Soft Collinear Effective Field Theory

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Applications to Beyond Standard Model Physics

Outline

- Effective Field Theory (EFT): Why do we need it?
    - Example: Fermi Theory
- Soft Collinear Effective Theory (SCET)
    - Basic ingredients
- How is SCET different from "conventional" EFT?
- Applications of SCET
    - Describe new heavy resonances

Why EFT?

There is a vast range of energy scales in particle interactions, where in different regimes there are *interesting* phenomena to be studied.
★ Ideal scenario: We know The One theory that *unifies* the description of these phenomena: The Theory of Everything!
★ Reality: We construct "*Effective Field Theories*" that appropriately describe the important degrees of freedom at certain energy scales.

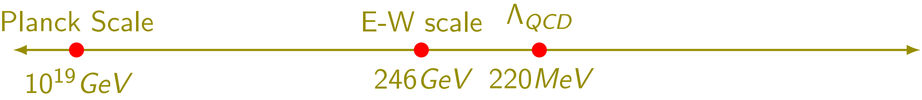
The Standard Model itself is an EFT of a more fundamental theory.

Two types of EFT's

- "Bottom-Up" : The underlying UV theory it not known
  ↪ The Standard Model
  ↪ Higher dimension gauge theories
  ↪ Einstein theory of Gravity
- "Top-down": The UV theory is known
  ↪ Four Fermi Theory
  ↪ Heavy Quark EFT
  ↪ Soft Collinear EFT

What is an EFT?

Effective Field Theory is a Quantum Field Theory that describes the Physics below some cut off scale Λ.



$*$ Expansion in the parameter $\lambda = \frac{E}{\Lambda}$. Write the needed operators at a certain order in λ.

$*$ There is a separation of long and short distance interactions:

$$\int_{p_0}^{M} d^d p f(p) = \underbrace{\int_{p_0}^{\mu} d^d p f(p)}_{\text{EFT operators}} + \underbrace{\int_{\mu}^{M} d^d p f(p)}_{\text{Wilson coeff.}}$$

The dependence on μ is controlled by RG equations.

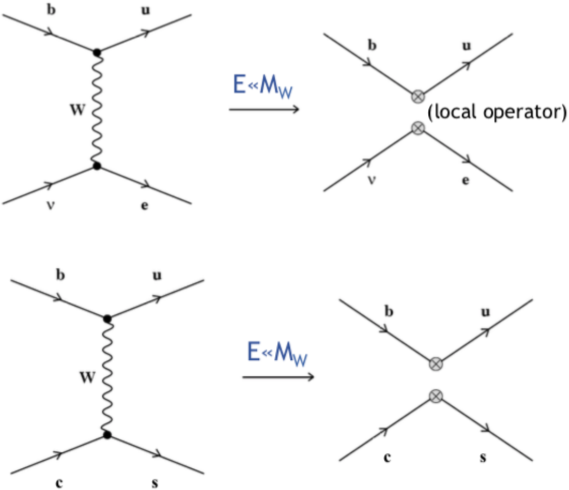
Why using an EFT

- Simplified calculations: Expanding in terms of the scale λ significantly simplifies the computations
- Factorization: Separate the Physics of different scales, separate the perturbative from non-perturbative Physics.
  Example: Multi-parton interactions
- Perturbation theory not disturbed. A consistent way of summing the large logarithms in multi-scale problems: $\alpha^n log^n(\lambda)$.
- Symmetries: Chiral symmetry, Heavy-Quark symmetry

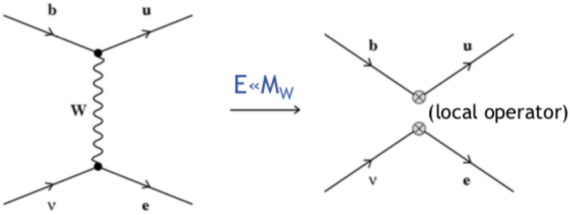
Prominent applications in QCD.

Example: Fermi Theory

Fermi Theory describes the exchange of W bosons at low energies ($E \ll M_W$) via a local operator.



Example: Fermi Theory



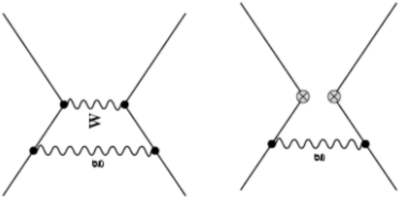
$$\frac{1}{p^2 - M_w^2} \xrightarrow{\lambda^2 = \frac{p^2}{M_w^2}} -\frac{1}{M_w^2}\left(1 + \frac{p^2}{M_w^2} + \cdots\right)$$

Interaction strength: $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_w^2} \Rightarrow$ weak interactions.

Fermi Theory

$$\frac{1}{p^2 - M_w^2} \xrightarrow{\lambda^2 = \frac{p^2}{M_w^2}} -\frac{1}{M_w^2}\left(1 + \frac{p^2}{M_w^2} + \cdots\right)$$

* This expansion does not work when loop momenta need to be integrated.

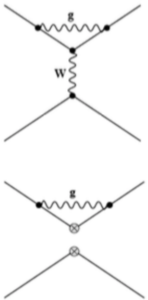


$$\int d^d p \frac{1}{p^2 - M_w^2} \mathrm{f}(p) \neq -\frac{1}{M_w^2} \int d^d p \left(1 + \frac{p^2}{M_w^2} + \cdots\right) \mathrm{f}(p)$$

Example: Fermi Theory

Write down the effective operator that describes this four-fermion interaction:

$$\mathcal{L}_{eff} \ni -\frac{4G_F}{\sqrt{2}} V_{ub} \underbrace{C_1(\mu)}_{=1} \bar{e}_L \gamma_\mu \nu_L \bar{u}_L \gamma^\mu b_L$$



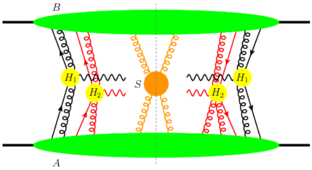
Soft Collinear EFT

- SCET is an EFT used to describe physical problems with large scale hierarchies.
- It is the appropriate frame work to describe the decay of a heavy particle into very energetic light particles.

  Bauer, Fleming, Pirjol, Stewart 2001; Bauer, Pirjol, Stewart 2002; Beneke, Chapovsky, Diehl, Feldmann 2002
- Consistent way of dealing with the large logs for a large scale separation. For instance: $M_{heavy} \gg \mathrm{v}$

Applications

- Multi-parton interactions and factorization theorem
  cross section $=$ hard $\otimes$ collinear $\otimes$ anti-collinear $\rightarrow$ it breaks down for a certain momentum scaling of radiated gluons $\rightarrow$ treat the problem in SCET



- Decay of a heavy particle into light *energetic* Standard Model particles $\rightarrow$ our example

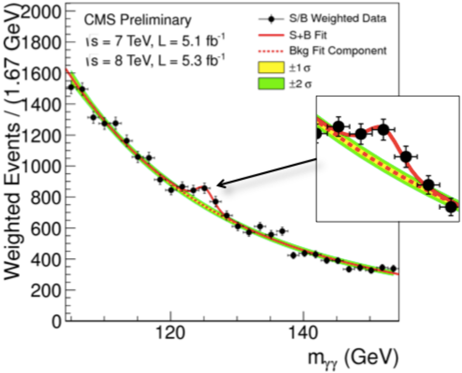
Scenario

Assume that a new **heavy** particle is discovered at LHC with mass M_{heavy} and assume other heavy particles could exist at order M energy scale with $M_{heavy} \sim M$.

Which is the *appropriate* frame work to describe it?

$\hookrightarrow$Effective Field Theory ☹

$\hookrightarrow$Soft Collinear EFT ☺



Scenario

Assume that a new **heavy** particle is discovered at LHC with mass M_{heavy} and assume other heavy particles could exist at order M energy scale with $M_{heavy} \sim M$. Which is the *appropriate* frame work to describe it?
↪Effective Field Theory ☹
In an EFT we integrate out the heavy degrees of freedom and not a huge separation of scales is assumed.
↪Soft Collinear EFT ☺
In our case the resonance is still a heavy degree of freedom and moreover there are also energetic particles in the final states.

- The relevant scales of our problem are v and M_{heavy}: $\lambda = \frac{v}{M_{heavy}} \ll 1$
- The large hierarchy between scales brings up problems with summing the large logs.
- An infinite tower of operators needed in EFT: $\frac{M_{heavy}}{M} \sim \mathcal{O}(1)$

Ingredients for SCET

- The final states define $\vec{n}_i$ directions of energy flow: jets. One jet may contain more than one particle.
- Work in light-cone coordinates:

$$p^\mu = \bar{n} \cdot p \frac{n^\mu}{2} + n \cdot p \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

- Different components of momenta scale differently with respect to λ.

$$\lambda = \frac{\mathrm{v}}{M_{heavy}}$$

  - The most important momenta scaling:

$$p_c \sim (1, \lambda^2, \lambda), p_{\bar{c}} \sim (\lambda^2, 1, \lambda), p_s \sim (\lambda^2, \lambda^2, \lambda^2), p_h \sim (1, 1, 1)$$

  - For each particle introduce a field per different momentum modes.

$$\Phi = \Phi_h + \Phi_c + \Phi_s$$

Particles in the same jet can interact with each other, while from different jets **only** via soft interactions.

Ingredients for SCET

The gauge invariant building blocks:

$$\Phi_{n_i}(x) = W^{\dagger}_{n_i}(x)\phi(x)$$

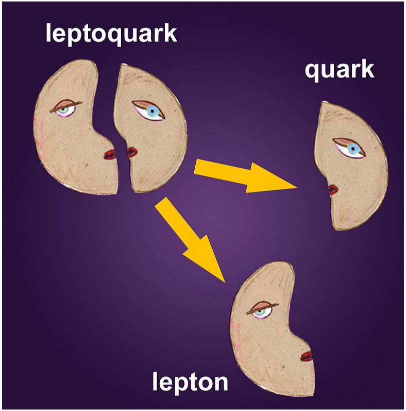
$$\chi_{n_i}(x) = \frac{\not{n}_i\bar{\not{n}}_i}{4} W^{\dagger}_{n_i}(x)\psi(x)$$

$$\mathcal{A}_{n_i}(x) = W_i^{(A)\dagger}(x)[iD^{\mu}_{n_i} W^{A}_{n_i}(x)]$$

The Wilson lines are a consequence of the non-locality that enters in SCET.
Power counting for the fields:

$$\chi(x) \sim \lambda, \;\; H(x) \sim \lambda, \;\; \Phi(x) \sim \lambda^0$$

Application to leptoquarks



Several BSM
Physics models predict that quarks and leptons merge together at very high energies to form leptoquarks.

- hypothetical heavy particles ($m > 740 GeV$) *arXiv:1806.03472*
- quark-lepton symmetry
- flavor anomalies *arXiv:1706.07808*
- Predicted in: $SU(5)$ GUT, Pati-Salam model, Technicolor, Composite models etc.

For application to
a heavy scalar see *arXiv:1806.01278*

The effective Lagrangian

Assume the heavy resonance Φ to be a leptoquark of spin-0 that under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ transforms as $(3, 1, -\frac{1}{3})$.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Φ	3	1	-$\frac{1}{3}$
H	1	2	$\frac{1}{2}$
$\begin{pmatrix} \nu_\ell \\ \ell_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$\frac{1}{6}$
ℓ_R	1	1	-1
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$

The gauge invariant operators

Assume the heavy resonance Φ to be a leptoquark of spin-0 that under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ transforms as $(3, 1, -\frac{1}{3})$.
The gauge invariant building blocks:

$$\chi(x) \sim \lambda, \;\; H(x) \sim \lambda, \;\; \Phi(x) \sim \lambda^0$$

The only gauge invariant operators built out of SM fields and the field Φ at leading order in λ:

$$\mathcal{O}_{RL} = \Phi \bar{\chi}_{d,R} \chi_{\ell,L} H$$

$$\mathcal{O}_{RR} = \bar{\chi}^c_{u,R} \chi_{\ell,R} \Phi^*$$

$$\mathcal{O}_{LL} = \bar{\chi}^c_{Q,L} i\sigma_2 \chi_{\ell,L} \Phi^*$$

$$H(x) \xrightarrow{\text{SSB}} \frac{1}{\sqrt{2}} \begin{pmatrix} o \\ v + h(x) \end{pmatrix}$$

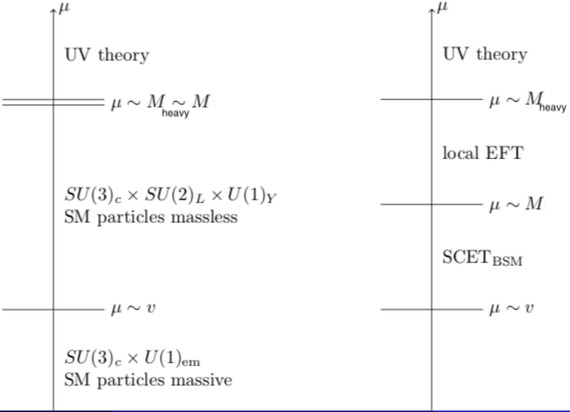
Note: No operator with gauge fields at order λ^2.

Effective Lagrangian

The effective Lagrangian at order λ^2 reads:

$$\mathcal{L}^{(\lambda^2)} = \mathrm{MC}_{RL}\mathcal{O}_{RL} + \mathrm{MC}_{RR}\mathcal{O}_{RR} + \mathrm{MC}_{LL}(M, M_\Phi, \mu)\mathcal{O}_{LL} + h.c,$$

where $\mathrm{C} \equiv \mathrm{C}(M, M_\Phi, \mu)$, are Wilson coefficients which will be matched to the UV completion at scale $\mu \sim M_{heavy} \sim M$.



Next: Run the Wilson coefficients to the electroweak scale v:

$$\mu \frac{d}{d\mu} \mathrm{C}(\mu) = \Gamma(\mu)\mathrm{C}(\mu)$$

$$\Gamma(\mu) = \sum_r \gamma_{cusp}^{(r)} \log \frac{Q^2}{\mu^2} + \sum_i \gamma_i$$

γ_{cusp} is needed to cancel all the divergences, including the ones in double ϵ

Calculation Γ in SCET usually not so trivial!

Conclusion

- EFT is a powerful tool to make predictions at certain energy scale.
- SCET is the appropriate frame-work to treat the decays of heavy particles into energetic final states: correctly treating the large scale separation.
- Systematically sum the large logs: $\log(\frac{M_{heavy}}{v}) \Rightarrow$ crucial to have predictive power of the theory at all orders.