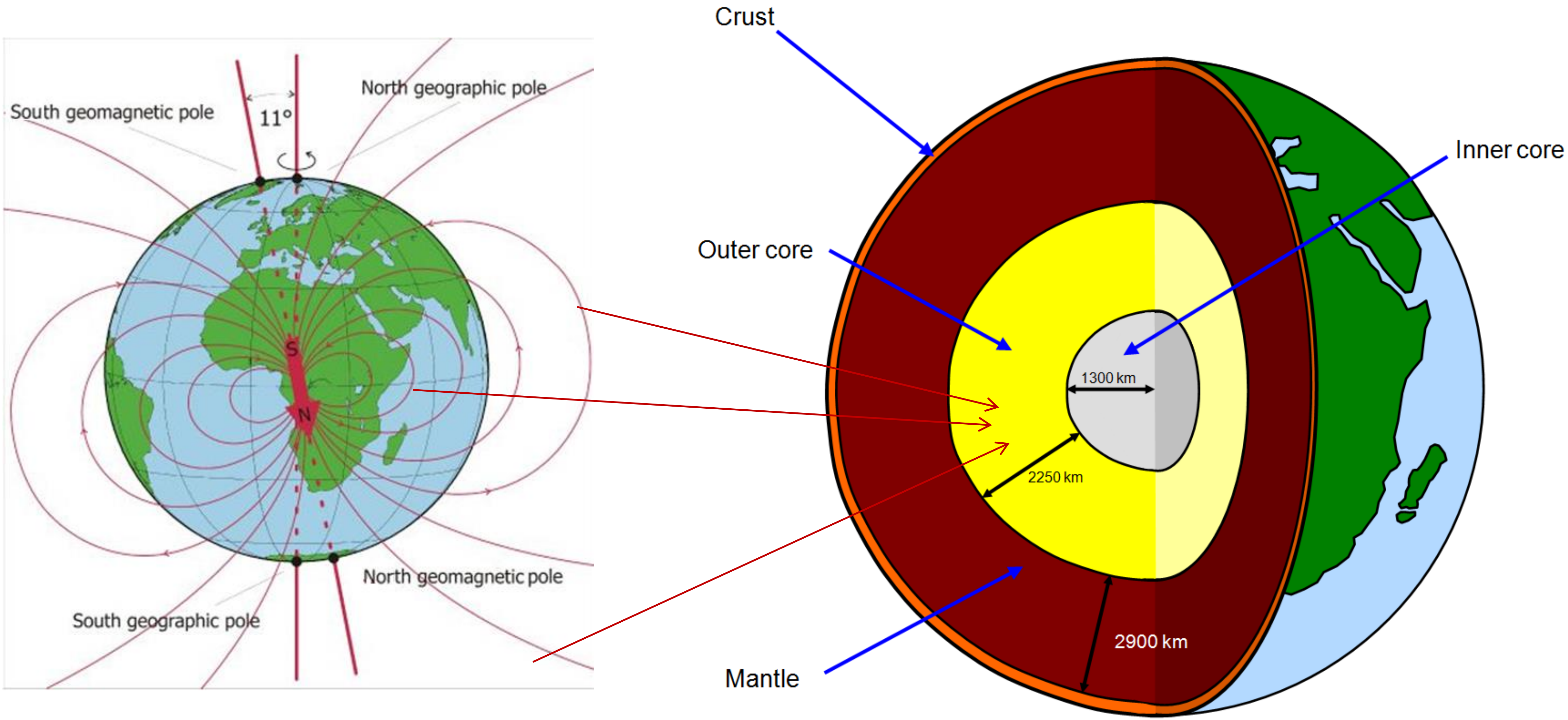
# Outline

- Introduction
- Why calculate the velocity at CMB?
- Magnetic field at CMB
- Velocity field at CMB
- Method
- Inversion of equations
- Results
- Conclusion

# Introduction



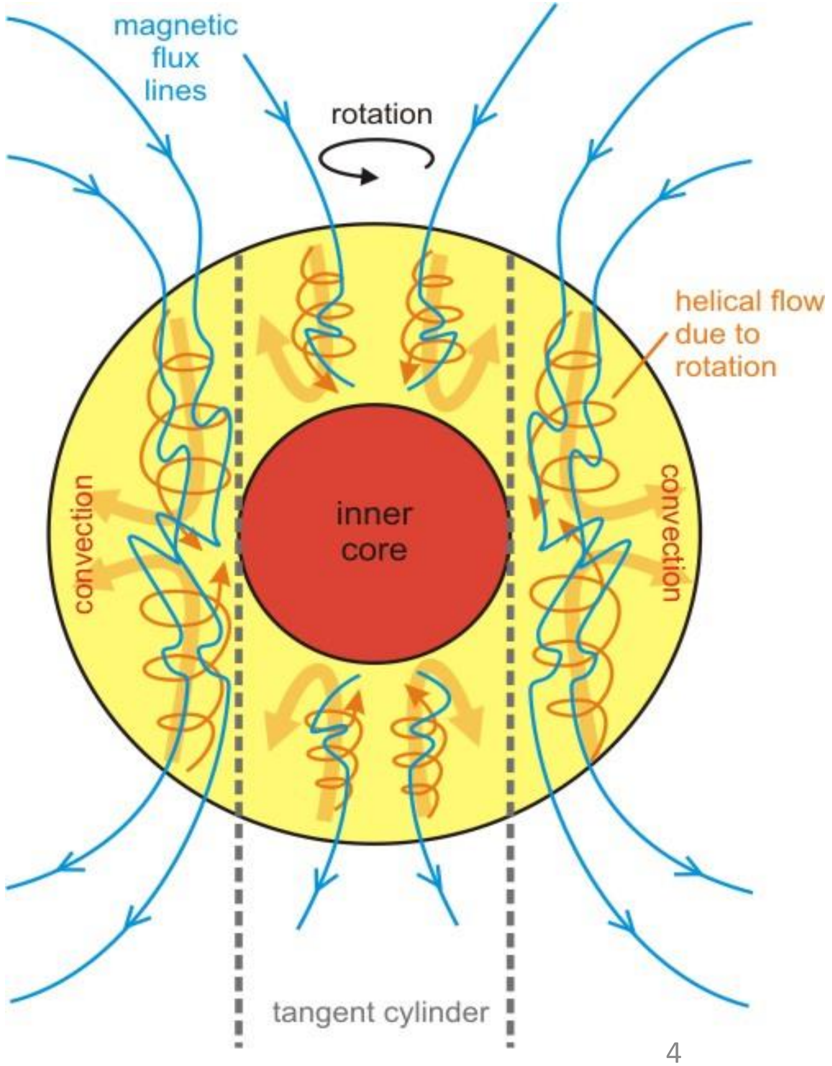
# Introduction (generation of magnetic field)

- The Earth's main magnetic field is generated and maintained against Ohmic loss by dynamo mechanism. This mechanism takes place in the outer core which contains liquid iron. The thermal and chemical convection of this electrically conducting fluid generated magnetic field described by the induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B}\right) + \eta \nabla^2 \vec{B}$$

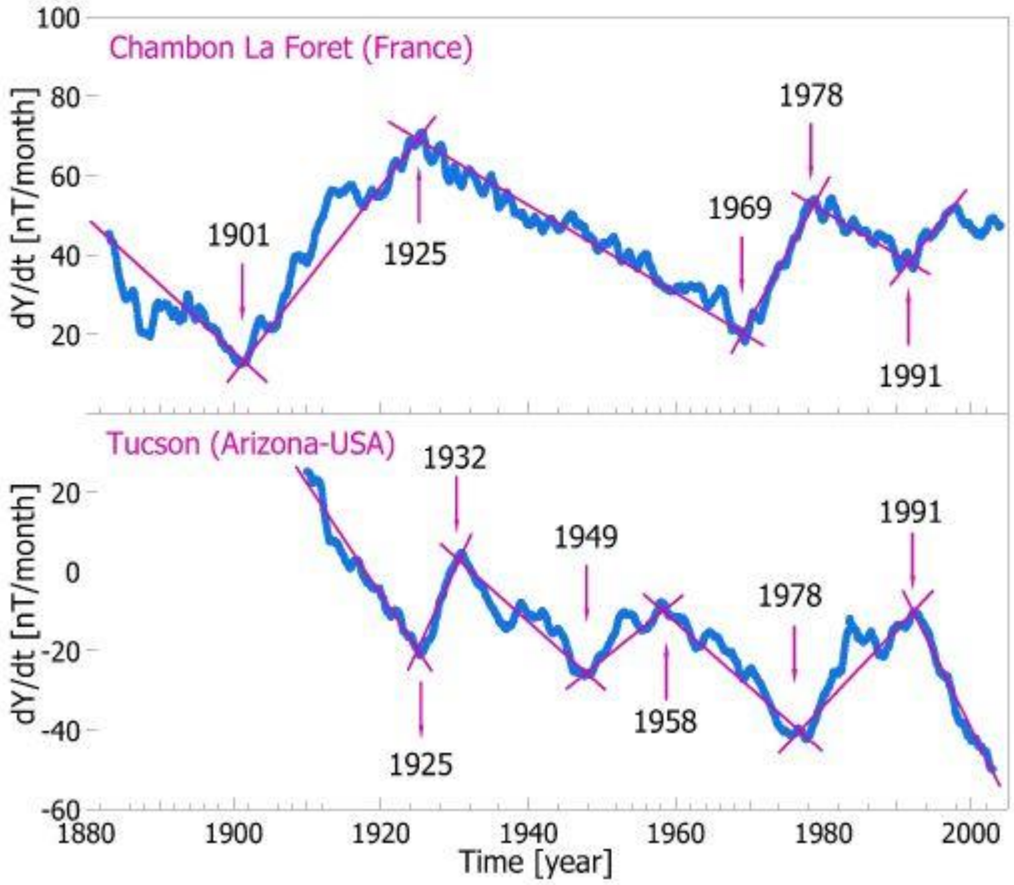
where the magnetic diffusivity $\eta = \frac{1}{\mu_0 \sigma}$ , $\sigma$ is electrical conductivity

- Other mechanisms that have their contribution are differential rotation, nutation of the Earth's axis and core-mantle tidal coupling
- The geomagnetic field exhibits temporal variation on different timescales: from fraction of a second to millions of years. The main field discussed here constitutes more than 90% of the total geomagnetic field and is mainly dipolar



# Introduction (time variation of magnetic field)

- The dipolar magnetic field generally varies in magnitude on a typical timescales from several years to centuries
- These gradual changes are collectively known as Secular Variation (SV)
- Over several years the SV exhibits a clear trend (purple solid lines)
- There are cases when the trend of SV changes abruptly in less than one year
- Such changes are known as jerks
- There is hypothesized the connection between jerks and corresponding abrupt changes in the velocity field at the CMB (Whaler et al., 2016)



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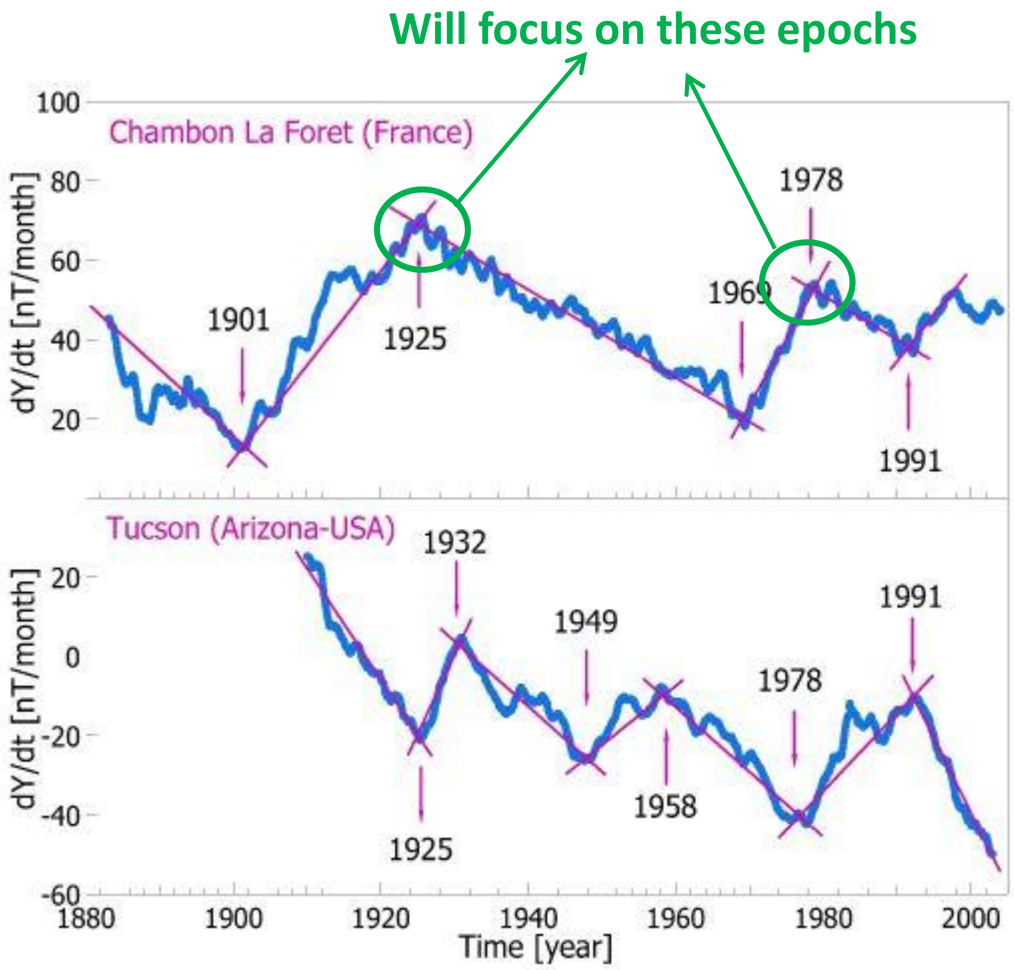

# Why calculate the velocity at CMB?

- To study the convection in the bulk of the outer core (Bloxham and Gubbins, 1991)
- To make short-range prediction of SV (Beggan and Whaler, 2010)
- To study the possible relation between the flow at CMB and the changes in Length of Day (LOD) (Hide et al., 1993)
- To study the mechanical coupling between the outer core and the mantle (Deleplace and Cardin, 2006)

# The induction equation (direct problem)

- The induction equation allows the calculation of $\vec{B}$ and its rate of change when the velocity field is known

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left( \vec{v} \times \vec{B} \right) + \eta \nabla^2 \vec{B}$$

- If $\vec{B}_i$ is the magnetic field in the *i*-th step then

$$\vec{B}_{i+1} = \vec{B}_i + \frac{\partial \vec{B}_i}{\partial t}$$

- The calculation is carried over in the next step

# The induction equation (inverse problem)

- The inverse calculation is possible: the induction equation allows the calculation of $\vec{v}$ when the magnetic field is known

- In the Earth's case, due to its internal structure, one cannot use all the information from the magnetic field

- Let discuss the characteristics of the magnetic field and velocity field at the CMB!

# Magnetic field at CMB

- The geomagnetic field outside the core is considered to be potential (the electric currents are negligible there)
- The geomagnetic field is expanded in spherical harmonics series

$$B_r = -\sum_{l=1}^{L_{\max}} \sum_{m=0}^{l} (l+1) \left( \frac{c}{r} \right)^{l+2} \left[ g_m^l \cos(m\varphi) + h_m^l \sin(m\varphi) \right] Y_m^l (\cos\theta)$$

$$B_\theta = -\sum_{l=1}^{L_{\max}} \sum_{m=0}^{l} \left( \frac{c}{r} \right)^{l+2} \left[ g_m^l \cos(m\varphi) + h_m^l \sin(m\varphi) \right] \frac{dY_m^l (\cos\theta)}{d\theta}$$

$$B_\varphi = -\sum_{l=1}^{L_{\max}} \sum_{m=0}^{l} m \left( \frac{c}{r} \right)^{l+2} \left[ -g_m^l \sin(m\varphi) + h_m^l \cos(m\varphi) \right] \frac{Y_m^l (\cos\theta)}{\sin\theta}$$

where $c$ is the radius of the core and spherical coordinates are employed throughout

- Only the radial component of the magnetic field is continuous through the CMB!

# Velocity field at CMB

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(hypothesis “frozen flux”)

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Really complicated dynamics!

# Method (first step)

- Radial induction equation

$$\dot{B}_r = -\nabla \cdot \left( B_r \vec{v} \right)$$

- Nabla expanded:

$$\nabla = \hat{\vec{r}} \left( \hat{\vec{r}} \cdot \nabla \right) + \nabla_H = \frac{\partial}{\partial r} \hat{\vec{r}} + \nabla_H$$

$$\nabla_H = \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\vec{\theta}} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \hat{\vec{\varphi}}$$

- The divergence yields: $\dot{B}_r + \nabla_H \cdot \left( B_r \vec{v} \right) = 0.$

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# Method (second step)

- The velocity is separated into a toroidal and poloidal constituent (Backus, 1986):

$$\vec{v}_T = \nabla\times\left(\vec{r}T\right) = \left(0, \frac{1}{\sin\theta}\frac{\partial T}{\partial\varphi}, -\frac{\partial T}{\partial\theta}\right),$$

$$\vec{v}_S = \nabla\times\left[\nabla\times\left(\vec{r}S\right)\right] = \nabla_H\left(rS\right) = \left(0, \frac{\partial S}{\partial\theta}, \frac{1}{\sin\theta}\frac{\partial S}{\partial\varphi}\right)$$

- Total velocity $\vec{v} = \left(0, \frac{1}{\sin\theta}\frac{\partial T}{\partial\varphi} + \frac{\partial S}{\partial\theta}, -\frac{\partial T}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial S}{\partial\varphi}\right)$
- After substitution

$$\dot{B}_r = -B_r\left(\frac{1}{r}\frac{\cos\theta}{\sin\theta}\frac{\partial S}{\partial\theta} + \frac{1}{r}\frac{\partial^2 S}{\partial\theta^2} + \frac{1}{r\sin^2\theta}\frac{\partial^2 S}{\partial\varphi^2}\right)$$

$$-\left(\frac{1}{r\sin\theta}\frac{\partial T}{\partial\varphi} + \frac{1}{r}\frac{\partial S}{\partial\theta}\right)\frac{\partial B_r}{\partial\theta} + \left(\frac{1}{r\sin\theta}\frac{\partial T}{\partial\theta} - \frac{1}{r\sin^2\theta}\frac{\partial S}{\partial\varphi}\right)\frac{\partial B_r}{\partial\varphi}$$

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$$-\left(\frac{1}{r\sin\theta}\frac{\partial T}{\partial\varphi} + \frac{1}{r}\frac{\partial S}{\partial\theta}\right)\frac{\partial B_r}{\partial\theta} + \left(\frac{1}{r\sin\theta}\frac{\partial T}{\partial\theta} - \frac{1}{r\sin^2\theta}\frac{\partial S}{\partial\varphi}\right)\frac{\partial B_r}{\partial\varphi}$$

# Method (third step)

- There is only one equation, hence there is inherent non-uniqueness
- The expansion of the velocity field in spherical harmonics series resolves the non-uniqueness
- In spherical harmonics with complex coefficients

$$B_r = \sum_{l_1,m_1} \left(\frac{a}{r}\right)^{l_1+2} \left(l_1+1\right) g_{l_1}^{m_1} Y_{l_1}^{m_1}\left(\theta,\varphi\right), \dot{B}_r = \frac{\partial B_r}{\partial t} = \sum_{l_1,m_1} \left(\frac{a}{r}\right)^{l_1+2} \left(l_1+1\right) \dot{g}_{l_1}^{m_1} Y_{l_1}^{m_1}\left(\theta,\varphi\right)$$

$$T = \sum_{l_2,m_2} t_{l_2}^{m_2} Y_{l_2}^{m_2}\left(\theta,\varphi\right), S = \sum_{l_3,m_3} s_{l_3}^{m_3} Y_{l_3}^{m_3}\left(\theta,\varphi\right)$$

- The coefficients *t* and *s* are unknown

# Method (third step)

- Substitution into the radial induction equation

$$\sum_{l_1,m_1}\left(\frac{a}{r}\right)^{l_1+2}\left(l_1+1\right)\dot{g}_{l_1}^{m_1}Y_{l_1}^{m_1}\left(\theta,\varphi\right)=\frac{1}{r}\sum_{l_2,m_2}\sum_{l_3,m_3}\left(\frac{a}{r}\right)^{l_2+2}\left(l_2+1\right)g_{l_2}^{m_2}\times$$

$$\times\left[\!\!\left[t_{l_3}^{m_3}\frac{1}{\sin\theta}\left[\frac{\partial Y_{l_2}^{m_2}}{\partial\varphi}\frac{\partial Y_{l_3}^{m_3}}{\partial\theta}-\frac{\partial Y_{l_2}^{m_2}}{\partial\theta}\frac{\partial Y_{l_3}^{m_3}}{\partial\varphi}\right]-\right.\right.$$

$$\left.\left.-s_{l_3}^{m_3}\left[\frac{\partial Y_{l_2}^{m_2}}{\partial\theta}\frac{\partial Y_{l_3}^{m_3}}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial Y_{l_2}^{m_2}}{\partial\varphi}\frac{\partial Y_{l_3}^{m_3}}{\partial\varphi}-l_3\left(l_3+1\right)Y_{l_2}^{m_2}Y_{l_3}^{m_3}\right]\right]\!\!\right].$$

- Integration over the CMB

$$\dot{g}_{l_1}^{m_1}=\sum_{l_3,m_3}\left[\frac{1}{r}\left(\frac{r}{a}\right)^{l_1+2}\frac{1}{\left(l_1+1\right)}\sum_{l_2,m_2}\left(\frac{a}{r}\right)^{l_2+2}\left(l_2+1\right)g_{l_2}^{m_2}\times\oint_\Omega\frac{1}{\sin\theta}Y_{l_1}^{m_1*}\left(\frac{\partial Y_{l_2}^{m_2}}{\partial\varphi}\frac{\partial Y_{l_3}^{m_3}}{\partial\theta}-\frac{\partial Y_{l_2}^{m_2}}{\partial\theta}\frac{\partial Y_{l_3}^{m_3}}{\partial\varphi}\right)d\Omega\right]t_{l_3}^{m_3}+$$

$$+\sum_{l_3,m_3}\left\{-\frac{1}{r}\left(\frac{r}{a}\right)^{l_1+2}\frac{1}{\left(l_1+1\right)}\sum_{l_2,m_2}\left(\frac{a}{r}\right)^{l_2+2}\left(l_2+1\right)g_{l_2}^{m_2}\times\oint_\Omega\left[Y_{l_1}^{m_1*}\frac{\partial Y_{l_2}^{m_2}}{\partial\theta}\frac{\partial Y_{l_3}^{m_3}}{\partial\theta}+\frac{1}{\sin^2\theta}Y_{l_1}^{m_1*}\frac{\partial Y_{l_2}^{m_2}}{\partial\varphi}\frac{\partial Y_{l_3}^{m_3}}{\partial\varphi}-l_3\left(l_3+1\right)Y_{l_1}^{m_1*}Y_{l_2}^{m_2}Y_{l_3}^{m_3}\right]d\Omega\right\}s_{l_3}^{m_3}$$

# Method (third step)

- Define the Elsasser and Gaunt matrices

$$E_{l_1 l_3}^{m_1 m_3} = \frac{1}{r}\left(\frac{r}{a}\right)^{l_1+2} \frac{1}{(l_1+1)} \sum_{l_2, m_2} \left(\frac{a}{r}\right)^{l_2+2} (l_2+1) g_{l_2}^{m_2} \times \oint_{\Omega} \left( \frac{\partial Y_{l_3}^{m_3}}{\partial \theta} \frac{\partial Y_{l_2}^{m_2}}{\partial \varphi} - \frac{\partial Y_{l_2}^{m_2}}{\partial \theta} \frac{\partial Y_{l_3}^{m_3}}{\partial \varphi} \right) Y_{l_1}^{m_1} d\Omega$$

$$G_{l_1 l_3}^{m_1 m_3} = \frac{2}{r}\left(\frac{r}{a}\right)^{l_1+2} \frac{1}{(l_1+1)} \sum_{l_2, m_2} \left(\frac{a}{r}\right)^{l_2+2} (l_2+1) g_{l_2}^{m_2} \times \left[ l_1 (l_1+1) + l_3 (l_3+1) - l_2 (l_2+1) \right] \oint_{\Omega} Y_{l_1}^{m_1} Y_{l_2}^{m_2} Y_{l_3}^{m_3} d\Omega$$

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Elsasser integral

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Gaunt integral

$$G_{l_1 l_3}^{m_1 m_3} = \frac{2}{r}\left(\frac{r}{a}\right)^{l_1+2} \frac{1}{(l_1+1)} \sum_{l_2, m_2} \left(\frac{a}{r}\right)^{l_2+2} (l_2+1)\, g_{l_2}^{m_2} \times \left[ l_1(l_1+1) + l_3(l_3+1) - l_2(l_2+1) \right] \oint_{\Omega} Y_{l_1}^{m_1} Y_{l_2}^{m_2} Y_{l_3}^{m_3} d\Omega$$

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- In matrix form: $\dot{\mathbf{g}} = \mathbf{E}\mathbf{t} + \mathbf{G}\mathbf{s}$

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- In matrix form: $\dot{\mathbf{g}} = \mathbf{E}\mathbf{t} + \mathbf{G}\mathbf{s}$

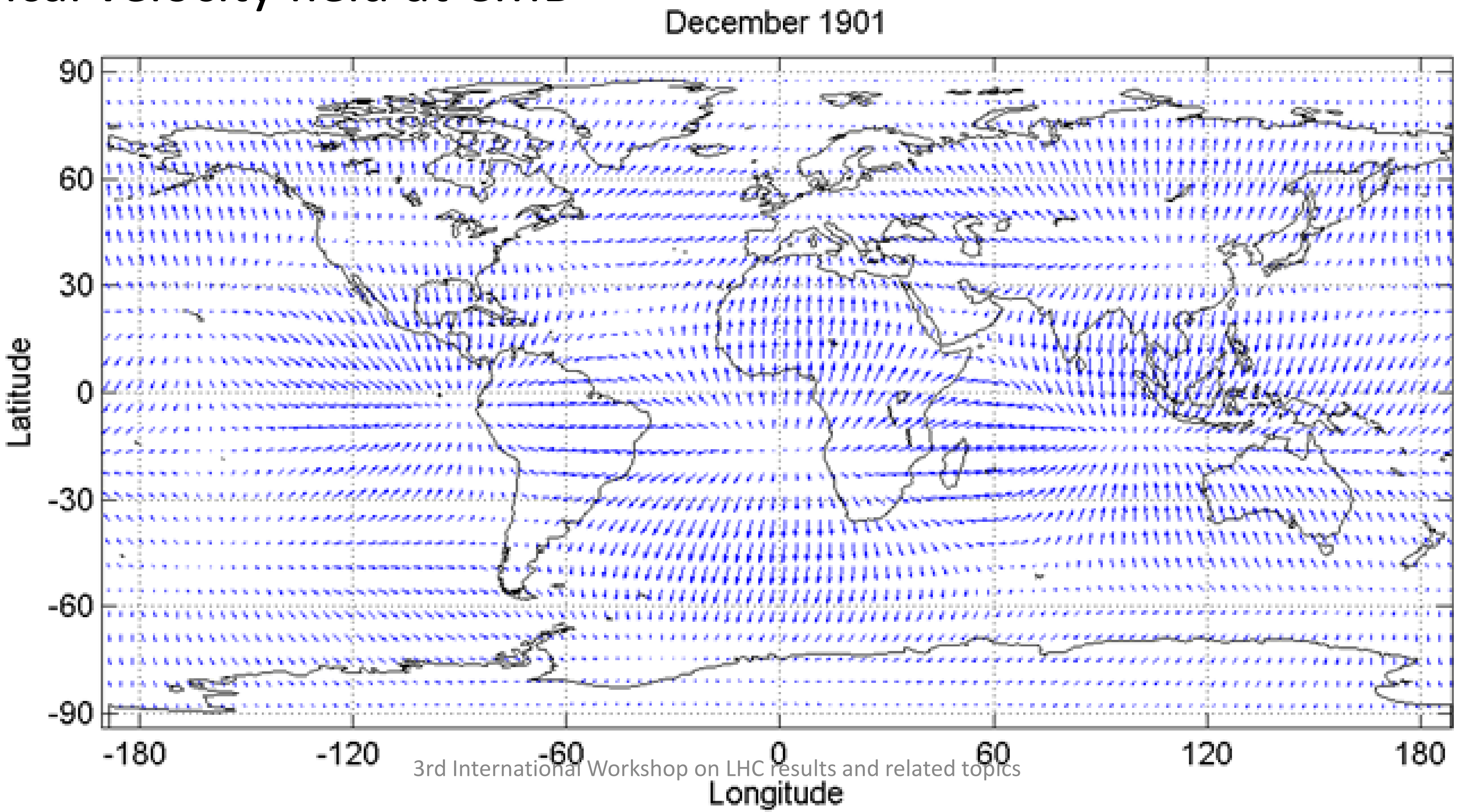
- Final version: $\dot{\mathbf{g}} = (\mathbf{E} : \mathbf{G}) \begin{pmatrix} \mathbf{t} \\ \hline \mathbf{s} \end{pmatrix}$

# Inversion of equations

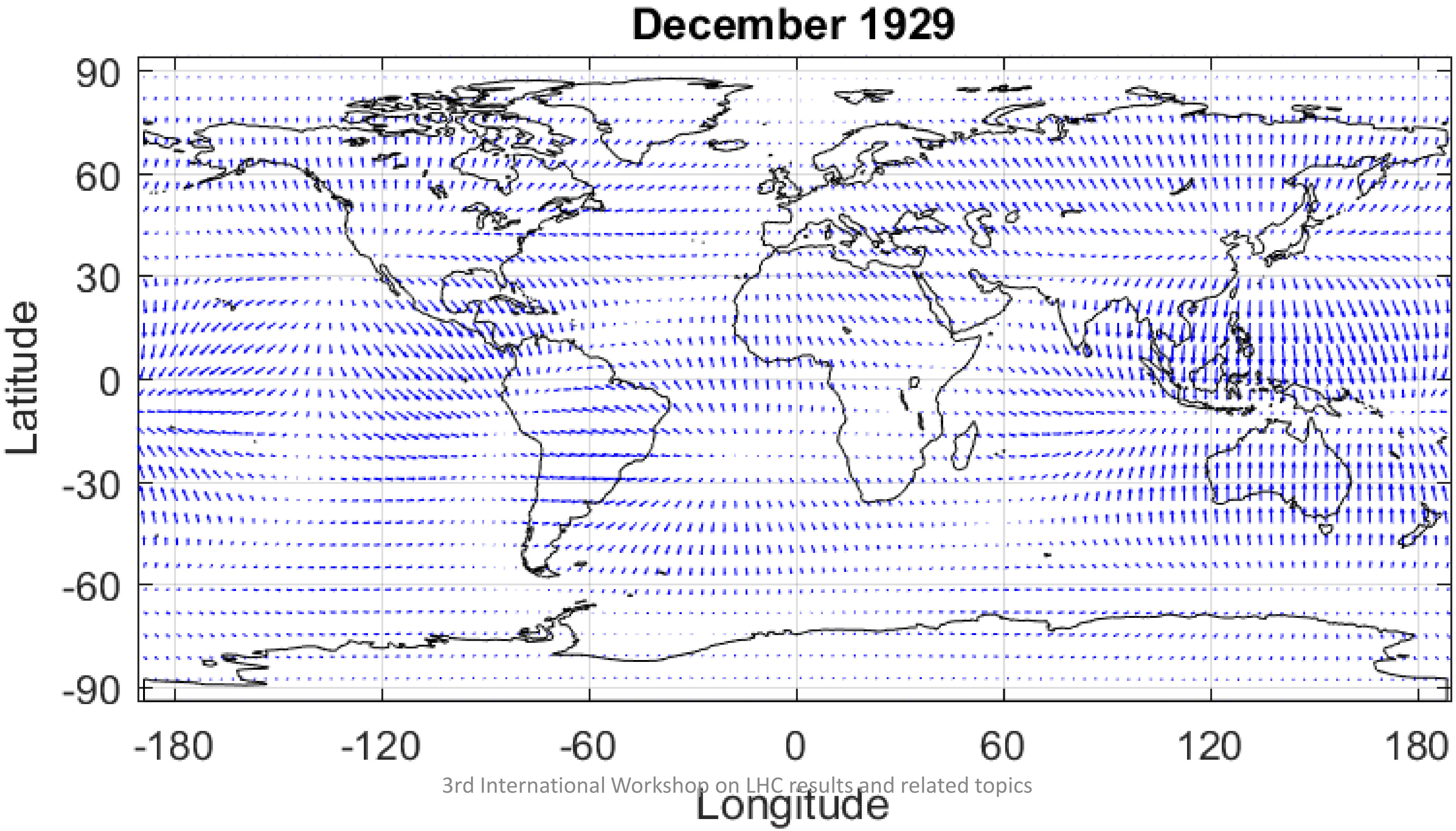
- The Gauss coefficients are recovered from **gufm1** model (Jackson et al., 2000). The model describes the magnetic field for the period 1590-1990
- The Gauss coefficients for SV are calculated as the differences of consecutive monthly values
- The magnetic field is expanded up to maximal degree $l_1 = 6$, while the toroidal and poloidal velocity fields up to $l_2 = l_3 = 4$
- The total number of equations is $l_1 (l_1 + 2) = 48$. the total number of unknowns is again 48 because $l_2 (l_2 + 2) = 24$ and $l_3 (l_3 + 2) = 24$
- The system is determined and the recovered velocity field is <u>unique</u>

# Results

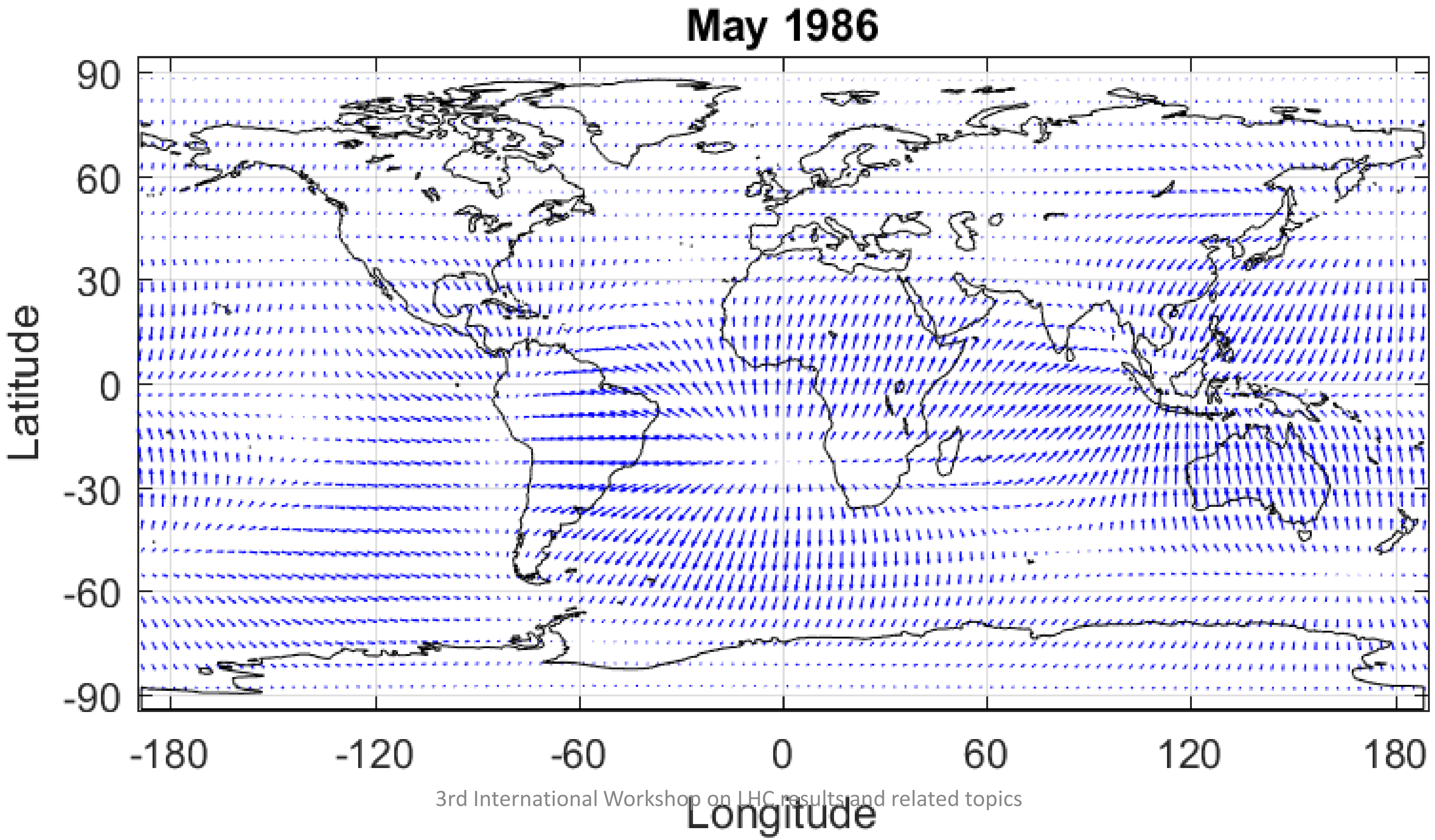
## A typical velocity field at CMB



# Results

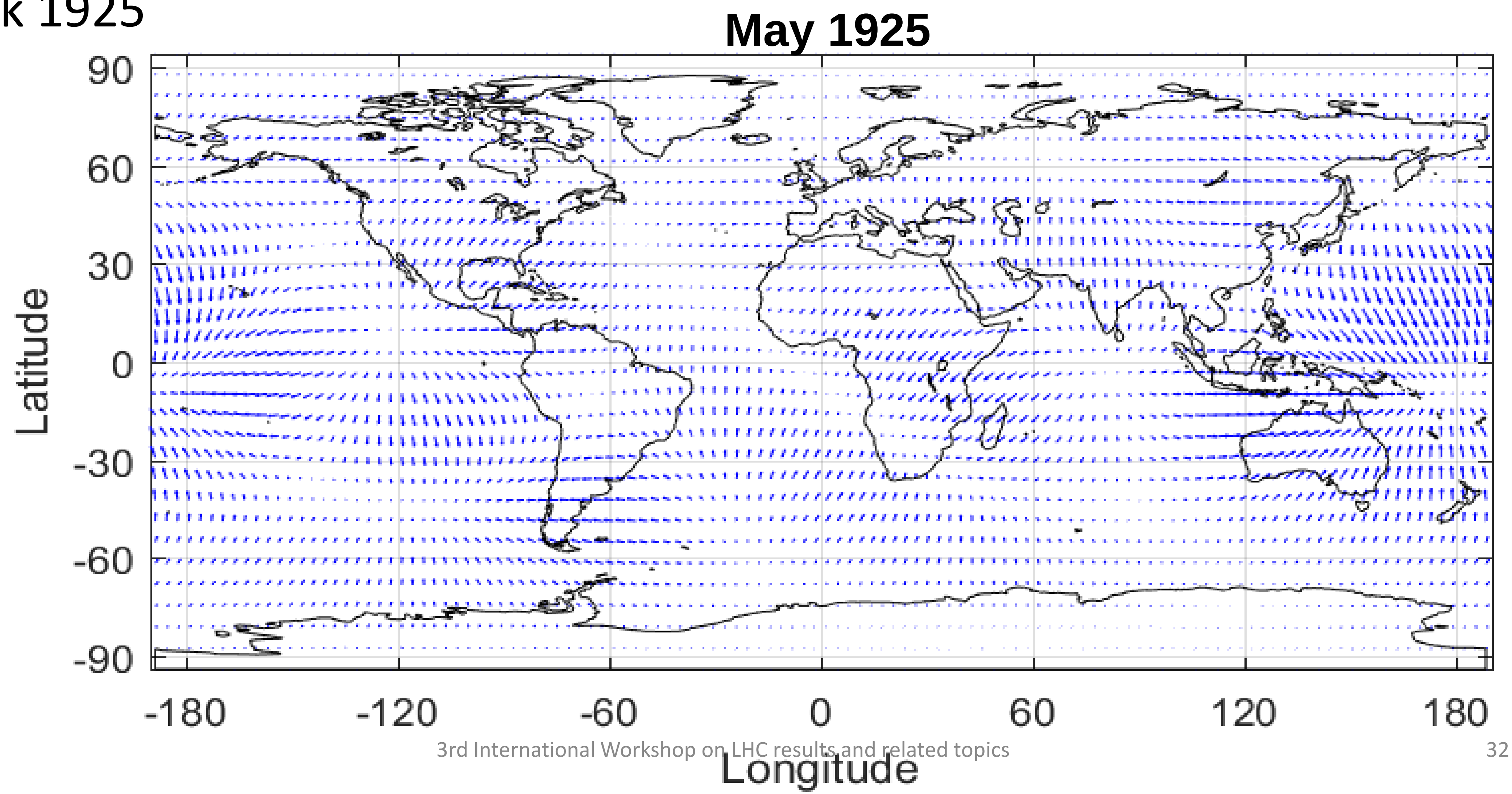

December 1929
90
60
30
0
-30
-60
-90
Latitude
-180
-120
-60
0
60
120
180
Longitude

# Results


May 1986
90
60
30
0
-30
-60
-90
Latitude
-180
-120
-60
0
60
120
180
Longitude

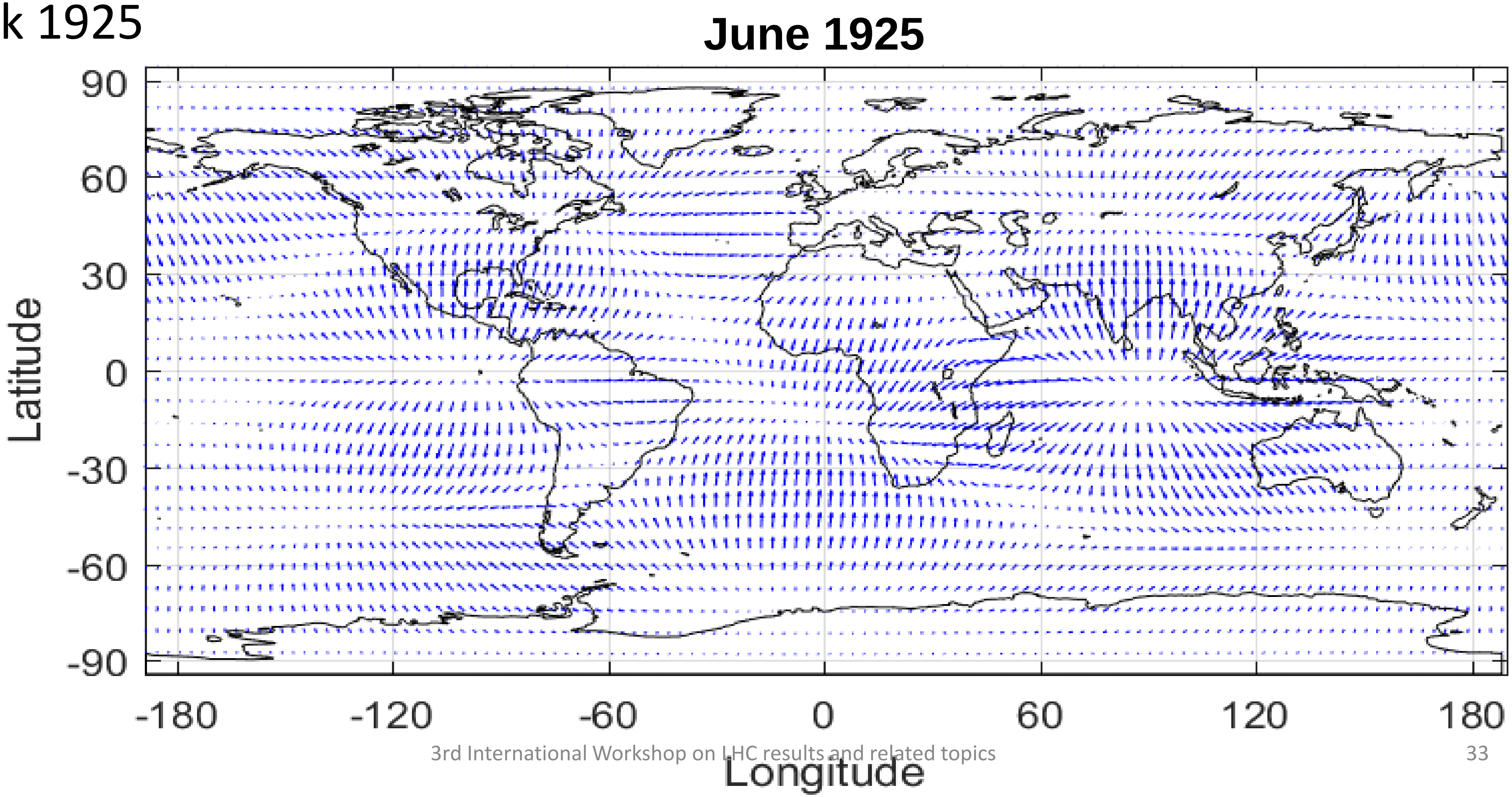
# Results

Jerk 1925



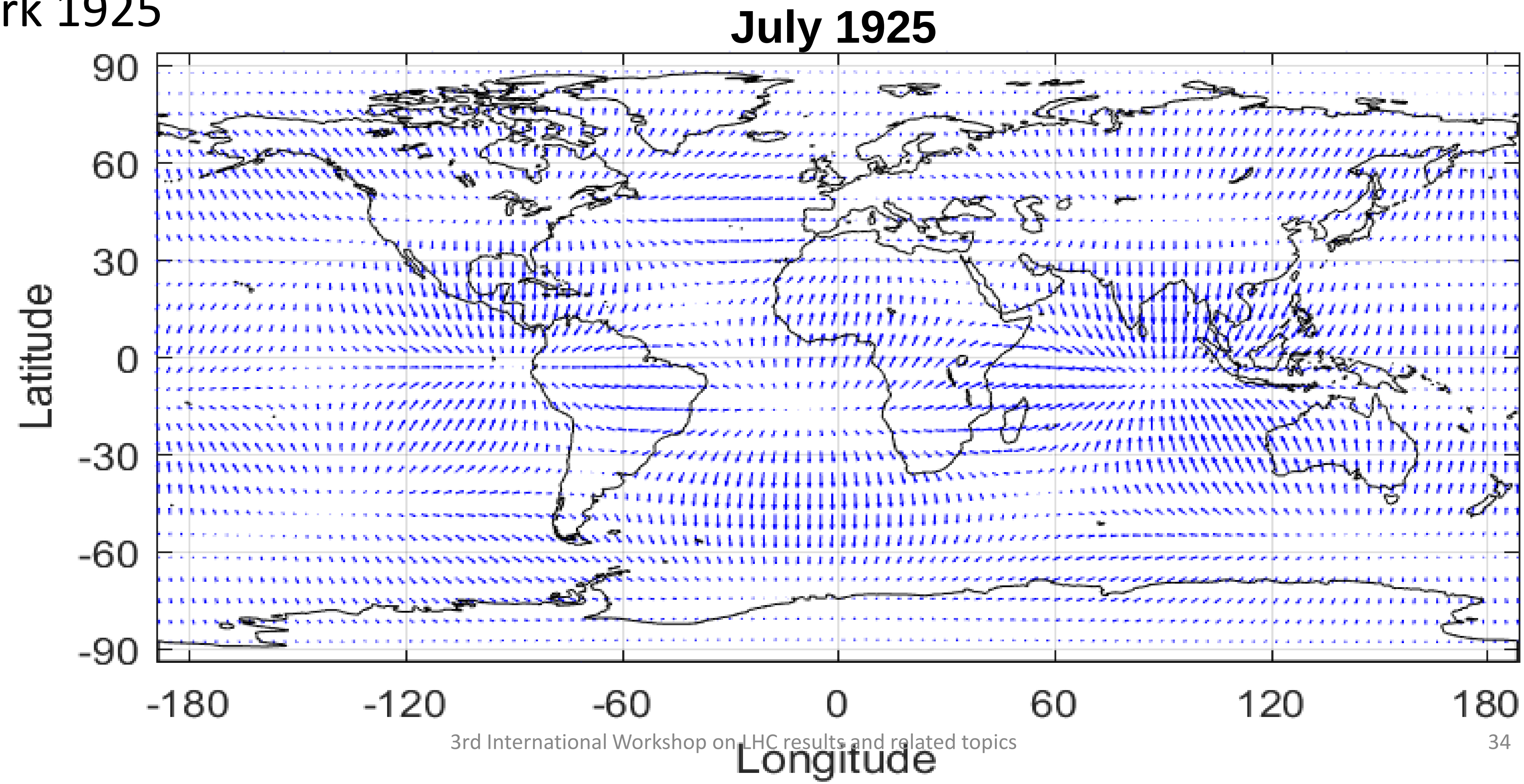
# Results

Jerk 1925



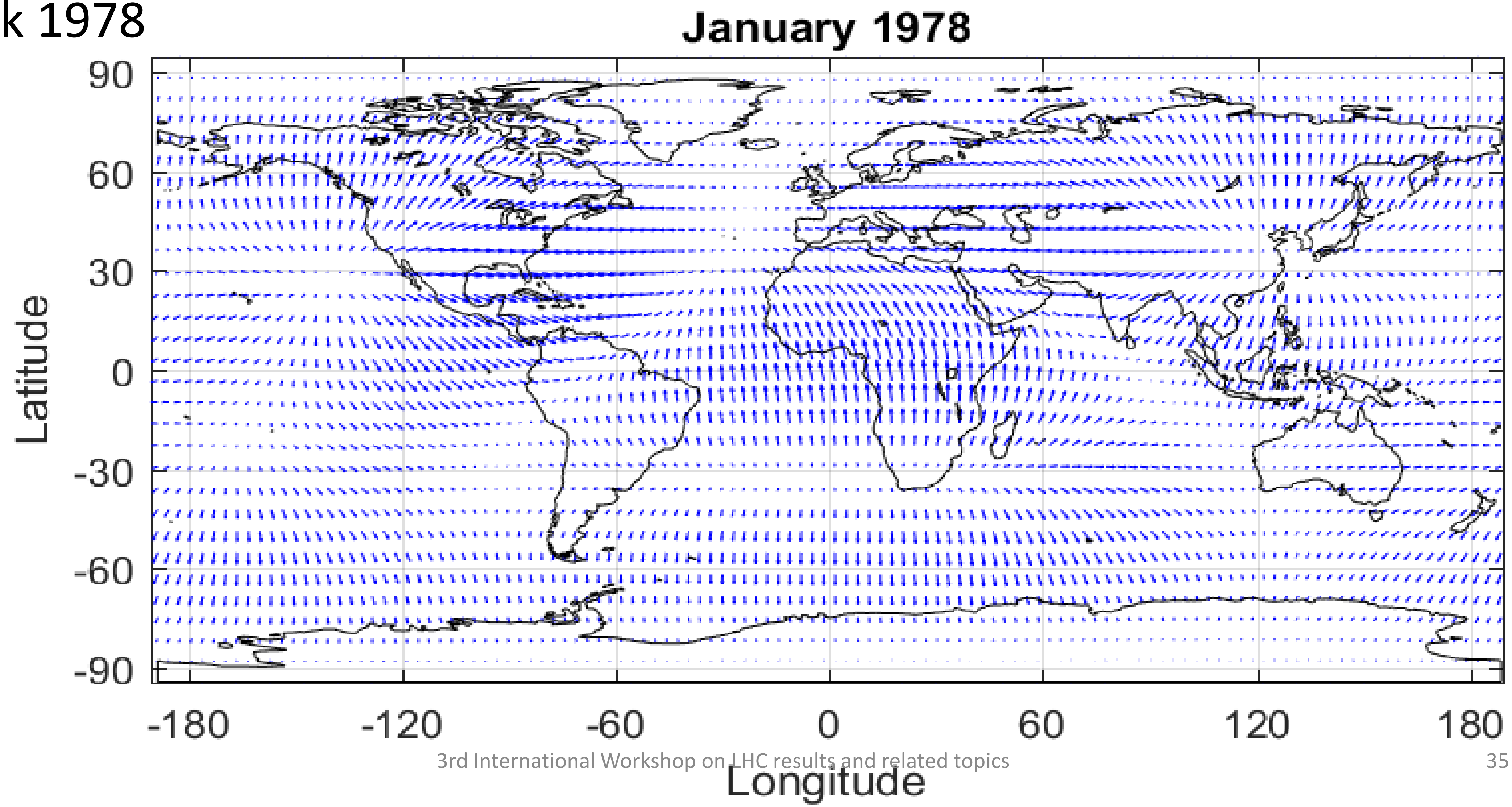
# Results

Jerk 1925



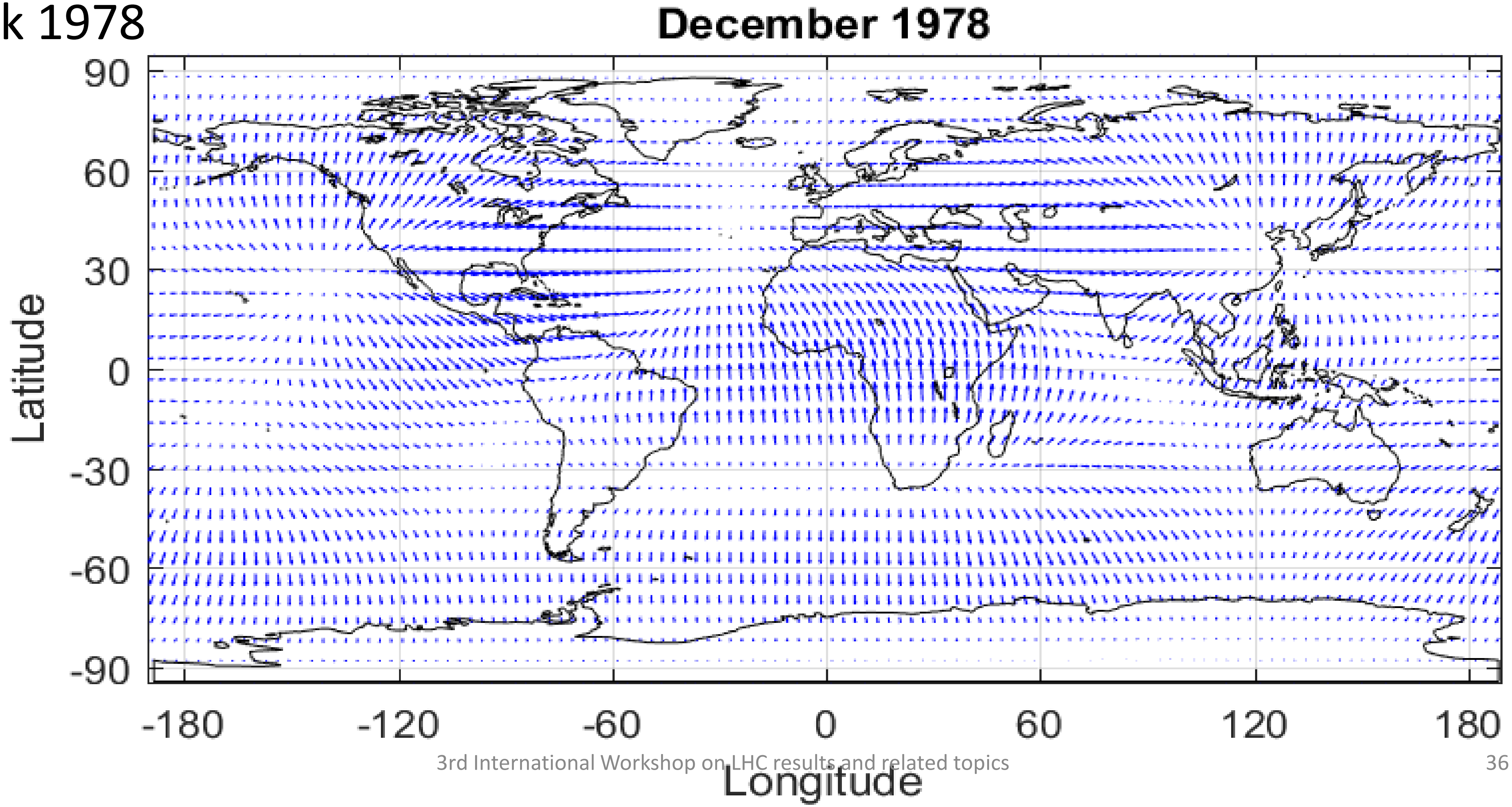
# Results

Jerk 1978



# Results

## Jerk 1978



# Conclusions

- Slow changes in the velocity field are observed reproducing the typical timescale of several years for the SV
- There are visible some structures especially under the South Atlantic and Indonesian Archipelago. Interestingly, these structures are related to well-known patterns of the geomagnetic field like the South Atlantic Anomaly (SAA)
- In some epochs when jerks have occurred, like 1925, 1969 and 1986 there are observed sharp changes in the velocity field at the CMB. In other epochs like 1933, 1958 or 1978 there are not observed large changes in the velocity field
- This fact indicates: 1) jerks may not be related to the velocity field at the CMB as previously believed, but sharp changes in the magnetic field may be the result of other processes that occur in the outer core; 2) jerks may be related to small scale flows at the CMB
- In order to analyze point 2 we have to increase the degree of spherical harmonic expansion $l$

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