Building a viable Asymptotically Safe SM

Steven Abel (Durham IPPP)

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w/ Sannino, ArXiV: 1707.06638 Phys.Rev. D96 (2017) no.5, 055021
w/ Molgaard,Sannino, ArXiV: 1812.04856
• Motivation; hierarchy versus triviality

• Asymptotically safe 4D QFTs

• Tetrad model for the ASSM

• Radiative symmetry breaking

• Relevant operators
Hierarchy versus triviality
The triviality problem:
Scalars lead to Landau poles:

=> the theory is UV incomplete

But trying to UV complete it results in the hierarchy problem

The triviality problem:
Hints from QCD

\[ \partial_t \alpha = -B \alpha^2 \]

\[ \alpha_* = 0 \]
Hints from QCD

QCD is (unlike SUSY) a UV complete theory. Why?

1. *There is no hierarchy problem:* quark masses are protected by chiral symmetry

2. *There is no triviality problem:* QCD is asymptotically free

\[ \partial_t \alpha = -B \alpha^2 \]

\[ \alpha_* = 0 \]
QCD is (unlike SUSY) a UV complete theory. Why?

1. *There is no hierarchy problem:* quark masses are protected by chiral symmetry

2. *There is no triviality problem:* QCD is asymptotically free

\[ \partial_t \alpha = -B \alpha^2 \]

But, we do not care about running masses because they do not change the Gaussian UV fixed point. We simply measure them and let them run. Or equivalently, relevant operators are anyway effectively zero in the UV.

*So we don’t even need the chiral symmetry: point 1 becomes irrelevant in this case.*
Asymptotic safety in 4D QFT
**The Basic Idea**

Weinberg used this as a basis for his proposal of UV complete theories.

Gaussian IR fixed point $\Rightarrow$ perturbative

Interacting UV fixed point $\Rightarrow$ finite anomalous dimensions

In a field theory replace $1/\epsilon$ with $1/\gamma$ $\Rightarrow$ divergences of marginal operators (which affect the fixed point), some cured

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Gastmans et al '78  
Weinberg '79  
Peskin  
Reuter, Wetterich  
Gawedski, Kupiainen  
Kawai et al,  
de Calan et al',  
Litim  
Morris


**Categorise the possible content of a theory as follows:**

**Irrelevant operators:** would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are assumed zero (exactly renormalizable trajectory)

**Marginal operators:** can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free) or irrelevant.

**Relevant operators:** become “irrelevant” in the UV but may determine the IR fixed point.

**Dangerously irrelevant operators:** grow in both the UV and IR (common in e.g. SUSY)

**Harmless relevant operators:** shrink in both the UV and IR
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Note relevant or marginally relevant operators still have “infinites” at the FP - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (By definition they become unimportant at in the UV.)
Caswell-Banks-Zaks fixed point:

Take QCD with $SU(N_C)$ and $N_F$ fermions but very large numbers of colours+flavours

\[ \partial_t \alpha = -B \alpha^2 + C \alpha^3 \]

Turns out $C > 0$, $B > 0$: theory has stable IR fixed point at $\alpha = B/C$ and unstable one in UV $\alpha = 0$

\[ B \propto \epsilon = \frac{N_F}{N_C} - \frac{11}{2} \]

Note perturbativity: $B \ll C$

requires many fields (Veneziano limit) with $N_F \approx 11N_C/2$

Familiar from weakly coupled supersymmetry where $N_F \lesssim 3N_C$ in $\mathcal{N} = 1$ case
Cartoon of a would-be Interacting UV FP:

Again would have …

\[
\partial_t \alpha = -B\alpha^2 + C\alpha^3
\]

But requires \( C < 0, B < 0 \), this theory has *stable* IR fixed point at \( \alpha = 0 \) and *unstable* UV one at \( \alpha = B/C \)

At \( t \to \infty \) the coupling ends up here (and fields have finite anomalous dimensions)

Again perturbativity would require

\[
N_F \approx 11N_C / 2
\]
Real situation requires several couplings to realise

Litim & Sannino ’14

Need to add scalars and Yukawa couplings:

\[ \mathcal{L} = -\frac{1}{2} \text{Tr} F^{\mu\nu} F_{\mu\nu} + \text{Tr} (\overline{Q} i D Q) + y \text{Tr} (\overline{Q} H Q) + \text{Tr} (\partial_\mu H^\dagger \partial^\mu H) \]
\[ -u \text{Tr} [(H^\dagger H)^2] - v (\text{Tr} [H^\dagger H])^2 , \]

\( H \) is an \( N_F \times N_F \) scalar
Initially have \( U(N_F)_L \times U(N_F)_R \) flavour symmetry
Quiver diagram for this model:

<table>
<thead>
<tr>
<th></th>
<th>$SU(N_C)$</th>
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Four 't Hooft-like couplings - flow could in principle be four dimensional

\[
\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \quad \alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}
\]

but driven by the Yukawa we find 1D trajectories…

\[
\beta_g = \alpha_g^2 \left[ \frac{4}{3} \epsilon + \left( \frac{25}{3} + \frac{26}{3} \epsilon \right) \alpha_g - 2 \left( \frac{11}{2} + \epsilon \right)^2 \alpha_y \right]
\]

\[
\beta_y = \alpha_y \left( 13 + 2 \epsilon \right) \alpha_y - 6 \alpha_g
\]
Along the critical-curve/exact-trajectory can parameterise the flow in terms of $\alpha_g(t)$

\[
\begin{align*}
\alpha_y(t) &= \frac{6}{13} \alpha_g(t), \\
\alpha_h(t) &= 3\frac{\sqrt{23} - 1}{26} \alpha_g(t), \\
\alpha_v(t) &= 3\frac{\sqrt{20} + 6\sqrt{23} - 6\sqrt{23}}{26} \alpha_g(t),
\end{align*}
\]

At the fixed point it is arbitrarily weakly coupled, $\alpha_g^* = 0.4561 \epsilon$, where $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$
Figure 1: The renormalisation group flow of the marginal couplings from the UV fixed point and around the critical curve, towards the Gaussian IR fixed point.

Values of the renormalisation time $t = \ln \mu/\mu_0$ by [3, 60]

\[
\begin{align*}
g(t) &= \left( \frac{\mu}{\mu_0} \right)^{\frac{1}{2} \frac{1}{W(e^{4/3})}} g(t), \quad (6a) \\
y(t) &= \frac{1}{3} g(t), \quad (6b) \\
h(t) &= \frac{3\sqrt{23}}{23} \frac{1}{26} g(t), \quad (6c) \\
v(t) &= \frac{3\sqrt{20} + 6\sqrt{23}}{26} g(t), \quad (6d)
\end{align*}
\]

Where $W$ is the Lambert W-function (a.k.a. the product log defined by $W(z)e^{W(z)} = z$) and $\epsilon$ is defined by the initial condition, $g(0) = g_1 + \frac{1}{W(e^{4/3})} g.$ \quad (7)

Perturbation theory is valid for all values of $t$ as long as $\epsilon$ is small.

Since we can access all scales through this set of solutions, the initial gauge coupling is the only free parameter distinguishing different physical systems that flow from the UV fixed point, and must be set by hand in accord with the measurement of the coupling at some scale. However, as mentioned above one can simply use the gauge coupling itself to parameterise the flow along the critical curve linking the UV interacting fixed point to the IR non-interacting one (also known as the separatrix): it is a monotonically increasing function of $\mu.$
Tetrad Model for the ASSM...
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Large UV Safe theory

SM
Tetrad Model - focus on breaking SU(Nc) to SU(3) colour with new scalars …

c.f. Gies, Jaeckel, Wetterich ‘04; Bond, Litim; Bond, Hiller, Kowalska, Litim; Gies, Rechenberger, Scherer, Zambelli; Pelaggi, Plascencia, Salvio, Sannino, Smirnov; Molinaro, Sannino, Wang; Mann, Meffe, Sannino, Steele, Wang, Zhang,

\[ SU(2)_R = [SU(2)_r \otimes SU(2)_S]_{\text{diag}} \]

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Tetrad Model - focus on breaking $SU(N_c)$ to $SU(3)$ colour with new scalars ...

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$$SU(2)_R = [SU(2)_r \otimes SU(2)_s]_{\text{diag}}$$

| $Q_{ai}$ | $SU(N_C)$ | $SU(N_F) \supset SU(2)_L \otimes SU(n_F)_L$ | $SU(N_F)_R \supset SU(2)_r \otimes SU(n_F)_r$ | $SU(N_S) = SU(N_C -{|N_F|}_2) \oplus SU(2)_S$ | spin |
| --- | --- | --- | --- | --- | --- |
| $Q^{ia}$ | | | | | |
| $H^i_j$ | 1 | | | | |
| $S_{a,\ell=1..N_S}$ | | | | | |
| $q^i_\ell$ | 1 | | | | |
| $q^i_\ell$ | 1 | | | | |

Extension of Pati-Salam - breaks to $SU(3)$ if we choose $N_S = N_C - 2$

$$\frac{N_S}{N_C} \rightarrow 1; \quad \frac{N_F}{N_C} \rightarrow \frac{21}{4} + \epsilon$$

$$\tilde{S} = \left( \begin{array}{cccc} \tilde{d}^c & \tilde{c}^c & \tilde{\phi}^{-1} & \cdots & \tilde{\phi}^{-\frac{1}{2}} \\ \tilde{u}^c & \tilde{d}^c & \tilde{\phi}^{-\frac{1}{2}} & \cdots & \tilde{\phi}^{\frac{1}{2}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{T}^{-\frac{1}{6}} & \tilde{\phi}^{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \\ \end{array} \right)$$

$$N_S = N_C - 2$$
Weak breaking must then occur along the H-Higgs directions:

\[
H = \begin{pmatrix}
\begin{pmatrix} h_u^0 & h_u^- \\ h_u^+ & h_u^d \end{pmatrix} & \begin{pmatrix} h_u^0 & h_u^- \\ h_u^+ & h_u^d \end{pmatrix} & \begin{pmatrix} h_u^0 & h_u^- \\ h_u^+ & h_u^d \end{pmatrix} & \cdots \\
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\vdots & \vdots & \vdots & \ddots \\
\end{pmatrix}
\]

Assignment implies 9 pairs of Higgses one for each Yukawa coupling.
Explicit embedding looks like P-S with \( SU(N_C) \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \)

\[
Q = \left( \begin{array}{cccc}
q_1 & \ell_1 & \cdots & \left( \begin{array}{c}
\psi_1^\frac{1}{2} \\
\psi_{-\frac{1}{2}}
\end{array} \right) \\
q_2 & \ell_2 & \cdots & \left( \begin{array}{c}
\psi_1^\frac{1}{2} \\
\psi_{-\frac{1}{2}}
\end{array} \right) \\
q_3 & \ell_3 & \cdots & \left( \begin{array}{c}
\psi_1^\frac{1}{2} \\
\psi_{-\frac{1}{2}}
\end{array} \right) \\
\vdots & \vdots & \ddots & \ddots
\end{array} \right)_{N_C}
\]

\( N_S = N_C - 2 \)

\[
q = \left( \begin{array}{cccc}
\psi_0 & \psi_1 & \cdots & \psi_1^\frac{1}{2} \\
\psi_{-1} & \psi_0 & \cdots & \psi_{-\frac{1}{2}} \\
\psi_0 & \psi_1 & \cdots & \psi_1^\frac{1}{2} \\
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\end{array} \right)
\]

\( \tilde{q} = \left( \begin{array}{cccc}
\tilde{\psi}_0 & \tilde{\psi}_{-1} & \tilde{\psi}_1^\frac{1}{2} & \tilde{\psi}_{-\frac{1}{2}} \\
\tilde{\psi}_1 & \tilde{\psi}_0 & \tilde{\psi}_1^\frac{1}{2} & \tilde{\psi}_{-\frac{1}{2}} \\
\tilde{\psi}_{-1} & \tilde{\psi}_0 & \tilde{\psi}_1^\frac{1}{2} & \tilde{\psi}_{-\frac{1}{2}} \\
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q_3 & \ell_3 & \cdots & \Psi_{\frac{1}{2}} & \Psi_{-\frac{1}{2}} & \cdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots 
\end{pmatrix}
\]

\[ N_S = N_C - 2 \]

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\psi_{-1} & \psi_0 \\
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\tilde{q} = \begin{pmatrix}
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\vdots & \vdots \\
\tilde{\psi}_0 & \tilde{\psi}_{-1} \\
\tilde{\psi}_1 & \tilde{\psi}_0 \\
\vdots & \vdots 
\end{pmatrix}
\]

Little q's required (by chiral symmetry) to remove the extra SU(2) doublets: (Nc-4) uncharged under SU(2)R
And the couplings that do this are as follows:

\[ \mathcal{L}_{\text{UVFP}} \supset \mathcal{L}_{\text{KE}} + \frac{y}{\sqrt{2}} \text{Tr} \left[ (QH) \cdot \tilde{Q} \right] + \frac{\tilde{y}}{\sqrt{2}} \text{Tr} \left[ qH^\dagger \tilde{q} \right] - \frac{\tilde{Y}}{\sqrt{2}} \text{Tr} \left[ (\tilde{S} \cdot Q) \tilde{q} \right] - \frac{Y}{\sqrt{2}} \text{Tr} \left[ (\tilde{Q} \cdot \tilde{S}^\dagger) q \right] \]

- \ u_1 \text{Tr} \left[ H^\dagger H \right]^2 - u_2 \text{Tr} \left[ H^\dagger H H^\dagger H \right] - v_1 \text{Tr} \left[ H^\dagger H \right] \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \right]
- w_1 \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \right]^2 - w_2 \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \tilde{S}^\dagger \cdot \tilde{S} \right]

Note expect relatively light (TeV scale) q-states looking like “higgsinos”
And the couplings that do this are as follows:

\[ L_{UVFP} \supset L_{KE} + \frac{y}{\sqrt{2}} \text{Tr} \left[ (QH) \cdot \tilde{Q} \right] + \frac{\tilde{y}}{\sqrt{2}} \text{Tr} [qH^\dagger \tilde{q}] - \frac{\tilde{Y}}{\sqrt{2}} \text{Tr}[\left( \tilde{S} \cdot Q \right) \tilde{q}] - \frac{Y}{\sqrt{2}} \text{Tr}[\left( \tilde{Q} \cdot S^\dagger \right) q] \]

- \[ u_1 \text{Tr} [H^\dagger H]^2 - u_2 \text{Tr} [H^\dagger H H^\dagger H] - v_1 \text{Tr} [H^\dagger H] \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \right] \]

- \[ w_1 \text{Tr} \left[ \tilde{S}^\dagger \cdot \tilde{S} \right]^2 - w_2 \text{Tr} \left[ \tilde{S}^\dagger \cdot S \tilde{S}^\dagger \cdot \tilde{S} \right] \]

For later use define rescaled c’pgs: \[ \alpha_g = \frac{N_C g^2}{(4\pi)^2}; \quad \alpha_y = \frac{N_C y^2}{(4\pi)^2}; \quad \alpha_{\tilde{y}} = \frac{N_C \tilde{y}^2}{(4\pi)^2}; \quad \alpha_Y = \frac{N_C Y^2}{(4\pi)^2}; \quad \alpha_{\tilde{Y}} = \frac{N_C \tilde{Y}^2}{(4\pi)^2}; \]

\[ \alpha_{u_1} = \frac{N_F^2 u_1}{(4\pi)^2}; \quad \alpha_{u_2} = \frac{N_F u_2}{(4\pi)^2}; \quad \alpha_{v_1} = \frac{N_F^2 v_1}{(4\pi)^2}; \quad \alpha_{w_1} = \frac{N_F^2 w_1}{(4\pi)^2}; \quad \alpha_{w_2} = \frac{N_C w_2}{(4\pi)^2} \]
In case you’re suffering from “expectation versus reality syndrome” …
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Or equivalently …
A quiver diagram is useful to see (at least some of) what we did:

Before:

\[ SU(N_c) \]

\[ SU(N_F)_L \to H \to SU(N_F)_R \]

Fig. 1: Quiver diagram of the fixed point theory of [7]. Solid lines represent fermions, dashed lines represent bosons.
After: (hence the name Tetrad)
As this model is based on LS, the same UVFP applies (see later). But what about AS for
the SU(2)xSU(2) electroweak gauge groups?

These see a large number of flavours (Nf (small f) of order order Nc)?

This gives UVFP behaviour with a fixed point at ’t Hooft couple ~ 1 … if Nf >>16:

Palanques Mestre, Pascual; Gracey; Holdom;
Shrock; Antipin, Pica, Sannino
As this model is based on LS, the same UVFP applies (see later). But what about AS for the SU(2)xSU(2) electroweak gauge groups?

These see a large number of flavours (Nf (small f) of order order Nc)?

This gives UVFP behaviour with a fixed point at ’t Hooft couple $\sim 1$ … if Nf $>>$ 16:

Resum first terms gives

$$\frac{3 \beta \tilde{\alpha}}{4 \tilde{\alpha}^2} = 1 + \frac{H(\tilde{\alpha})}{N_f} + O(N_f^{-2})$$

$$H(\tilde{\alpha}) = \frac{1}{4} \log |3 - 2\tilde{\alpha}| + \text{constant}$$

Palanques Mestre, Pascual; Gracey; Holdom; Shrock; Antipin, Pica, Sannino
Overall the picture is ...
Can show by power counting that the two kinds of UVFP decouple.
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In the Veneziano limit the corrections to the weak FP go like epsilon. Can neglect everything but SU(2) gauge couplings when determining the SU(2) fixed points.
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i.e.

\[ \alpha \sim \frac{1}{N_f} \alpha^2 \sim \frac{1}{N_f} \alpha^3 \sim \frac{1}{N_f} \alpha^{(L-1)} \]

\[ \alpha_g \tilde{\alpha} \sim \epsilon \tilde{\alpha} \]

\[ \alpha_y \tilde{\alpha} \sim \epsilon \tilde{\alpha} \]

\[ \frac{\epsilon}{N_f} \tilde{\alpha}^{(L-1)} \]

\[ \frac{\epsilon}{N_f} \tilde{\alpha}^{(L-1)} \]
Conversely for the SU(Nc) fixed point ...

\[ x_F \alpha_g \]

\[ x_F \alpha_g^2 \sim \epsilon^2 \]

\[ x_F \alpha_y \alpha_g \sim \epsilon^2 \]

\[ \frac{1}{N_f N} \tilde{\alpha} \alpha_g \sim \frac{\epsilon}{N_f^2} \tilde{\alpha} \lessapprox \epsilon^3 \tilde{\alpha} \]
Radiative symmetry breaking
• Suppose that the classically relevant operators are negligible. (compared to the scales we are about to generate.)

• Then Coleman-Weinberg radiative symmetry breaking is induced along the flow.

• First look at Yukawas:

![Graph showing nearby fixed points and true UV fixed point with cross-over induced by Y couplings]
• This in turn induces a flow in the quartic couplings driving them negative: we essentially have Gildener-Weinberg breaking of the extended PS symmetry.

• Note that the H mass-squareds are all positive at this scale.
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• Note that the H mass-squareds are all positive at this scale.
Adding relevant operators (mass-squareds)
Organize relevant operators in terms of the $U(N_F) \times U(N_F)$ flavour symmetry that we break with the mass-squareds (closed under RG):

$$H = \frac{(h_0 + i p_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + i p_a) T_a$$

$$\mathcal{L}_{\text{Soft}} = -m_{h_0}^2 \text{Tr} [H^\dagger H] - \sum_{a=1}^{N_F^2-1} \Delta_a^2 \text{Tr} [HT^a] \text{Tr} [H^\dagger T^a]$$

*Then solve Callan Symanzik eqn for them as usual =>*
Non-trivial simple example...

Consider case where the trace component has a slightly smaller mass-squared:

\[
V_{class}^{(2)} = m_0^2 \text{Tr}(H^\dagger H) + 2\Delta^2 \sum_a \text{Tr}(T_a H^\dagger)\text{Tr}(T_a H)
\]

\[
\begin{align*}
m^2 &
\Delta^2 \begin{cases}
m_{ha}^2 &= m_{pa}^2 = m_0^2 + \Delta^2 \\
m_{h0}^2 &= m_{p0}^2 = m_0^2
\end{cases}
\end{align*}
\]
Non-trivial simple example...

After some work find the following answer in terms of two RG invariants, one for each independent (non-predicted) relevant operator (where \( \nu = (1 - 1/N_F^2) \)):

\[
m_0^2 = \tilde{m}_*^2 \left( \frac{\alpha_{g}^*}{\alpha_{g}} - 1 \right)^{-\frac{3f_{m_0}}{4\epsilon}} - \Delta_{*}^2 \nu \left( \frac{\alpha_{g}^*}{\alpha_{g}} - 1 \right)^{-\frac{3f_{\Delta}}{4\epsilon}},
\]

\[
m_{a=1...N_F^2-1}^2 = \tilde{m}_*^2 \left( \frac{\alpha_{g}^*}{\alpha_{g}} - 1 \right)^{-\frac{3f_{m_0}}{4\epsilon}} + \Delta_{*}^2 (1 - \nu) \left( \frac{\alpha_{g}^*}{\alpha_{g}} - 1 \right)^{-\frac{3f_{\Delta}}{4\epsilon}}
\]

\[f m_0 > f \Delta\]

Dies away quickly in the IR  
Dies away slowly in the IR
Starting values get relatively closer in UV (note the masses are all shrinking in absolute terms in the IR) - full flavour symmetry restored precisely at fixed point.

The sum of the mass-squareds quickly dies to zero in IR.

$$m^2$$

$$\Delta_{(0)}^2$$

$$m_{(0)}^2$$

$$\sim \Delta_*^2 / N_F^2 \sim \Delta_*^2$$
Induces radiative breaking for H...

\[
\frac{m_0^2}{m_*^2} = 0.99 - 0.025 \exp(-t)
\]

\[
\alpha_{g, \text{min}} \xrightarrow{\epsilon \to 0} \frac{1}{2} \alpha_g
\]

\[
m_{0, \text{min}}^2 \sim -\tilde{m}_*^2
\]
Generally in IR find flavour hierarchies grow ...

\[ V \to \sum_{n > 1} \Delta_n^2 \left[ \text{Tr}_n (h^2 + p^2) - n \left( (\text{Tr}_n h)^2 + (\text{Tr}_n p)^2 \right) \right] \]

where \( \text{Tr}_n \) is the trace over the SU(n) sub-matrix
• Adapted perturbative asymptotically safe QFTs (gauge-Yukawa theories)
• A minimal embedding of the SM within this set-up straightforward within an extended PS structure
• Radiative symmetry breaking can be driven by Coleman-Weinberg or running mass-terms
• Overall now has the “feel of” other RG systems with large numbers of degrees of freedom in the UV: simpler dual way to understand this type of theory?
• It would be very nice to have a better lattice handle on large Nf UV fixed points