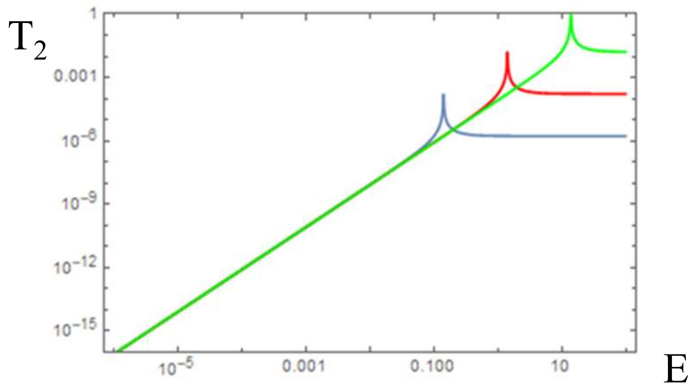
Gauge Assisted Quadratic Gravity

Can a renormalizeable QFT be the UV completion of gravity beyond Planck scale?

This proposed variant keeps gravity weakly coupled at all relevant scales



Need to carefully explore theories with quartic propagators

Work with Gabriel Menezes

arXiv:1712.04468 , arXiv:1804.04980, arXiv:1812.03603…

John Donoghue
CERN

Jan 29, 2019

Connections:

Early pioneers: Stelle, Fradkin-Tsetlyn, Adler, Zee, Smilga,
Tomboulis, Hasslacher-Mottola,
Lee-Wick, Coleman, Boulware-Gross….

Present activity: Einhorn-Jones, Salvio-Strumia, Holdom-Ren,
Donoghue-Menezes, Mannheim, Anselmi
Odintsov-Shapiro, Mazumdar, Narain-Anishetty…

In the neighborhood: Lu-Perkins-Pope-Stelle, ‘t Hooft,
Grinstein-O’Connell-Wise ….

Overview:

Consider Yang-Mills plus quadratic gravity:

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{4g^2}g^{\mu\alpha}g^{\nu\beta}F^a_{\mu\nu}F^a_{\alpha\beta} - \frac{1}{2\xi^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \frac{1}{6f_0^2}R^2\right]$$

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta} - \frac{1}{2}\left(R_{\mu\alpha}g_{\nu\beta} - R_{\nu\alpha}g_{\mu\beta} - R_{\mu\beta}g_{\mu\alpha} + R_{\nu\beta}g_{\mu\alpha}\right) \\ &+ \frac{R(g)}{6}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}\right) \quad . \end{aligned}$$

Classically scale invariant

"Helper" YM becoming strong will define Planck scale

Take quadratic gravity to be weakly coupled at Planck scale
- in particular $\xi^2 \ll g^2$

if $\xi = 0.1$ at the Planck scale, it would become strong at $\Lambda_\xi = 10^{-1006}$ eV.)

Topics to discuss:

1) Inducing the Einstein action

2) Quartic propagators and Lee-Wick

3) The spin 2 graviton propagator

4) Unitarity at leading order

5) Modified Lehmann representation

6) Unitarity beyond leading order – Lee Wick contour

Helper YM gauge interaction

Helper YM will become strong at some scale
- call this the Planck scale
- will generate non-scale invariant terms in gravity action

Most particularly, can generate *G*

Low energy gravity turns into Einstein gravity

Adler-Zee formula:

$$\frac{1}{16\pi G} = \frac{i}{96} \int d^4x\ x^2 < 0|T\ T(x)T(0)|0 >$$

where $T(x) = \eta_{\mu\nu} T^{\mu\nu}(x)$

Quick derivation:

Zee

Weak field limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Gravitaional coupling:

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} T_{\mu\nu}$$

To second order in the gravitational field:

$$i \int d^4x \mathcal{L}_{eff} = \frac{1}{2} \left(\frac{-i}{2} \right)^2 \int d^4x \; d^4y \; h_{\mu\nu}(x) h_{\alpha\beta}(y) < 0|T \; T^{\mu\nu}(x) T^{\alpha\beta}(y)|0 >$$

For convenience, consider special case

(see also Brown, Zee)

$$h_{\mu\nu}(x) = \frac{1}{4} \eta_{\mu\nu} h(x)$$

Slowly varying fields (on QCD scale):

$$h(y) = h(x) + (y-x)^\mu \partial_\mu h(x) + \frac{1}{2} (y-x)^\mu (y-x)^\nu \partial_\mu \partial_\nu h(x) + \ldots.$$

This results in the effective Lagrangian:

$$
\begin{aligned}
i\mathcal{L}_{eff}(x) &= -\frac{1}{128}h^2(x)\int d^4z \ <0|T\ T(z)T(0)|0> \\
&+ \frac{1}{1024}(\partial_\mu h(x))^2 \int d^4z\ z^2 \ <0|T\ T(z)T(0)|0>
\end{aligned}
$$

The first term is part of the cosmological constant, the second is the Einstein action. Identify via:

$$
\sqrt{-g}R = -\frac{3}{32}(\partial_\mu h(x))^2
$$

This gives the Adler-Zee formula:

$$
\frac{1}{16\pi G} = \frac{i}{96}\int d^4x\ x^2 <0|T\ T(x)T(0)|0>
$$

Induced G – in SU(N) theories

$$T^{\mu}_{\ \mu} = \frac{\beta(g)}{2g} F^a_{\mu\nu} F^{a\mu\nu}$$

Construct the sum rule for QCD – lattice data, OPE and pert. theory

Separate long and short distance techniques at $x=x_0$

$$\psi(x) =< 0|T\, T(x)T(0)|0 >= [\psi_{pert}(x) + \psi_{OPE}(x)]\, \Theta(x_0 - x) + \psi_{lattice}(x)\Theta(x - x_0)$$

Perturbative:

$$\psi_{pert} = \frac{C_\psi}{x^8(\log(1/\Lambda^2 x^2))^2}\ , \qquad C_\psi = \frac{96}{\pi^4}$$

(Adler)

OPE:

$$\begin{aligned}\psi_{OPE} &= \left(\frac{b}{8\pi}\right)^2 \left[\frac{\alpha_s^2 b}{\pi^3 x^4} < \alpha_s G^2 > \right. \\ &+ \left. \frac{2\alpha_s^2}{\pi^2 x^2} < gG^3 > + \frac{29\alpha_s^3 \log(\mu^2 x^2)}{2\pi^2 x^2} < gG^3 > \right]\end{aligned}$$

(NSVZ, Bagan, Steele)

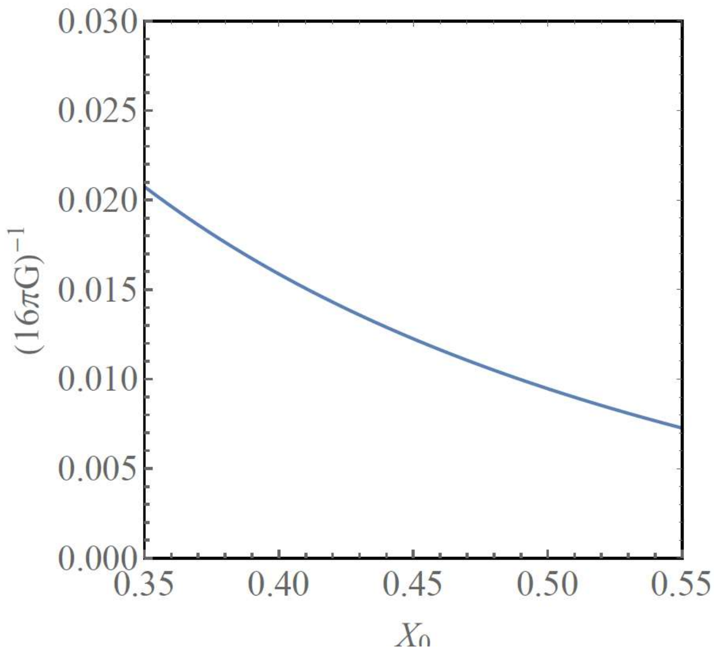
Lattice: $\psi_{lattice} = \frac{t_0^2 M_g}{4\pi^2 x} K_1(M_g x)$ (Wightman function)

$t_0 = 1.1 \pm 0.22\ \ \mathrm{GeV}^3$ $\qquad M_g = 1.71 \pm 0.05 \pm 0.08\ \ \mathrm{GeV}$

(Chen, et al.2006)

Results

Pure QCD – **Induced G is positive**



Matching at $X_0^{-1} = 2\text{ GeV}$:

$$\frac{1}{16\pi G_{\text{ind}}} = 0.095 \pm 0.030\ \text{GeV}^2$$

Scale up by 10^{19} to get correct Planck scale

Note also:

Cosmological constant:

$$\Lambda_{\text{ind}} = \frac{1}{4}\langle 0|T^{\mu}_{\ \mu}|0\rangle = \frac{1}{4}\left\langle 0\left|\frac{\beta(g)}{2g}F^{a}_{\mu\nu}F^{a\mu\nu}\right|0\right\rangle$$

With standard values, Λ = -0.0044 GeV^4 (return to this later)

Induced R^2 term

Brown-Zee

$$\frac{1}{6f_{\text{ind}}^2} = \frac{i}{13824}\int d^4z\,(z^2)^2\langle T\{\bar{T}(z)\bar{T}(0)\}\rangle.$$

We find (low energy only):

$$\frac{1}{6f_{\text{ind}}^2} = 0.00079 \pm 0.00030.$$

Quartic propagators:

$$R^2 \sim \partial^2 g \partial^2 g$$

Inevitable in a renormalizable gravity QFT:

- normal matter loops need renormalization at order R^2 (at all orders)
- gravity loops stop at R^2 iff fundamental quadratic propagators

Comes with expectations of ghosts:

$$\frac{-i}{q^4} \sim \frac{-i}{q^2(q^2-\mu^2)} = \frac{1}{\mu^2}\left(\frac{i}{q^2} - \frac{i}{q^2-\mu^2}\right)$$

For quadratic gravity, dangerous ghost in spin-two propagator

The Lee-Wick mechanism

~1969

Apparently consistent finite higher derivative theories
- extra state with negative norm
- modifies propagators

$$iD_{F\mu\nu}(q) = -ig_{\mu\nu}\left[\frac{1}{q^2} - \frac{1}{q^2-\Lambda^2}\right] = -ig_{\mu\nu}\frac{-\Lambda^2}{q^2(q^2-\Lambda^2)} = -ig_{\mu\nu}\frac{1}{q^2(1-\frac{q^2}{\Lambda^2})}$$

Key is that coupling to light states leads to unstable ghost
- does not appear in the asymptotic spectrum

$$G(q^2) = \frac{1}{(q^2+i\epsilon)\left[1+\Pi(q^2)-\frac{q^2}{\Lambda^2}\right]}$$

$$\Pi(q^2) = q^2\frac{\alpha}{3\pi}\int_{4m_e^2}^{\infty} ds \frac{1}{s(s-q^2-i\epsilon)}\sqrt{1-\frac{4m_e^2}{s}}\left(1+\frac{2m_e^2}{s}\right)$$

LW theories seem mostly healthy

$$G(q) = \frac{1}{(q^2 + i\epsilon)\left(1 - \frac{N\alpha}{3\pi}[\log|q^2|/m_r^2 - i\pi] - \frac{q^2}{m_r^2}\right)}$$

Unitarity seems satisfied – direct calculation

But "micro-causality" is violated
- non-causal propagation of order the ghost width

Modern overview of unitarity and causality
-Grinstein, O'Connell, Wise (2009)

Strong similarity to quadratic gravity

Loops do the same in the graviton propagator

Both matter and graviton loops
Effect of unitarity

Tomboulis 1977

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right]}{2q^2\left(1 - \frac{N_s G_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}\,.$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2.$$

Decay of high mass ghost states could be a common feature

JFD
PRD 2017

Aside: Test case

Triplet "colored" scalar in SU(2) with quartic propagator

SU(2) valued field

$U = e^{i\frac{\tau^a\phi^a}{f}}$ with $U \to V(x)UV^\dagger(x)$ $V(x)$ in SU(2).

with scale invariant Lagrangian

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{f^2}{4}Tr\left[(D^\mu D_\mu U)^\dagger(D^\nu D_\nu U)\right] \\
&+ d_1\left(Tr\left[D^\mu U^\dagger D_\mu U\right]\right)^2 + d_2 Tr\left[D^\mu U^\dagger D^\nu U\right]Tr\left[D_\mu U^\dagger D_\mu U\right] \\
&+ d_3 Tr\left[D^\mu U^\dagger D^\nu U D_\mu U^\dagger D_\mu U\right] + d_4 Tr\left[U^\dagger D^2 U\right]Tr\left[D_\mu U^\dagger D^\mu U\right] \\
&+ d_5 Tr\left[U^\dagger D^2 U D_\mu U^\dagger D^\mu U\right] + d_6 Tr\left[U^\dagger D^2 U\right]Tr\left[U^\dagger D^2 U\right] \\
&+ d_7 F^i_{\mu\nu} Tr\left[\tau^i D^\mu U^\dagger D^\nu U\right] + d_8 F^i_{\mu\nu}F^{j\mu\nu}Tr\left[\tau^i U^\dagger \tau^j U\right]
\end{aligned}
$$

Scalar will be confined – usual discussions of g.s. not relevant.
Should be possible to simulate on lattice

Background field renormalization can be accomplished

- for suitably general Lagrangian

With $\mathcal{L}(U) = \mathcal{L}(\bar{U}) + \Delta^a \mathcal{O}^{ab} \Delta^b + ...$

$$\mathcal{O}^{ab} = \left[D^2 D^2 + A^{\alpha\beta\gamma} D_\alpha D_\beta D_\gamma + B^{\alpha\beta} D_\alpha D_\beta + C^\alpha D_\alpha + E\right]^{ab}$$

The divergences are captured in the heat-kernel coefficient:

$$Tr < x| \log \mathcal{D} |x > |_{div} = \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \frac{d}{2}) \; Tr \; a_2(x)$$

Barvinsky and Vilkovisky have worked this out

$$\begin{aligned}
a_2 \; = \; & \frac{1}{6} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8} F_{\mu\nu} [D^\mu, A^\nu] + \frac{9}{80} F_{\mu\nu} A^{\mu\alpha\beta} A^\nu{}_{\alpha\beta} + \frac{9}{160} F_{\mu\nu} A^\mu A^\nu + \frac{1}{8} C_\mu A^\mu + \frac{1}{24} B_{\mu\nu} B^{\mu\nu} + \frac{1}{48} B^2 \\
- \; & \frac{1}{16} B D_\mu A^\mu + \frac{1}{8} B^{\mu\nu} D_\mu A_\nu - \frac{1}{8} B^{\mu\nu} D^\alpha A_{\mu\nu\alpha} + \frac{9}{80} A^{\mu\nu\alpha} D_\mu D_\nu A_\alpha - \frac{3}{80} A^\mu D_\mu D_\nu A^\nu - \frac{3}{160} A^\mu D^2 A_\mu \\
- \; & \frac{3}{40} A^{\mu\nu\alpha} D_\mu D_\beta A^\beta{}_{\nu\alpha} - \frac{1}{80} A^{\mu\nu\alpha} D^2 A_{\mu\nu\alpha} - \frac{1}{640} B (2 A^{\mu\nu\alpha} A_{\mu\nu\alpha} + 3 A^\mu A_\mu) \\
- \; & \frac{3}{320} B^{\mu\nu} (2 A_{\mu\alpha\beta} A_\nu{}^{\alpha\beta} + A_\mu A_\nu + A_{\mu\nu\alpha} A^\alpha + A^\alpha A_{\mu\nu\alpha}) - \frac{1}{640} A^\beta D_\beta (2 A^{\mu\alpha\beta} A_{\mu\alpha\beta} + 3 A^\mu A_\mu) \\
- \; & \frac{1}{640} A^\beta [2 (D_\beta A^{\mu\nu\alpha}) A_{\mu\nu\alpha} + 3 (D_\beta A^\mu) A_\mu] - \frac{3}{160} A^{\mu\nu\alpha} D_\mu (A_\nu A_\alpha + 2 A_{\nu\beta\gamma} A_\alpha{}^{\beta\gamma} + A_{\nu\alpha\beta} A^\beta + A^\beta A_{\nu\alpha\beta}) \\
+ \; & \frac{3}{320} A^{\mu\nu\alpha} (A_\nu D_\mu A_\alpha + 2 A_{\nu\beta\gamma} D_\mu A_\alpha{}^{\beta\gamma} + A_{\nu\alpha\beta} D_\mu A^\beta + A^\beta D_\mu A_{\nu\alpha\beta}) \\
+ \; & \frac{1}{960 \times 32 \times 42} A^{\mu\nu\alpha} A^{\beta\gamma\delta} A^{\epsilon\sigma\rho} A^{\lambda\omega\eta} g_{\mu\nu\alpha\beta\gamma\epsilon\sigma\rho\lambda\omega\eta}
\end{aligned}$$

Quadratic gravity

$$S_{\text{quad}} = \int d^4x\sqrt{-g}\left[\frac{1}{6f_0^2}R^2 - \frac{1}{2\xi^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - \eta G\right]$$

Free-field mode decomposition depends on gauge fixing.

-all contain a scalar mode and tensor mode

Scalar has massive non-ghost and massless ghost – due to R^2

$$D^{(0)}_{\mu\nu\alpha\beta}(q^2) = \left(\frac{q^4}{f_0^2} - \frac{2q^2}{\kappa^2}\right)^{-1}\mathcal{P}^{(0)}_{\mu\nu\alpha\beta} = \frac{\kappa^2}{2}\left(\frac{1}{q^2 - M_0^2} - \frac{1}{q^2}\right)\mathcal{P}^{(0)}_{\mu\nu\alpha\beta}$$

Spin 2 mode has massive ghost

$$D^{(2)}_{\mu\nu\alpha\beta}(q^2) = \left(\frac{q^2}{\kappa^2} - \frac{q^4}{2\xi^2}\right)^{-1}\mathcal{P}^{(2)}_{\mu\nu\alpha\beta} = \kappa^2\left(\frac{1}{q^2} - \frac{1}{q^2 - M_2^2}\right)\mathcal{P}^{(2)}_{\mu\nu\alpha\beta}$$

Spin 2 propagator after induced G:

- one loop order in light fields - logs from vacuum polarization

$$
\begin{aligned}
iD_{\mu\nu\alpha\beta} &= i\mathcal{P}^{(2)}_{\mu\nu\alpha\beta} D_2(q) \\
D_2^{-1}(q) &= \frac{q^2+i\epsilon}{\tilde{\kappa}^2(q)} - \frac{q^4}{2\xi^2(\mu)} - \frac{q^4 N_{\rm eff}}{640\pi^2} \ln\left(\frac{-q^2-i\epsilon}{\mu^2}\right) - \frac{q^4 N_q}{1280\pi^2} \ln\left[\frac{(q^2)^2}{\mu^4}\right]
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{1}{\tilde{\kappa}^2(q)} &\to \frac{1}{\kappa^2} \qquad q \ll M_P \\
&\to 0 \qquad q \gg M_P
\end{aligned}
\qquad\qquad \kappa^2 = 32\pi G
$$

and $N_{\rm eff}$ = active d.o.f. with quadratic propagators
N_q = active d.o.f. with quartic propagators

Here ignoring cosmological constant

Three regions:

$$
\begin{aligned}
iD_{\mu\nu\alpha\beta} &= i\mathcal{P}^{(2)}_{\mu\nu\alpha\beta} D_2(q) \\
D_2^{-1}(q) &= \frac{q^2 + i\epsilon}{\tilde{\kappa}^2(q)} - \frac{q^4}{2\xi^2(\mu)} - \frac{q^4 N_{\mathrm{eff}}}{640\pi^2} \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right) - \frac{q^4 N_q}{1280\pi^2} \ln\left[\frac{(q^2)^2}{\mu^4}\right]
\end{aligned}
$$

1) Beyond Planck mass:

YM plus quadratic gravity

$$
N_{\mathrm{eff}} = N_\infty = D + N_{SM}, \quad N_q = 199/3
$$

2) Intermediate energy

- interplay of quadratic and quartic terms
- dominated by resonance

3) Low energy EFT region

quartic terms subdominant

$$
\begin{aligned}
N_{\mathrm{eff}} &= N_V + \frac{1}{4}N_{1/2} + \frac{1}{6}N_S + \frac{21}{6} \\
&= \frac{21}{6} + N_{SM} + N_{BSM}
\end{aligned}
$$

First: Vacuum polarization of normal light fields

- the EFT regime

$$\Pi_{\mu\nu,\alpha\beta}(q^2) = -\frac{N_{\rm eff}^{(0)}}{640\pi^2} q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right) \mathcal{P}^{(2)}_{\mu\nu\alpha\beta}.$$

- with usual prescription

$$\ln(-q^2 - i\epsilon) = \ln(|q^2|) - i\pi\theta(q^2)$$

Leads to propagator

$$D^{-1}(q^2) = \frac{q^2 + i\epsilon}{\tilde{\kappa}^2} - \frac{q^4}{2\xi^2(\mu)} - \frac{N_{\rm eff}}{640\pi^2} q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right)$$

Next, the very high energy regime

- non-gravitational light fields are the same
- gravity now quartic propagators not quadratic

No imaginary part generated from quadratic gravity

-on shell triple graviton coupling vanishes in Minkowski

$$\sqrt{-g}\ R^2 = ... + \frac{1}{2} h^\lambda_\lambda R^{(1)} R^{(1)} + 2R^{(1)} R^{(2)} + ...$$

-on shell condition is R=0

Or regularize with $D \sim \frac{1}{q^4 + \epsilon^2}$

Logs then come with $-\frac{q^4 N_q}{1280\pi^2} \ln\left(\frac{(q^2)^2}{\mu^4}\right)$ $\qquad N_q = 199/3$

Verified by complete calculation

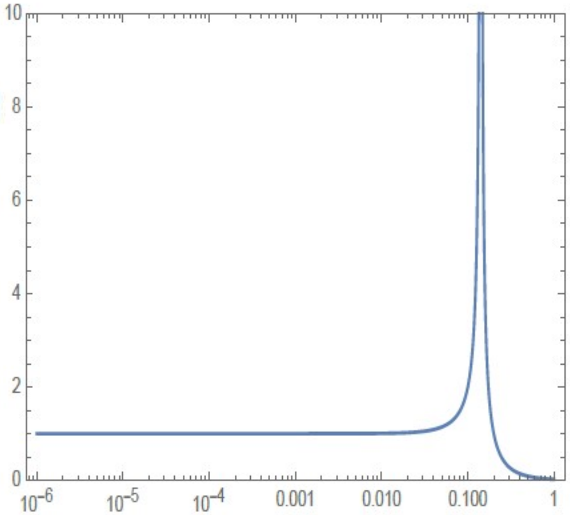
$$
\begin{aligned}
\mathcal{T}^{\mu\nu}{}_{\alpha\beta\gamma\delta}(k,q) \; = \; & -\frac{i}{2\xi^2}\Bigg\{\Big[\eta^{\mu\nu} q_\lambda q_\kappa + I^{\mu\nu}{}_{\lambda\kappa} q^2 - q^\sigma\Big(I^{\mu}{}_{\sigma\lambda\kappa} q^\nu + I^{\nu}{}_{\sigma\lambda\kappa} q^\mu\Big)\Big]\bar{\mathcal{R}}^{\lambda\kappa}{}_{\alpha\beta\gamma\delta}(k,q) \\
& - \frac{2}{3}\Big(\eta^{\mu\nu}q^2 - q^\mu q^\nu\Big)\bar{\mathcal{R}}_{\alpha\beta\gamma\delta}(k,q) - 2\mathcal{P}^{\mu\nu\sigma\tau} I_\sigma{}^\lambda{}_{\gamma\delta} I_{\tau\lambda\alpha\beta} p^2 k^2 \\
& + \Big(\mathcal{P}^{\mu\nu}{}_{\sigma\lambda} - \frac{1}{4}\eta^{\mu\nu}\eta_{\sigma\lambda}\Big) q^\sigma \Big(p^\lambda k^2 + k^\lambda p^2\Big) I_{\alpha\beta\gamma\delta} - \Big(p^\mu p^\nu k^2 + k^\mu k^\nu p^2\Big) I_{\alpha\beta\gamma\delta} \\
& - \frac{1}{2} q^2 \Big[I_\lambda{}^\nu{}_{\gamma\delta} I^{\lambda\mu}{}_{\alpha\beta} k^2 + I_\lambda{}^\nu{}_{\alpha\beta} I^{\lambda\mu}{}_{\gamma\delta} p^2 + (\mu \leftrightarrow \nu)\Big] \\
& + q_\rho \Big[k^2\Big(I_\lambda{}^\nu{}_{\gamma\delta} I^{\lambda\mu}{}_{\alpha\beta} p^\rho + p^\mu (I^{\nu\lambda}{}_{\gamma\delta} I^{\rho}{}_{\lambda\alpha\beta} - I^{\rho\lambda}{}_{\gamma\delta} I^{\nu}{}_{\lambda\alpha\beta}) + \frac{1}{2} q^\mu \Big(I^{\rho\lambda}{}_{\gamma\delta} I^{\nu}{}_{\lambda\alpha\beta} - I^{\nu\lambda}{}_{\gamma\delta} I^{\rho}{}_{\lambda\alpha\beta}\Big)\Big) \\
& + p^2\Big(I_\lambda{}^\nu{}_{\alpha\beta} I^{\lambda\mu}{}_{\gamma\delta} k^\rho + k^\mu (I^{\nu\lambda}{}_{\alpha\beta} I^{\rho}{}_{\lambda\gamma\delta} - I^{\rho\lambda}{}_{\alpha\beta} I^{\nu}{}_{\lambda\gamma\delta}) + \frac{1}{2} q^\mu \Big(I^{\rho\lambda}{}_{\alpha\beta} I^{\nu}{}_{\lambda\gamma\delta} - I^{\nu\lambda}{}_{\alpha\beta} I^{\rho}{}_{\lambda\gamma\delta}\Big)\Big) + (\mu \leftrightarrow \nu)\Big] \\
& + q_\lambda q_\sigma \Big[k^2\Big(I^{\lambda\nu}{}_{\gamma\delta} I^{\mu\sigma}{}_{\alpha\beta} - \frac{1}{2}\Big(I^{\mu\nu}{}_{\gamma\delta} I^{\sigma\lambda}{}_{\alpha\beta} + I^{\mu\nu}{}_{\alpha\beta} I^{\lambda\sigma}{}_{\gamma\delta}\Big)\Big) + p^2\Big(I^{\lambda\nu}{}_{\alpha\beta} I^{\mu\sigma}{}_{\gamma\delta} - \frac{1}{2}\Big(I^{\mu\nu}{}_{\alpha\beta} I^{\sigma\lambda}{}_{\gamma\delta} + I^{\mu\nu}{}_{\gamma\delta} I^{\lambda\sigma}{}_{\alpha\beta}\Big)\Big) \\
& + \frac{1}{2} q^\sigma \Big(k_\kappa\Big(I^{\mu\nu}{}_{\gamma\delta} I^{\kappa\lambda}{}_{\alpha\beta} - I^{\lambda\nu}{}_{\gamma\delta} I^{\mu\kappa}{}_{\alpha\beta}\Big) + p_\kappa\Big(I^{\mu\nu}{}_{\alpha\beta} I^{\kappa\lambda}{}_{\gamma\delta} - I^{\lambda\nu}{}_{\alpha\beta} I^{\mu\kappa}{}_{\gamma\delta}\Big)\Big) \\
& + \frac{1}{2} q^\nu \Big(k_\kappa\Big(I^{\lambda\sigma}{}_{\gamma\delta} I^{\mu\kappa}{}_{\alpha\beta} - I^{\mu\lambda}{}_{\gamma\delta} I^{\kappa\sigma}{}_{\alpha\beta}\Big) + p_\kappa\Big(I^{\lambda\sigma}{}_{\alpha\beta} I^{\mu\kappa}{}_{\gamma\delta} - I^{\mu\lambda}{}_{\alpha\beta} I^{\kappa\sigma}{}_{\gamma\delta}\Big)\Big) + (\mu \leftrightarrow \nu)\Big]\Bigg\} \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\mathcal{R}}^{\lambda\kappa}{}_{\alpha\beta\gamma\delta}(k,q) \; = \; & -I_{\alpha\beta\gamma\delta} q^\lambda q^\kappa + q^\sigma\Big[I_{\sigma\tau\gamma\delta}\Big(I^{\tau\kappa}{}_{\alpha\beta} k^\lambda - I^{\lambda\kappa}{}_{\alpha\beta} k^\tau\Big) + I_{\sigma\tau\alpha\beta}\Big(I^{\tau\kappa}{}_{\gamma\delta} p^\lambda - I^{\lambda\kappa}{}_{\gamma\delta} p^\tau\Big)\Big] \\
& + \Big(I^{\tau\lambda}{}_{\gamma\delta} p_\sigma + I^{\tau}{}_{\sigma\gamma\delta} p^\lambda - I^{\lambda}{}_{\sigma\gamma\delta} p^\tau\Big)\Big(I^{\sigma\kappa}{}_{\alpha\beta} k_\tau + I^{\sigma}{}_{\tau\alpha\beta} k^\kappa - I_\tau{}^\kappa{}_{\alpha\beta} k^\sigma\Big) + k^2 I^{\kappa}{}_{\tau\gamma\delta} I^{\lambda\tau}{}_{\alpha\beta} + p^2 I^{\kappa}{}_{\tau\alpha\beta} I^{\lambda\tau}{}_{\gamma\delta} \\
& - 2k_\sigma k_\tau \Big(\frac{1}{3} I^{\lambda\kappa}{}_{\gamma\delta} I^{\sigma\tau}{}_{\alpha\beta} + \frac{1}{2} I^{\kappa\sigma}{}_{\gamma\delta} I^{\tau\lambda}{}_{\alpha\beta}\Big) - 2p_\sigma p_\tau \Big(\frac{1}{3} I^{\lambda\kappa}{}_{\alpha\beta} I^{\sigma\tau}{}_{\gamma\delta} + \frac{1}{2} I^{\kappa\sigma}{}_{\alpha\beta} I^{\tau\lambda}{}_{\gamma\delta}\Big) \\
& - q^\lambda\Big(I^{\kappa\sigma}{}_{\gamma\delta} I_{\tau\sigma\alpha\beta} k^\tau + I^{\kappa\sigma}{}_{\alpha\beta} I_{\tau\sigma\gamma\delta} p^\tau\Big) \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\mathcal{R}}_{\alpha\beta\gamma\delta}(k,q) \; = \; & -q^2 I_{\alpha\beta\gamma\delta} + \Big(I^{\lambda}{}_{\tau\gamma\delta} p_\sigma + I^{\lambda}{}_{\sigma\gamma\delta} p_\tau - I_{\tau\sigma\gamma\delta} p^\lambda\Big)\Big(I_\lambda{}^\tau{}_{\alpha\beta} k^\sigma + I^{\tau\sigma}{}_{\alpha\beta} k_\lambda - I_\lambda{}^\sigma{}_{\alpha\beta} k^\tau\Big) \\
& + I^{\lambda}{}_{\sigma\gamma\delta} k_\tau \Big(I^{\tau}{}_{\lambda\alpha\beta} k^\sigma + I^{\tau\sigma}{}_{\alpha\beta} k_\lambda - I^{\sigma}{}_{\lambda\alpha\beta} k^\tau\Big) + I^{\lambda}{}_{\sigma\alpha\beta} p_\tau \Big(I^{\tau}{}_{\lambda\gamma\delta} p^\sigma + I^{\tau\sigma}{}_{\gamma\delta} p_\lambda - I^{\sigma}{}_{\lambda\gamma\delta} p \\
& + \frac{1}{2} q^\lambda \Big(I_{\lambda\sigma\gamma\delta} I^{\kappa\sigma}{}_{\alpha\beta} k_\kappa + I_{\lambda\sigma\alpha\beta} I^{\kappa\sigma}{}_{\gamma\delta} p_\kappa\Big).
\end{aligned}
$$

Resonance in spin-two propagator

$q^2\ \mathrm{Abs} D(q^2)$



q^2

Figure units: $\kappa^2 = 1$

$\xi^2 = 0.1$

Narrow width pole position: $$M^2 = m_r^2 + i\gamma = \frac{2\xi^2(m_r)}{\kappa^2} + i\frac{2\xi^2 m_r^2 N_{eff}}{640\pi}$$

Written as "ghost-like" plus "opposite sign width"

$$D(q) = \left[\frac{\kappa^2}{1+\frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}}\right]\ \frac{1}{-\delta q^2 + i\gamma} = \left[\frac{\kappa^2}{1+\frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}}\right]\ \frac{-1}{\delta q^2 - i\gamma}$$

$$q^2 = m_r^2 + \delta q^2$$

vs $$D(q) = \frac{1}{q^2 - (M - i\frac{\Gamma}{2})^2} = \frac{1}{\delta q^2 + iM\Gamma}$$

Unusual features:

Norm and sign of imaginary part are opposite normal

$$D(q) = \left[\frac{\kappa^2}{1+\frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}}\right]\frac{1}{-\delta q^2+i\gamma} = \left[\frac{\kappa^2}{1+\frac{N_{\rm eff}\xi^2(m_r)}{320\pi^2}}\right]\frac{-1}{\delta q^2-i\gamma}$$

vs

$$D(q) = \frac{1}{q^2-(M-i\frac{\Gamma}{2})^2} \qquad\qquad D(q) = \frac{1}{\delta q^2+iM\Gamma} \ .$$

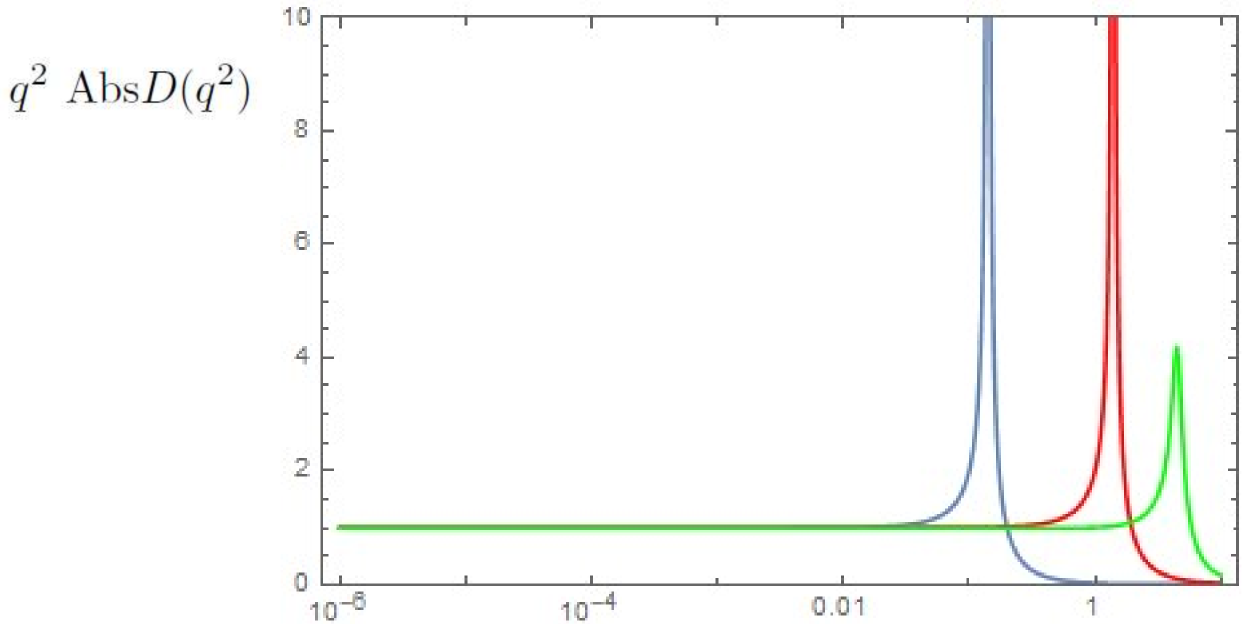
The two signs are important:

$$\frac{1}{i\gamma} = \frac{1}{iM\Gamma} \qquad\qquad iD(q^2) \sim \frac{iZ}{q^2-m_r^2-iZ\gamma}$$

Equivalently stated, overall imaginary propagator carries same sign

$$\mathrm{Im}D(q) = -\frac{\gamma}{(q^2-m^2)^2+\gamma^2} \quad vs \quad -\frac{M\Gamma}{(q^2-M^2)^2+M^2\Gamma^2} \ .$$

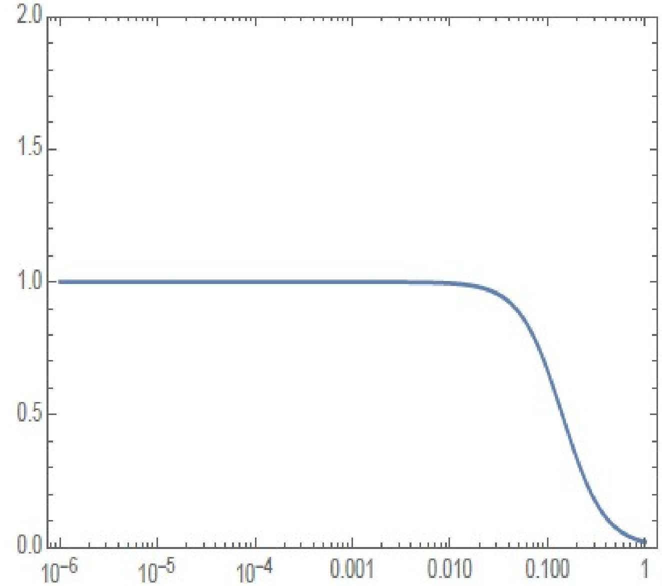
Resonance in propagator:



$\xi^2 = 0.1, 1, 10$

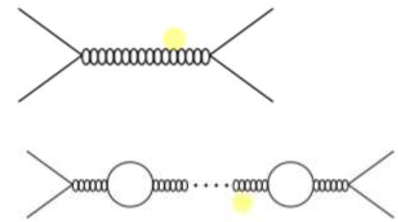
Euclidean/spacelike propagator:

$q^2 \ \mathrm{Abs}D(q^2)$



Unitarity in the spin two channel

Do these features cause trouble in scattering?
- consider scattering in spin 2 channel



First consider single scalar at low energy:

$$i\mathcal{M} = \left(\frac{1}{2}V_{\mu\nu}(q)\right)\left[iD^{\mu\nu\alpha\beta}(q^2)\right]\left(\frac{1}{2}V_{\alpha\beta}(-q)\right)$$

$$\mathcal{M} = 16\pi\sum_{J=0}^{\infty}(2J+1)T_J(s)P_J(\cos\theta)$$

Results in

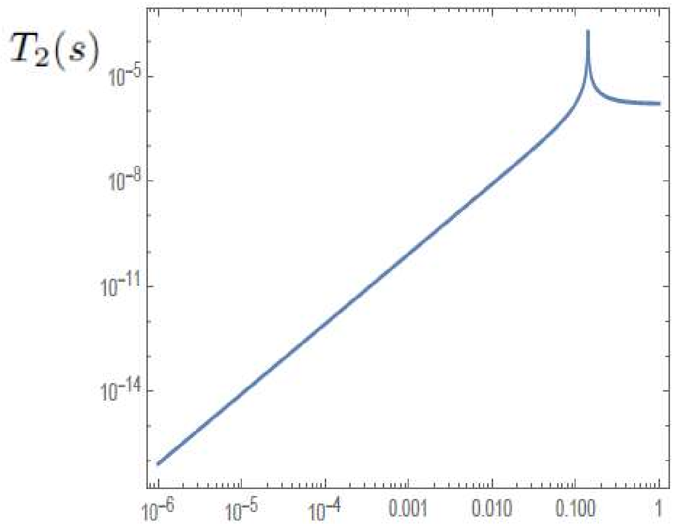
$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\,\bar{D}(s).$$

$N_{\rm eff} = 1/6$ for a single scalar field

$$\bar{D}^{-1}(s) \;=\; \frac{1}{\tilde{\kappa}^2}\left\{1 - \frac{\tilde{\kappa}^2 s}{2\xi^2(\mu)} - \frac{\tilde{\kappa}^2 s N_{\rm eff}}{640\pi^2}\ln\left(\frac{s}{\mu^2}\right) + \frac{i\tilde{\kappa}^2 s N_{\rm eff}}{640\pi}\right\}$$

J= 2 scattering is unitary:

- and weakly coupled:



Elastic unitarity $$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

Satisfied

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\left\{\frac{1}{\kappa^2} - \frac{s}{2\xi^2(\mu)} - \frac{sN_{\rm eff}}{640\pi^2}\left[\ln\left(\frac{s}{\mu^2}\right) - i\pi\right] - \frac{sN_q}{1280\pi^2}\ln\left(\frac{s^2}{\mu^4}\right)\right\}^{-1} .$$

Note for later: R^2 terms do not contribute to scattering

Satisfies elastic unitarity:

$$\mathrm{Im}T_2 = |T_2|^2.$$

This implies the structure

$$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

for any real *f(s)*

Signs and magnitudes work out for $A(s) = -\frac{N_{\mathrm{eff}}s}{640\pi}.$

Multi-particle problem:

- just diagonalize the J=2 channel
- same result but with general N

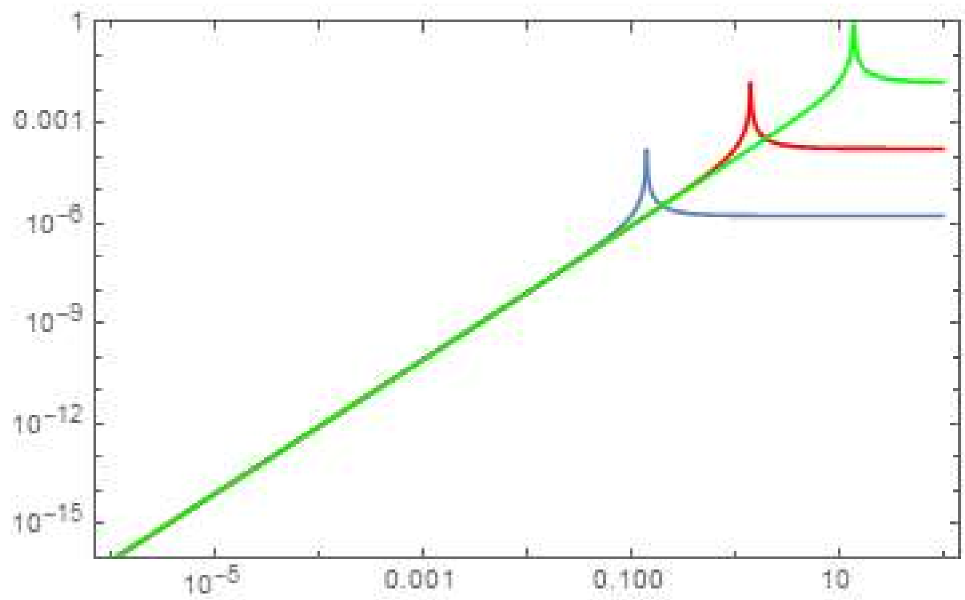
Very high energy region:

- quartic propagators for gravity
- does not couple to on-shell gravitons
- also does not contribute to imaginary parts

- unitarity still works

$$T_2(s) = -\frac{N_{\rm eff}s}{640\pi}\left\{\frac{1}{\kappa^2} - \frac{s}{2\xi^2(\mu)} - \frac{sN_{\rm eff}}{640\pi^2}\left[\ln\left(\frac{s}{\mu^2}\right) - i\pi\right] - \frac{sN_q}{1280\pi^2}\ln\left(\frac{s^2}{\mu^4}\right)\right\}^{-1}.$$

Scattering amplitude:



Modified Lehman and higher order loops

Physical propagator evaluated with $q^2 + i\epsilon$

$$D^{-1}(q^2) \;=\; \frac{q^2 + i\epsilon}{\tilde{\kappa}^2} - \frac{q^4}{2\xi^2(\mu)} - \frac{N_{\rm eff}}{640\pi^2} q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right)$$

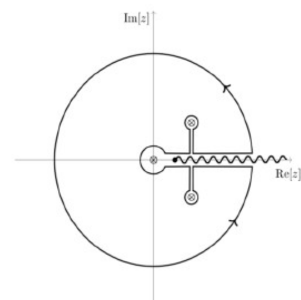
Cut along real axis

Another pole found on other side of cut using $q^2 - i\epsilon$

Then

$$f(a) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{z-a}\, dz$$

plus



leads to a representation

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

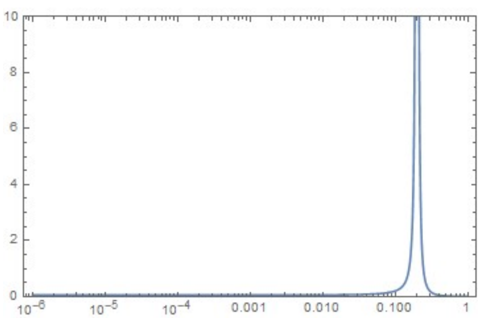
Modified Lehman representation:

$$D(q) = \frac{1}{q^2 + i\epsilon} \underbrace{- \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}}}_{\text{Sum is real}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Imaginary parts live here

Spectral function has resonant behavior:

$$\rho(s) = \frac{\frac{N_{eff}\kappa^2}{640\pi}}{\left(1 - \frac{s}{m_r^2} - \frac{N_{eff}\kappa^2 s}{640\pi^2}\log\frac{s}{m_r^2}\right)^2 + \left(\frac{N_{eff}\kappa^2 s}{640\pi}\right)^2}$$



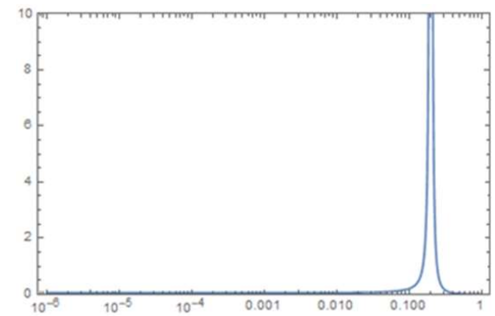
Currently studying loops with full propagator

- unitarity seems to work out using spectral integral
- new imaginary parts from resonant production (like G. O'C. W.)

This is really three resonance structures

Fix spectral function by matching

$$\rho(s) = \frac{\frac{N_{eff}\kappa^2}{640\pi}}{\left(1 - \frac{s}{m_r^2} - \frac{N_{eff}\kappa^2 s}{640\pi^2}\log\frac{s}{m_r^2}\right)^2 + \left(\frac{N_{eff}\kappa^2 s}{640\pi}\right)^2}$$



Previously we found a single resonance in propagator

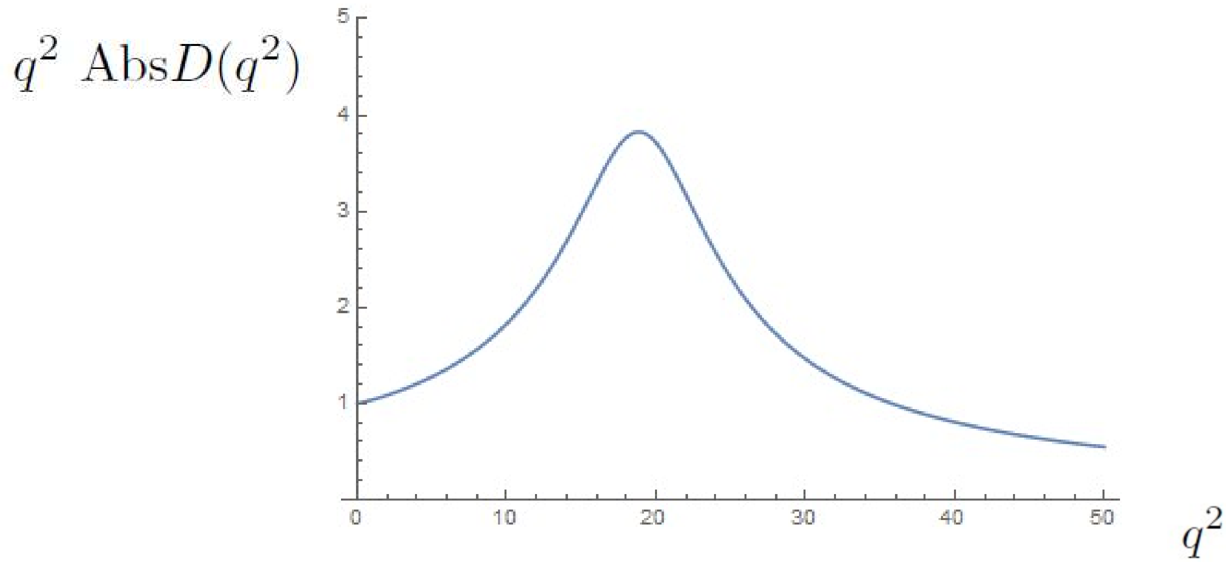
In Lehman representation we have three

The spectral portion and the cc pole essentially cancel
- exact in narrow width approximation

G.O'C.W

Beyond the narrow width approximation

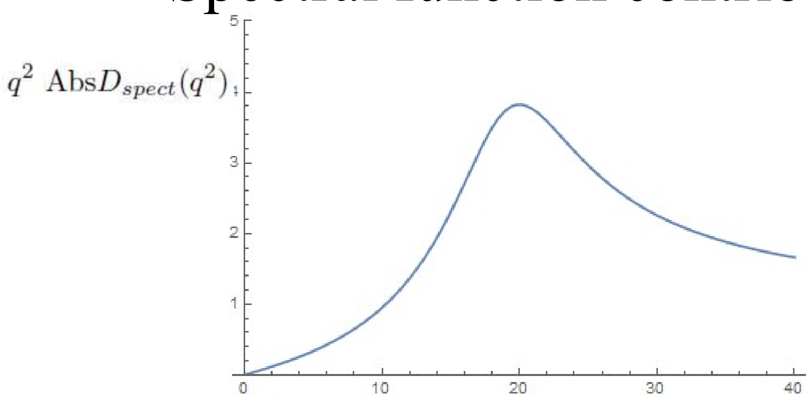
$\xi^2 = 10$



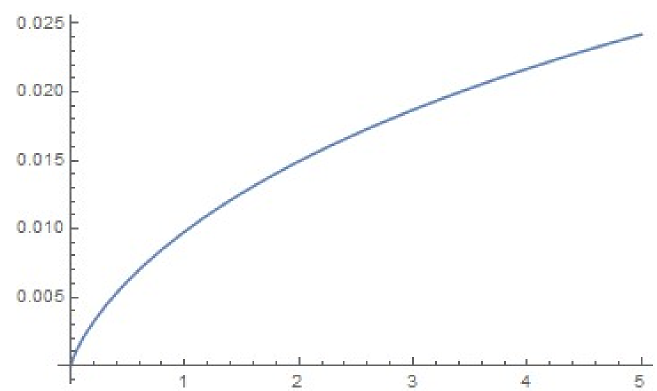
$M^2 = 18.9 + 4.7i$

$\beta = .925 + 0.17i$

Spectral function contribution



Diff. (spect. – cc pole)



Note change in scale

Demonstration of cancelation within three pole representation

$\xi^2 = 10$

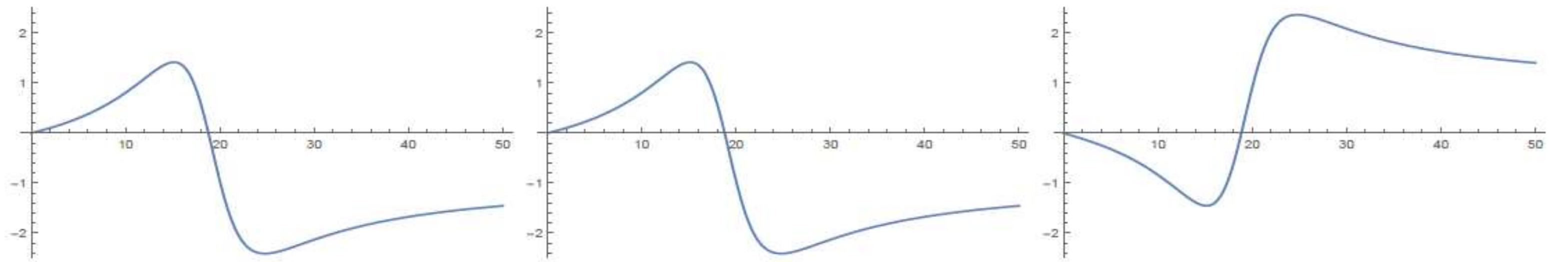


FIG. 7: Real parts of the three components of the propagator (left to right), the pole, the complex congugate pole and the spectral contribution.

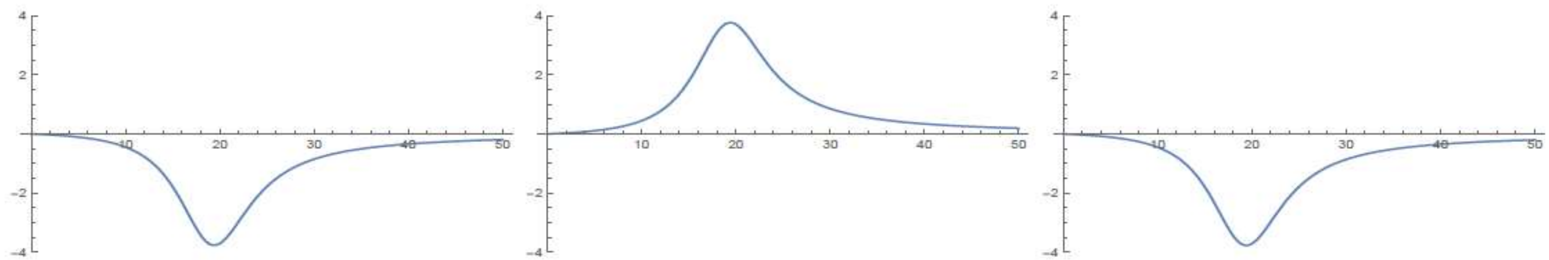
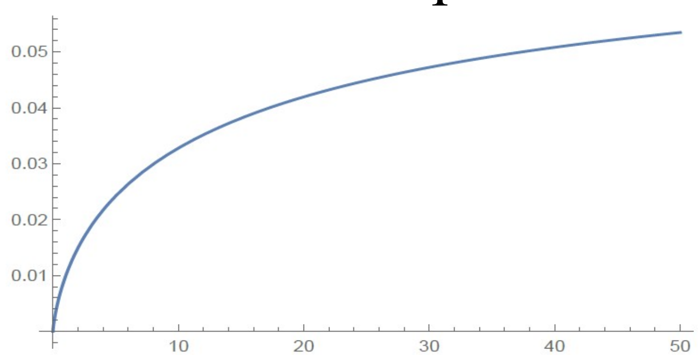


FIG. 8: Imaginary parts of the three components of the propagator (left to right), the pole, the complex congugate pole and the spectral contribution.

The sum of the c.c.-pole and the spectral function has no pole left over



Note change in scale

Lehman representation explains a lot of the physics

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M^2} - \frac{\beta^*}{q^2 - M^{*2}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Imaginary parts only arise in "normal" parts of the propagator

Unitarity according to G. O'C.W

Imaginary parts from:

-massless x massless

- massless x spectral
- spectral x spectral

These are the normal particles in the theory

Two thresholds are new, but expected

- producing the heavy unstable ghost state

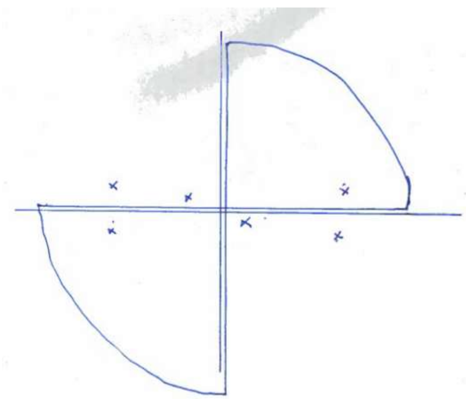
Unstable ghosts in loops

BUT:

- this is not enough in loops

$\pm i\,\gamma$ interacts differently with $+i\epsilon$

- calculational technique of rotating contour to Euclidean



Specific example:

- usual treatment of bubble diagram
- implies no imaginary part for 1 massless and 1 ghost state

Vanishing of "normal plus ghost" cut:

$$i\mathcal{A} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \frac{-1}{(k-p)^2 - m^2 - i\gamma}$$

Write in terms of retarded propagators:

**

$$i\mathcal{A} = \int \frac{d^4k}{(2\pi)^4} \left[\Delta_R(k) - i\frac{\pi}{\omega_k}\delta(k_0 - \omega_k)\right] \left[-\Delta_R(k-p) - i\frac{\pi}{E_k}\delta(k_0 - p_0 + E_{k-p})\right]$$

Only the cross terms contribute. After some manipulation

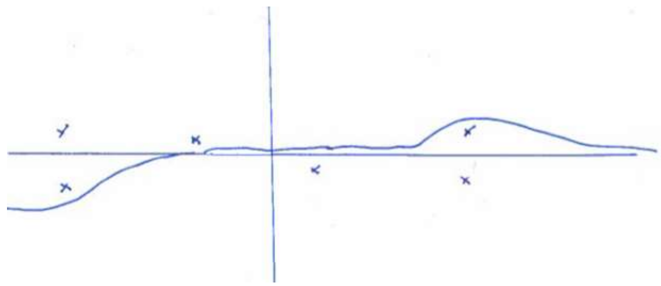
**

$$\mathrm{Im}\mathcal{A} = \int \frac{d^4k}{(2\pi)^4} \left[-\frac{\pi}{\omega_k}\delta(k_0 - \omega_k)\ \pi\delta((k-p)^2 - m^2) + \pi\delta(k^2)\ \frac{\pi}{E_k}\delta(k_0 - p_0 + E_{k-p})\right]$$

Normally these terms add to produce the Cutkosky rule.
Here they are equal and cancel.

Solution from Lee and Wick:

Deformed contour – "LW contour"



Anselmi talk

Equivalently – theory defined by Euclidean PI, rotated

This is NEW physics ingredient
(although also present in Asymptotic Safety program)

With this, unitarity seems to work:

In scalar bubble diagram:

$$
\begin{aligned}
F(p) &= -\frac{i\kappa^4}{16\pi^2}\left\{-2+\ln\left(\frac{-p^2-i\epsilon}{\mu^2}\right)+\frac{2}{\pi}\int_0^\infty ds\,\rho(s)\int_0^1 dx\ln\left(\frac{(1-x)s-i\epsilon-x(1-x)p^2}{\mu^2}\right)\right.\\
&+ \frac{1}{\pi^2}\int_0^\infty ds\,\rho(s)\int_0^\infty ds'\,\rho(s')\int_0^1 dx\ln\left(\frac{xs'+(1-x)s-i\epsilon-x(1-x)p^2}{\mu^2}\right)\\
&+ \left[(\mathcal{R}^2)^*\int_0^1 dx\ln\left(\frac{M^{2*}-x(1-x)p^2}{\mu^2}\right)-2\mathcal{R}^*\int_0^1 dx\ln\left(\frac{(1-x)M^{2*}-i\epsilon-x(1-x)p^2}{\mu^2}\right)\right.\\
&+ |\mathcal{R}|^2\int_{1/2}^1 dx\ln\left(\frac{xM^{2*}+(1-x)M^2-x(1-x)p^2}{\mu^2}\right)+|\mathcal{R}|^2\int_0^{1/2} dx\ln\left(\frac{xM^2+(1-x)M^{2*}-x(1-x)p^2}{\mu^2}\right)\\
&- \left.\left.\frac{2\mathcal{R}^*}{\pi}\int_0^\infty ds\,\rho(s)\int_0^1 dx\ln\left(\frac{(1-x)M^{2*}+xs-i\epsilon-x(1-x)p^2}{\mu^2}\right)+(\mathcal{R}^*\leftrightarrow\mathcal{R};M^{2*}\leftrightarrow M^2)\right]\right\} \qquad (117)
\end{aligned}
$$

And with all tensor indices:

$$
\begin{aligned}
\mathcal{F}^{\mu\nu\alpha\beta}(p) &= \mu^{4-d}\int\frac{d^dk}{(2\pi)^d}i\tau^{\mu\nu}_{\rho\tau\gamma\delta}(k,p)iD^{\rho\tau\sigma\epsilon}(k)\,i\tau^{\alpha\beta}_{\sigma\epsilon\lambda\kappa}(k,p)iD^{\gamma\delta\lambda\kappa}(p-k)=D_0(\eta^{\mu\nu}\eta^{\alpha\beta}+\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\nu\alpha}\eta^{\mu\beta})\\
&+ D_1(\eta^{\mu\nu}p^\alpha p^\beta+\eta^{\alpha\beta}p^\mu p^\nu+\eta^{\nu\alpha}p^\mu p^\beta+\eta^{\nu\beta}p^\mu p^\alpha+\eta^{\mu\alpha}p^\nu p^\beta+\eta^{\mu\beta}p^\nu p^\alpha)+D_2p^\mu p^\nu p^\alpha p^\beta. \qquad (123)
\end{aligned}
$$

The cosmological constant problem

Embarrassment for general picture

- vacuum energy generated from scale invariant start

Some options:

1) Unimodular gravity – vacuum energy is not relevant

2) Caswell-Banks-Zaks – beta function vanishes in IR

3) Strong coupled gravity – perhaps gravity cancels gauge

4) Spin connection as helper gauge theory – no vev JFD'16

5) Supersymmetric helper gauge theory – no c.c. at this scale

6) Abandon our principles – just add a constant to cancel it

<u>Summary:</u>

Gravity kept weakly coupled

Planck scale associated with helper gauge theory

Narrow, unstable ghost

Likelihood of micro-causality violation

Unitarity satisfied in simplest approximation

Loops of unstable ghosts being studied
- unitarity can be made to work
- but need Lee-Wick contour