Scale invariance and symmetries in inflation

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Scale invariance in particle physics and cosmology
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Outline

Why are universal relations important?

- Relations to underlying principles behind phenomenology
- Independent of model detail
- Symmetries behind

Strong discriminator for different classes of models

Examples

- Squeezed limit of scalar bispectrum & power spectrum

$$\lim_{q\to0}\frac{B_{\mathscr{R}}(q,k_1,k_2)}{P_{\mathscr{R}}(q)} = (1-n_{\mathscr{R}})P_{\mathscr{R}}(k) \quad (k_1\approx k_2\equiv k)$$

- Similar relations also hold

Are such relations from symmetry ?

Examples

- Squeezed limit of scalar bispectrum & power spectrum

$$\lim_{q\to 0} \frac{B_{\mathscr{R}}(q,k_1,k_2)}{P_{\mathscr{R}}(q)} = (1-n_{\mathscr{R}})P_{\mathscr{R}}(k) \quad (k_1 \approx k_2 \equiv k)$$

- Similar relations also hold

Are such relations from ~~symmetry~~ scale invariance?

What am I going to discuss?

Scale invariance in particle physics and cosmology

- Scale invariance during inflation?
- Relation to symmetries during inflation?
- Consequences and further idea?

Scale invariance with gravity

Inclusion of gravity requires a dimensionful coupling

$$S = \int d^4x \sqrt{-g} \frac{m_{\rm Pl}^2}{2} R + S_{\rm matter}$$

Scale invariance is broken from the beginning!

Quadratic action for cosmological perturbations

Spatial metric $g_{ij} = a^2(t)\left(e^h\right)_{ij}$ with $h_{ij} = 2H_L\delta_{ij} + \gamma_{ij} + \cdots$

$$S_2^{(s)} = \int d^4x a^3 m_{\rm Pl}^2 \epsilon \left[\dot{\mathscr{R}}^2 - \frac{(\nabla\mathscr{R})^2}{a^2}\right] \qquad \left(\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1\right)$$

Canonical form with $d\tau = adt$, $u \equiv z\mathscr{R}$ and $z \equiv a\dot{\phi}_0/H$

$$S_2^{(s)} = \int d\tau d^3x \frac{1}{2}\left[u'^2 - (\nabla u)^2 + \underbrace{\frac{z''}{z}}_{\equiv -m^2} u^2\right]$$

Breaking of scale invariance

(Global) rescaling of coordinates x^μ and u (N.B. u has mass dim 1)

$$x^\mu \to e^\alpha x^\mu \quad \text{and} \quad u \to e^{-\alpha} u$$

Associated current is **not** conserved, but slightly broken

$$\partial_\mu j^\mu = \underbrace{\left(m^2 + \frac{1}{2} \frac{dm^2}{d\log\tau} \right)}_{=\mathcal{O}(\epsilon)} u^2$$

Current and Ward-Takahashi identities

- Action as a function of ϕ and $\partial_\mu\phi$: $S = \int d^4x \mathscr{L}(\phi, \partial_\mu\phi)$
- Under $\phi \to \phi + \delta\phi$, $\delta S = \int d^4x\Delta$ with $\partial_\mu j^\mu = \Delta$

WT identities i.t.o. connected Green's functions reads (e.g. Coleman 1985)

$$\frac{\partial}{\partial y^\mu}\left\langle T\left[j^\mu(y)\phi(x_1)\cdots\phi(x_n)\right]\right\rangle = \left\langle T\left[\Delta(y)\phi(x_1)\cdots\phi(x_n)\right]\right\rangle - i\delta^{(4)}(y-x_1)\left\langle T\left[\delta\phi(x_1)\cdots\phi(x_n)\right]\right\rangle - \cdots$$

Symmetries during inflation

During inflation, space-time is almost de Sitter

1. Very special gravitational background
2. Symmetries we enjoy in dS:
   - GR: general coordinate invariance $x^\mu \to x^\mu + \xi^\mu$
   - Time-dependent: time translational symmetry is broken (EFT of inflation with Goldstone mode $\pi = -\mathscr{R}/H$)
   - dS isometries: transformations that leave dS as dS

Isometries of dS

Special coordinate transformations that leave dS as dS

1. Translation: $x^i \to x^i + a^i$
2. Rotation: $x^i \to x^i + \omega^i{}_j x^j$ with $\omega_{ij} = -\omega_{ji}$
3. Special conformal transformations:

$$t \to t - 2H^{-1}(\boldsymbol{b}\cdot\boldsymbol{x}) \quad \text{and}$$
$$x^i \to x^i - b^i\left(-H^{-2}e^{-2Ht} + x^2\right) + 2(\boldsymbol{b}\cdot\boldsymbol{x})x^i$$

At the end of inflation ($t \to \infty$) time is not affected

✓ Dilatation: both time and spatial coord change as

$$t \to t - H^{-1}\log(1+\lambda) \quad \text{and} \quad x^i \to (1+\lambda)x^i$$

Again as $t \to \infty$ only spatial coordinates matter

Scale invariance as a dS isometry

- Scale invariance is one of dS isometries, dilatation symmetry
- Departure from scale inv amounts to departure from perfect dS
- Scale inv manifests itself through appropriate perturbation

We need a good gauge choice

Gauge choice

Different perturbations transform differently under $x^\mu \to x^\mu + \xi^\mu$, e.g. $\gamma_{ij} \to \gamma_{ij}$, so we can choose ξ^μ by setting specific variables zero

$$\text{Comoving gauge: } \delta\phi = 0 \text{ and } \partial_j\left(h_{ij} - 2H_L\delta_{ij}\right) = 0$$

Why dilatation is special in comoving gauge (or vice versa)

Under dilatation $x^i \to (1+\lambda)x^i$, gauge conditions are invariant:

$$\delta\phi \to \left(1 - \lambda x^k \partial_k\right)\delta\phi$$

$$h_{ij} - 2H_L\delta_{ij} \to \left(1 - \lambda x^k \partial_k\right)\left(h_{ij} - 2H_L\delta_{ij}\right)$$

Form of metric remains intact under dilatation: residual symmetry, and e.g. $H_L = 0$ (flat gauge condition) is not preserved

$$H_L \to \begin{cases} H_L - \lambda\left(1 + x^i\partial_i H_L\right) & \text{(dilatation)} \\ H_L + \left[x^2 b^i - 2(\boldsymbol{b}\cdot\boldsymbol{x})x^i\right]\partial_i H_L - 2(\boldsymbol{b}\cdot\boldsymbol{x}) & \text{(SCTs)} \end{cases}$$

WT identities for $\mathscr{R}$ with scale invariance

Integrating WT identity for n-point correlation function of $\mathscr{R}$

- Dilatation is an exact symmetry, so $\Delta = 0$
- Evaluated at the same time, $x_1^0 = x_2^0 = \cdots = x_n^0 \equiv t$

$$\left\langle \left[Q, \mathscr{R}(\boldsymbol{x}_1) \cdots \mathscr{R}(\boldsymbol{x}_n) \right] \right\rangle = -i \left\langle \delta \mathscr{R}(\boldsymbol{x}_1) \cdots \mathscr{R}(\boldsymbol{x}_n) \right\rangle \cdots - i \left\langle \mathscr{R}(\boldsymbol{x}_1) \cdots \delta \mathscr{R}(\boldsymbol{x}_n) \right\rangle$$

with $Q \equiv \int d^3x \, j^0(t, \boldsymbol{x})$: dilatation charge

Why curvature perturbation is special

Q is the generator of transformation under dilatation: $\delta\phi = i[Q,\phi]$

$$\left.\begin{array}{l} \delta\mathscr{R} = -1 - x^i\partial_i\mathscr{R} \\ \delta\phi = -x^i\partial_i\phi \text{ for } \phi \in \{\gamma_{ij}\cdots\} \end{array}\right\} \to \langle Q\mathscr{R}(\boldsymbol{k})\rangle = \frac{i}{2}(2\pi)^3\delta^{(3)}(\boldsymbol{k}) + \text{real part}$$

1. -1 is the Goldstone nature of $\mathscr{R}$
2. Dilatation charge creates $\mathscr{R}$ out of vacuum with $\boldsymbol{k} \approx 0$

Lowest-order relation

Specifying to $n = 2$

- LHS: inserting 1-ptl excited state $P_{\mathscr{R}}^{-1/2}\mathscr{R}(\boldsymbol{k})|\Omega\rangle$ gives

$$-i\left\langle\Omega\middle|\left[Q,\mathscr{R}(\boldsymbol{k}_1)\mathscr{R}(\boldsymbol{k}_2)\right]\middle|\Omega\right\rangle = \lim_{q\to 0}\frac{B_{\mathscr{R}}(q,k_1,k_2)}{P_{\mathscr{R}}(q)}$$

- RHS: using $\delta\mathscr{R} = -1 - x^i\partial_i\mathscr{R}$ gives (with $|\boldsymbol{k}_1| \approx |-\boldsymbol{k}_2| \equiv k$)

$$-\left(3+\boldsymbol{k}_2\cdot\nabla_{\boldsymbol{k}_2}\right)\langle\mathscr{R}(\boldsymbol{k}_1)\mathscr{R}(\boldsymbol{k}_2)\rangle = (1-n_{\mathscr{R}})P_{\mathscr{R}}(k)$$

Equating LHS and RHS gives lowest-order nG consistency relation

WT identities for $\mathscr{R}$ with broken scale invariance

Dilatation charge is not conserved so now Δ term is included:

$$\left\langle \left[i\int_{\tau_0}^{\tau} d\tau' \int d^3y \left(m^2 + \frac{1}{2}\frac{dm^2}{d\log\tau'} \right) u^2(\tau',\boldsymbol{y}), u(\tau,\boldsymbol{x}_1) u(\tau,\boldsymbol{x}_2) \right] \right\rangle$$

This gives

$$\underbrace{n_{\mathscr{R}} = 3 + \frac{d\log|u_k|^2}{d\log\tau}}_{\text{definition of } n_{\mathscr{R}}} + \underbrace{4\Im\left[\frac{u_k^{*2}}{|u_k|^2} \int_{\tau_0}^{\tau} d\tau' \left(m^2 + \frac{1}{2}\frac{dm^2}{d\log\tau'} \right) u_k^2(\tau') \right]}_{=2\left[2-\log 2-\gamma-\log(-k\tau)\right]\epsilon\eta \text{ with } \eta\equiv\dot{\epsilon}/(H\epsilon)}$$

Comoving gauge in multi-field inflation

1. Spatial condition: $\partial_j\left(h_{ij}-2H_L\delta_{ij}\right)=0$, the same
2. Temporal condition: $\dot\phi_{0a}\delta\phi^a=0$ (equiv to $T^0{}_i=0$)
   - Decompose $\delta\phi^a$ along and orthogonal to time evolution

$$\delta\phi^a=\delta\phi^a_\perp+\dot\phi^a_0\pi \quad \text{with} \quad \dot\phi_{0a}\delta\phi^a_\perp=0$$

   - $\dot\phi_{0a}\delta\phi^a=\dot\phi_0^2\pi=0$ means simply $\pi=0$, so MS variables becomes

$$\delta\phi^a-\frac{\dot\phi^a_0}{H}\left(H_L-\frac{\Delta}{3}H_T\right)=\delta\phi^a_\perp-\frac{\dot\phi^a_0}{H}\Bigg(\underbrace{H_L-\frac{\Delta}{3}H_T-H\pi}_{\equiv\mathcal{R}\text{ with }H_T=0\,\&\,\pi=0}\Bigg)$$

1 d.o.f. in gravity, and $n-1$ d.o.f. in matter

Lowest-order relation with multiple fields

While $\delta\phi_\perp$ is orthogonal to time evolution, it can interact with $\mathscr{R}$

$$\lim_{q\to 0}\frac{B_{\mathscr{R}}(k_1,k_2,q)}{P_{\mathscr{R}}(q)} = (1-n_{\mathscr{R}})P_{\mathscr{R}}(k)$$
$$+\lim_{q\to 0}\left[P_a(q)-\frac{P^2_{a\mathscr{R}}(q)}{P_{\mathscr{R}}(q)}-\sum_{b\neq a}\frac{P^2_{ab}(q)}{P_b(q)}\right]^{-1}\frac{P_{a\mathscr{R}}(q)}{P_{\mathscr{R}}(q)}$$
$$\times\left[B_{\mathscr{R}\mathscr{R}a}(k_1,k_2,q)-\frac{P_{a\mathscr{R}}(q)}{P_{\mathscr{R}}(q)}B_{\mathscr{R}}(k_1,k_2,q)\sum_{b\neq a}\frac{P_{ab}(q)}{P_b(q)}B_{\mathscr{R}\mathscr{R}b}(k_1,k_2,q)\right]$$

- Existence of interactions imposes normalization factor
- No assumption except for weak interactions between $\delta\phi^a_\perp$'s

1. Introduction
2. Scale invariance during inflation
3. Symmetries during inflation
4. Consequences and beyond
   - Non-Gaussian consistency relation
   - Spectral tilt of power spectrum
   - Extension to multi-field case
5. Conclusions

Conclusions

- Scale invariance during inflation
  - Scale invariance is broken
  - But the breaking is not strong
- Symmetries during inflation
  - dS isometries that leave dS as dS
  - Scale invariance as one of dS isometries
  - Good gauge choice is necessary: comoving gauge
- Consequences and beyond
  - WT identities based on symmetries
  - Various consequences follow