

Building a viable Asymptotically Safe SM

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w/ Sannino, [ArXiv:1704.00700](https://arxiv.org/abs/1704.00700) Phys.Rev. D96 (2017) no.10, 106013

w/ Sannino, [ArXiv: 1707.06638](https://arxiv.org/abs/1707.06638) Phys.Rev. D96 (2017) no.5, 055021

w/ Molgaard, Sannino, [ArXiv: 1812.04856](https://arxiv.org/abs/1812.04856)

Outline

- Motivation; hierarchy versus triviality
- Asymptotically safe 4D QFTs
- Tetrad model for the ASSM
- Radiative symmetry breaking
- Relevant operators

Hierarchy versus triviality

The triviality problem:

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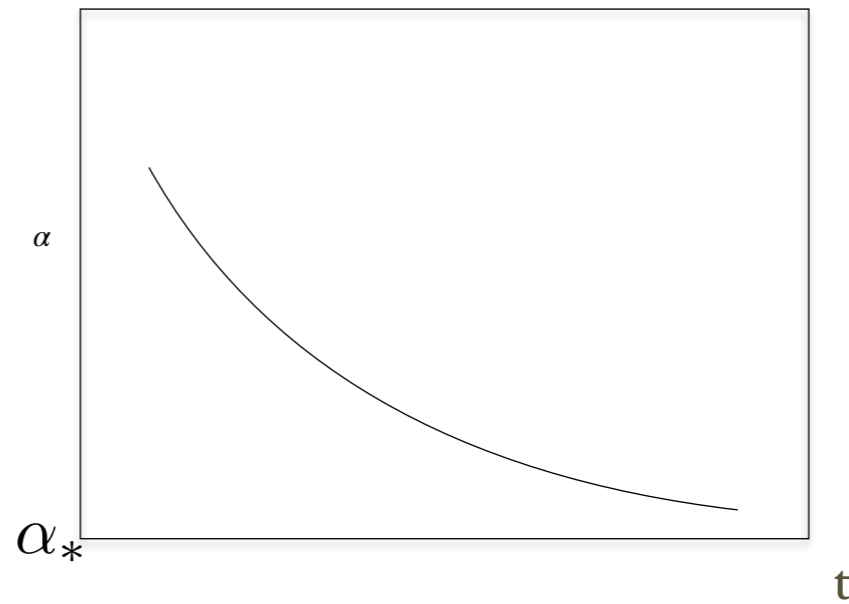
Scalars lead to Landau poles:

=> the theory is UV incomplete

But trying to UV complete it results in the hierarchy problem

Hints from QCD

$$\partial_t \alpha = -B\alpha^2$$



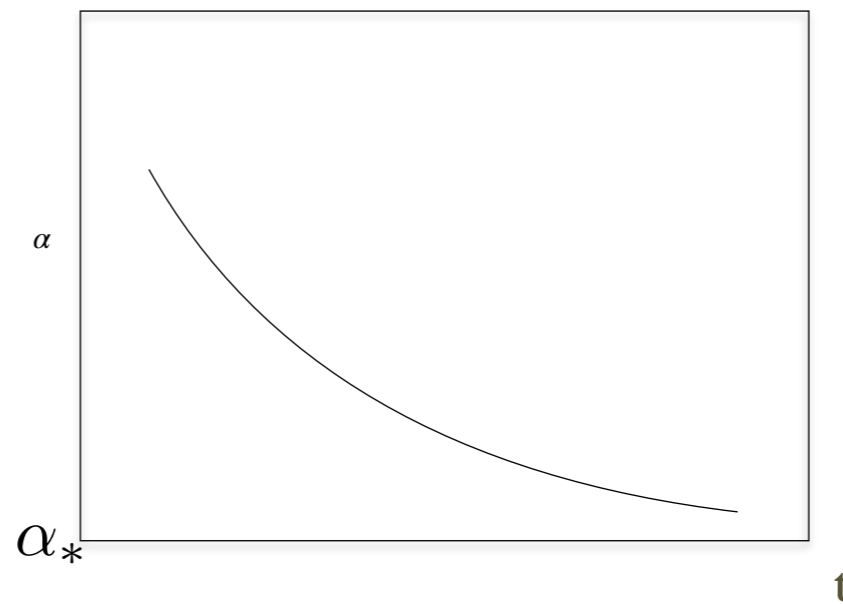
$$\alpha_* = 0$$

Hints from QCD

QCD is (unlike SUSY) a UV complete theory. Why?

1. *There is no hierarchy problem:* quark masses are protected by chiral symmetry
2. *There is no triviality problem:* QCD is asymptotically free

$$\partial_t \alpha = -B\alpha^2$$



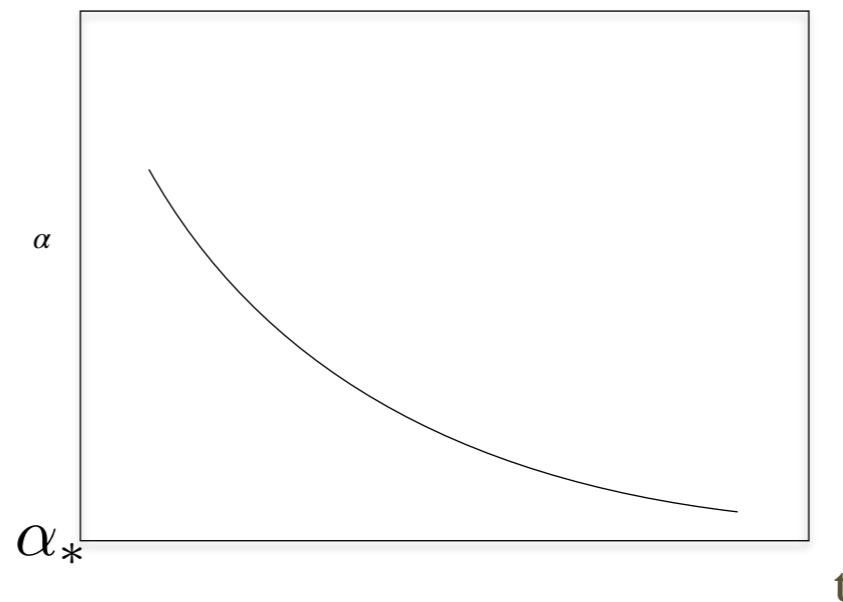
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1. *There is no hierarchy problem:* quark masses are protected by chiral symmetry
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But, we do not care about running masses because they do not change the Gaussian UV fixed point. We simply measure them and let them run. Or equivalently, relevant operators are anyway effectively zero in the UV.

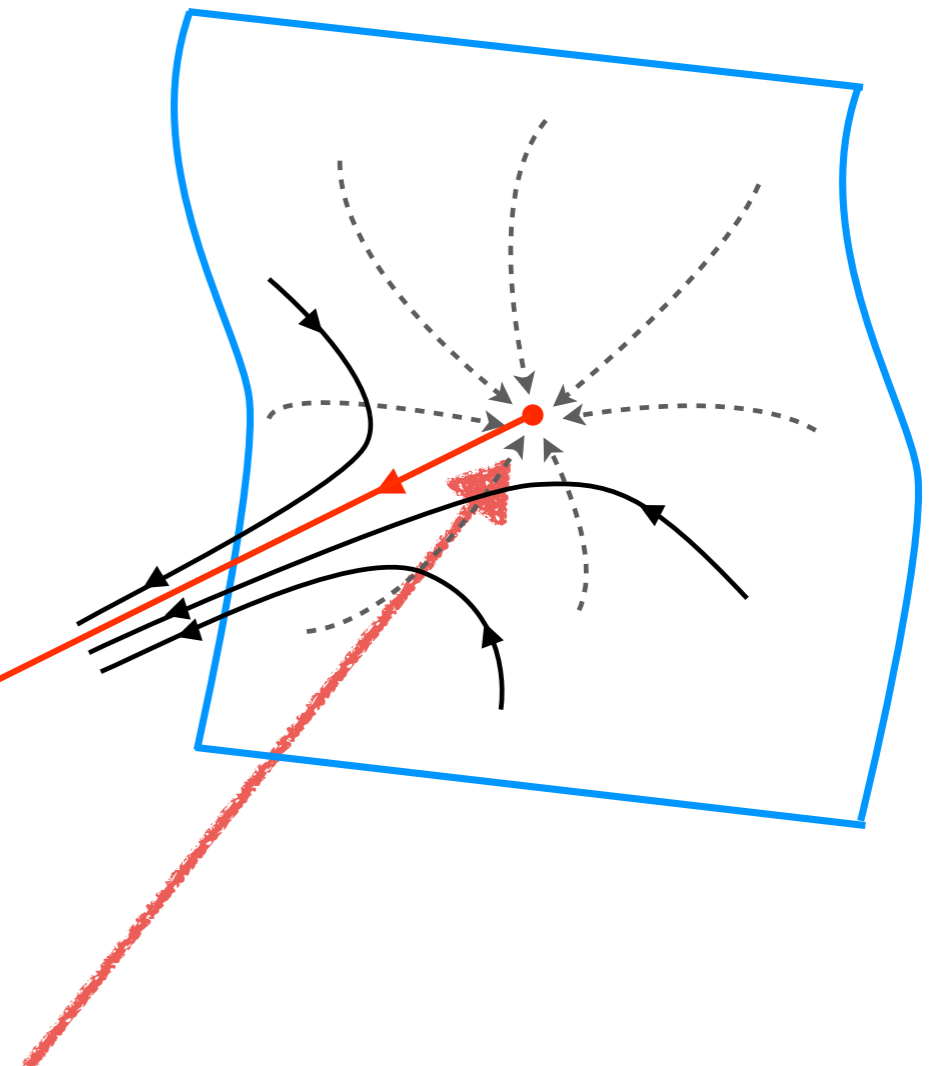
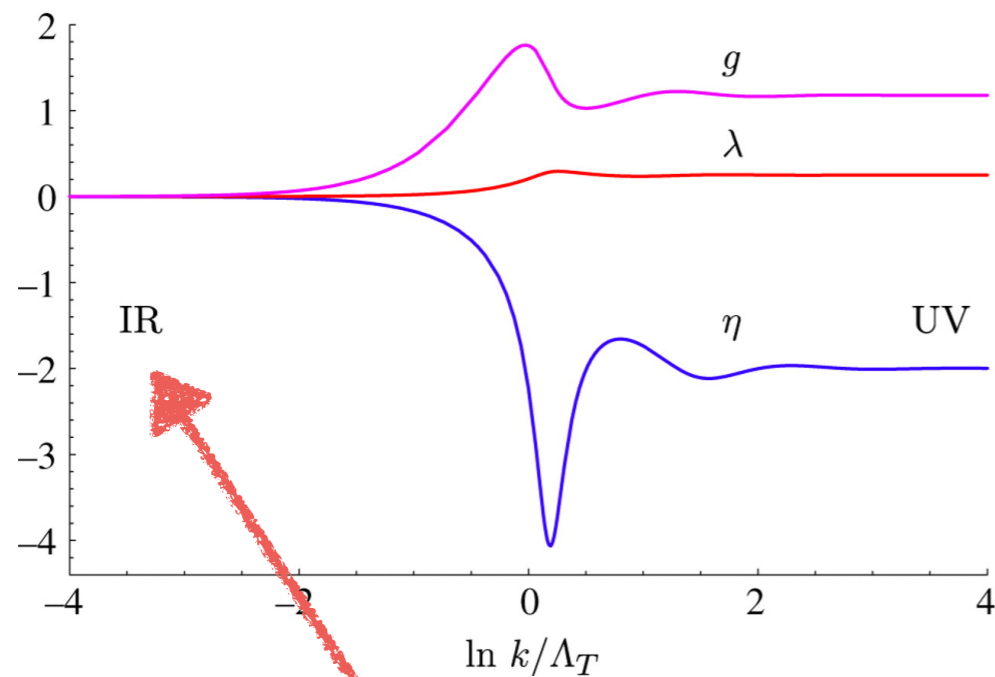
So we don't even need the chiral symmetry: point 1 becomes irrelevant in this case.

Asymptotic safety in 4D QFT

Gastmans et al '78
 Weinberg '79
 Peskin
 Reuter, Wetterich
 Gawedski, Kupiainen
 Kawai et al,
 de Calan et al ',
 Litim
 Morris

The Basic idea

Weinberg used this as a basis for his proposal of UV complete theories



Gaussian IR fixed point => perturbative

Interacting UV fixed point => finite anomalous dimensions

In a field theory replace $1/\epsilon$ with $1/\gamma$ => divergences of marginal operators (which affect the fixed point), some cured

Categorise the possible content of a theory as follows:

Irrelevant operators: would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are assumed zero (exactly renormalizable trajectory)

Marginal operators: can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free) or irrelevant.

Relevant operators: become “irrelevant” in the UV but may determine the IR fixed point.

Dangerously irrelevant operators: grow in both the UV and IR (common in e.g. SUSY)

Harmless relevant operators: shrink in both the UV and IR

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Note relevant or marginally relevant operators still have “infinities” at the FP - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (By definition they become unimportant at in the UV.)

U.V. v. I.R. F.P.

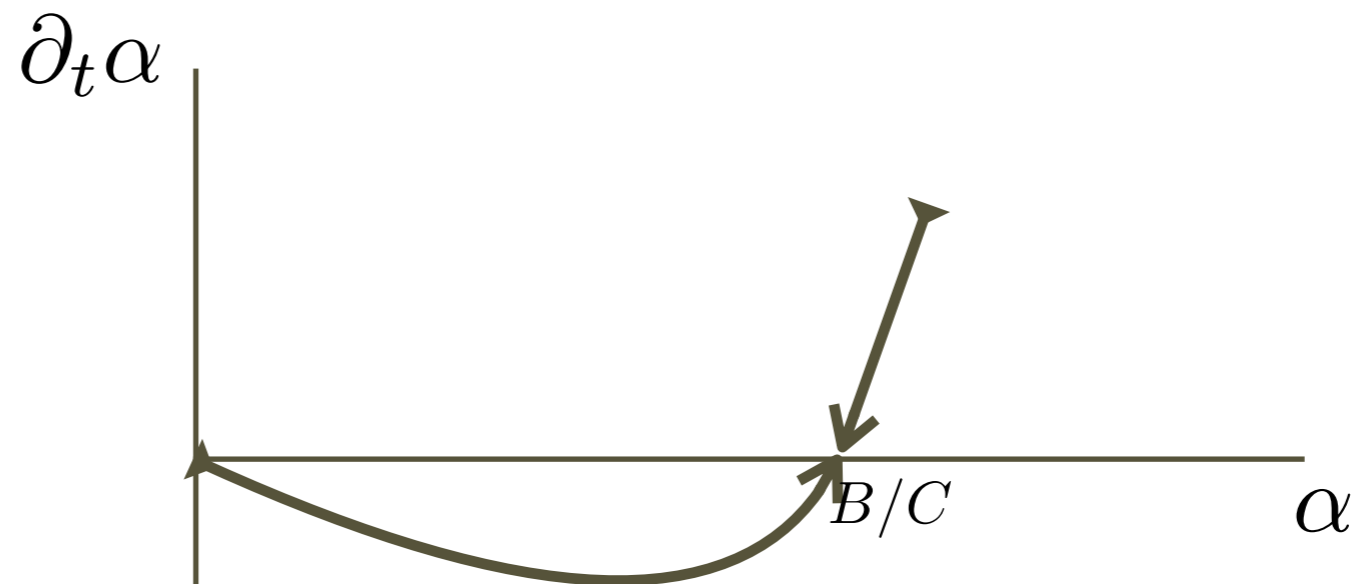
Caswell-Banks-Zaks fixed point:

Take QCD with $SU(N_C)$ and N_F fermions but very large numbers of colours+flavours

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$

$$B \propto \epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

Turns out $C > 0$, $B > 0$: theory has *stable* IR fixed point at $\alpha = B/C$ and *unstable* one in UV $\alpha = 0$



Note perturbativity: $\implies B \ll C$

requires many fields (Veneziano limit) with $N_F \approx 11N_C/2$

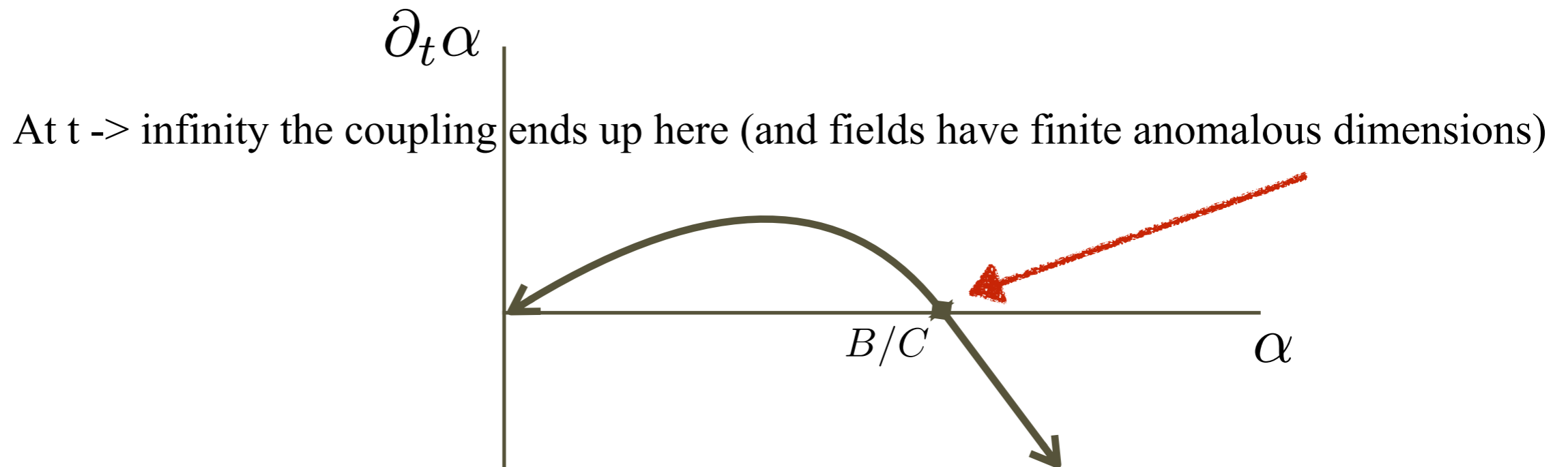
Familiar from weakly coupled supersymmetry where $N_F \lesssim 3N_C$ in $\mathcal{N} = 1$ case

Cartoon of a would-be Interacting UV FP:

Again would have ...

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$

But requires $C < 0$, $B < 0$, this theory has *stable* IR fixed point at $\alpha = 0$ and *unstable* UV one at $\alpha = B/C$



Again perturbativity would require

$$N_F \approx 11N_C/2$$

Real situation requires several couplings to realise

Litim & Sannino '14

Need to add **scalars** and **Yukawa couplings**:

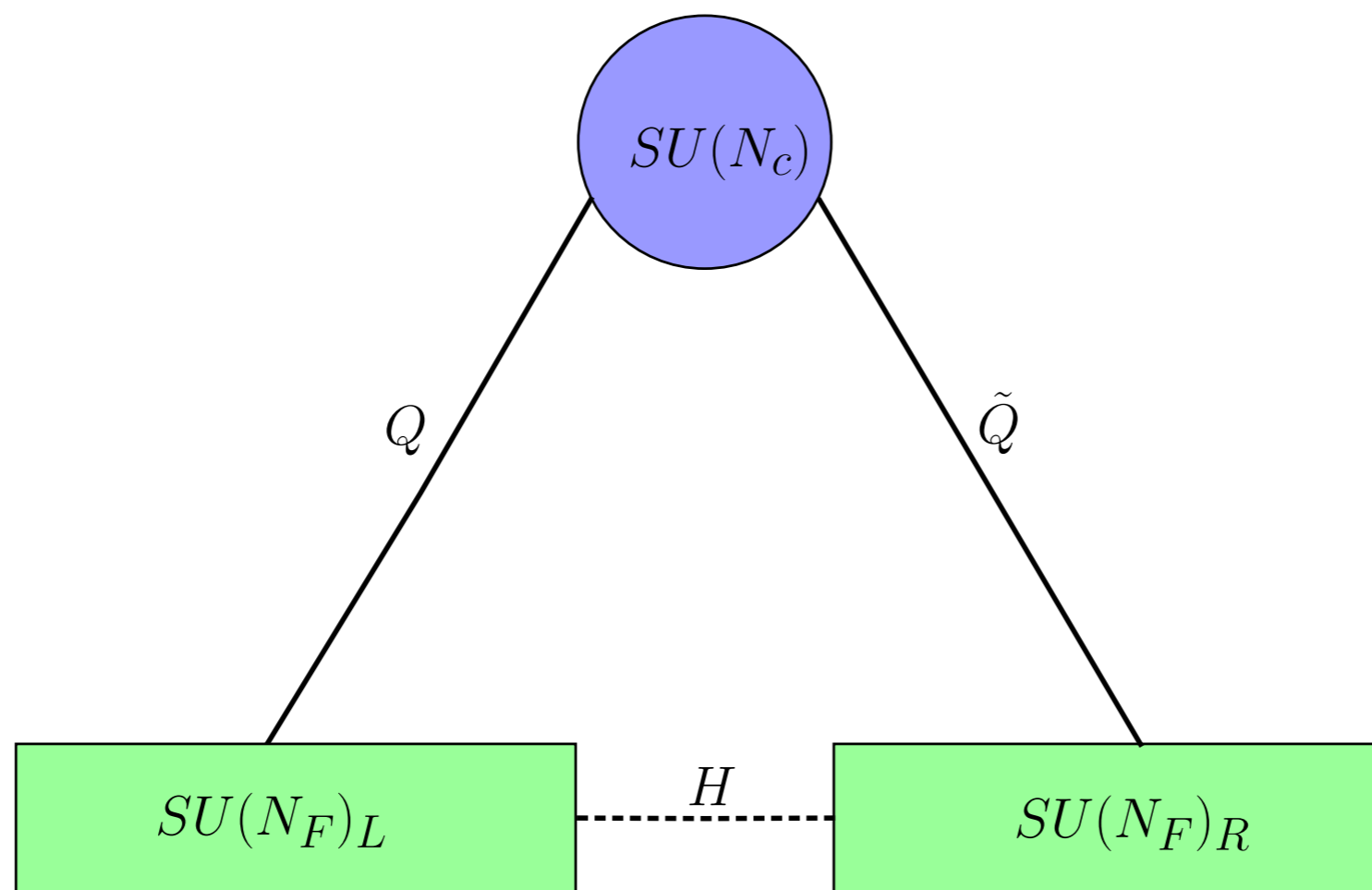
$$\mathcal{L} = -\frac{1}{2} \text{Tr} F^{\mu\nu} F_{\mu\nu} + \text{Tr} (\bar{Q} i \not{D} Q) + y \text{Tr} (\bar{Q} H Q) + \text{Tr} (\partial_\mu H^\dagger \partial^\mu H) \\ - u \text{Tr} [(H^\dagger H)^2] - v (\text{Tr} [H^\dagger H])^2,$$

H is an $N_F \times N_F$ scalar

Initially have $U(N_F)_L \times U(N_F)_R$ flavour symmetry

Quiver diagram for this model:

	$SU(N_C)$	$SU(N_F)_L$	$SU(N_F)_R$	spin
Q_{ai}	\square	\square	1	1/2
\tilde{Q}^{ia}	$\tilde{\square}$	1	$\tilde{\square}$	1/2
H_j^i	1	$\tilde{\square}$	\square	0



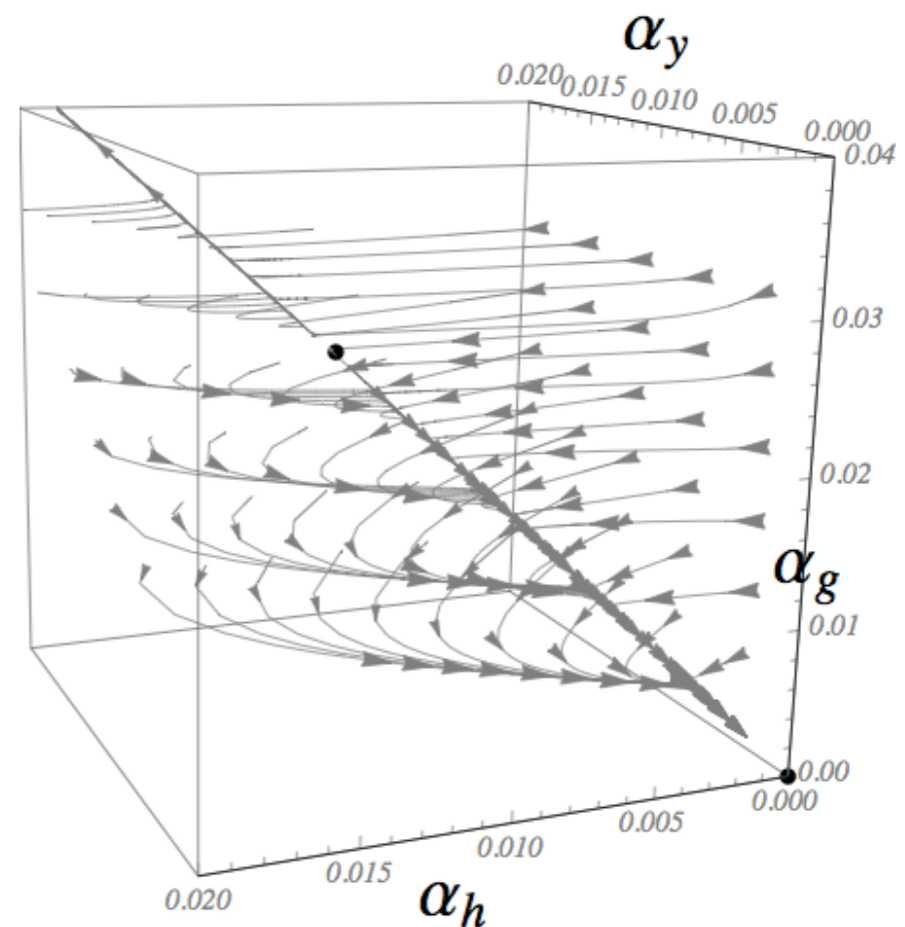
Four 't Hooft-like couplings - flow could in principle be four dimensional

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \quad \alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

but driven by the Yukawa we find 1D trajectories...

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right) \alpha_g - 2 \left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right]$$

$$\beta_y = \alpha_y \left[(13 + 2\epsilon) \alpha_y - 6 \alpha_g \right]$$

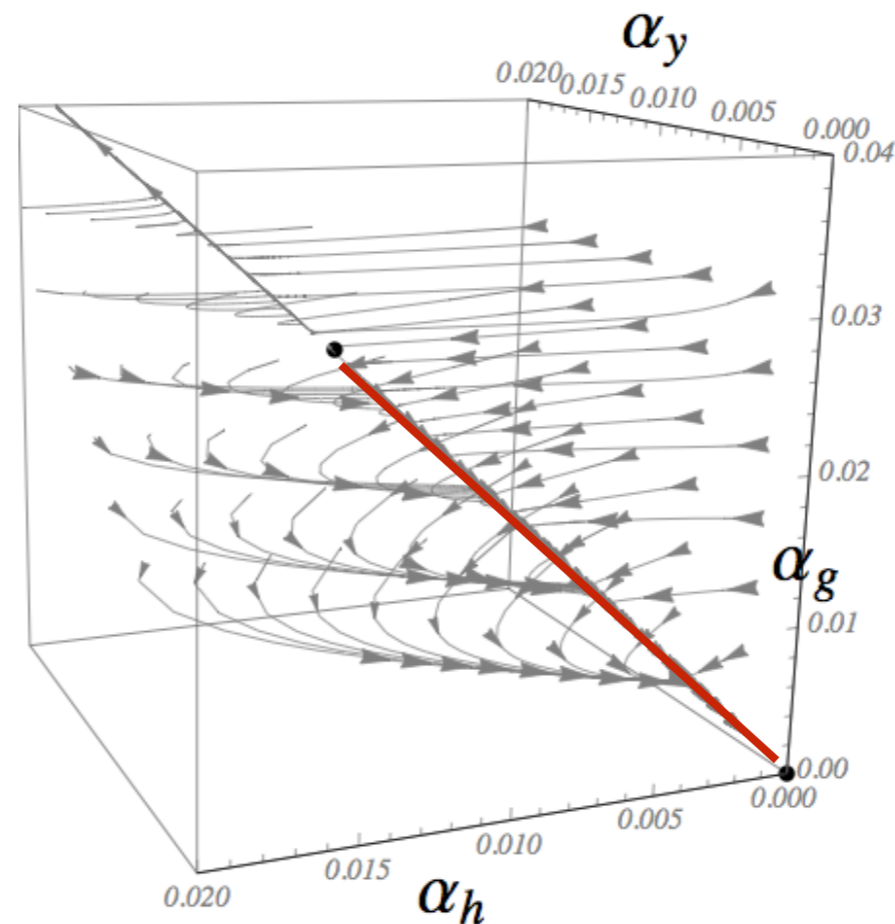


Along the critical-curve/exact-trajectory can parameterise the flow in terms of $\alpha_g(t)$

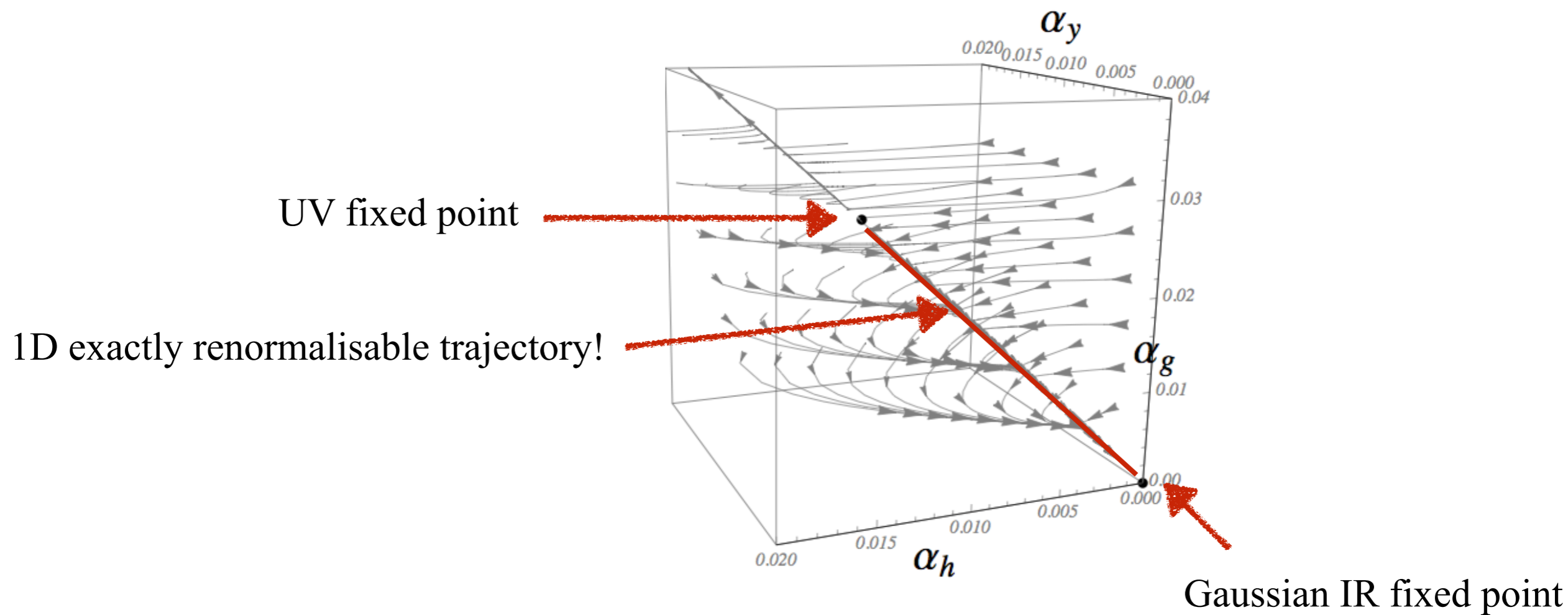
$$\alpha_y(t) = \frac{6}{13}\alpha_g(t) ,$$

$$\alpha_h(t) = 3\frac{\sqrt{23}-1}{26}\alpha_g(t) ,$$

$$\alpha_v(t) = \frac{3\sqrt{20+6\sqrt{23}}-6\sqrt{23}}{26}\alpha_g(t) ,$$



At the fixed point it is arbitrarily weakly coupled, $\alpha_g^* = 0.4561 \epsilon$, where $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$



Tetrad Model for the ASSM...

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Large UV Safe theory

SM

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graph TD; A[Large UV Safe theory] --- B[SM]
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• **Tetrad Model - focus on breaking $SU(N_C)$ to $SU(3)$ colour with new scalars ...**

c.f. Gies, Jaeckel, Wetterich '04; Bond, Litim; Bond, Hiller, Kowalska, Litim; Gies, Rechenberger, Scherer, Zambelli; Pelaggi, Plascencia, Salvio, Sannino, Smirnov; Molinaro, Sannino, Wang; Mann, Meffe, Sannino, Steele, Wang, Zhang,

$$SU(2)_R = [SU(2)_r \otimes SU(2)_S]_{\text{diag}}$$

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \otimes SU(n_g)_L$	$SU(N_F)_R \supset SU(2)_r \otimes SU(n_g)_r$	$SU(N_S) = SU(N_C - 4)_S \oplus SU(2)_S$	spin
Q_{ai}	\square	$\square \supset (\square, \square)$	1	1	1/2
\tilde{Q}^{ia}	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1	1/2
H_j^i	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1	0
$\tilde{S}_{a,\ell=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	0
\tilde{q}_ℓ^i	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1	$\square = \square_{N_C-4} \oplus \square_2$	1/2
q_j^ℓ	1	1	$\square \supset (\square, \square)$	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	1/2

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H_j^i	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1	0
$\tilde{S}_{a,\ell=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	0
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Extension of Pati-Salam - breaks to $SU(3)$ if we choose $N_S = N_C - 2$

$$\frac{N_S}{N_C} \rightarrow 1; \quad \frac{N_F}{N_C} \rightarrow \frac{21}{4} + \epsilon$$

$$\tilde{S} = \left(\overbrace{\begin{pmatrix} \begin{pmatrix} \tilde{d}^c \\ \tilde{u}^c \end{pmatrix} & \begin{pmatrix} \tilde{e}^c \\ \tilde{\nu}^c \end{pmatrix} & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} & \dots & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \dots & \tilde{\phi}_0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \dots & \tilde{\phi}_0 \end{pmatrix} \right) \left. \vphantom{\tilde{S}} \right\} N_S = N_C - 2$$

- Weak breaking must then occur along the H-Higgs directions:

$$H = \left(\begin{array}{c} \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{11} \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{21} \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{31} \\ \vdots \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{12} \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{22} \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{32} \\ \vdots \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{13} \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{23} \\ \left(\begin{array}{c} h_u^0 \\ h_u^+ \end{array} \begin{array}{c} h_d^- \\ h_d^0 \end{array} \right)_{33} \\ \vdots \\ H_0 \end{array} \right)$$

- Assignment implies 9 pairs of Higgses one for each Yukawa coupling

- Explicit embedding looks like P-S with $SU(N_C) \times SU(2)_L \times SU(2)_R \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

$$Q = \left(\overbrace{\begin{matrix} q_1 & \ell_1 & \cdots & \left(\begin{matrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{matrix} \right) & \cdots \\ q_2 & \ell_2 & \cdots & \left(\begin{matrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{matrix} \right) & \cdots \\ q_3 & \ell_3 & \cdots & \left(\begin{matrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{matrix} \right) & \cdots \\ \vdots & \vdots & & \ddots & \end{matrix}}^{N_C} \right) \Bigg\} N_F ; \quad \tilde{Q} = \left(\begin{matrix} \left(\begin{matrix} u^c \\ d^c \end{matrix} \right) & \left(\begin{matrix} \nu_e^c \\ e^c \end{matrix} \right) & \cdots & \left(\begin{matrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{matrix} \right) & \cdots \\ \left(\begin{matrix} s^c \\ c^c \end{matrix} \right) & \left(\begin{matrix} \nu_\mu^c \\ \mu^c \end{matrix} \right) & \cdots & \left(\begin{matrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{matrix} \right) & \cdots \\ \left(\begin{matrix} b^c \\ t^c \end{matrix} \right) & \left(\begin{matrix} \nu_\tau^c \\ \tau^c \end{matrix} \right) & \cdots & \left(\begin{matrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{matrix} \right) & \cdots \\ \vdots & \vdots & & \ddots & \end{matrix} \right)$$

$$q = \left(\overbrace{\begin{matrix} \left(\begin{matrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{matrix} \right) & \left(\begin{matrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{matrix} \right) & \cdots & \left(\begin{matrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{matrix} \right) \\ \left(\begin{matrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{matrix} \right) & \left(\begin{matrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{matrix} \right) & \cdots & \left(\begin{matrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{matrix} \right) \\ \left(\begin{matrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{matrix} \right) & \left(\begin{matrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{matrix} \right) & \cdots & \left(\begin{matrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{matrix} \right) \\ \vdots & \vdots & & \vdots \end{matrix}}^{N_S = N_C - 2} \right) ; \quad \tilde{q} = \left(\begin{matrix} \left(\begin{matrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{matrix} \right) & \left(\begin{matrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{matrix} \right) & \cdots & \left(\begin{matrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{matrix} \right) \\ \left(\begin{matrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{matrix} \right) & \left(\begin{matrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{matrix} \right) & \cdots & \left(\begin{matrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{matrix} \right) \\ \left(\begin{matrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{matrix} \right) & \left(\begin{matrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{matrix} \right) & \cdots & \left(\begin{matrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{matrix} \right) \\ \vdots & \vdots & & \vdots \end{matrix} \right)$$

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$$\begin{array}{c}
 \begin{array}{c}
 \left. \begin{array}{c}
 \overbrace{\left(\begin{array}{ccc} q_1 & \ell_1 & \cdots \end{array} \right)}^{N_C} \\
 \left(\begin{array}{ccc} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{array} \right) \cdots \\
 q_2 & \ell_2 & \cdots \\
 \left(\begin{array}{ccc} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{array} \right) \cdots \\
 q_3 & \ell_3 & \cdots \\
 \left(\begin{array}{ccc} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{array} \right) \cdots \\
 \vdots & \vdots & \ddots
 \end{array} \right\} N_F ; \quad \tilde{Q} = \left(\begin{array}{ccc}
 \left(\begin{array}{c} u^c \\ d^c \end{array} \right) & \left(\begin{array}{c} \nu_e^c \\ e^c \end{array} \right) & \cdots \\
 \left(\begin{array}{c} s^c \\ c^c \end{array} \right) & \left(\begin{array}{c} \nu_\mu^c \\ \mu^c \end{array} \right) & \cdots \\
 \left(\begin{array}{c} b^c \\ t^c \end{array} \right) & \left(\begin{array}{c} \nu_\tau^c \\ \tau^c \end{array} \right) & \cdots \\
 \vdots & \vdots & \cdots \\
 \left(\begin{array}{c} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{array} \right) \cdots \\
 \left(\begin{array}{c} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{array} \right) \cdots \\
 \left(\begin{array}{c} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{array} \right) \cdots \\
 \vdots & \vdots & \cdots
 \end{array} \right)
 \end{array} \\
 \\
 \begin{array}{c}
 N_S = N_C - 2 \\
 \left. \left(\begin{array}{ccc}
 \left(\begin{array}{cc} \psi_0 & \psi_1 \end{array} \right) \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \\
 \left(\begin{array}{cc} \psi_{-1} & \psi_0 \end{array} \right) \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \\
 \left(\begin{array}{cc} \psi_0 & \psi_1 \end{array} \right) \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \\
 \left(\begin{array}{cc} \psi_{-1} & \psi_0 \end{array} \right) \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{array} \right) \\
 \vdots & \vdots & \vdots
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 \left(\begin{array}{cc} \tilde{\psi}_0 & \tilde{\psi}_{-1} \end{array} \right) \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \\
 \left(\begin{array}{cc} \tilde{\psi}_1 & \tilde{\psi}_0 \end{array} \right) \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \\
 \left(\begin{array}{cc} \tilde{\psi}_0 & \tilde{\psi}_{-1} \end{array} \right) \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \\
 \left(\begin{array}{cc} \tilde{\psi}_1 & \tilde{\psi}_0 \end{array} \right) \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \cdots \left(\begin{array}{c} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{array} \right) \\
 \vdots & \vdots & \vdots
 \end{array} \right)
 \end{array}
 \end{array}
 \end{array}$$

- Little q's required (by chiral symmetry) to remove the extra SU(2) doublets: $(N_C - 4)$ uncharged under SU(2)_R

And the couplings that do this are as follows:

Standard Yukawas

masses remove 2 q's

removes excess quark colours:
S locks colour/flavour

$$\begin{aligned}
 \mathcal{L}_{\text{UVFP}} \supset \mathcal{L}_{\text{KE}} &+ \frac{y}{\sqrt{2}} \text{Tr} \left[(QH) \cdot \tilde{Q} \right] + \frac{\tilde{y}}{\sqrt{2}} \text{Tr} \left[qH^\dagger \tilde{q} \right] - \frac{\tilde{Y}}{\sqrt{2}} \text{Tr} \left[(\tilde{S} \cdot Q) \tilde{q} \right] - \frac{Y}{\sqrt{2}} \text{Tr} \left[(\tilde{Q} \cdot \tilde{S}^\dagger) q \right] \\
 &- u_1 \text{Tr} \left[H^\dagger H \right]^2 - u_2 \text{Tr} \left[H^\dagger H H^\dagger H \right] - v_1 \text{Tr} \left[H^\dagger H \right] \text{Tr} \left[\tilde{S}^\dagger \cdot \tilde{S} \right] \\
 &- w_1 \text{Tr} \left[\tilde{S}^\dagger \cdot \tilde{S} \right]^2 - w_2 \text{Tr} \left[\tilde{S}^\dagger \cdot \tilde{S} \tilde{S}^\dagger \cdot \tilde{S} \right] ,
 \end{aligned}$$

Note expect relatively light (TeV scale) q-states looking like “higgsinos”

And the couplings that do this are as follows:

Standard Yukawas

masses remove 2 q's

removes excess quark colours:
S locks colour/flavour

$$\begin{aligned}
 \mathcal{L}_{\text{UVFP}} \supset \mathcal{L}_{\text{KE}} &+ \frac{y}{\sqrt{2}} \text{Tr} [(QH) \cdot \tilde{Q}] + \frac{\tilde{y}}{\sqrt{2}} \text{Tr} [qH^\dagger \tilde{q}] - \frac{\tilde{Y}}{\sqrt{2}} \text{Tr} [(\tilde{S} \cdot Q) \tilde{q}] - \frac{Y}{\sqrt{2}} \text{Tr} [(\tilde{Q} \cdot \tilde{S}^\dagger) q] \\
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 \end{aligned}$$

For later use define rescaled c'pgs:

$$\begin{aligned}
 \alpha_g &= \frac{N_C g^2}{(4\pi)^2}; \quad \alpha_y = \frac{N_C y^2}{(4\pi)^2}; \quad \alpha_{\tilde{y}} = \frac{N_C \tilde{y}^2}{(4\pi)^2}; \quad \alpha_Y = \frac{N_C Y^2}{(4\pi)^2}; \quad \alpha_{\tilde{Y}} = \frac{N_C \tilde{Y}^2}{(4\pi)^2}; \\
 \alpha_{u_1} &= \frac{N_F^2 u_1}{(4\pi)^2}; \quad \alpha_{u_2} = \frac{N_F u_2}{(4\pi)^2}; \quad \alpha_{v_1} = \frac{N_C^2 v_1}{(4\pi)^2}; \quad \alpha_{w_1} = \frac{N_C^2 w_1}{(4\pi)^2}; \quad \alpha_{w_2} = \frac{N_C w_2}{(4\pi)^2}
 \end{aligned}$$

In case you're suffering from "expectation versus reality syndrome" ...



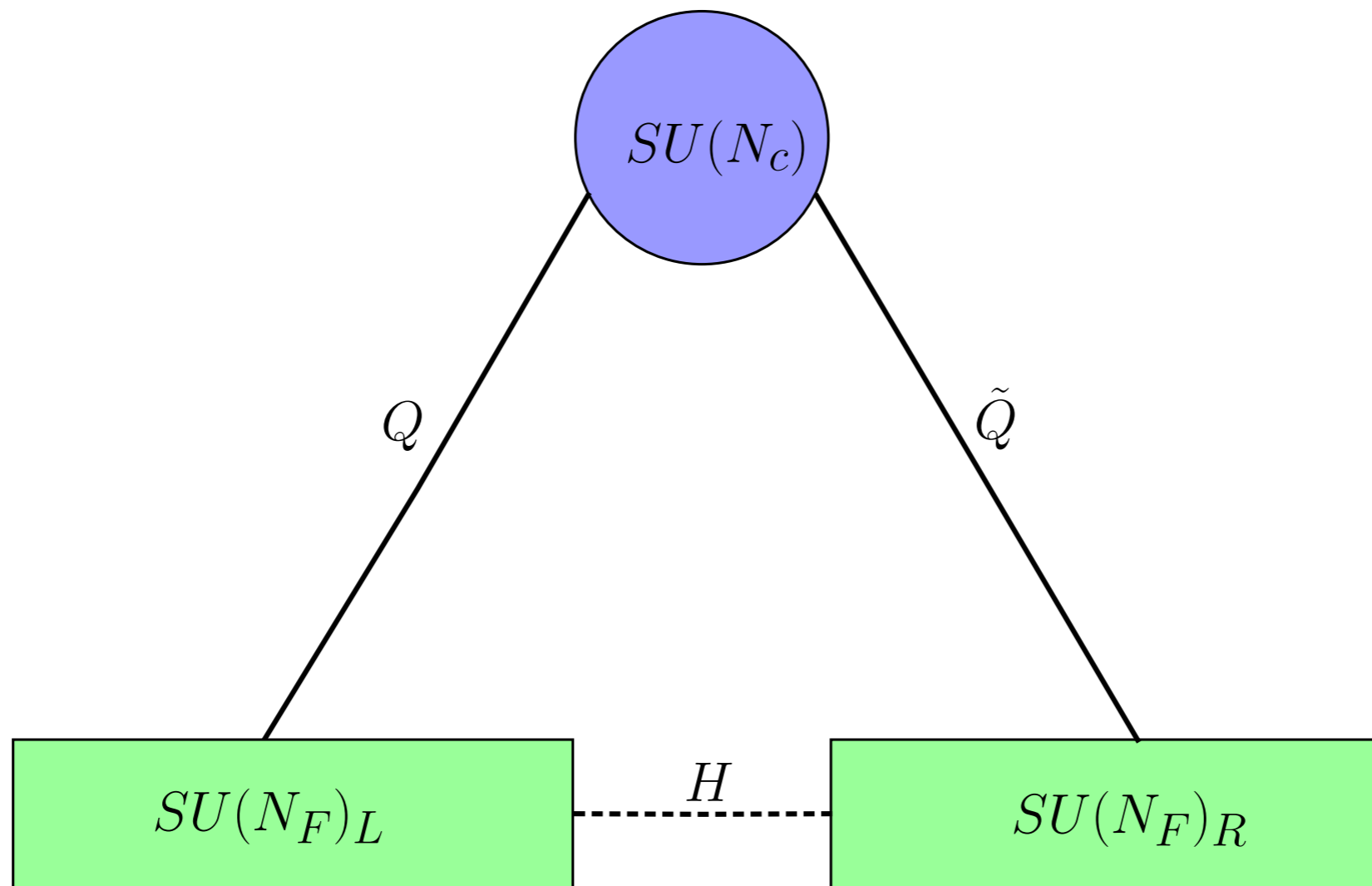
In case you're suffering from "expectation versus reality syndrome" ...

Or equivalently ...

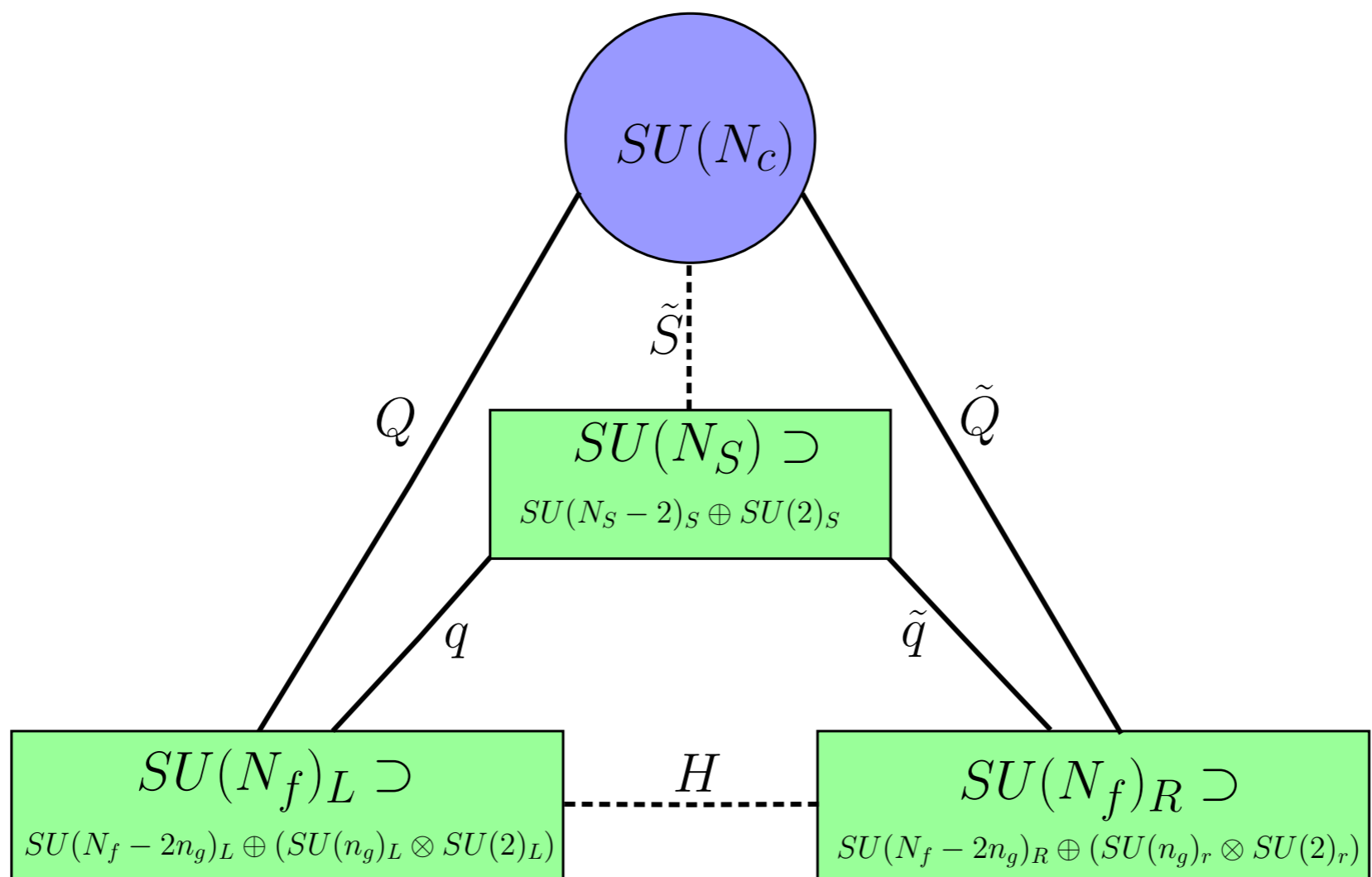


A quiver diagram is useful to see (at least some of) what we did:

Before:



After: (hence the name Tetrad)



- **As this model is based on LS, the same UVFP applies (see later). But what about AS for the $SU(2) \times SU(2)$ electroweak gauge groups?**

These see a large number of flavours (N_f (small f) of order order N_c)?

- **This gives UVFP behaviour with a fixed point at 't Hooft couple $\sim 1 \dots$ if $N_f \gg 16$:**

Palanques Mestre, Pascual; Gracey; Holdom;
Shrock; Antipin, Pica, Sannino

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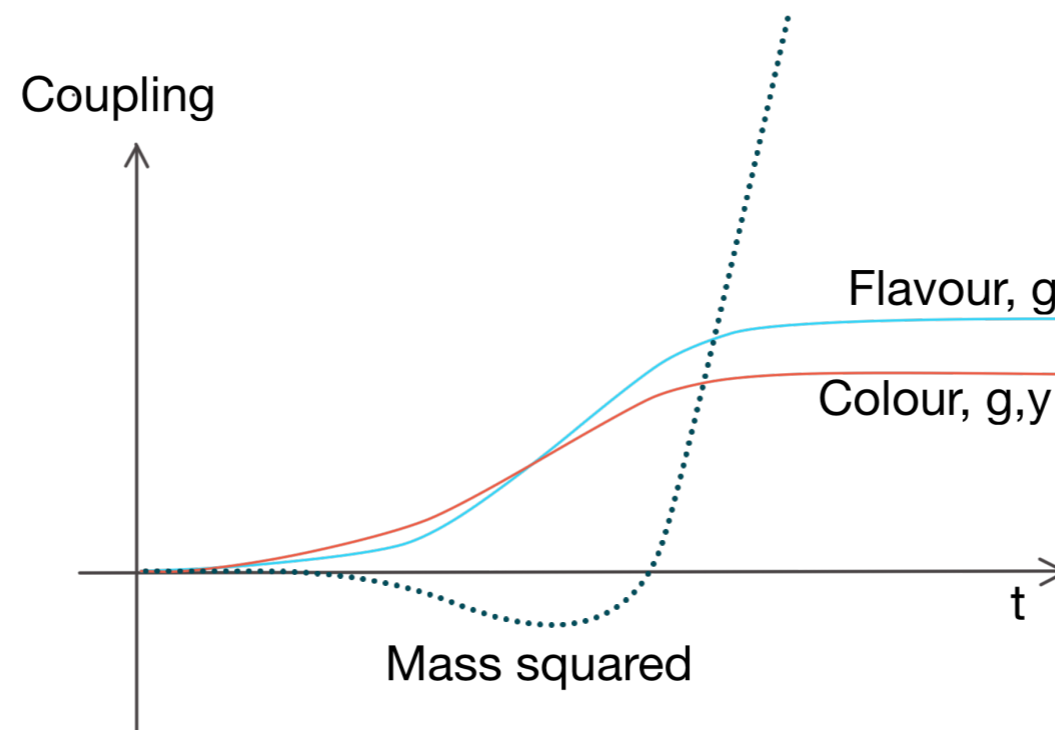
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Resum first terms gives

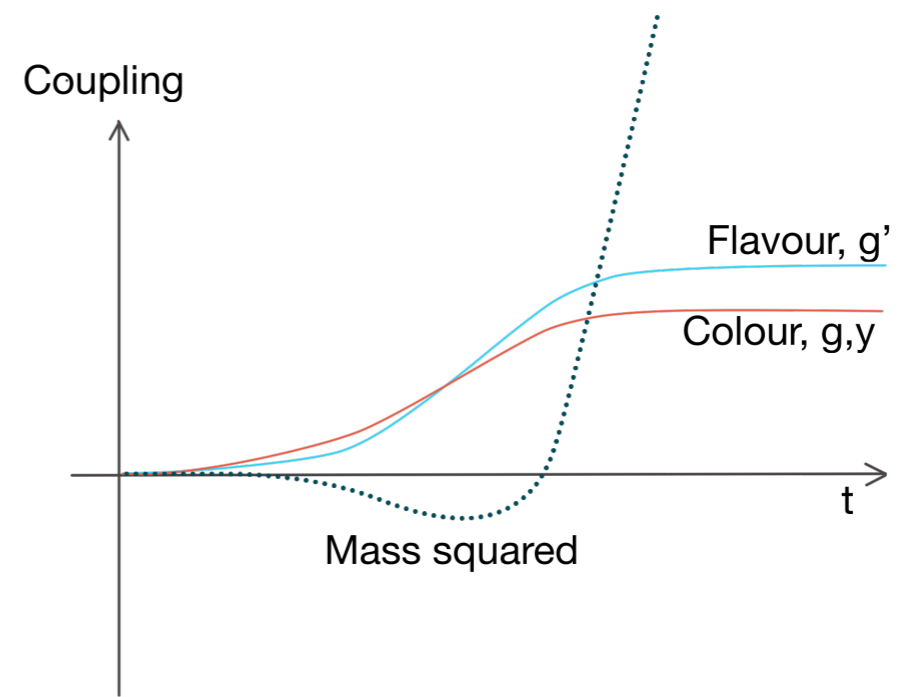
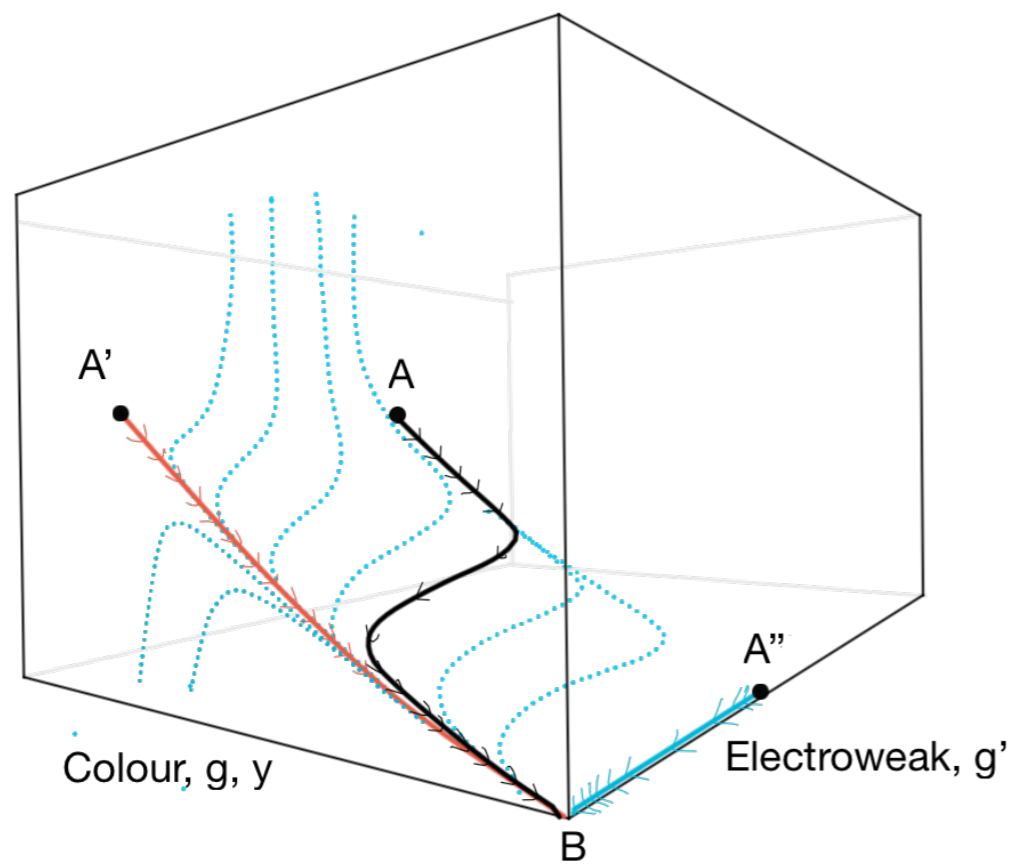
$$\frac{3}{4} \frac{\beta_{\tilde{\alpha}}}{\tilde{\alpha}^2} = 1 + \frac{H(\tilde{\alpha})}{N_f} + \mathcal{O}(N_f^{-2})$$

$$H(\tilde{\alpha}) = \frac{1}{4} \log |3 - 2\tilde{\alpha}| + \text{constant}$$



$$\tilde{\alpha}_* = \frac{3}{2} - C e^{-4N_f}$$

- Overall the picture is ...

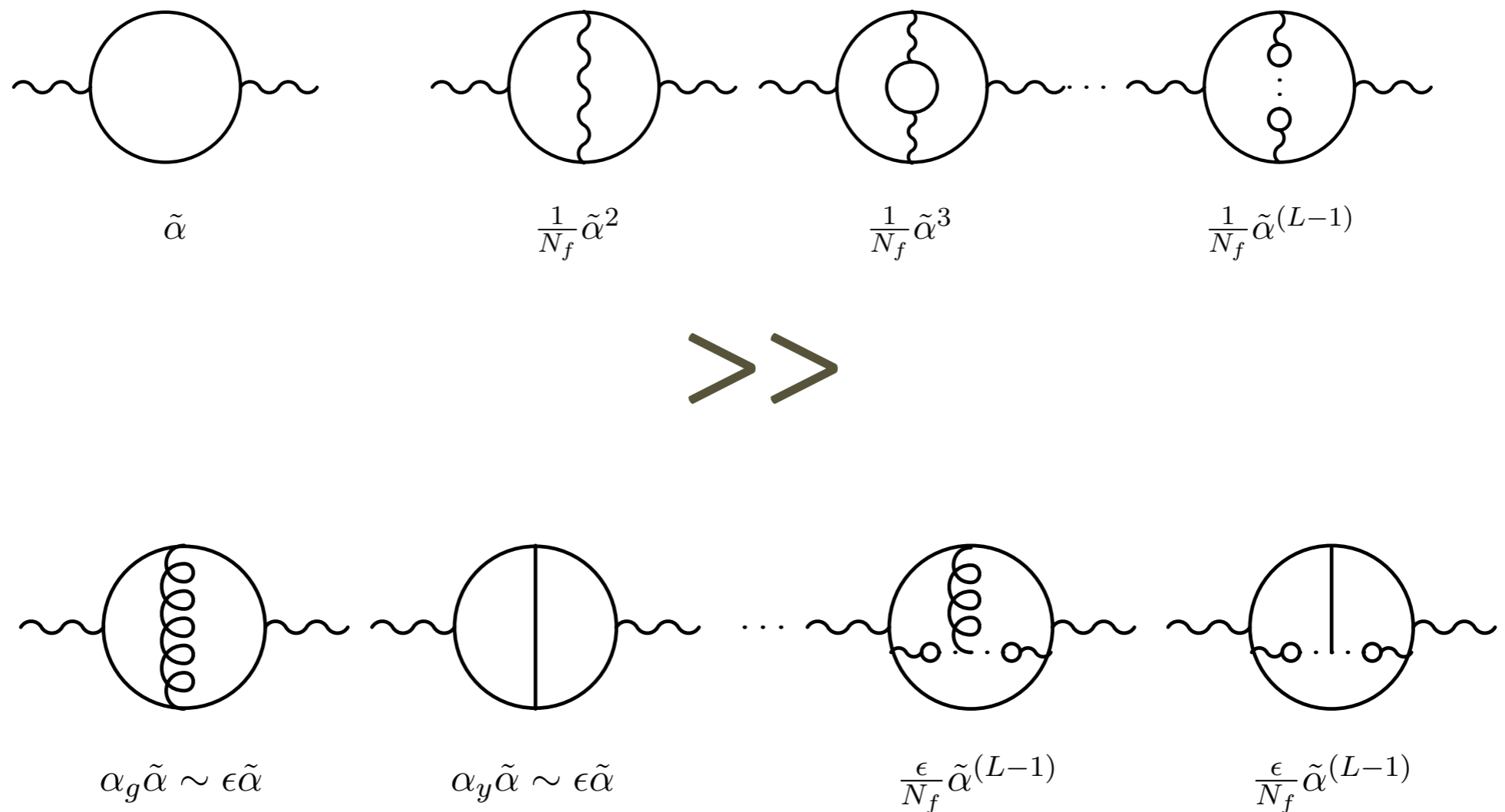


- **Can show by power counting that the two kinds of UVFP decouple.**

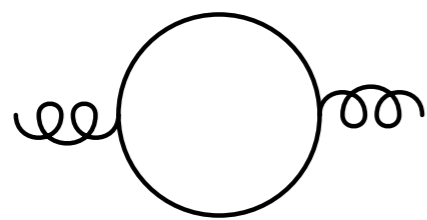
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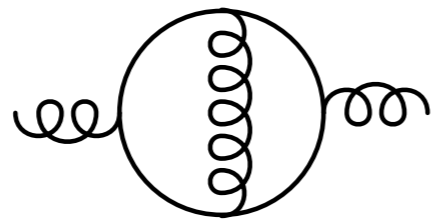
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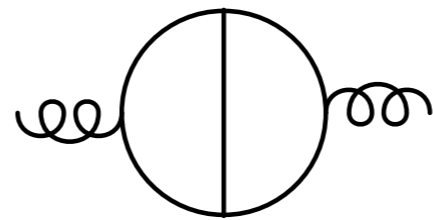
- **Conversely for the SU(Nc) fixed point ...**



$$x_F \alpha_g$$

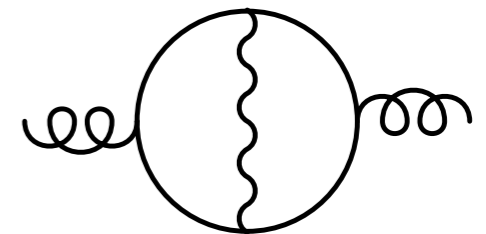


$$x_F \alpha_g^2 \sim \epsilon^2$$



$$x_F \alpha_y \alpha_g \sim \epsilon^2$$

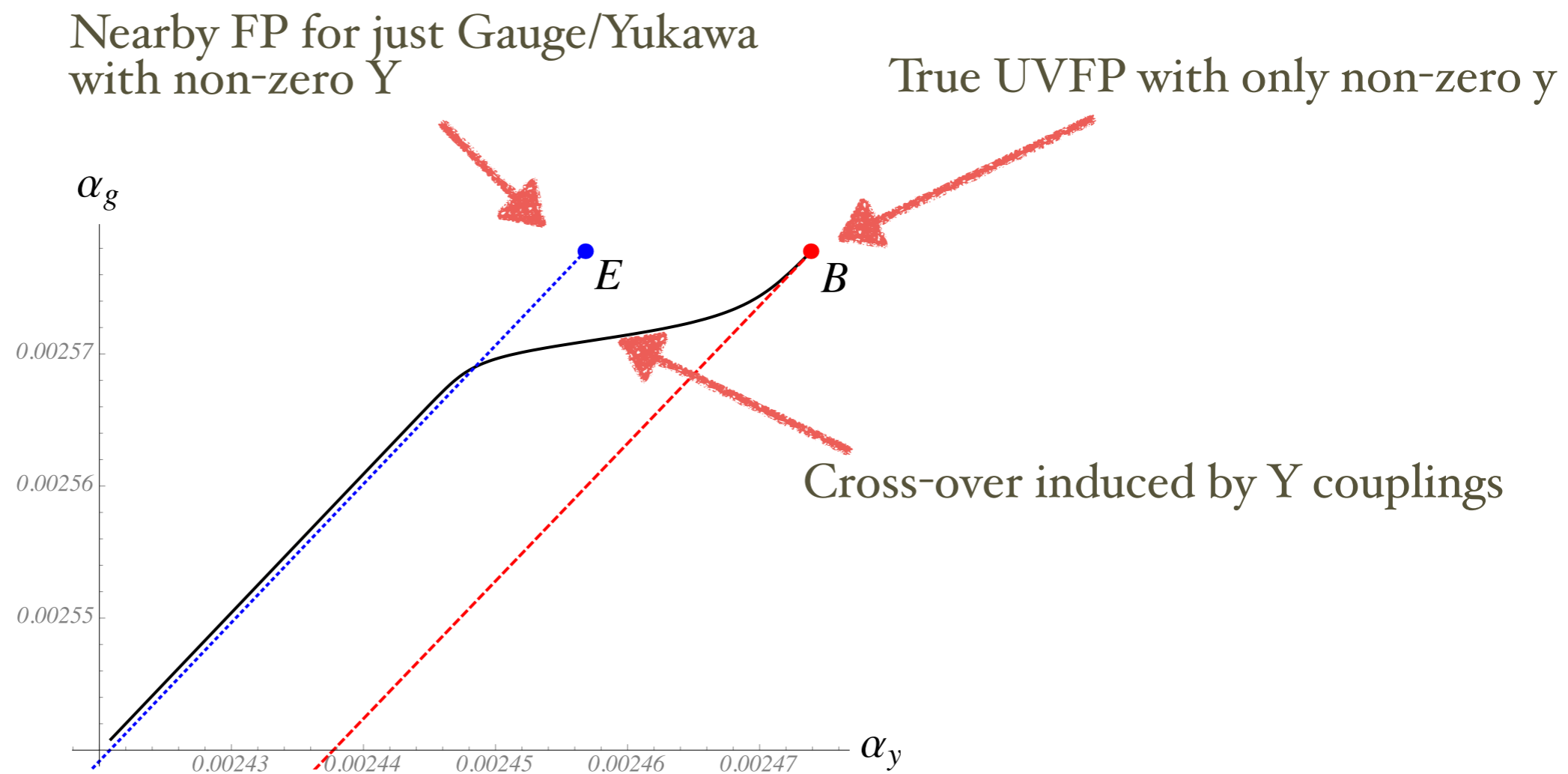
$\gg \gg$



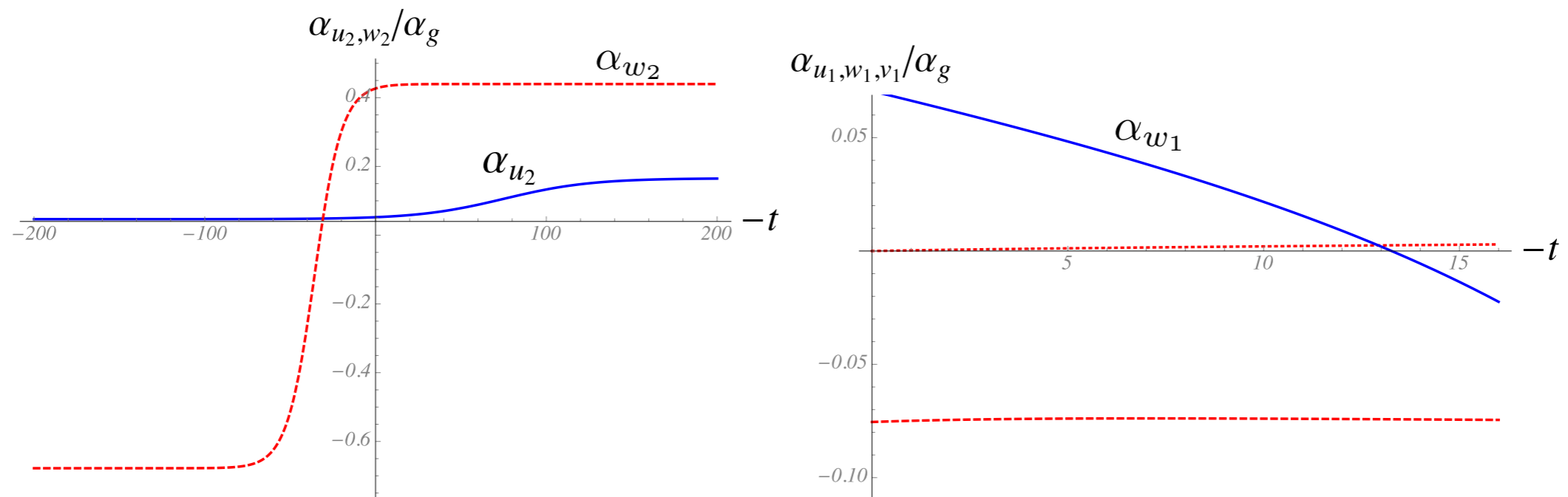
$$\frac{1}{N_f N} \tilde{\alpha} \alpha_g \sim \frac{\epsilon}{N_f^2} \tilde{\alpha} \lesssim \epsilon^3 \frac{\tilde{\alpha}}{25}$$

Radiative symmetry breaking

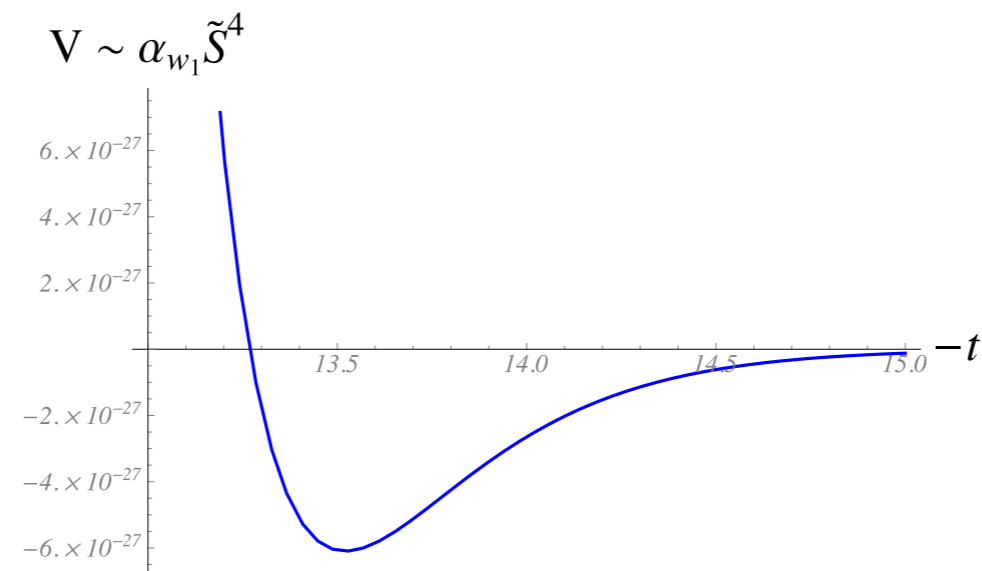
- Suppose that the classically relevant operators are negligible. (compared to the scales we are about to generate.)
- Then Coleman-Weinberg radiative symmetry breaking is induced along the flow.
- First look at Yukawas:



- This in turn induces a flow in the quartic couplings driving them negative: we essentially have Gildener-Weinberg breaking of the extended PS symmetry.
- Note that the H mass-squareds are all positive at this scale.



- This in turn induces a flow in the quartic couplings driving them negative: we essentially have Gildener-Weinberg breaking of the extended PS symmetry.
- Note that the H mass-squareds are all positive at this scale.



Adding relevant operators (mass-squareds)

Organize relevant operators in terms of the $U(N_F) \times U(N_F)$ flavour symmetry that we break with the mass-squareds (closed under RG):

$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a) T_a$$

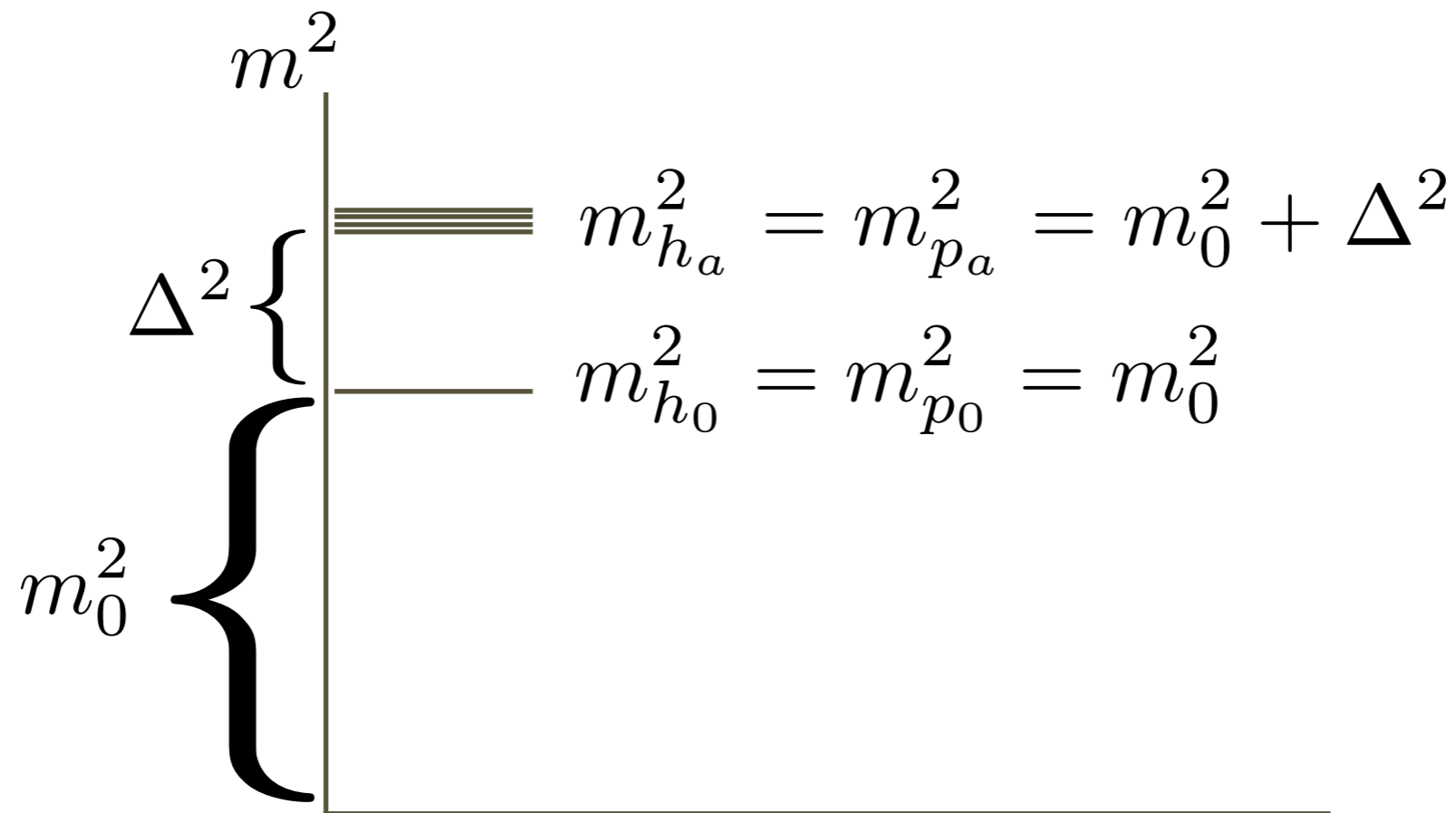
$$\mathcal{L}_{Soft} = -m_{h_0}^2 \text{Tr} [H^\dagger H] - \sum_{a=1}^{N_F^2 - 1} \Delta_a^2 \text{Tr} [HT^a] \text{Tr} [H^\dagger T^a]$$

Then solve Callan Symanzik eqn for them as usual =>

Non-trivial simple example...

Consider case where the trace component has a slightly smaller mass-squared:

$$V_{class}^{(2)} = m_0^2 \text{Tr}(H^\dagger H) + 2\Delta^2 \sum_a \text{Tr}(T_a H^\dagger) \text{Tr}(T_a H)$$



Non-trivial simple example...

After some work find the following answer in terms of two RG invariants, one for each independent (non-predicted) relevant operator (where $\nu = (1 - 1/NF^2)$):

$$m_0^2 = \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3fm_0}{4\epsilon}} - \Delta_*^2 \nu \left(\frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3f\Delta}{4\epsilon}},$$
$$m_{a=1 \dots N_F^2 - 1}^2 = \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3fm_0}{4\epsilon}} + \Delta_*^2 (1 - \nu) \left(\frac{\alpha_g^*}{\alpha_g} - 1 \right)^{-\frac{3f\Delta}{4\epsilon}}$$

$$f_{m_0} > f_\Delta$$

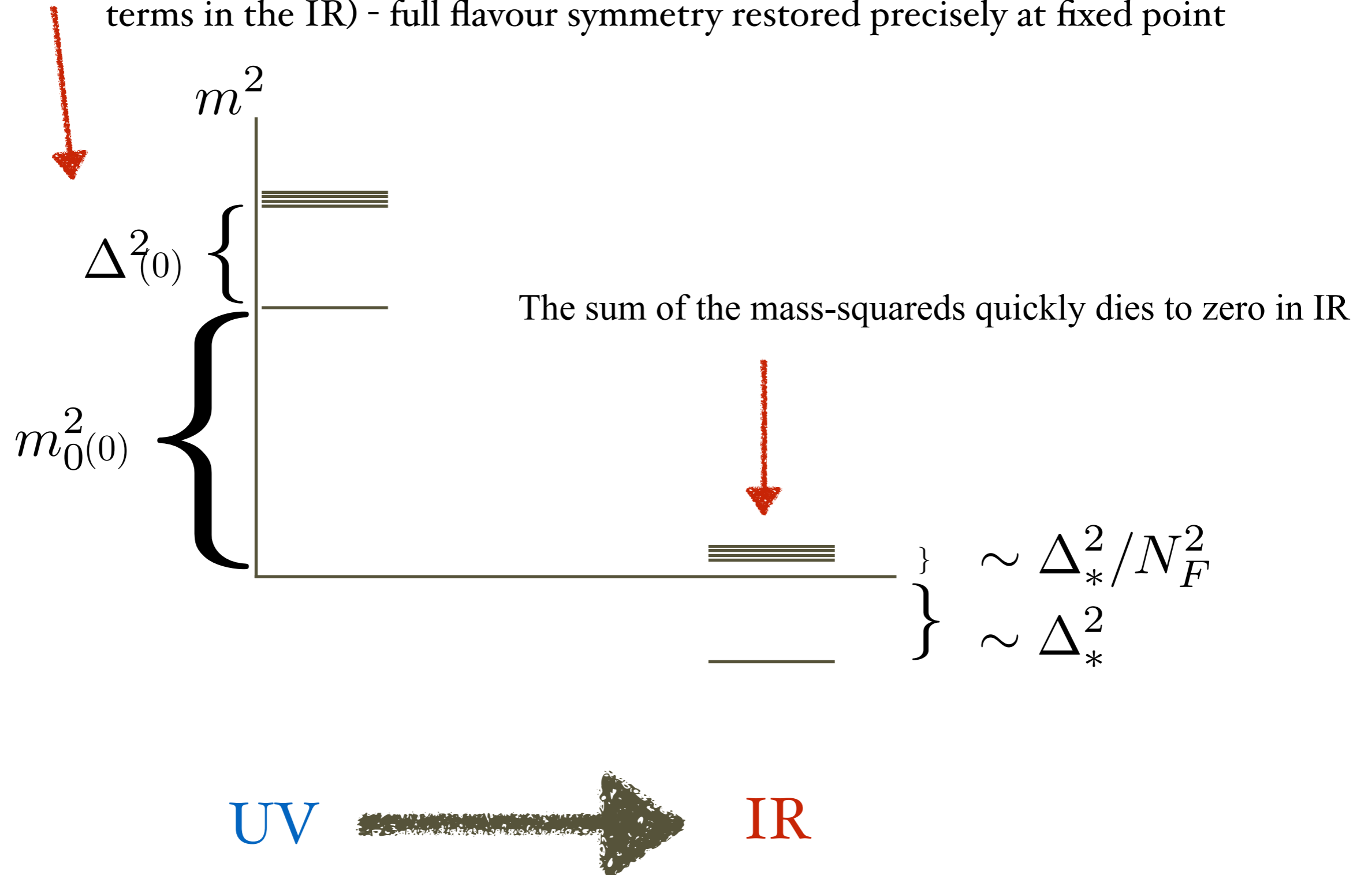


Dies away quickly *in the IR*

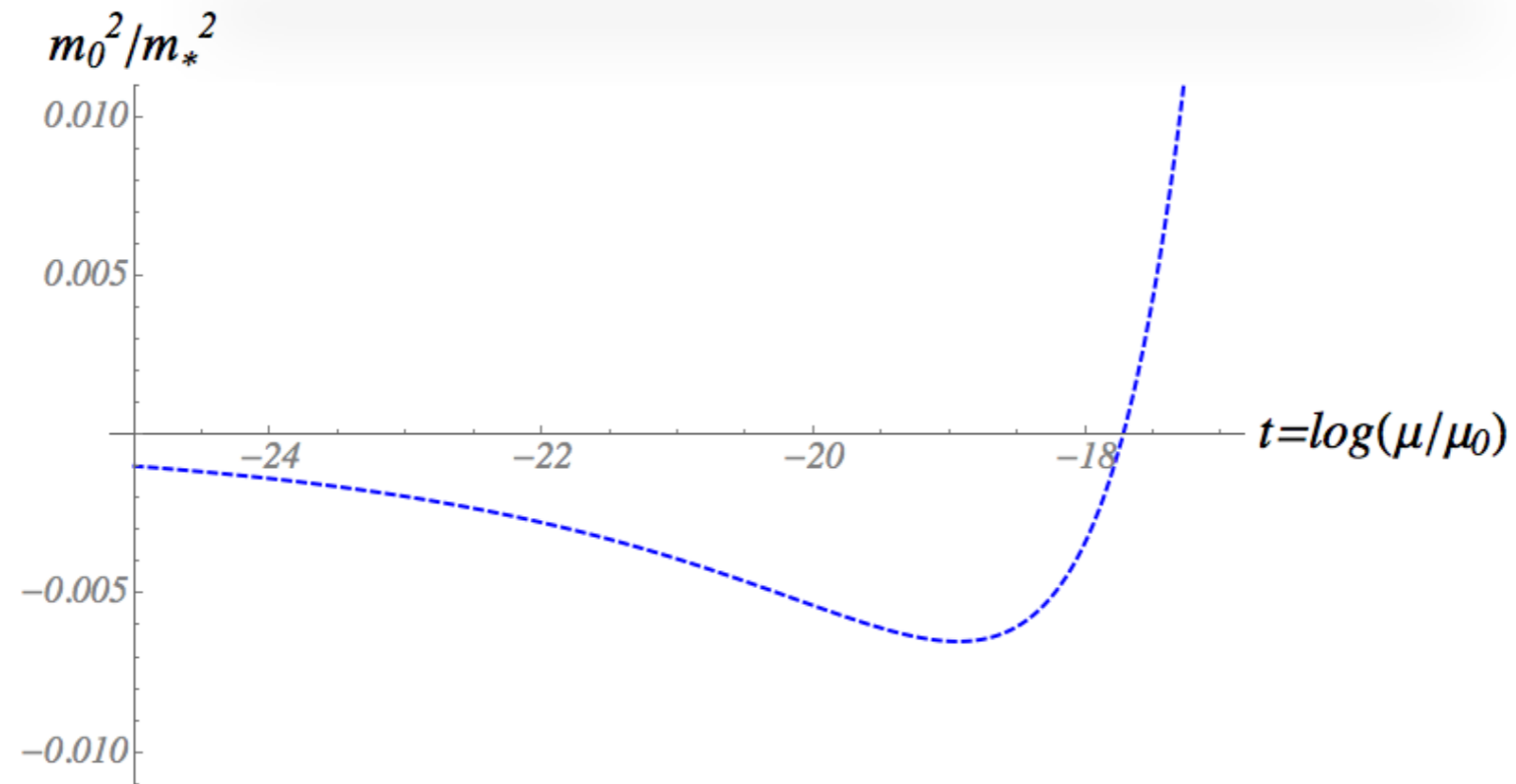


Dies away slowly *in the IR*

Starting values get relatively closer in UV (note the masses are all shrinking in absolute terms in the IR) - full flavour symmetry restored precisely at fixed point



Induces radiative breaking for H...



$$\alpha_{g,min} \xrightarrow{\epsilon \rightarrow 0} \frac{1}{2} \alpha_g^*$$

$$m_{0,min}^2 \sim -\tilde{m}_*^2$$

Generally in IR find flavour hierarchies grow ...

$$V \rightarrow \sum_{n>1} \Delta_n^2 \left[\text{Tr}_n (h^2 + p^2) - n \left((\text{Tr}_n h)^2 + (\text{Tr}_n p)^2 \right) \right]$$

where Tr_n is the trace over the SU(n) sub-matrix

Summary

- Adapted perturbative asymptotically safe QFTs (gauge-Yukawa theories)
- A minimal embedding of the SM within this set-up straightforward within an extended PS structure
- Radiative symmetry breaking can be driven by Coleman-Weinberg or running mass-terms
- Overall now has the “feel of” other RG systems with large numbers of degrees of freedom in the UV: simpler dual way to understand this type of theory?
- It would be very nice to have a better lattice handle on large N_f UV fixed points