What do we know about quantum corrections to Higgs Inflation?

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1 Introduction

2 Quantum corrections
   - Scale invariance at high scales
   - Corrections at intermediate scales
   - UV-completions?

3 Conclusions
Lesson from LHC so far – Standard Model is good

- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim 125\,\text{GeV} –$ weak coupling! Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}\,\text{GeV}$ as the highest energy to worry about
Lesson from LHC so far – Standard Model is good

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- LHC 2012 – final piece of the model discovered – Higgs boson
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- Add gravity
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  - get Planck scale $M_P \sim 1.22 \times 10^{19}$ GeV as the highest energy to worry about
Many things in cosmology are not explained by SM

Experimental observations
- Dark Matter
- Baryon asymmetry of the Universe
- Inflation (nearly scale invariant spectrum of initial density perturbations)

Laboratory also asks for SM extensions
- Neutrino oscillations
CMB observations favour flat potentials
Chaotic inflation—a scalar field

\[ H^2 \simeq \frac{1}{3M_P^2} \left( V(\phi) + \dot{\phi}^2/2 \right), \quad \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0 \]

\[ \frac{\lambda (20M_P)^4}{4} \]

\[ 3M_P \quad \text{Slow roll inflation} \quad 20M_P \]

\[ \frac{\lambda}{4} \phi^4 \]

\[ \delta T/T \sim 10^{-5} \text{ normalization} \]

quartic coupling: \( \lambda \sim 10^{-13} \) (or mass: \( m \sim 10^{13} \) GeV)

Can not be the SM Higgs field?
Non-minimal coupling to gravity solves the problem

Quite an old idea

For a scalar field coupling to the Ricci curvature is possible (actually *required* by renormalization)

- [A.Zee’78, L.Smolin’79, B.Spokoiny’84]
- [D.Salopek J.Bond J.Bardeen’89]

Scalar part of the (Jordan frame) action

\[
S_J = \int d^4x \sqrt{-g} \left\{ - \frac{M_P^2}{2} R - \frac{\xi}{2} h^2 R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\}
\]

- $h$ is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$GeV
- SM higgs vev $v \ll M_P / \sqrt{\xi}$ – can be neglected in the early Universe
- At $h \gg M_P / \sqrt{\xi}$ all masses are proportional to $h$ – scale invariant spectrum!

[FB, Shaposhnikov’08]
Conformal transformation – nice way to calculate

It is possible to get rid of the non-minimal coupling by the conformal transformation (change of variables)

\[ \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 \equiv 1 + \frac{\xi h^2}{M_P^2} \]

Redefinition of the Higgs field to get canonical kinetic term

\[ \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2}{M_P^2}} \Omega^4 \implies \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ \Omega^2 \simeq \exp\left(\frac{2\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\xi \end{cases} \]

Resulting action (Einstein frame action)

\[ S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2} \hat{R} + \frac{\partial_{\mu} \chi \partial^{\mu} \chi}{2} - \frac{\lambda}{4} \frac{h(\chi)^4}{\Omega(\chi)^4} \right\} \]
Potential – different stages of the Universe

\[ \frac{\lambda M_P^4}{4\xi^2} \]

\[ \frac{\lambda M_P^2 \chi^2}{6\xi^2} \]

\[ \frac{\lambda M_P^4}{4\xi^2} \left( 1 - e^{-2\chi/\sqrt{6}M_P} \right)^2 \]

\[ \chi_{\text{WMAP}} \simeq 5.4M_P \]

Hot Big Bang

Preheating

Slow roll inflation

\[ \frac{\delta T}{T} \sim 10^{-5} \] normalization

\[ \frac{\xi}{\sqrt{\lambda}} \simeq 47000 \] – at inflation

Small \( \lambda \) is traded for large \( \xi \)
CMB parameters are predicted
Exactly like preferred by CMB

For large $\xi$ Higgs inflation

spectral index $n \sim 1 - \frac{8(4N+9)}{(4N+3)^2} \sim 0.97$

tensor/scalar ratio $r \sim \frac{192}{(4N+3)^2} \sim 0.0033$

$\delta T / T \sim 10^{-5} \Rightarrow \frac{\xi}{\sqrt{\lambda}} \sim 47000$

Note: for very near critical top quark/Higgs masses results change and allow for larger $r$
Consistency

Up to now we neglected the quantum effects, assuming they do not spoil the story.

Is this really the case?
Let us work in the Einstein frame for simplicity.

Change of variables:

\[
\frac{d\chi}{dh} = \frac{M_P \sqrt{M_P^2 + (\xi + 6\xi^2)h^2}}{M_P^2 + \xi h^2}
\]

leads to the higher order terms in the potential (expanded in a power law series)

\[
V(\chi) = \lambda \frac{h^4}{4\Omega^4} \approx \lambda \frac{h^4}{4} \approx \lambda \frac{\chi^4}{4} + \# \frac{\chi^6}{(M_P/\xi)^2} + \cdots
\]

Unitarity is violated at tree level in scattering processes (eg. $2 \rightarrow 4$) with energy above the "cut-off"

\[
E > \Lambda_0 \sim \frac{M_P}{\xi}
\]

Hubble scale at inflation is $H \sim \lambda^{1/2} \frac{M_P}{\xi}$ – not much smaller than the today cut-off $\Lambda_0$ :

[Burgess, Lee, Trott’09, Barbon, Espinosa’09, Hertzberg’10]
"Cut off" is background dependent!

Classical background \[ \chi(x, t) \rightarrow \bar{\chi}(t) + \delta \chi(x, t) \]

leads to background dependent suppression of operators of \( \text{dim } n > 4 \)

\[
\frac{\mathcal{O}(n)(\delta \chi)}{[\Lambda(n)(\bar{\chi})]^{n-4}}
\]

Example

Potential in the inflationary region \( \chi > M_P \):

\[
U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left( 1 - e^{-\frac{2\chi}{\sqrt{6}M_P}} \right)^2
\]

leads to operators of the form:

\[
\mathcal{O}(n)(\delta \chi) = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta \chi)^n}{M_P^n}
\]

Leading at high \( n \) to the "cut-off"

\[ \Lambda \sim M_P \]
Cut-off grows with the field background

Jordan frame

Einstein frame

Relation between cut-offs in different frames:
\[ \Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}} \Omega \]

Relevant scales

Hubble scale \( H \sim \lambda^{1/2} \frac{M_P}{\xi} \)

Energy density at inflation \( V^{1/4} \sim \lambda^{1/4} \frac{M_P}{\sqrt{\xi}} \)

Reheating temperature \( M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi} \)

Problems during reheating – Kohei Kamada’s talk yesterday

[FB, Magnin, Shaposhnikov, Sibiryakov’11, FB, Gorbunov, Shaposhnikov’11]
Below $M_P/\xi$
- Renormalizable $\phi^4$-like Standard Model

Above $M_P/\sqrt{\xi}$
- Non-renormalizable, but the potential nicely arranges

$$e^{-\chi/M} + e^{-2\chi/M} + e^{-3\chi/M} + \ldots$$

Higher terms are irrelevant

Can we go “through” the $M_P/\xi$ scale?
Shift symmetric UV completion allows to have effective theory during inflation

\[ \mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left( 1 + \sum u_n e^{-n \cdot \chi / M} \right) \]

Effective action (from quantum corrections of loops of \( \delta \chi \))

\[ \mathcal{L}_{\text{eff}} = f^{(1)}(\chi) \frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi) \frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi) \frac{(\partial \chi)^4}{M^4} + \ldots \]

All the divergences are absorbed in \( u_n \) and in \( f^{(n)} \sim \sum f_l e^{-n \chi / M} \)

**Required (asymptotically at large fields)**

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<th>Shift symmetry (Einstein frame)</th>
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Should be true in quantum case!
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How to get scale invariant quantum theory

Regularization breaks scale invariance

Classically fully scale invariant action

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} (\partial_\mu \phi)^2 - \lambda_0 (h^2 - \alpha_0 \phi^2)^2 \]

c.f. previous talk by J. Rubio

In the dimensional regularization the renormalized parameters \( \lambda, \alpha \)

\[ \lambda_0 = \mu^{4-n} \left( \lambda + \sum C_k(\lambda, \alpha) \frac{1}{(n-4)^k} \right) \]

\[ \alpha_0 = \left( \alpha + \sum D_k(\lambda, \alpha) \frac{1}{(n-4)^k} \right) \]

Scale \( \mu \) explicitly breaks scale invariance on quantum level
How to get scale invariant quantum theory
Scale invariant regularization at the cost of renormalizability

Generate scale $\mu$ out of the fields

Replace

$$\mu^{4-n} \rightarrow (\phi^2)^{\frac{4-n}{n-2}} F(h/\phi)$$

- The renormalized action is explicitly scale invariant
- At the same time, it does not have the same form, as the original – it has arbitrary powers of the fields (when expanded around some $\phi = \tilde{\phi} + \delta \phi$). non-renormalizable

[Shaposhnikov, Zenhausern’09], also
[Englert, Truffin, Gastmans’76, Wetterich’88]
Just asymptotically scale invariant is also possible

- Actually, no need to introduce $\phi$
- Start from original *asymptotically* scale invariant theory

**Generate scale $\mu$ out of the fields**

Replace

$$\mu^{4-n} \rightarrow (\mu^2)^{\frac{4-n}{n-2}} F(h/\mu_*)$$

with some $\mu_*$ and $F$ of the form:

[Shaposhnikov, Shimada’18]
Quantum scale invariance at the cost of renormalizability

- Not losing too much – the theory with gravity was not renormalizable in any case
- However – there are infinitely many counterterms

Difference between any two choices of $F(h)$ can be absorbed in redefinition of tree level (counterterm) potential

<table>
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<tr>
<th>Popular choices:</th>
<th>choice I</th>
<th>choice II</th>
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<tbody>
<tr>
<td>Jordan frame</td>
<td>$M_P^2 + \xi h^2$</td>
<td>$M_P^2$</td>
</tr>
<tr>
<td>Einstein frame</td>
<td>$M_P^2$</td>
<td>$\frac{M_P^4}{M_P^2 + \xi h^2}$</td>
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[Hamada, Kawai, Nakanishi,, Oda’17, Mooij, Shaposhnikov, Voumard’18]
Further assumptions

1. Assume something about subtraction rules
   - strong: Only add infinite part of counterterms ("MS")
   - weaker: Add only required operators, but with some finite coefficients

2. Try to embed this in something with better UV behaviour ("UV-complete")
1. Adding only required counterterms to the action

- Generic – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the required counterterms at each order in loop expansion

\[
\mathcal{L} = \frac{(\partial \chi)^2}{2} - \frac{\lambda}{4} F^4(\chi) + i \bar{\psi}_t \partial \psi_t + \frac{y_t}{\sqrt{2}} F(\chi) \bar{\psi}_t \psi_t
\]

\[
F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \begin{cases} 
\chi, & \chi < \frac{M_P}{\sqrt{\xi}} \\
\frac{M_P}{\sqrt{\xi}} \left(1 - e^{-\sqrt{2/3} \chi / M_P}\right)^{1/2}, & \chi > \frac{M_P}{\sqrt{\xi}}
\end{cases}
\]

Doing quantum calculations we should add

\[
\mathcal{L} + \mathcal{L}_{\text{1-loop}} + \delta \mathcal{L}_{\text{1-loop c.t.}} + \cdots
\]
Counterterms: $\lambda$ modification

Calculating vacuum energy

\[
\delta \mathcal{L}_{ct} = \frac{1}{2} \text{Tr} \ln \left[ \Box - \left( \frac{\lambda}{4} (F^4)'' \right)^2 \right]
\]

\[
= \frac{9 \lambda^2}{64 \pi^2} \left( \frac{2}{\bar{\epsilon}} - \ln \frac{\lambda (F^4)''}{4 \mu^2} + \frac{3}{2} \right) \left( F'^2 + \frac{1}{3} F'' F \right)^2 F^4,
\]

\[
\delta \mathcal{L}_{ct} = - \text{Tr} \ln \left[ i \partial + y_t F \right]
\]

\[
= - \frac{y_t^4}{64 \pi^2} \left( \frac{2}{\bar{\epsilon}} - \ln \frac{y_t^2 F^2}{2 \mu^2} + \frac{3}{2} \right) F^4
\]
Counterterms: $\lambda$ modification

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\[
\delta \mathcal{L}_{ct} = \frac{9\lambda^2}{64\pi^2} \left( \frac{2}{\bar{\epsilon}} + \delta\lambda_{1a} \right) \left( F'^2 + \frac{1}{3} F'' F \right)^2 F^4,
\]

\[
\delta \mathcal{L}_{ct} = -\text{Tr} \ln \left[ i\partial + y_t F \right]
\]

\[
\delta \mathcal{L}_{ct} = -\frac{y_t^4}{64\pi^2} \left( \frac{2}{\bar{\epsilon}} + \delta\lambda_{1b} \right) F^4
\]

Small $\chi$: $F'^4 F^4 \sim \chi \sim F^4$

Large $\chi$: $F'^4 F^4 \sim e^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

$\delta\lambda_{1b}$ – just $\lambda$ redefinition, while $\delta\lambda_{1a}$ is not!
Modified “evolution” of $\lambda(\mu)$

For RG we should in principle write infinite series

$$
\frac{d\lambda}{d\ln \mu} = \beta_\lambda(\lambda, \lambda_1, a\ldots)
$$

$$
\frac{d\lambda_1}{d\ln \mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \ldots)
$$

... 

- Assuming $\delta_i$ are small and have the same hierarchy, as the loop expansion, we truncate this to just first equation.
- Neglect change of $\delta \lambda_1$ between $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$

$$
\lambda(\mu) \rightarrow \lambda(\mu) + \delta \lambda \left[ \left( F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right],
$$
Counterterms: Top Yukawa coupling

Calculating propagation of the top quark in the background $\chi$

\[
\delta \mathcal{L}_{ct} \sim \left( \# \frac{y_t^3}{\bar{\epsilon}} + \delta y_{t1} \right) F' F \bar{\psi} \psi \\
+ \left( \# \frac{y_t \lambda}{\bar{\epsilon}} + \delta y_{t2} \right) F'' (F^4)'' \bar{\psi} \psi
\]

\[
y_t(\mu) \rightarrow y_t(\mu) + \delta y_t \left[ F'^2 - 1 \right]
\]
Threshold effects at $M_P/\xi$ summarized by two new arbitrary constants $\delta\lambda$, $\delta y_t$

\[
\lambda(\mu) \rightarrow \lambda(\mu) + \delta\lambda \left[ \left( F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right]
\]

\[
y_t(\mu) \rightarrow y_t(\mu) + \delta y_t \left[ F'^2 - 1 \right]
\]
Large $\xi$ – return to tree level predictions

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4} \frac{M_P^4}{\xi^2} \left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

- If $\xi \sim 5 \times 10^4 \sqrt{\lambda} \gg 1$
  - logarithmic RG running of $\lambda$ is negligible
  - threshold “jumps” at $\mu \sim M_P/\xi$ are below inflationary scale
    - irrelevant for inflationary observables.
- All this story is not needed – we are in general attractor class of inflationary models

**spectral index**

$$n \approx 1 - \frac{8(4N + 9)}{(4N + 3)^2} \approx 0.97$$

**tensor/scalar ratio**

$$r \approx \frac{192}{(4N + 3)^2} \approx 0.0033$$
Small $\xi$ – critical HI

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4} \frac{M_P^4}{\xi^2} \left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

- Small $\xi \lesssim 10 - \lambda$ vs. $\delta\lambda$ significant, may give interesting “features” in the potential (“critical inflation”, large $r$)
- However – tend to get both inflation and $\delta\lambda$ “jumps” in the same scale around $M_P/\xi$
- Loop corrections change result – harder to control

Bezrukov, Pauly, Rubio’17
2) UV completions
Embed into something with better behaviour

- $R^2$-higgs inflation
  [Ema’17, Gorbunov, Tokareva’19]

$$S = \int d^4 x \sqrt{-g} \left\{ - \left( \frac{M_P^2}{2} + \frac{\xi h^2}{2} \right) R + \frac{M_P}{12M^2} R^2 - \frac{(\partial h)^2}{2} - \frac{\lambda h^4}{4} \right\}$$

K. Kohei’s talk yesterday

- $R^2$ is complete up to $M_P$
- If scalaron is below the problematic scale the whole theory is weakly coupled up to $M_P$

$$M \lesssim \frac{M_P}{\xi}$$

- More generic additional scalar below $M_P/\xi$
  [Giudice, Lee’11]
Relation between low energy and inflationary dynamics change

- **Basic HI**
  - Low energy: $\lambda$, and high order operators controlled by $\xi$
  - Inflation: perturbations fixed by $\sqrt{\xi}/\lambda$

- **$R^2$-Higgs**
  - Low energy: $\lambda$, and high order operators controlled by $\xi, M$
  - Inflation: perturbations fixed by $(\sqrt{\xi}/\lambda + M_P^2/3M^2)$

No known UV completion without a state with $M < M_P/\xi$

See, however [Calmet, Casadio’14]
Further note on variable choice:

- How do we interpret the gravity action:
  - Metric – $g_{\mu \nu}(x)$ is an independent field, Connection – $\Gamma_{\mu \nu}^\lambda \equiv g_{\lambda \rho}^2 \left( g_{\rho \mu, \nu} + g_{\rho \nu, \mu} - g_{\mu \nu, \rho} \right)$
  - Palatiny – $g_{\mu \nu}(x)$, $\Gamma_{\mu \nu}^\lambda(x)$ are independent fields

- Different classical dynamics if $\xi \neq 0$
  Can be seen as different transformation under $g_{\mu \nu} \to \Omega(x) g_{\mu \nu}$

Rather different inflationary predictions!

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<tr>
<td>$\xi \sim 5 \times 10^4 \sqrt{\lambda}$</td>
<td>$\xi \sim 1.5 \times 10^{10} \lambda$</td>
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<tr>
<td>$r \sim 3.2 \times 10^{-3}$</td>
<td>$r \sim 3.5 \times 10^{-14} \lambda^{-1}$</td>
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e.g. Rasanen, Wahlman’17; Järv, Racioppi, Tenkanen’17
Conclusions

- Scale invariance is indispensable to make sense of non-minimally coupled inflation
- Much more assumptions needed to relate low and high energy physics
  - intermediate $M_P/\xi$ scale
- Still lacking
  - UV completion without even more scales?
  - Arguments how to be fully predictive without one?


