Three Scale Invariant Tales

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CERN 28 January 2019

Outline

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- Conformal/scale invariance and the cosmological constant/ vacuum energy.

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- On the road to removing the string scale.

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- On the road to removing the string scale.
- Time translational invariance and SUSY breaking in a conformal setup.

Conformal/scale invariance and the cosmological constant/ vacuum energy

- A Zel'dovich type argument for a problem:

  Consider QED in the 40s. One has a proton, an electron and a photon. One understands physics up to the Rydberg scale R_Y. Ergo in an effective field theory, valid up to R_Y, the value of the cosmological constant would be at least R_Y^4.

  This is a problem….

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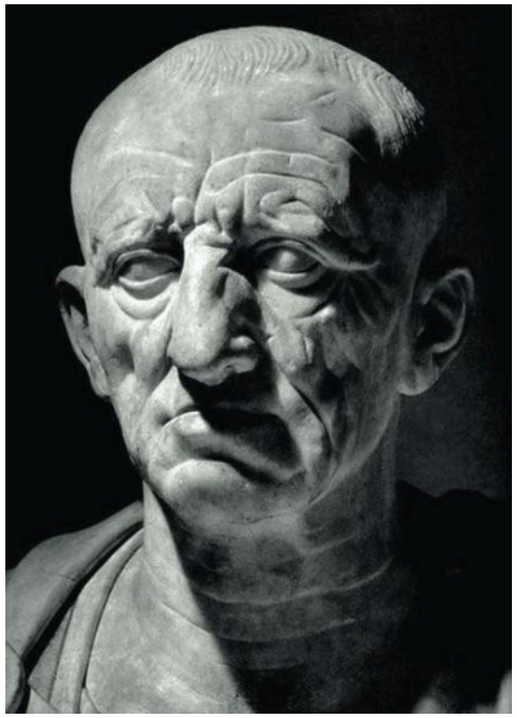

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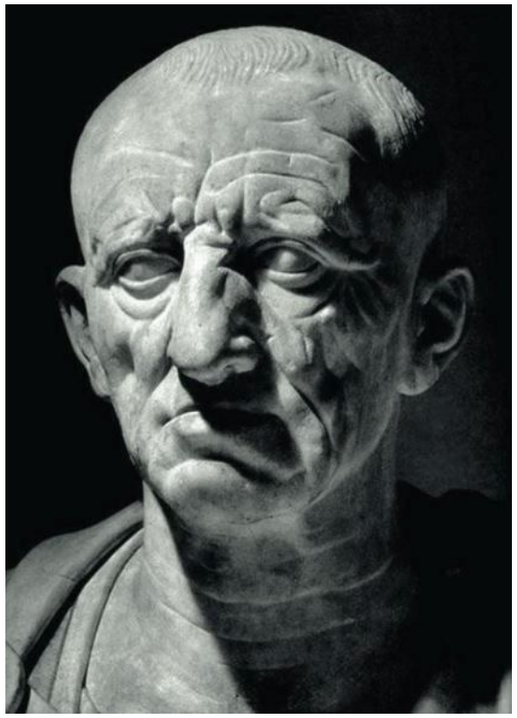

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Chiral Lagrangian

$$
\begin{aligned}
\mathcal{L}_4 = {} & L_1 \left\{ \mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] \right\}^2 + L_2 \mathrm{Tr}\left[D_\mu U (D_\nu U)^\dagger \right] \mathrm{Tr}\left[D^\mu U (D^\nu U)^\dagger \right] \\
& + L_3 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger D_\nu U (D^\nu U)^\dagger \right] + L_4 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger \right] \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger \right) \\
& + L_5 \mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger) \right] + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger \right) \right]^2 \\
& + L_7 \left[\mathrm{Tr}\left(\chi U^\dagger - U \chi^\dagger \right) \right]^2 + L_8 \mathrm{Tr}\left(U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger \right) \\
& - i L_9 \mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + f^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U \right] + L_{10} \mathrm{Tr}\left(U f^L_{\mu\nu} U^\dagger f_R^{\mu\nu} \right) \\
& + H_1 \mathrm{Tr}\left(f^R_{\mu\nu} f_R^{\mu\nu} + f^L_{\mu\nu} f_L^{\mu\nu} \right) + H_2 \mathrm{Tr}\left(\chi \chi^\dagger \right) \textbf{ + ...}
\end{aligned}
$$

Where $U(x) = \frac{1}{F_0} \left[\sigma(x) + i \vec{\tau} \cdot \vec{\pi}(x) \right]$

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- Claim: in a conformal/scale invariant theory the value of the vacuum energy does not depend on the theory being spontaneously broken or not.

- The scale of the symmetry breaking does not affect the vacuum energy.

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There is nothing like solving a concrete model

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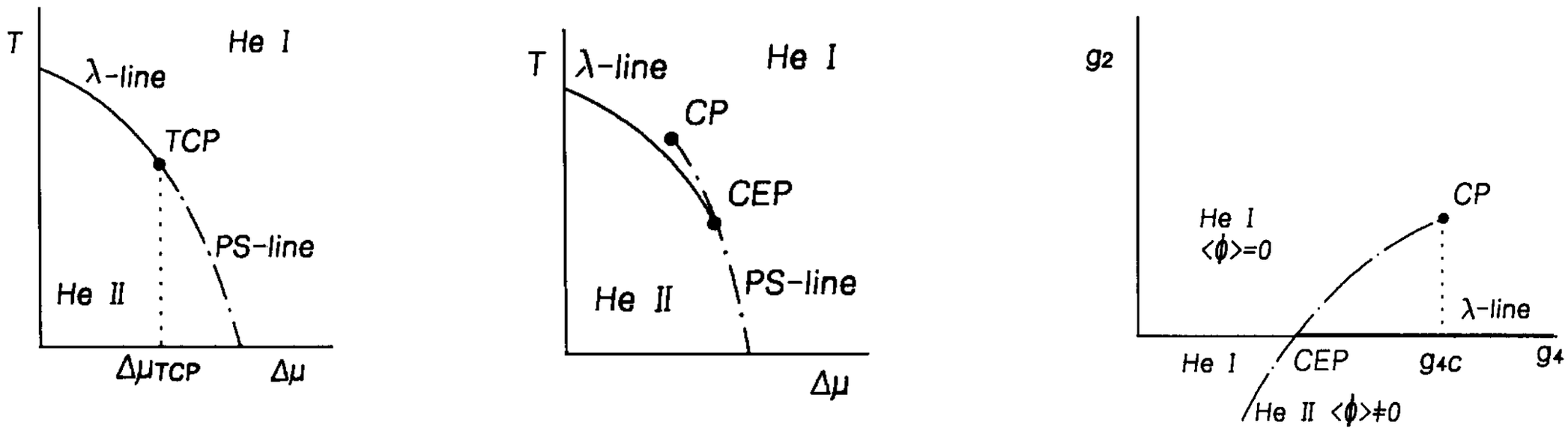

Illustrative Example in d=3

Consider

$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\phi} \partial_\nu \vec{\phi} + \frac{1}{6N^2} g_6 (\vec{\phi}^2)^3$$

where the fields $\vec{\phi}$ are in the vector representation of *O(N)*.

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$$f(g_6) = g_c - g_6 \qquad\qquad g_c = (4\pi)^2 \ .$$

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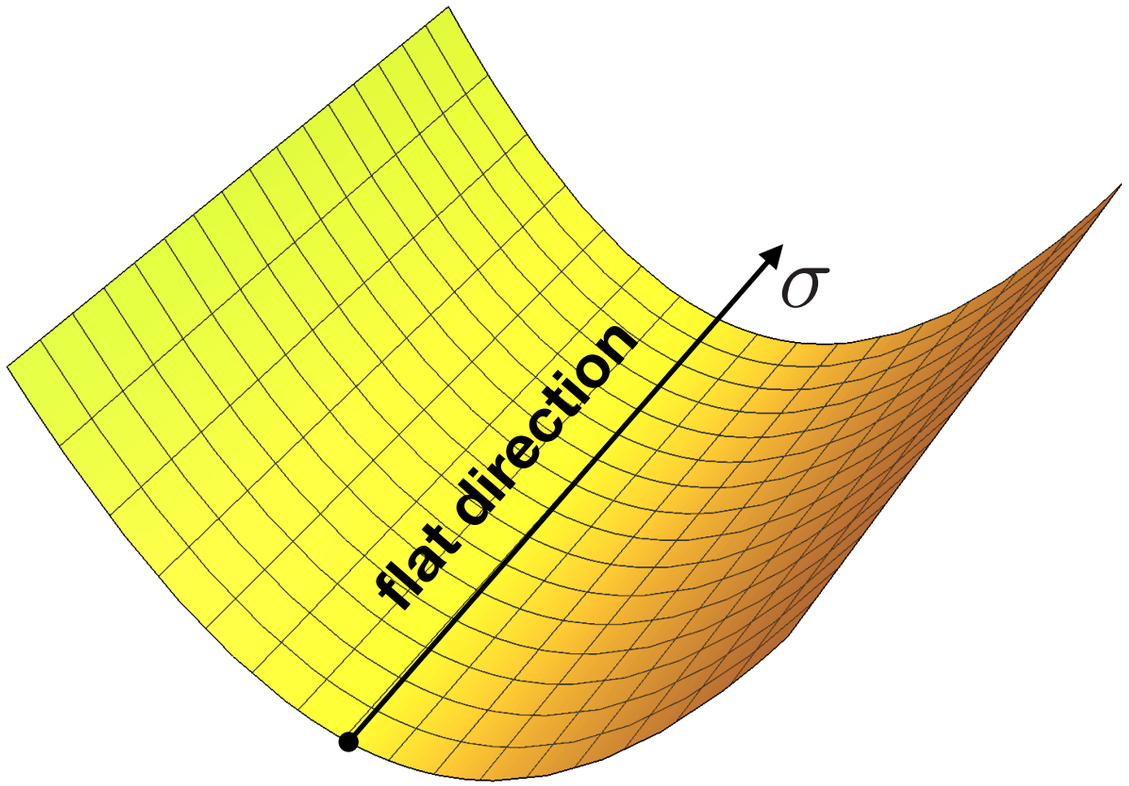

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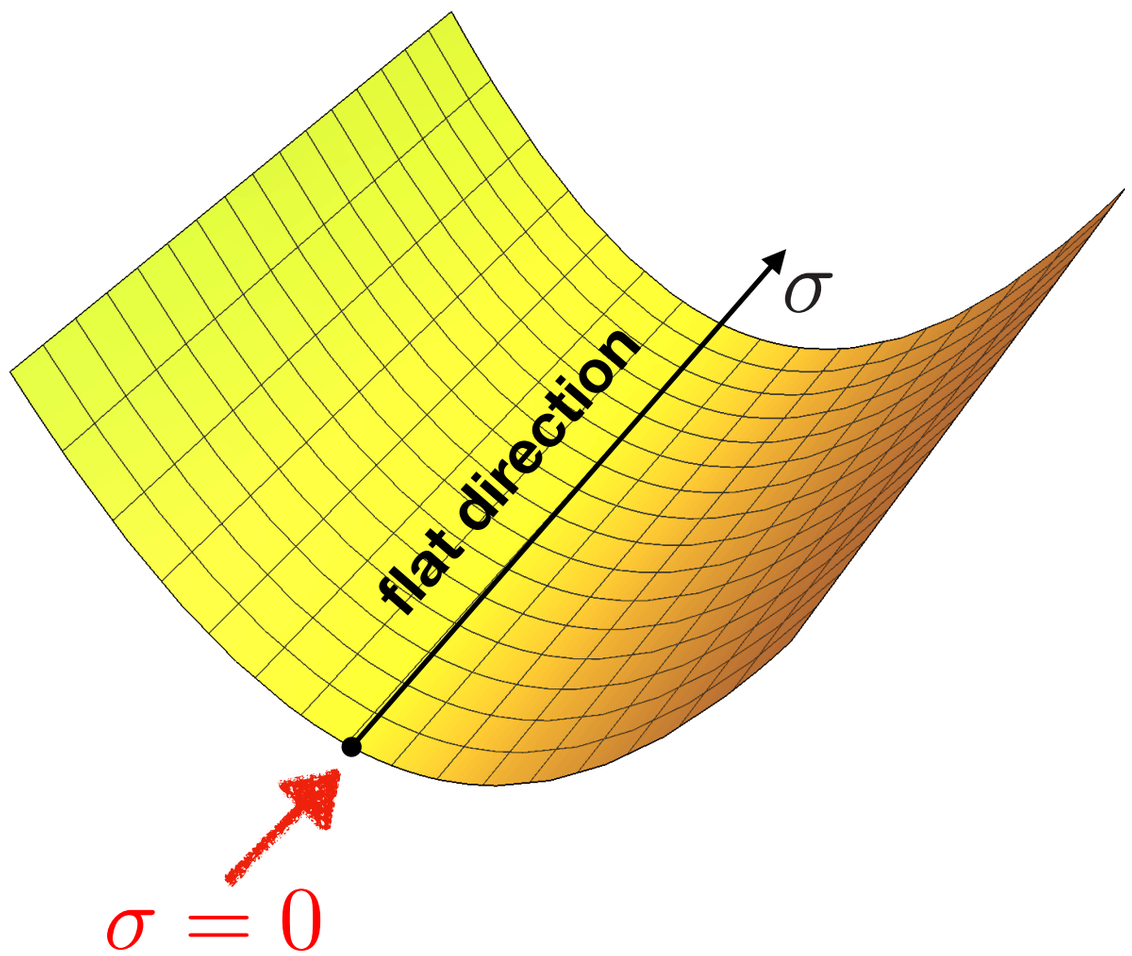

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- The effective potential from which the full results are derived becomes non-local after integrating out the original degrees of freedom.

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- The full overall *N*-dependence is now an explicit factor of *N*, and the integral over t and m^2 is performed by the method of steepest descent.

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- The free energy is non-local.

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$$V_{eff} = |\sigma|^{d/(d-2)} \left(\frac{d}{|K_d|}\right)^{2/(d-2)} \left[\left(\frac{d-2}{d} + \frac{1}{3}(-1)^{d/(d-2)} g \left(\frac{1}{2}|K_d|\right)^{2/(d-2)}\right)\right]$$

$$K_d = \frac{\Gamma(1-d/2)}{2^{d-1}\pi^{d/2}} < 0$$

This potential has a flat direction for $g=g_c$

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$$m^2 \vec{\phi} = 0$$

$$2U'(\tau) - m^2 = 0$$

$$\vec{\phi}^2 - \tau + \int \frac{dk}{k^2 + m^2} = 0$$

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- This example has only one scale.

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- The vacuum energy is independent of both scales. The ratio between the scales is NOT fixed.

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- But here SUSY plays a role in maintaining quantum mechanically a classical flat direction.

Flat Potential

Where is the Dilaton ?

Breakdown of the Equivalence principle?

BEH like?

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Caveat: In a super-selected sector with a non-vanishing value of the central charge (i.e. some dyonic charge) the ground state energy will reflect the energy of the dyonic distribution.

Including Quantum Gravity

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α' *scale delenda est*

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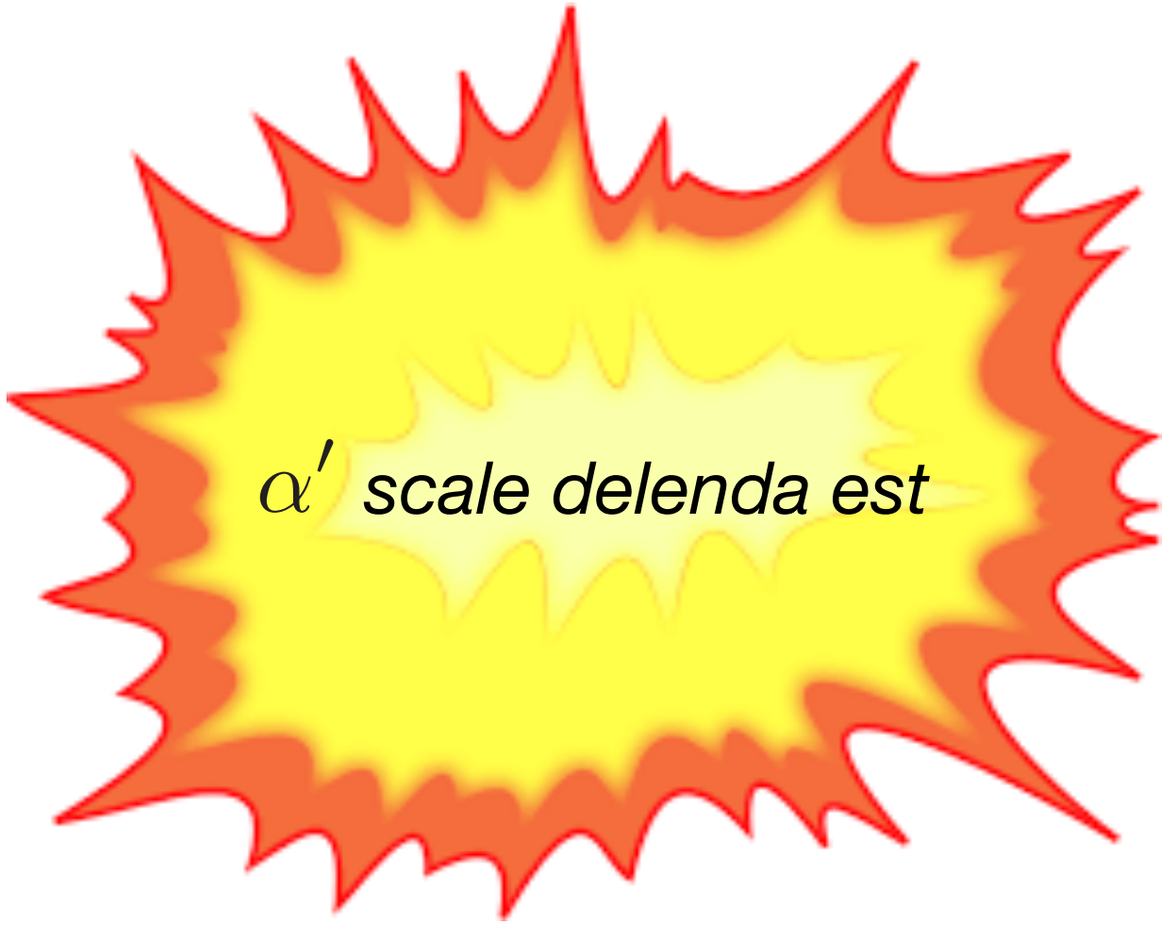

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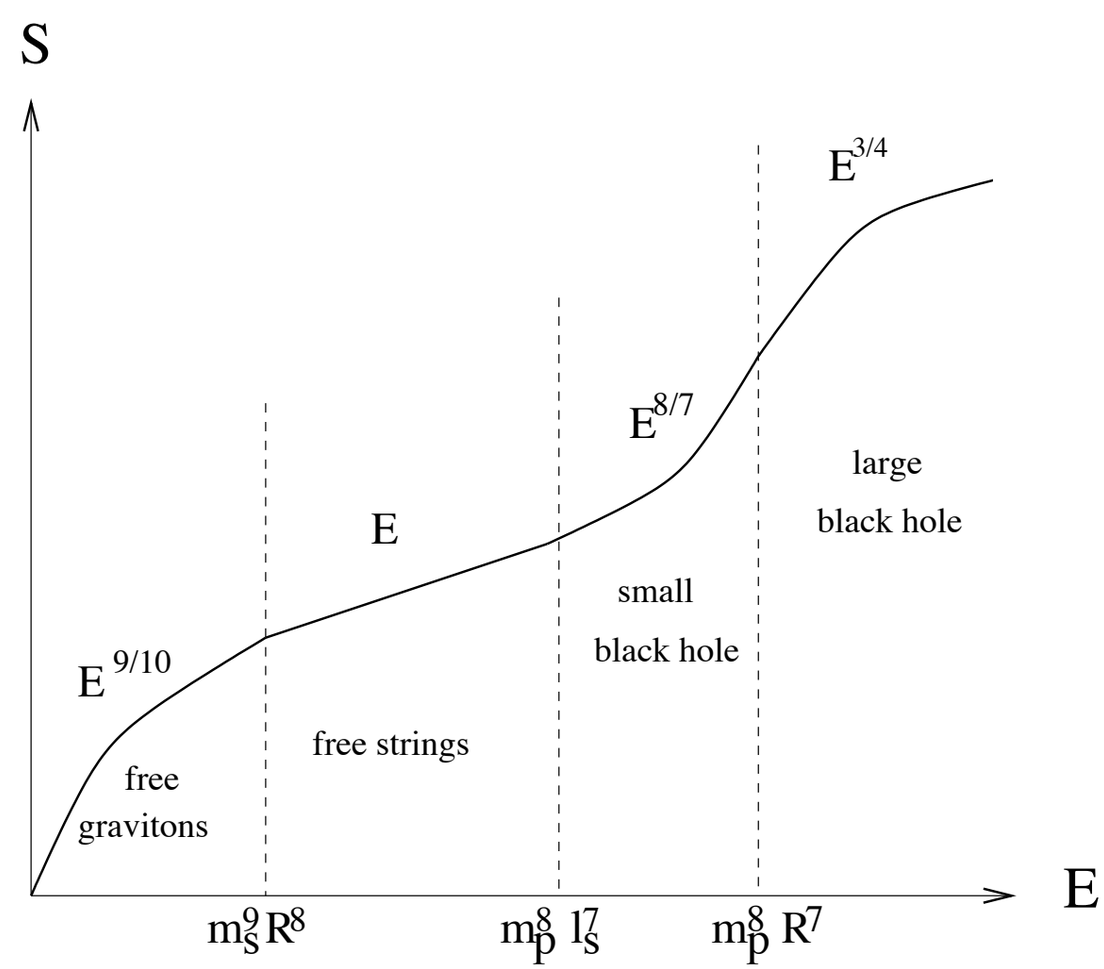

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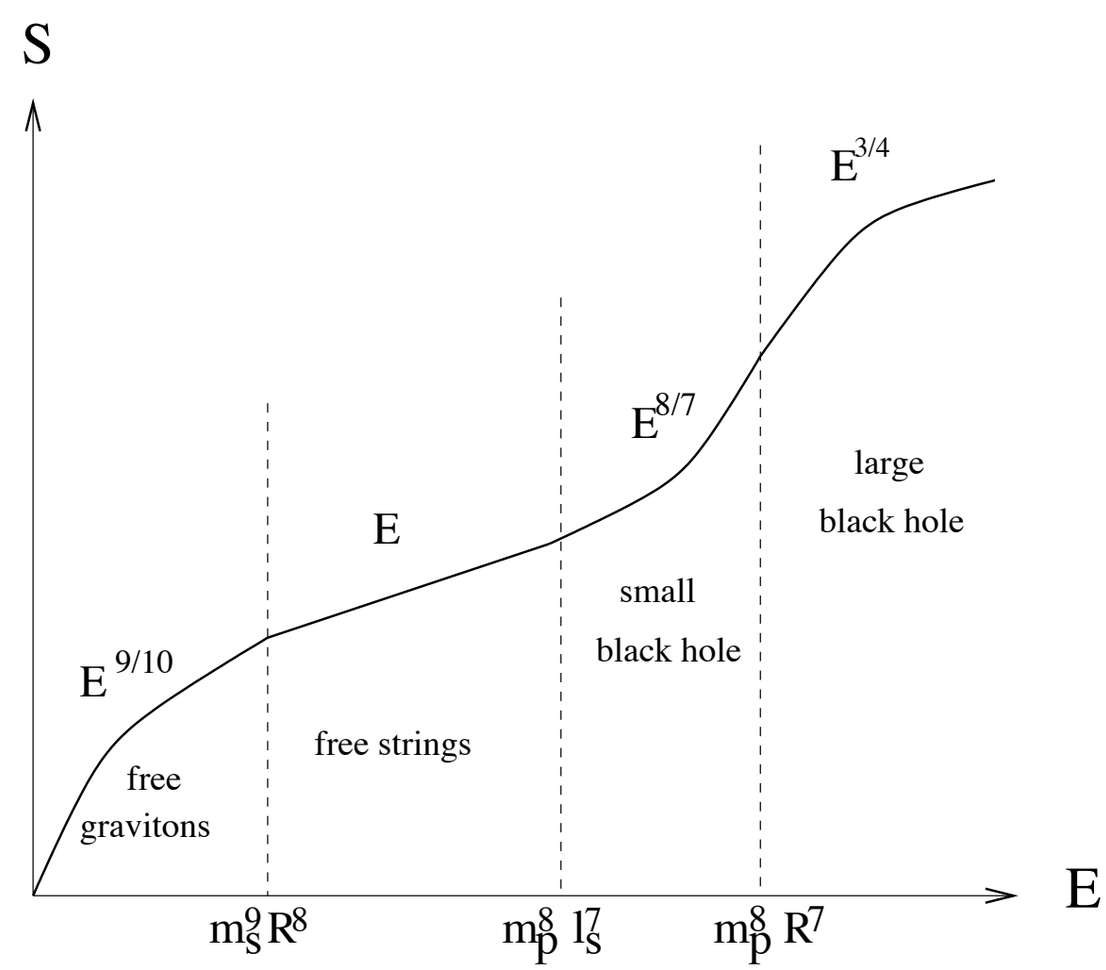

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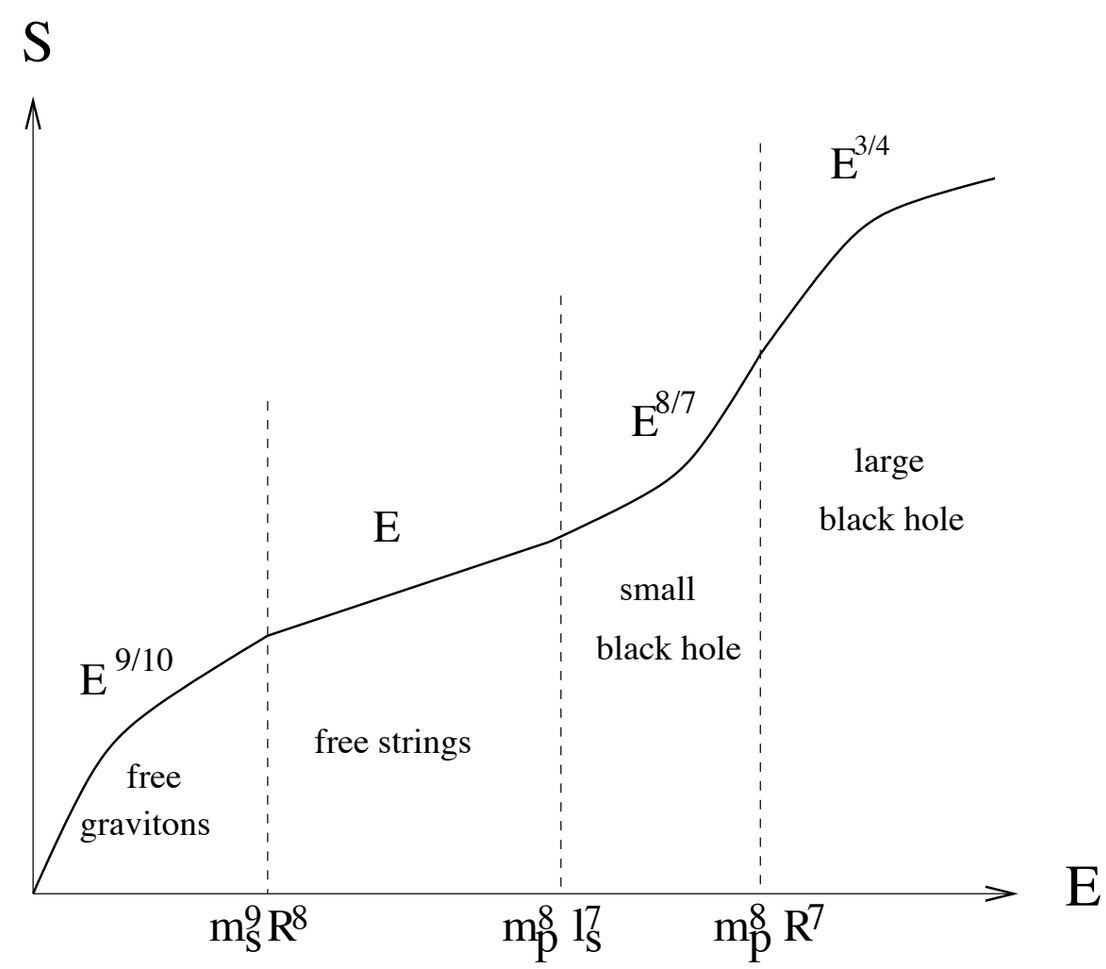

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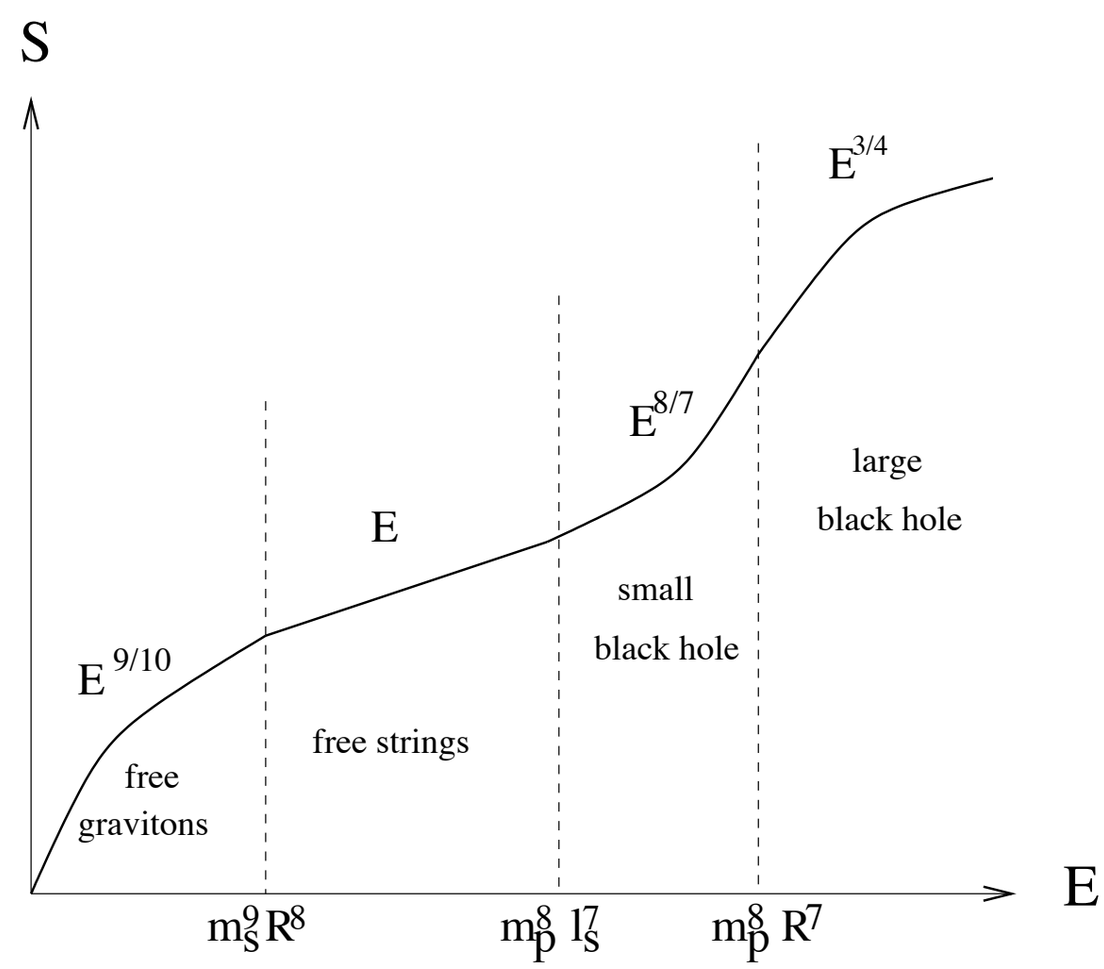

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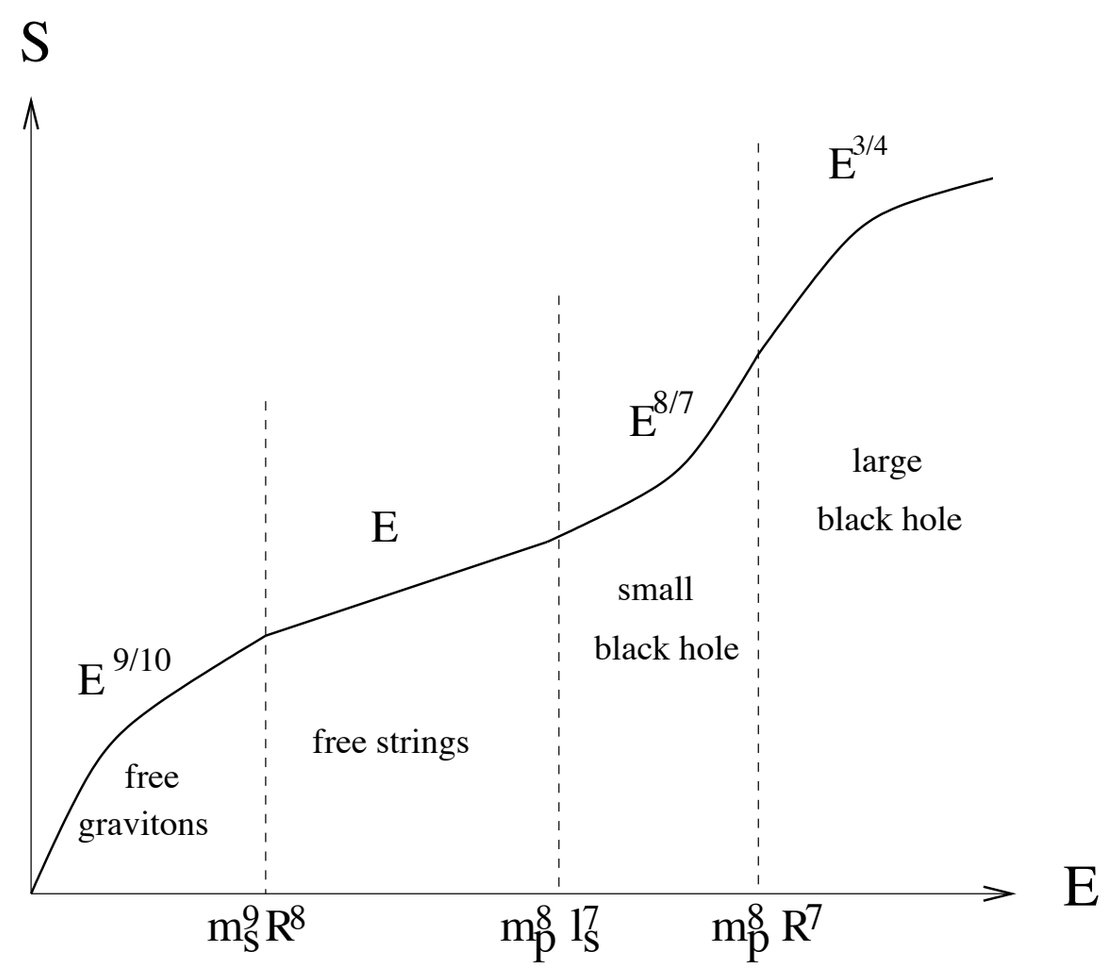

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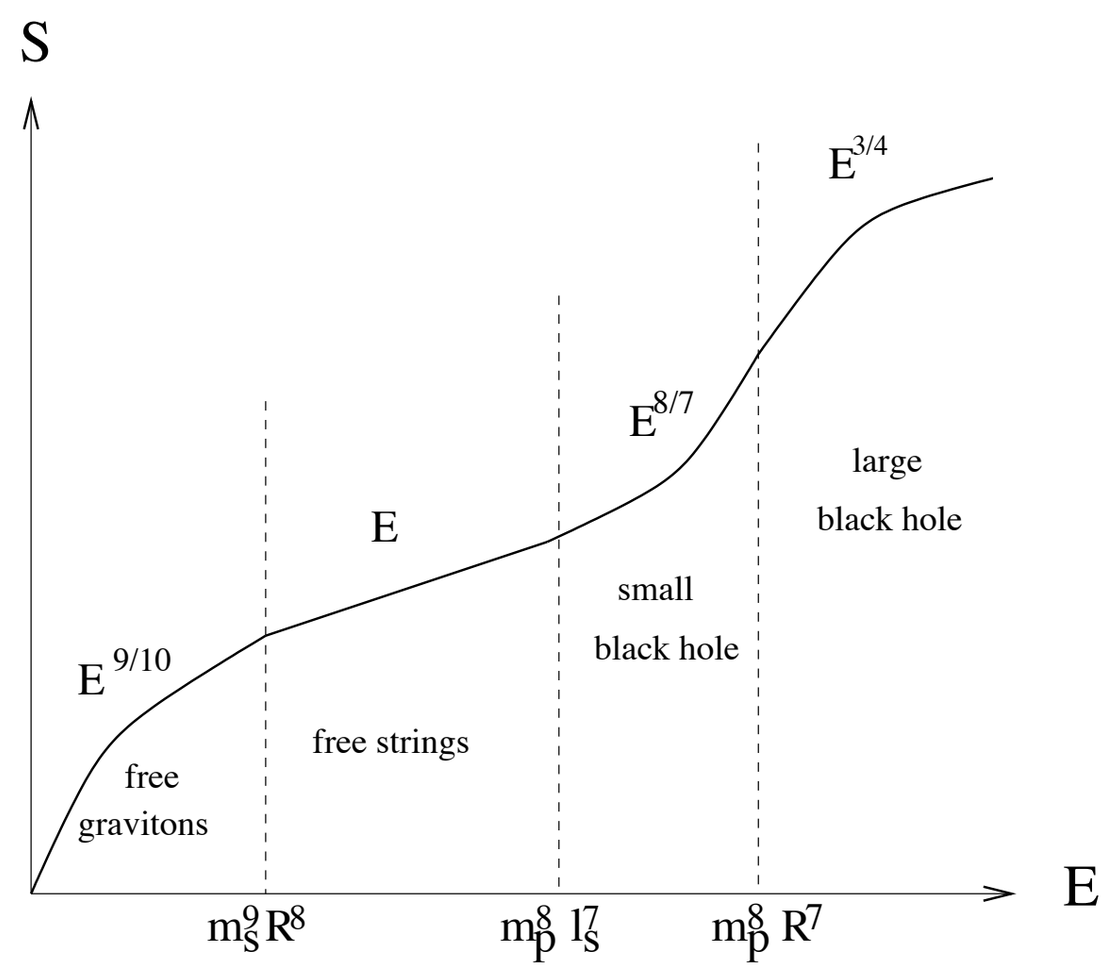
- AdS black holes
- Quite a few phases of gravity…



BH-String Transition

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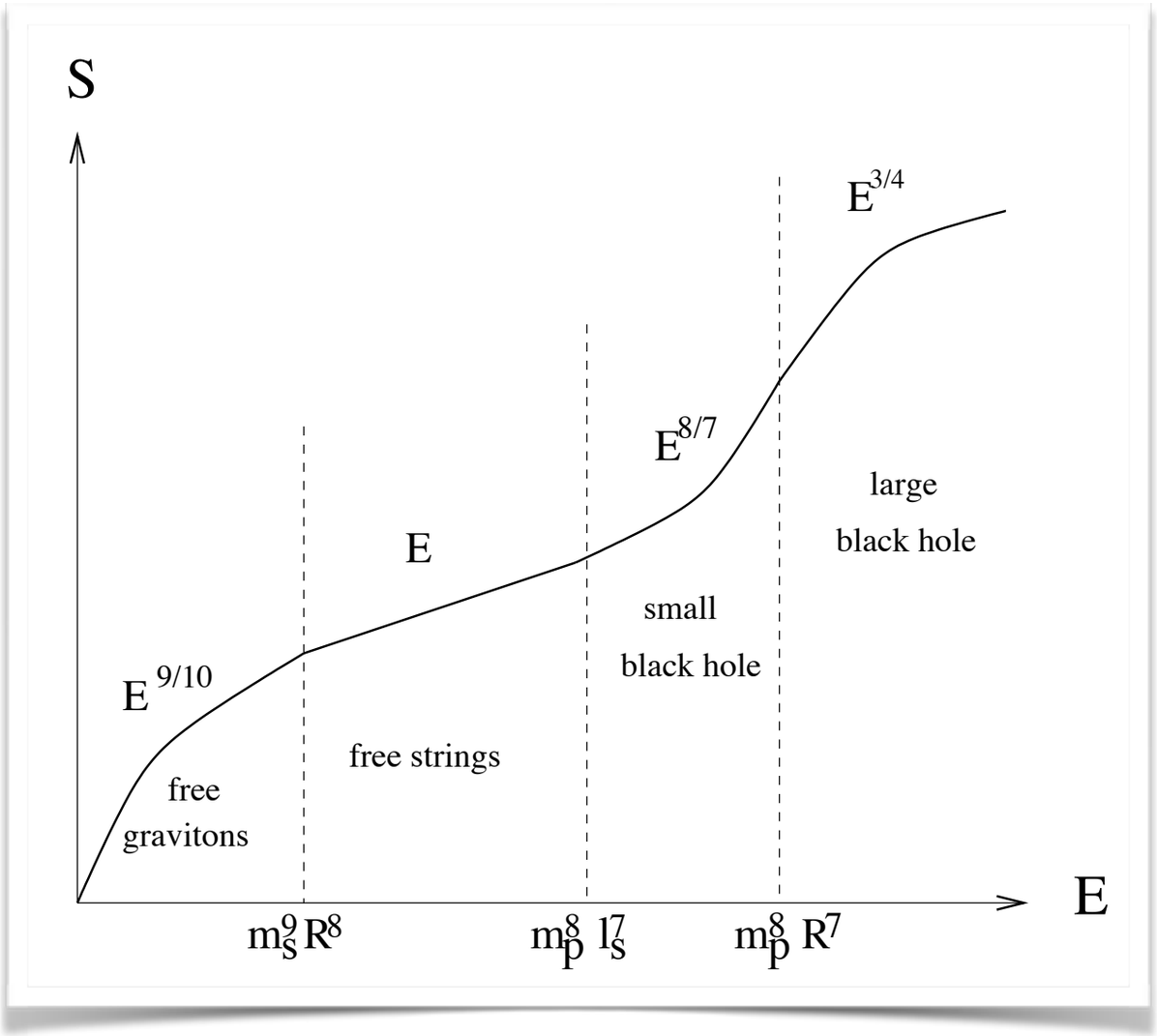

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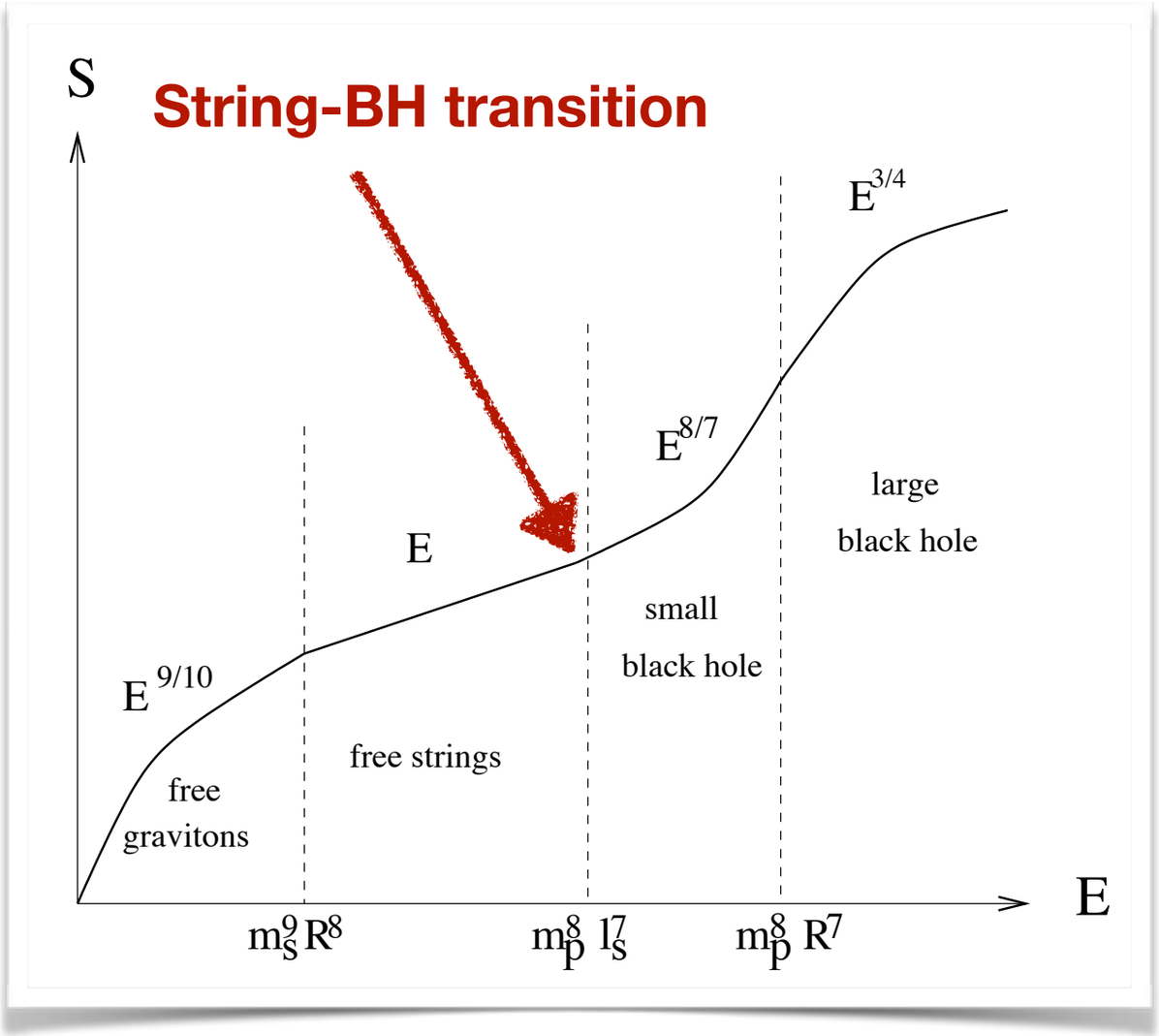

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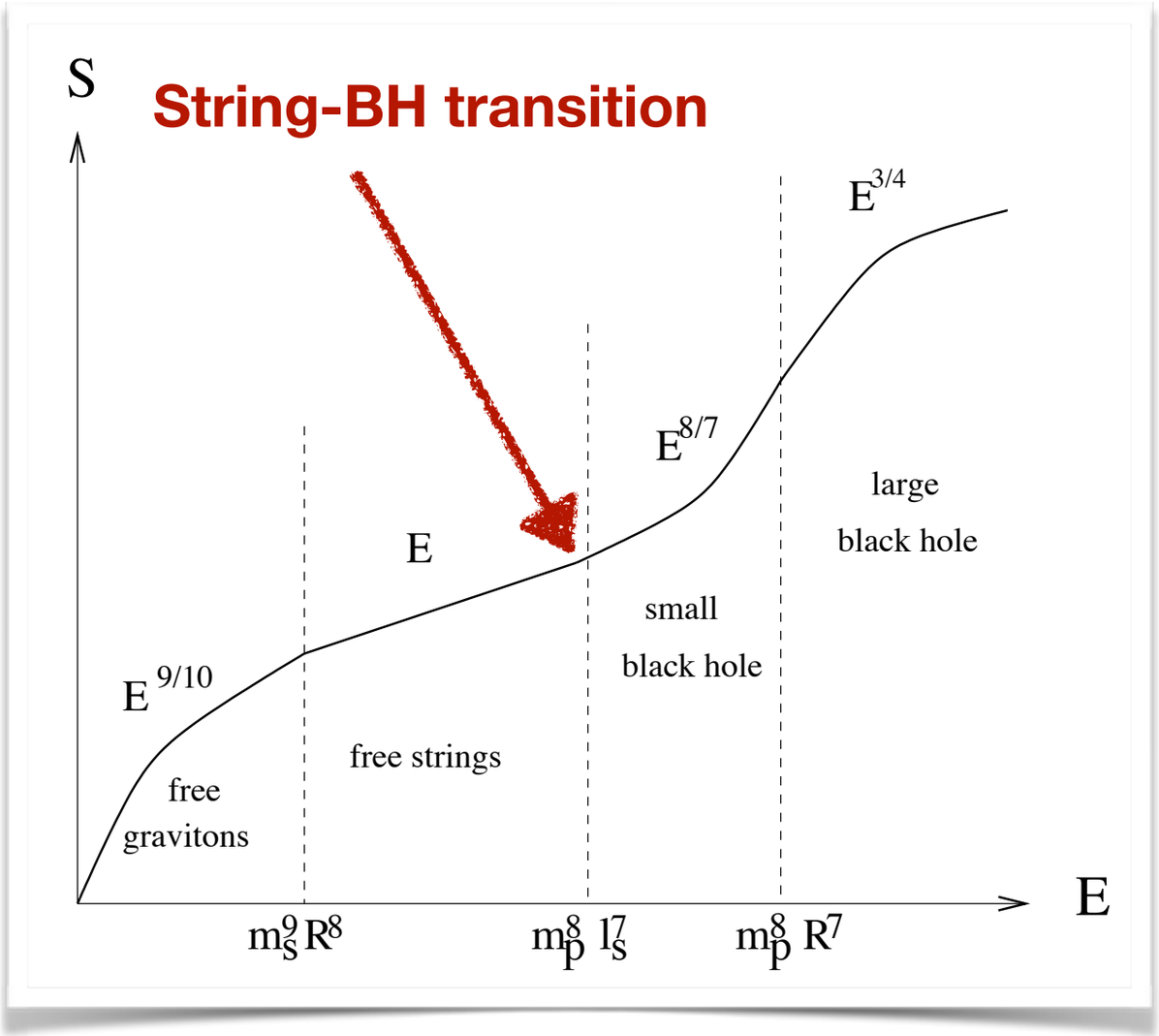

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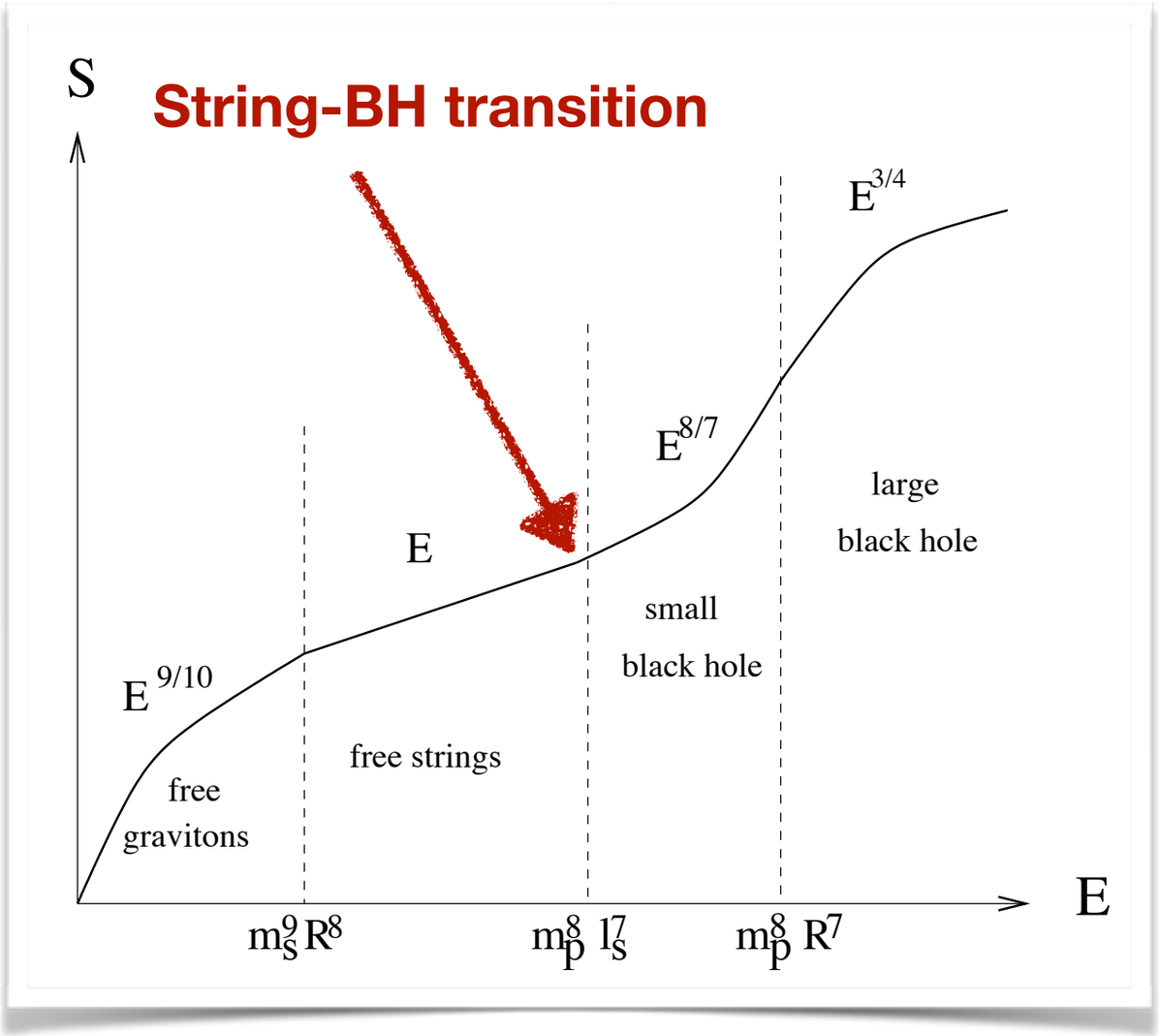

- *"Quark-Hadron transition"*
- One can also engineer a "transition as a number of flavors" in string theory.

On The Road to Tensionless Strings

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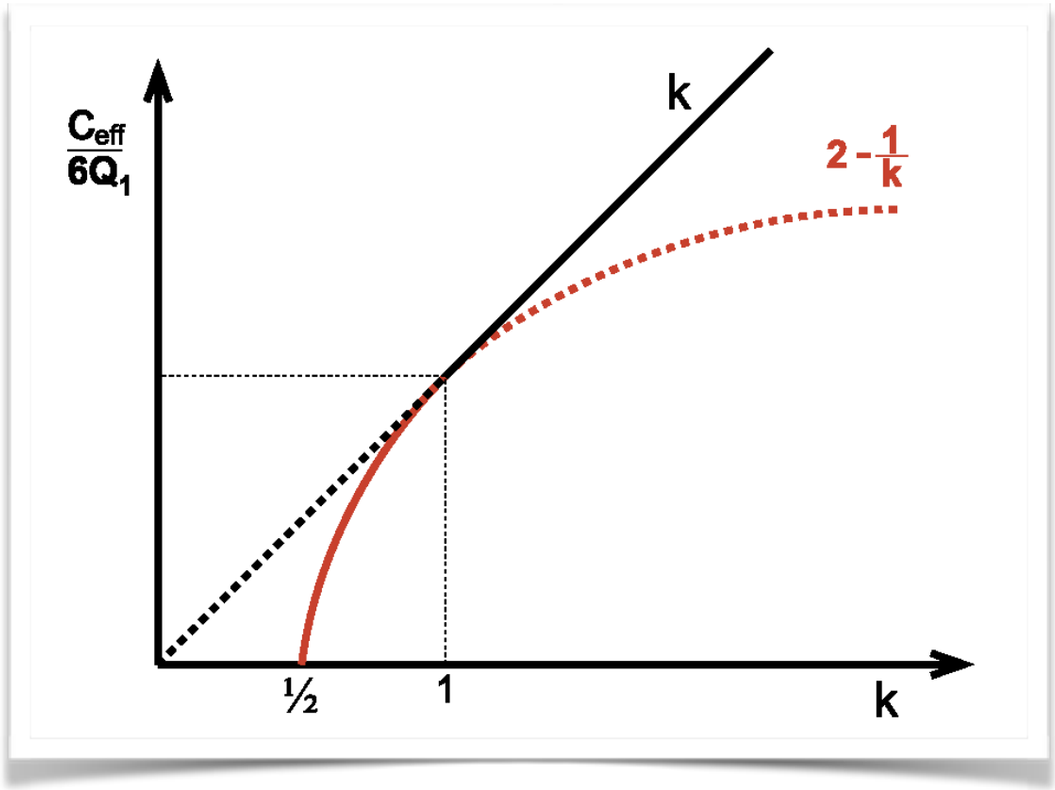

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- An island on the way of having a theory with no scale [Giribet-Kleban-Hull-Porrati-Rabinovici] at k=1
- This is a NEW type of phase transition with a discrete infinity of massless particles.
  More massless particles than at the BEH and Goldstone transitions…

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$$\frac{1}{2}(HK + KH) - D^2 = \frac{g}{4} - \frac{3}{16}$$

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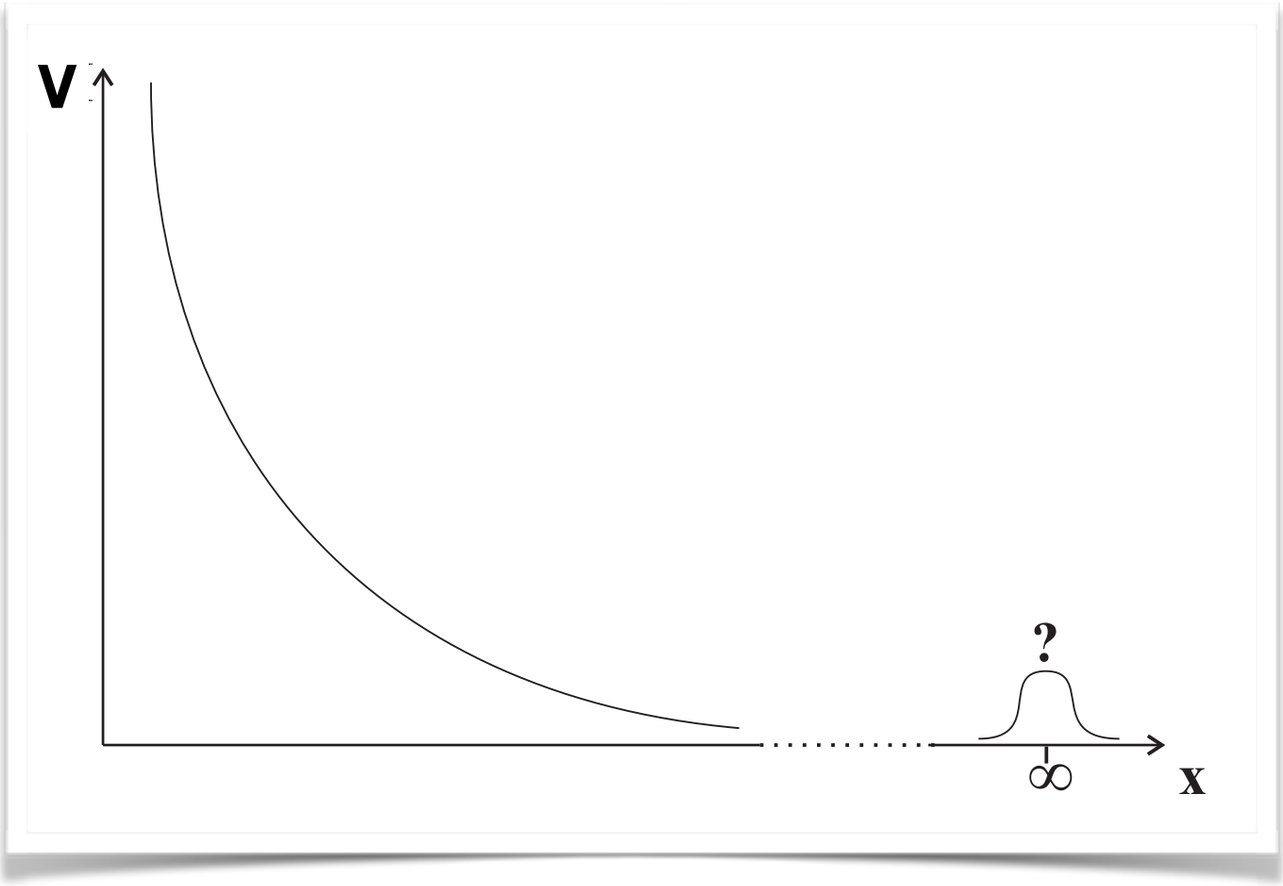
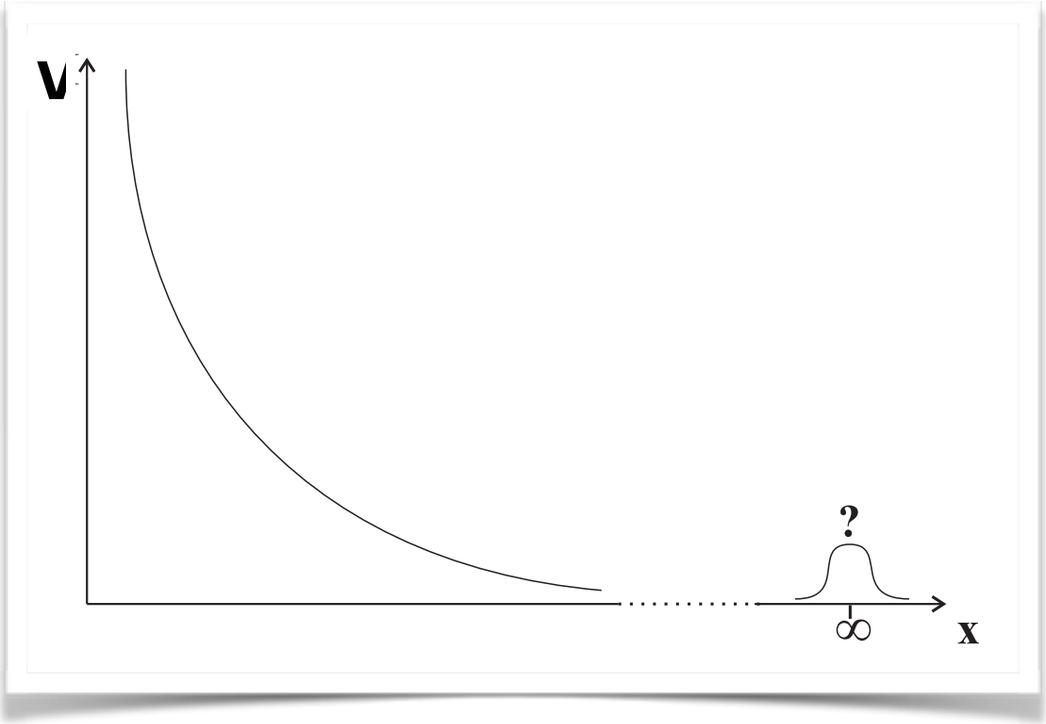
Note that for n=-2 the coupling g has a meaning

g/x^{-2} potential, g>0



Breaking of time translation invariance and SUSY



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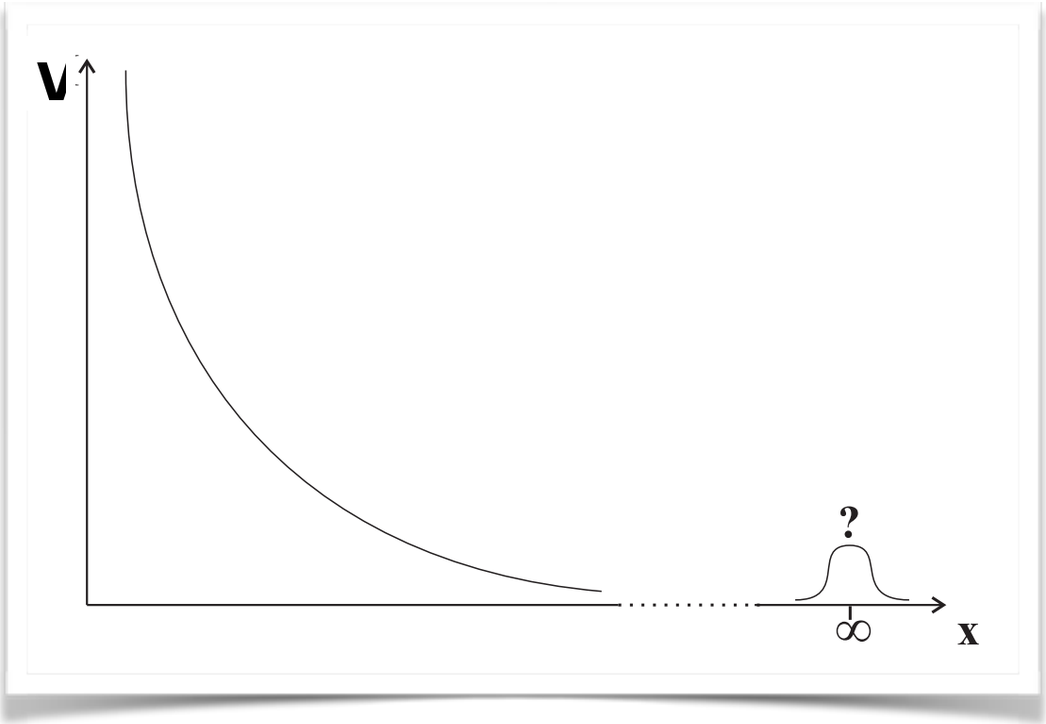

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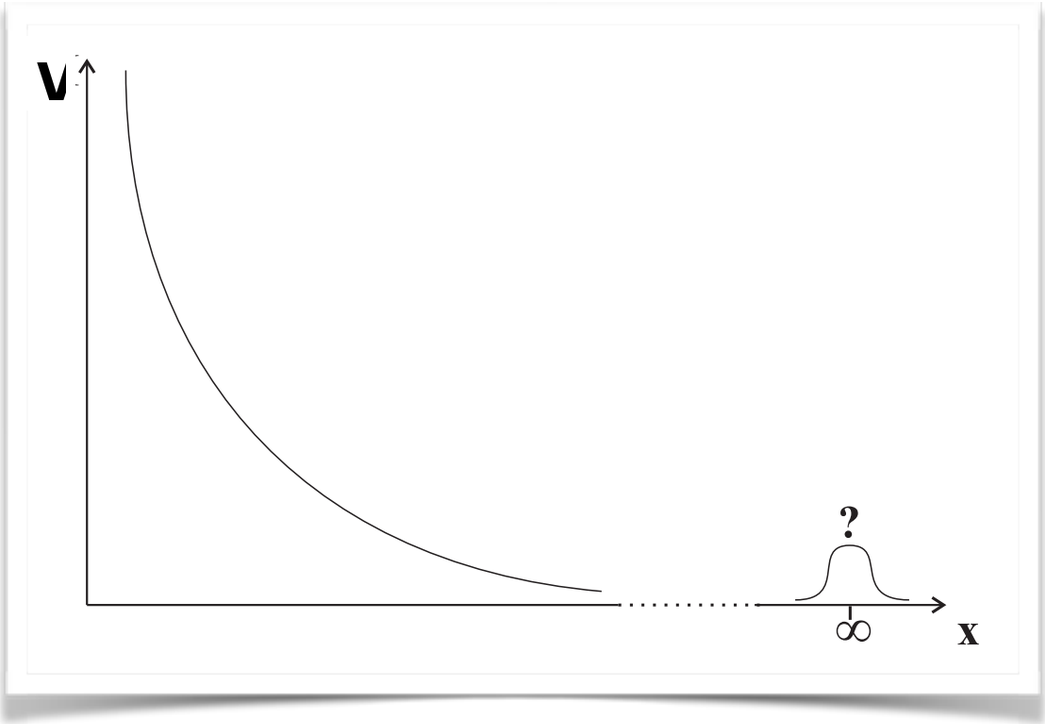

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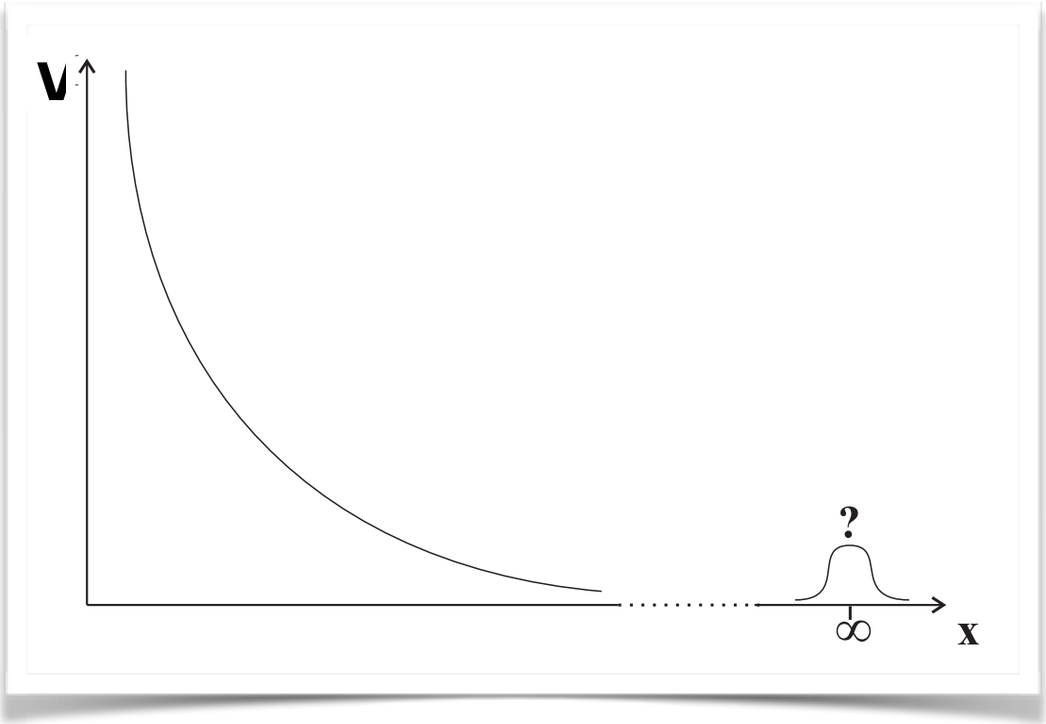

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- The system has an open positive energy spectrum bounded from below but the potential ground state at energy 0 is not plane wave normalizable.
- In the SUSY case, SUSY can't be broken, BUT SUSY will be spontaneously broken by this new mechanism. Also an example of $\mathcal{N}$=2 SUSY breaking down to $\mathcal{N}$=1 SUSY.

Summary

Are there D=4
scale invariant NON SUSY systems
With flat quantum directions?

Are all scales Spontaneously Generated?

Does Nature have runaway potentials of some kind?

g/x^{-2} potential, g>0