Scale Invariance in Particle Physics and Cosmology
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Scale invariance and its breaking in cosmology

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Introduction

- Gravity clashes with quantum mechanics on several occasions.

- This is very prominent in (observational) cosmology.

- The problems range from deeply conceptual to comparisons with data:

a partial list:

♠ The cosmological constant problem.

♠ The concept of the number of degrees of freedom in gravity and in QFT coupled to gravity:

(a) dS seems to have a finite number of degrees of freedom.

(b) The counting is "holographic"

Holography

• Holography is a generalization of the AdS/CFT correspondence: a conjectured non-perturbative duality between 4d N=4 sYM and IIB string theory on $AdS_5 \times S^5$.

• Although it remains a conjecture, it has been tested in many contexts and few scientists in the field doubt its validity.

• We know now (more or less) constructively, the way this duality may emerge from the Schwinger Source Functional of QFT.

E. Kiritsis, S. S. Lee, Razamat+Douglas, Leigh

• This leads to the (rough) claim that :

"(Generalized) String Theory is the dynamics of sources of QFT"

• It is a gross overstatement to say that we understand the above in all their details. We only have hints from simple situations that have been analyzed. But the hints seem convincing.

Emergent gravity

• Above all, holography suggests a credible picture for gravity that was suspected since the seventies.

• In that picture, the graviton is composite, made of standard quantum fields.

• This did not work properly, until holography came to the fore.

• To obtain local (effective) gravity and a semiclassical regime one needs "holographic QFTs" (large N, near-infinite coupling).

• It bypasses the Witten-Weinberg theorem in surprising ways.

• It is higher-dimensional.

• One can bring this idea to it general conclusion and show that "emergent" gravity is generic in QFTs (but is not generically semiclassical) and evades the WW theorem, as it is massless gravity with a cosmological constant.

Baggiolli+Betzios+Kiritsis+Niarchos

• This suggests that the gravity we feel and measure may be due to the presence of a huge (large N) UV QFT, that is communicated to the SM.

E. Kiritsis

• The most radical hints from holography pertain to:

(1) the information paradox (which however is not yet solved but has been refined)

(2) the notion of what is natural in gravity and what is fine tuning

• Example: A large or small cosmological constant in AdS/CFT is technically natural as it is determined by the number of dof of the UV CFT.

T. Banks

• My goal: to use holographic ideas in order to understand gravity and cosmology.

QFTs and CFTs

• Holography provided a geometric/gravitational picture of the whole Wilsonian philosophy and practice:

• In QFT, the crucial points on the map are CFTs*.

• In that sense, scale invariance and its intertwinement with other symmetries, is the crucial classifying property of the QFT landscape.

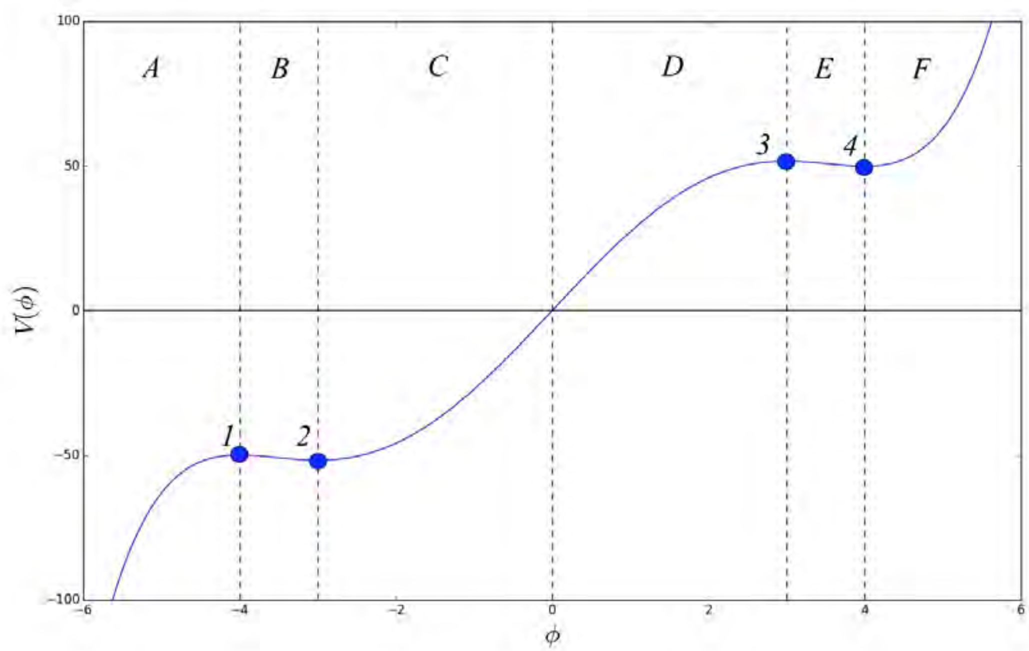
• The rest of the map is constructed from RG flows between the fixed points (CFTs).

• Navigation rules can be constructed but flows are difficult to follow if the theories are not weakly coupled.

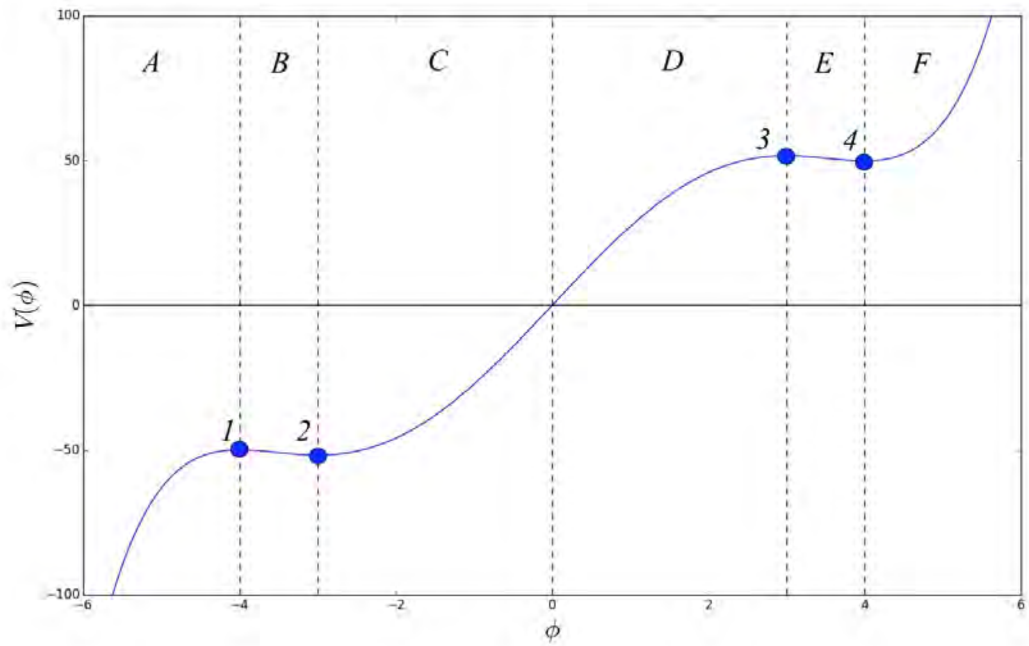
• There are global constrains (C-theorems) that reflect the notion of reduction of d.o.f. along a RG trajectory.

The (dual) gravitational picture

• In the space of holographic QFTs, the dual to the Wilsonian picture is that of the string landscape



$$S_{d+1} = M^{d-1} \int d^{d+1}x \sqrt{g} \left[R - \frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + V(\phi^i) \right] + \mathcal{O}(\partial^4)$$



♠ Maxima of the potential are UV fixed points.

♠ Minima of the potential are typically (but not always) IR fixed points.

♠ Interpolating gravity+scalar solutions are (holographic) RG flows.

• C-theorems are easier to construct in this picture, even for cases which was not obvious in the QFT picture (ie odd dimensions).

Holography/Cosmology correspondence

• The earliest idea attempts to built a map between gravitational duals of holographic QFTs and cosmological flows.

• This does not look like a duality but a correspondence between semiclassical dynamics.

• Various forms have already been discussed (dS/CFT correspondence, cosmology from weak-coupling QFT etc)

Strominger, Nojiri+Odintsov, Anninos+Hartmann+Strominger, Skenderis+McFadden, Kiritsis

$$S_{d+1} = M^{d-1} \int d^{d+1}x\sqrt{g}\left[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

• We are in the AdS regime when $V < 0$.

• This (effective) theory could be a holographic gravity theory dual to a QFT_d, or a model for standard inflation in $d+1$ dimensions.

• A holographic solution dual to a Poincaré-invariant state (like the vacuum) in QFT_d is given by an ansatz

$$\phi(u) \quad , \quad ds^2 = du^2 + e^{2A(u)} dx_\mu dx^\mu$$

• A cosmological solution of the inflationary type in (d+1) dimensions is obtained by replacing $u \leftrightarrow t$

$$\phi(t) \quad , \quad ds^2 = -dt^2 + e^{2A(t)} dx_\mu dx^\mu$$

• For every holographic solution, we can obtain a cosmological solution (including the perturbations) by

$$M^{d-1} \to -M^{d-1} \quad , \quad V \to -V \quad , \quad p^2 \to -p^2$$

• The first two can be obtained by the analytic continuation $N \to iN$ in the large-N QFT.

The Cosmology/Holography correspondence revisited

• We now do our Holography → Cosmology map, and translate the QFT language to cosmological solutions:

♠ CFTs ↔ fixed-points ↔ AdS geometry → dS ↔ eternally inflating universe.

♠ The AdS curvature scale ℓ → Inverse of the Hubble constant H in the "nearby" deSitter.

♠ A RG flow → a shrinking Universe

♠ An inverse RG flow → an expanding universe.

♠ The asymptotically AdS boundary $\to$ a universe of infinite (spatial) size

♠ The IR fixed point of the RG flow $\to$ the beginning of the universe (ie the initial singularity is replaced by the $\mathcal{I}^-$ boundary of deSitter space).

♠ The relation $(M\ell)^2 \sim N^2$ becomes $\frac{H^2}{M_p^2} \sim \frac{1}{N^2}$. Cosmological data indicate that:

Skenderis+McFadden

$$N \simeq 10^{3-4}$$

• The hierarchy $\frac{H^2}{M_P^2} \simeq 10^{-10}$ seems (technically) <u>un-natural</u> in gravity.

It is technically <u>natural</u> in QFT.

• Inflationary period/slow roll $\to$ two conditions:

(a) Evolution in the scaling region (neighborhood of a fixed point)

(b) The driving operator is nearly marginal ($\Delta \simeq d$).

• An exactly marginal operator does not disturb deSitter.

• A nearly marginal operator takes a long time until the vev is of the order of the source

$$\phi(t) \sim \phi_0 e^{(d-\Delta)Ht} + \phi_1 e^{\Delta Ht} + \cdots$$

• An inflating universe $\to$ RG flow towards a UV fixed point (by a relevant operator)

or

$\to$ RG flow away from an IR fixed point (by an irrelevant operator).

The cosmo picture in Wilsonian terms

• A relevant operator $\to$ exponentially evolving inflaton moving away from a minimum of the potential (UV fixed point)

• An irrelevant operator $\to$ exponentially evolving inflaton moving away from a maximum of the potential (IR fixed point)

• Marginally relevant or irrelevant operators $\to$ power law (slow) evolution.

• Scaling solutions with $\phi \to \infty \leftrightarrow$ Super-Planckian inflation $\to$ scaling QFT regimes with hyperscaling violation.

Gouteraux+Kiritsis, Huijse+Sachdev

• The scalar and tensor perturbations in cosmology $\to$ scalar and tensor fluctuations that determine the two point functions of the stress tensor and perturbing scalar operator dual to $\phi \sim T_\mu{}^\mu$.

• Exit from inflation $\to$ exit from the scaling region. It translates into two possibilities in QFT:

(a) The vev (subleading) part of the scalar solution becomes of the same order of the source (= slow roll stops)

(b) Another scalar operator becomes relevant/gets turned-on, backreacts on the flow and the system leaves the scaling region.
(This is known as Hybrid Inflation and this happens in "walking" QFTs)

Summary: In the QFT language to inflate = we are in one of the following three cases.

- SuperPlanckian $\to$ hyperscaling violating (approximate) fixed points.

- Starobinsky-like $\to$ Marginally relevant/irrelevant operators (like in asymptotic freedom in QFT).

- Quadratic potential $\to$ Nearly marginal operators.

Beyond the dS/CFT correspondence

• The dS/CFT correspondence is suggestive but we would like a more direct connection to the standard AdS/CFT correspondence for cosmology.

• It would be ideal if we can find situations where an AdS boundary coexists with a cosmological interior in semiclassical (super)-gravity solutions.

• The AdS boundary and the related b.c. can be associated in a standard way to a standard QFT using AdS/CFT rules.

• The cosmological evolution will appear in the interior of the geometry.

• If this is possible , it might relate cosmological evolution to the realm of time-dependent dynamics of QFT vevs.

• A solution of this type was found recently in 2d Jackiw-Teitelboim gravity ("Centaur" solution)

Anninos+Hofman

• Our next goal will be to find solutions (flows) in d>2 (super)gravity that interpolate between

♠ the "dS regime" of the landscape $(V(\phi) > 0)$ and

♠ the "AdS regime" $(V(\phi) < 0)$

• We also need ansatze that interpolate between the dS and AdS metrics:

♠ Flat $d - dimensional$ slices:

$$ds^2 = \frac{du^2}{f(u)} + e^{2A(u)} \left[-f(u)dt^2 + dx_i dx^i\right].$$

Here AdS/dS in Poincaré coordinates is given by $f = \pm 1$ and $e^{-\frac{u}{\ell}}$.

♠ Spherical slices:

$$ds^2 = \frac{du^2}{f(u)} + e^{2A(u)} \left[-f(u)dt^2 + d\Omega^2_{d-1}\right]$$

AdS and dS in this ansatz appear for

$$e^A = e^{-\frac{u}{\ell}} \quad , \quad f = \pm 1 + e^{2\frac{u}{\ell}}$$

Basic properties of dS flows

$$ds^2 = -du^2 + e^{2A(u)}\left[dt^2 + dx_i dx^i\right] \quad , \quad \phi(u)$$

with u as the time coordinate.

• We always look for globally regular solutions.

By a careful study of the gravitational equations we find:

• Near de Sitter minima of the potential, we have large/diverging scale factors that correspond to a large universe.

• Maxima of the potential in the dS regime are associated with a universe that shrinks to zero size in Poincaré coordinates and therefore signal the "beginning" of the universe.

• Here, Hawking's singularity theorem fails. The "big bang" is a coordinate singularity and is part of the $\mathcal{I}^-$ boundary of dS (which is an infinite size sphere.

• In the dS regime there are “bouncing solutions” as those we found earlier in the AdS regime.

Kiritsis+Nitti+Silva-Pimenta

• Such bounces are places in the cosmic evolution where the scalar field changes direction without any singularity.

• Such solutions correspond to cosmic clocks that change direction in time, in a fully regular fashion.

• Like in the AdS regime, regular flows in dS start and end at neighboring extrema of the bulk scalar potential.

dS/AdS Interpolating flows

• We now turn to interpolating flows:

$$ds^2 = \frac{du^2}{f(u)} + e^{2A(u)}\left[-f(u)dt^2 + dx_i dx^i\right].$$

• All interpolating solutions must have a horizon along the flow,

Freivogel+Hubeny+Maloney+Myers+Rangamani+Shenker, Lowe+Roy

• We can show that the null energy condition in the bulk implies that no such (regular) solutions exist.

Kiritsis+Tsouros

• The same analysis can be done for the ansatz with spherical slices

$$ds^2 = \frac{du^2}{f(u)} + e^{2A(u)}\left[-f(u)dt^2 + d\Omega_{d-1}^2\right]$$

• We can again show that the null energy condition does not allow regular interpolating flows.

Kiritsis+Tsouros

• There can be other possibilities:

♠ There is a third interpolating ansatz but it does not end on fixed points.

♠ There can be dS/AdS flows that are not captured by our setup.

They will involve a more than one coordinate flow, which would correspond in this setup to an infinite number of scalar fields.

♠ There can be other ways of realizing de Sitter in the AdS arena.

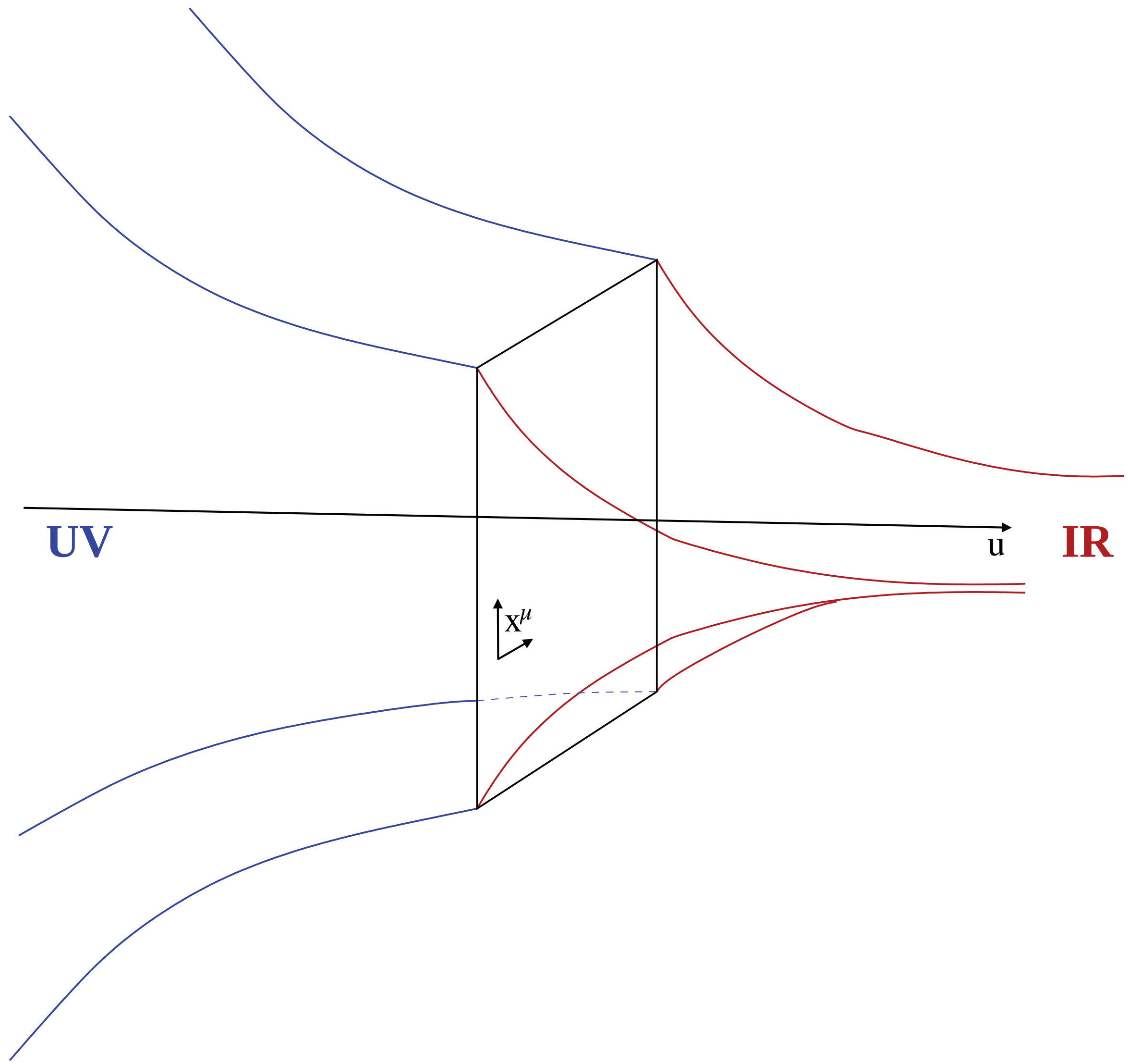
de Sitter brane-worlds in AdS

• String theory allows the existence of branes with dimension lower than the ambient dimension.

• The QFT dual is non-holographic theories (like the SM) coupled to holographic theories.

• A simple geometrical/gravitational picture is that of a codimension 1 brane in a 5d bulk.

• The 5d bulk gravitational theory is dual to a holographic 4d QFT and has therefore an asymptotically AdS boundary.



- As simple bulk action is

$$S_5 = M^3 \int d^5x \sqrt{g} \left[R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right]$$

with the scalar φ driving a standard RG Flow in the bulk QFT.

• The coupling of the brane theory to the bulk theory is summarized by

$$S_{brane} = M^2 \int d^4x\sqrt{-\gamma}\left(-W(\varphi) - \frac{1}{2}Z(\varphi)\gamma^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + U(\varphi)R_B + \cdots\right)$$

It can be shown that

• If the QFT dual to the bulk theory is defined on Minkowski space with

$$ds^2 = du^2 + e^{2A(u)}\ \eta_{\mu\nu}\ dx^\mu dx^\nu$$

• For generic potential functions, $V(\varphi), W(\varphi), Z(\varphi), U(\varphi)$ there are self-tuning solutions to the bulk+brane equations of motion so that the induced metric on the brane is flat.

Charmousis+Kiritsis+Nitti

• In such solutions, the brane is fixed to a single* point in the radial/holographic direction.

• If the dual (bulk) QFT is defined on de Sitter space then

$$ds^2 = du^2 + e^{2A(u)} \left(-d\tau^2 + \frac{\cosh^2(H\tau)}{H^2} d\Omega_d^2\right)$$

• There are solutions to the bulk and brane equations where again the brane is localized in the radial direction.

Ghosh+Kiritsis+Nitti+Witkowski

• The induced metric on the brane is now de Sitter.

• It relies on the fact that the associated string theory has AdS boundary conditions associated to a de Sitter metric.

• This realization bypasses swampland constraints.

Conclusions

• A holographic map of Inflationary scenarios and the general cosmological history, provides a radically different view of the effects and the notion of fine-tuning in cosmology

• It allows to classify inflationary scenarios using the Wilsonian language, in terms of scaling regions, and relevant, irrelevant and marginal operators.

• Regularity in holography is a crucial dynamical ingredient.

• It allows a global view of cosmology along the lines of QFT RG flows.

• Taken at phase value this approach demands what appears as fine tuning of "final" conditions at the future screen.

• This suggest that the right way to see cosmology is "starting from the future and looking back".

• To proceed to a more concrete contact between AdS and QFTs to cosmology, interpolating solutions between AdS and dS should exist.

• In a rather general context, we have shown that such regular solutions do not exist, but there are loopholes.

• In one of them, de Sitter on the brane seems possible in an asymptotically AdS bulk.

• Another possibility is Coleman-de Luccia tunneling.

• Our results seem to indicate that the CdL tunneling scenario needs revision.

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THANK YOU

The QFT Picture

- The eoms with the LI ansatz are

$$d(d-1)A'^2 - \frac{1}{2}\phi'^2 - V = 0 \quad , \quad 2(d-1)A'' + \phi'^2 = 0$$

- In holography, e^A is interpreted as a RG scale and the evolution in r is the (holographic) RG flow.

- ϕ is dual to a scalar operator $O(x)$ in the dual QFT. Its source is the coupling multiplying O in the QFT action.

- The flow can be made explicit by introducing the "superpotential" $W(\phi)$

Verlinde[2]+DeBoer

$$\frac{dW^2}{2(d-1)} - W'^2 = 2V$$

$$A' = -\frac{W(\phi)}{2(d-1)} \quad , \quad \phi' = \frac{dW}{d\phi} \equiv W'$$

- We can now calculate the holographic β-function

Gursoy+Kiritsis+Nitti

$$\frac{d\phi}{dA} = -2(d-1)\frac{W'(\phi)}{W(\phi)} \equiv \beta(\phi) \qquad \leftrightarrow \qquad \frac{dg}{d\log\mu} = \beta(g)$$

• The "fixed-point" solution

$$\phi = \phi_* \quad , \quad \beta(\phi_*) = 0 \quad , \quad e^A = e^{\frac{r}{\ell}} \quad , \quad V(\phi_*) = \frac{d(d-1)}{\ell^2}$$

is AdS_{d+1} space.

• Minima are UV fixed points, maxima are IR fixed points.

• The whole RG framework, the characterization of scaling regions etc, now applies to the gravitational solutions above.

• The flows can be generalized to space-time dependent couplings and metrics

$$\frac{d\phi}{dA} = \beta(\phi) + \beta_1(\phi)R + \beta_2(\phi)(\partial\phi)^2 + \cdots$$

Kiritsis+Li+Nitti

The basic concepts:

(0) $$(M\ell)^{d-1} \sim N^2$$

(1) The asymptotically AdS boundary $\to$ UV asymptotics of QFT.

(2) The scaling dimension Δ of $O(x)$ $\to$ mass of ϕ near ϕ_*

$$\frac{\Delta(d-\Delta)}{2d(d-1)} = \frac{V''(\phi_*)}{V(\phi_*)}$$

(3) The RG flow near the fixed point is scaling

$$\phi(u) \sim \phi_0(x)\ u^{d-\Delta} + \langle O(x)\rangle u^{\Delta} + \cdots \quad , \quad u \simeq e^{\frac{r}{\ell}}$$

(4) All flows end in a fixed-point in the UV and at a fixed point in the IR. They may wonder near fixed points in between.

(5) The global regularity of the semiclassical solutions is crucial: It is this that determines $\langle O(x)\rangle$ as a function of $\phi_0(x)$.

(6) Global regularity is (in principle) equivalent to an infinite number of "tunings". There is an infinite number of fields dual to operators and each has to be "fine tuned". Only EXACT AdS does not need tuning.

Global Regularity = a crucial dynamical principle in holography

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Inflation and Planck

• Planck has given data that narrow considerably the errors of observations especially for the scalar index

• This seems to favor models, with a very flat (effective) potential, in the class of Starobinsky (that also contains among others the B-Sh model)

• Such models seem to have several "problems" from the gravitational point of view

P. Steinhard

(a) A more serious initial conditions problem.

(b) It is an even more "unlikely potential".

(c) It gives little info on the "multiverse".

• I would like to argue that the holographic thinking gives a different picture

• It provides a general way of classifying inflationary models. This new way has universality features and is better suited to comparison with data.

• The universal characteristics are the same as those that Wilson postulated for QFT.

• They involve a scale-invariant fixed point and scaling exponents around the fixed point

• Perturbation spectra map to (pseudo)-QFT correlators of scalar operators and the stress tensor.

• The general class of inflation (that contains the Starobinsky and related models) corresponds to something that in QFT we call asymptotic freedom (or RG flow by marginally relevant operators)

A Wilsonian Classification

We can classify the types of β-functions:

I) $\beta \simeq b_0(\phi - \phi_*)^q$ with $q \geq 1$. The fixed point is at $\phi = \phi_*$.

$$V \simeq 3\frac{H^2}{\kappa^2}\left[1 - \text{constant}(\phi - \phi_*)^{q+1} + \cdots\right]$$

- $q = 1$ is the case of relevant operators with $b_0 \sim m^2 \sim \Delta(3 - \Delta)$.

- In this case we obtain

$$n_s - 1 \simeq 2(\Delta - 3) \quad , \quad r \simeq (n_s - 1)^2 \; e^{(n_s - 1)N}$$

- It agrees with cosmological data if $\Delta - 3 \simeq -0.02$, $N \simeq 120$.

- A β-function appearing in L-loop order is $\beta \sim (\phi)^{L+1}$.

II) $\beta \simeq \frac{b_0}{(\phi - \phi_*)^q}$ with $q > 1$. The fixed point is at $\phi \to +\infty$.

$$V \simeq 3\frac{H^2}{\kappa^2}\left[1 - \frac{\text{constant}}{\phi^{q-1}} + \cdots\right]$$

• In QFT this corresponds to a non-perturbative fixed point. Such fixed points are accessible only via duality symmetries. An example is Seiberg duality.

III) $\beta \simeq \frac{b_0}{(\phi-\phi_*)^q}$ with $0 < q < 1$. The fixed point is at $\phi \to +\infty$.

$$V(\phi) \simeq e^{\text{constant } \phi^{1-p}}$$

• This is a special class that seems to have no clear analogue in QFT although it looks similar to "hyperscaling violation" (see below).

III_0) This is III with $p = 0$. The scalar is running as a log to $+\infty$.

$$V(\phi) \simeq e^{2a\ \phi}$$

This is the type of potential that is generic in gauged supergravities obtained from string theory.

• Here the β-function is constant, as well as all other slow-roll parameters:

$$n_s - 1 = -a^2 \quad , \quad r = 8a^2 = -8(n_s - 1) \quad , \quad a_s = 0$$

• It is describing scaling physics that violates hyperscaling.

Kiritsis+Gouteraux

• All exponents are independent on N. This class is quite close to data if $a^2 \simeq 0.025$.

• It is realized among others in non-critical YM theories.

Kanitseider+Skenderis+Taylor

III_1) This is III with $p = 1$. The scalar is running off to $+\infty$.

$$V(\phi) \sim \phi^Q$$

0) Asymptotically Flat Inflationary Models (AFIM):

$\beta \sim e^{-2a\phi} \sim \lambda^2$ as $\lambda \equiv e^{-a\phi} \to 0$ or $a\phi \to \infty$.

$$V(\phi) \simeq 3\frac{H^2}{\kappa^2}\left[1 - \text{constant}\ e^{-a\phi} + \cdots\right]$$

- It corresponds to asymptotically free QFTs (as will be discussed later)

- It has a single parameter a.

The Principle of regularity in holography

In an asymptotically-AdS scaling region, a scalar dual to an operator of dimension Δ has a solution:

$$\phi(r) \simeq C_0 \, r^{d-\Delta} + C_1 \, r^{\Delta} + \cdots$$

with an AdS metric in Poincaré coordinates

$$ds^2 = \frac{\ell^2}{r^2}(dr^2 + dx_\mu dx^\mu)$$

with $r = 0$ being the boundary (UV) and $r \to \infty$ the Poincaré horizon (IR).

• C_0 and C_1 are the two arbitrary integration constants of the second order gravity equations.

• Near the UV, C_0 multiplies the dominant solutions and is interpreted as the coupling constant of the dual operator O.

• C_1 is the vev of O in the RG flow in question.

$$\phi(r) \simeq C_0\, r^{d-\Delta} + C_1\, r^{\Delta} + \cdots$$

- In the UV: For relevant operators $\phi \to 0$. For irrelevant operators $\phi \to \infty$.

Therefore: for irrelevant operators $C_0 = 0$ for UV regularity (as is known from QFT).

- In the IR:

For irrelevant operators, one of the two solutions diverges. (the UV data must be chosen so that it vanishes: one fine tuning)

- This tuning fixes the vev as a function of the source (coupling constant).

For relevant operators, both solutions diverge: the solution must be identically zero (or this IR fixed point cannot be reached)

Conclusion: For each dual relevant operator whose scalar solution is non-zero, there is at least one tuning (boundary condition).

It is imposed in the UV (boundary) so that solution is regular in the IR.

Regularity in Cosmology

• We now rotate 90^o and translate the holographic regularity requirements to cosmology.

• Scalars with small masses corresponding to relevant operators, can be singular in infinite past. The regularity condition therefore is imposed there. No boundary condition is needed in the future.

• Most other scalars with larger masses corresponding to irrelevant operators must be set to zero in future infinity.

• The infinite past is deSitter (regular)

• There is an infinite number of tunings: it is natural however in CFT.

• All the "data" and other information are at future infinity: In an expanding universe, one starts from today, and can determine the past.

• It is not clear whether there is a further justification for the suggestions above beyond the correspondence with holography.

Marginally-relevant operators

- I will be in 5d to be dual to a 4d CFT.

- Consider the Asymptotically Flat scalar potential

$$V(\phi) = \frac{12}{\ell^2}\left[1 + \sum_{n=1}^{\infty} V_n \lambda^n\right] \quad , \quad \lambda = e^{a\phi}$$

- It has an AdS minimum ($V_1 > 0$) at $\lambda = 0$.

- It is the softest possible extremum:

$$\left.\frac{d^n V}{d\phi^n}\right|_{\lambda=0} = 0 \qquad \forall n > 0$$

- We can calculate perturbatively in λ the superpotential

$$W = -\frac{6}{\ell}\left[1 + \sum_{n=1}^{\infty} W_n \lambda^n\right], W_1 = \frac{V_1}{2}, W_2 = \frac{V_2}{2} - \left(1 - \frac{3}{2}a^2\right)\frac{V_1^2}{8}$$

• We can calculate the β-function

$$\frac{d\lambda}{dA} \equiv \beta = -6a\lambda\frac{W'}{W} = -b_0\lambda^2 - b_1\lambda^3 + \cdots \quad , \quad b_0 = 6a^2 W_1, \cdots$$

• This is the β function of an asymptotically free (AF) theory.

• An AF potential corresponds to an AF scaling regime.

The solution for ϕ is what we know from QCD:

$$\lambda \sim \frac{1}{A} \sim \frac{1}{\log r}$$

with a metric that is nearly AdS.

• As expected from QFT arguments, there is an approximate scale invariance in this scaling regime, broken only logarithmically.

• All higher terms in the potential are not relevant for the main properties, only the leading exponential.

• This provides a large class of theories whose scaling properties are controlled from one real parameter: a.

Naturalness

• How "natural" (or fine tuned) are such potentials?

• From a gravitational point of view they are highly tuned, much more than other inflationary potentials

• From a holographic (QFT) Point of view they are technically natural.

• Their choice corresponds to choosing a QFT that has marginal but no relevant operators.

• There is a very common example: Yang Mills.

• Once we add quarks to YM, there is a scalar relevant operator: the quark mass operator. Even if we set it to zero in the UV, chiral symmetry breaking turns it on!. However, in the conformal window, it can be set to zero, and stay as such during the whole RG flow.

• Surprisingly, supersymmetry seems to work the opposite way: susy theories have more chances of having relevant operators.

Cosmology: AFIM

- The whole set of calculations for AF potentials can be run through in the cosmological setup.

- Many known inflationary models have such potentials: Starobinsky, BSh are among them.. They are not described always in term of potentials.

- The β-function formalism of holography is known as the Hamilton-Jacobi formalism in Inflation.

Salopek+Bond

- The solutions depend to leading non-trivial order on the parameter a:

$$A = -Ht + \frac{1}{4a^2 \log t} + \left[\left(\frac{4a^2-3}{96a^4} + \frac{V_2/V_1^2}{8a^4} \right) + \left(\frac{4a^2-3}{48a^4} + \frac{V_2/V_1^2}{4a^4} \right) \log t \right] \frac{1}{Ht} + \cdots$$

$$V_1 \lambda = -\frac{1}{2a^2 Ht} - \left[\frac{4a^2-3}{24a^4} + \frac{V_2/V_1^2}{2a^4} \right] \frac{\log t}{H^2 t^2} + \cdots$$

- A calculation of the spectral index gives

$$n_s - 1 \simeq -\frac{2}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

which is universal. (N=number of e-foldings)

- The power of gravitational waves depends on a:

$$r \simeq \frac{4}{a^2 N^2} + \mathcal{O}\left(\frac{1}{N^3}\right) \simeq \frac{(n_s - 1)^2}{a^2}$$

- The tilt is also universal:

$$a_s \equiv \frac{dn_s}{d\log k} \simeq \frac{2}{N^2} + \mathcal{O}\left(\frac{1}{N^3}\right)$$

- It agrees with data if $a \simeq \frac{9}{100}$.

Induced gravity realizations

• Can AFIM effective potentials be realized in "controlable" theories of gravity?

• In gravity/supergravity, we can write down more or less what we want.

• The Starobinsky model is using an R^2 term to do this. Why stop at R^2?

• If gravity, is the induced gravity from a (large N) CFT, there is a general argument from holography that this is all there is.

Hertog+Hawking, E. Kiritsis

Consider a CFT with cutoff Λ (RS-like setup) coupled to classical 4d gravity.

The effective gravity action is

$$S_{eff} \sim \Lambda_4 + N^2\Lambda^4 + (M_P^2 + N^2\Lambda^2)R + N^2 \log\Lambda \;\; R^2 + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

• RS "tuning": $\Lambda_4 + \Lambda^4 \simeq 0$.

String Theory: tree level

- None of the standard maximally supersymmetric string compactifications, including the associated gauged supergravities seem to give such potentials.

- It is easy to get the exponentials but not the constant part.

- All one-loop corrections to the vacuum energy are of the exponential type (and not constants)

- Conlon and Quevedo have argued that in large volume CY compactifications one can obtain such potentials iff:

(a) one loop contributions are included.

(b) the overall volume is fixed.

(c) The uplifting term conjectured by KKLT is present.

- It is not clear even today if this is realizable.

String Theory: curvature corrections

• In 5d string theory, including curvature corrections one can obtain such potentials.

E. Kiritsis

• This was done for the purpose of motivating IHQCD, an asymptotically free string-theoretic background.

• The essential points of the argument:

(a) 5d string theory

(b) The 5-form of string theory is non-propagating, but gives a non-linear potential for the dilaton at tree level.

$$S_{tree} = M^3 \int d^5x \sqrt{g} e^{-2\phi} \left[4(\partial\phi)^2 + F(R,\xi) \right] \quad , \quad \xi = -e^{2\phi} \frac{F_5^2}{5!}$$

Tseytlin

The equation of motion for F_5 can be solved algebraically for $\xi(R,\lambda)$ as a function of R and λ:

$$\xi \left(\frac{\partial F}{\partial \xi} \right)^2 = \lambda^2 \quad , \quad \lambda = e^{\phi}$$

The rest of the dynamics is obtained from the equivalent action

$$S_{tree} = M^3 \int d^5x \sqrt{g} \frac{1}{\lambda^2} \left[4\frac{(\partial\lambda)^2}{\lambda^2} + F(R,\xi) - 2\xi\frac{\partial F}{\partial \xi} \right]$$

- Two possibilities to get AFIM: (asymptotic deSitter solution, and $\lambda \to 0$)

(a) As $\lambda \to 0 \to \xi(R,\lambda) \to 0$. This is inconsistent with the the structure of the action.

(b) As $\lambda \to 0 \to \xi(R,\lambda) \to \xi_*$ with $F_\xi(\xi_*) = 0$.

This gives AFIM!!!

The only condition needed is very "weak":

that $\partial_\xi F(R,\xi)$ has a non-trivial zero.

- This is a non-perturbative in α' solution.

- A similar argument works in 4d string theory using a RR 3-form

Conformal couplings to scalars

- A locally favorite coupling is

$$\delta S \sim \xi \int d^4x \sqrt{g}\; R\; \phi^2$$

- This gives an AFIM with

$$\frac{H^2}{M_P^2} \simeq \frac{\lambda_4}{4\xi^2} \quad , \quad a = \sqrt{\frac{2\xi}{1+24\xi}}$$

- For Induced gravity, $M^2 \sim N_c^2$, $\xi \sim N_c^2$ so that

$$\frac{H^2}{M_P^2} \simeq \frac{1}{N^2} \quad , \quad a \simeq \frac{1}{2\sqrt{3}}$$

Basic properties of dS flows

$$ds^2 = -du^2 + e^{2A(u)}\left[dt^2 + dx_i dx^i\right] \quad , \quad \phi(u)$$

with u as the time coordinate.

• We always look for globally regular solutions.

By a careful study of the gravitational equations we find:

• Near de Sitter minima of the potential, we have large/diverging scale factors that correspond to a large universe. In this regime, the solutions have a free parameter (integration constant) that indicates a continuous family of locally regular solutions.

• Maxima of the potential in the dS regime are associated with a universe that shrinks to zero size in Poincaré coordinates and therefore signal the "beginning" of the universe.

• Here, Hawking's singularity theorem fails. The "big bang" is a coordinate singularity and is part of the $\mathcal{I}^-$ boundary of dS (which is an infinite size sphere.

• In this regime, the solutions are unique after imposing regularity, and therefore have no adjustable parameters.

• In the dS regime there are "bouncing solutions" as those we found earlier in the AdS regime.

Kiritsis+Nitti+Silva-Pimenta

• Such bounces are places in the cosmic evolution where the scalar field changes direction without any singularity.

• They can appear for relatively steep potentials. Such solutions correspond to cosmic clocks that change direction in time, in a fully regular fashion.

• Like in the AdS regime, regular flows in dS start and end at neighboring extrema of the bulk scalar potential.

• As is known from the AdS regime, there can be isolated regular solutions for special (tuned) bulk potentials. Such solutions are such that in the dS regime, they start at a maximum and end up at another maximum of the potential.

• There is an analogous Breitenlohner-Freedman (BF) bound in the dS regime as in the AdS regime. In the AdS regime the BF-violating extrema are unstable under small perturbations.

• In AdS regime there is a kind of censorship of BF-violating extrema: flows that depart of arrive there, are always subleading in Free-energy terms.

• In the AdS regime, the BF bound forbids scalar directions with sufficiently negative masses. This implies that the bound is relevant for maxima of the potential.

• The situation in dS is the inverse. Sufficiently massive scalars violate the BF bound. This implies that here it applies to minima of the potential.

• However, unlike AdS, it is not clear whether violations of the BF bound in dS implies instabilities or the necessity to turn on the relevant scalar solutions.

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 0 minutes
- Introduction 1 minutes
- Holography 2 minutes
- Emergent Gravity 4 minutes
- QFTs and CFTs 5 minutes
- The dual gravitational picture 7 minutes
- The cosmology/holography correspondence 9 minutes
- The cosmology/holography correspondence revisited 12 minutes
- The cosmo picture in Wilsonian terms 15 minutes
- Beyond the dS/CFT correspondence 17 minutes
- Basic properties of dS flows 19 minutes
- dS/AdS interpolating flows 21 minutes

- de Sitter brane-worlds in AdS 24 minutes
- Conclusions 25 minutes