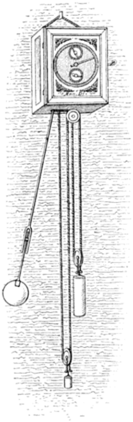
The Period of Simple Pendulum



Sergei Çelaj
University of Tirana

Content

Introduction

Galileo Galilei

$$\ddot{\theta} + \omega_0^2 sin(\theta) = 0$$

Estimating the Pendulum Integral

Perturbation solution of the elliptic integral

$$T = 4\sqrt{\frac{l}{g}}\int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-k^2 sin^2\phi}}$$

$$T = 4\sqrt{\frac{l}{g}}\int_0^{\frac{\pi}{2}} (1+\frac{1}{2}k^2 sin^2\phi + \ldots + \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot 2n} k^{2n} sin^{2n}\phi) d\phi.$$

We stop at the first order of the serie, also with k we indicate the amplitude of the pendulum.

Convergent of John Wallis series

$$\lim_{N\to+\infty} \frac{1}{N} \sum_{n=1}^{N} k^{2n} \cdot \frac{\pi}{2^{2n}} \cdot \frac{(2n)!}{(n!)^2} = \lim_{N\to+\infty} \frac{\pi}{2N} = 0$$

We prove that the limit converge so we can get a approximate value for the period.

Chaos

$$\tau = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 sin^2 \phi}}$$

1. With k we indicate $\sin\frac{\theta_o}{2}$. We take $\theta_0 = \pi$.
2. For $k = 1$ the form $1 - k^2 \sin^2\phi$ transforms into $1 - sin^2\phi = \cos^2\phi$.
3. Now we will have a new integral as $\tau = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{\cos^2\phi}} = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\cos\phi}$.
   The solution of wich will be $\sec\frac{\pi}{2} - \sec 0$.
4. But the $\sec\frac{\pi}{2} = \frac{1}{\cos\frac{\pi}{2}} = \lim\frac{1}{0}$ the integral diverge. So here we prove the chaos.

Trapezoidal Rule

1. **Trapezoidal rule is a technique for approximating the definite integral.**

2. **The elliptic integral** $\tau = \frac{T}{T_0} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}(1-k^2 sin^2\phi)^{-1/2}d\phi$ **can calculated based on this rule.**

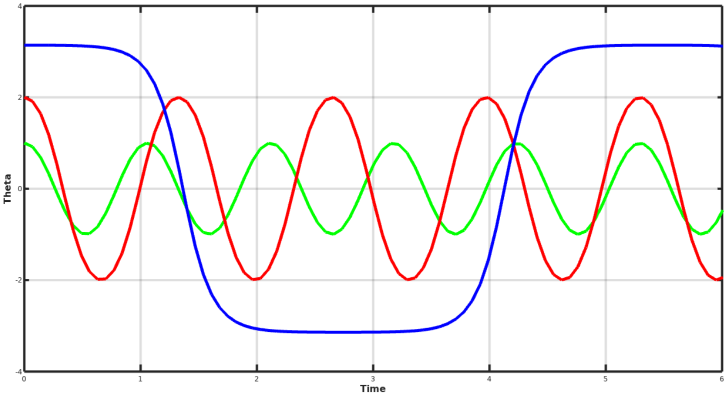
3. **This rule has an error of** $O(N^3)$**, where N is number of parts.**

Arithmetic-Geometrical Mean

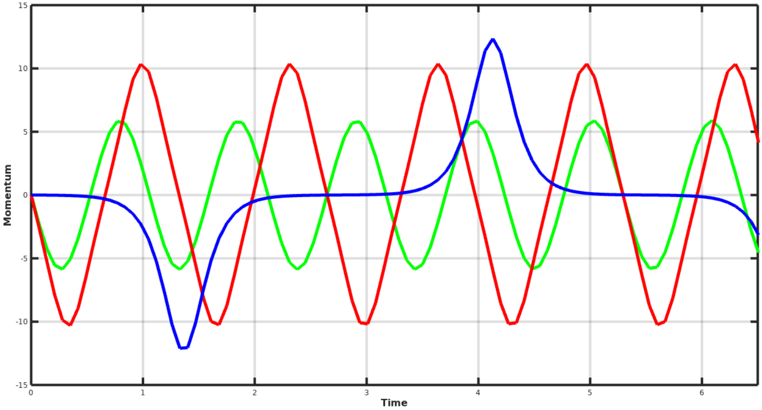
$$a_n = \frac{1}{2}(a_{n-1} + b_{n-1})$$
$$b_n = \sqrt{a_{n-1}b_{n-1}}$$

Graphic Intepretation

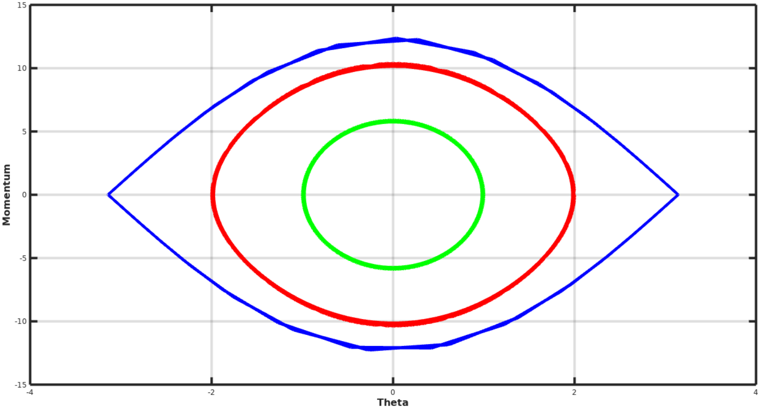
Interpretation of different amplitudes and large angles



Dependence of momentum in function of time



Variation of momentum by the angle



Conclusions

1. **Trapezoidal rule is a numerical method wich gives us a soluion to harmonic motion equation.**
2. **Arithmetic-Geometrical Mean gives a precised value of period with at least 10 digits.**
3. **Trapezoidal rule is less accurate then AGM method, with an difference of 4 digits.**
4. **Chaos is reached at $\theta_0 = \pi$ and the period can not be measured.**