What do we know about quantum corrections to Higgs Inflation?

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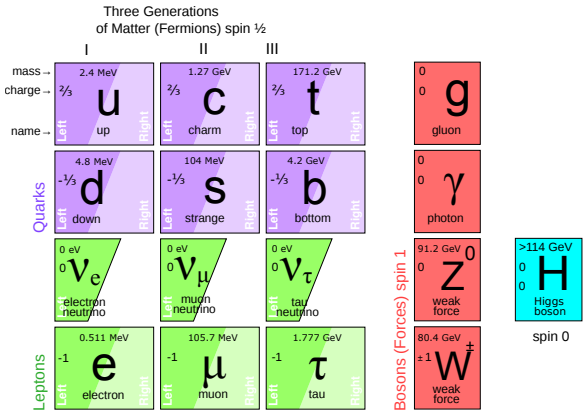
Outline

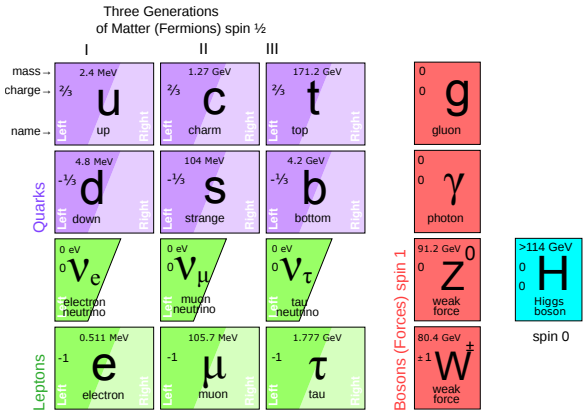
1. Introduction
2. Quantum corrections
   - Scale invariance at high scales
   - Corrections at intermediate scales
   - UV-completions?
3. Conclusions

Lesson from LHC so far – Standard Model is good



- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim 125\,\mathrm{GeV}$ – weak coupling! Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}\,\mathrm{GeV}$ as the highest energy to worry about

Lesson from LHC so far – Standard Model is good



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Einstein gravity

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Many things in cosmology are not explained by SM

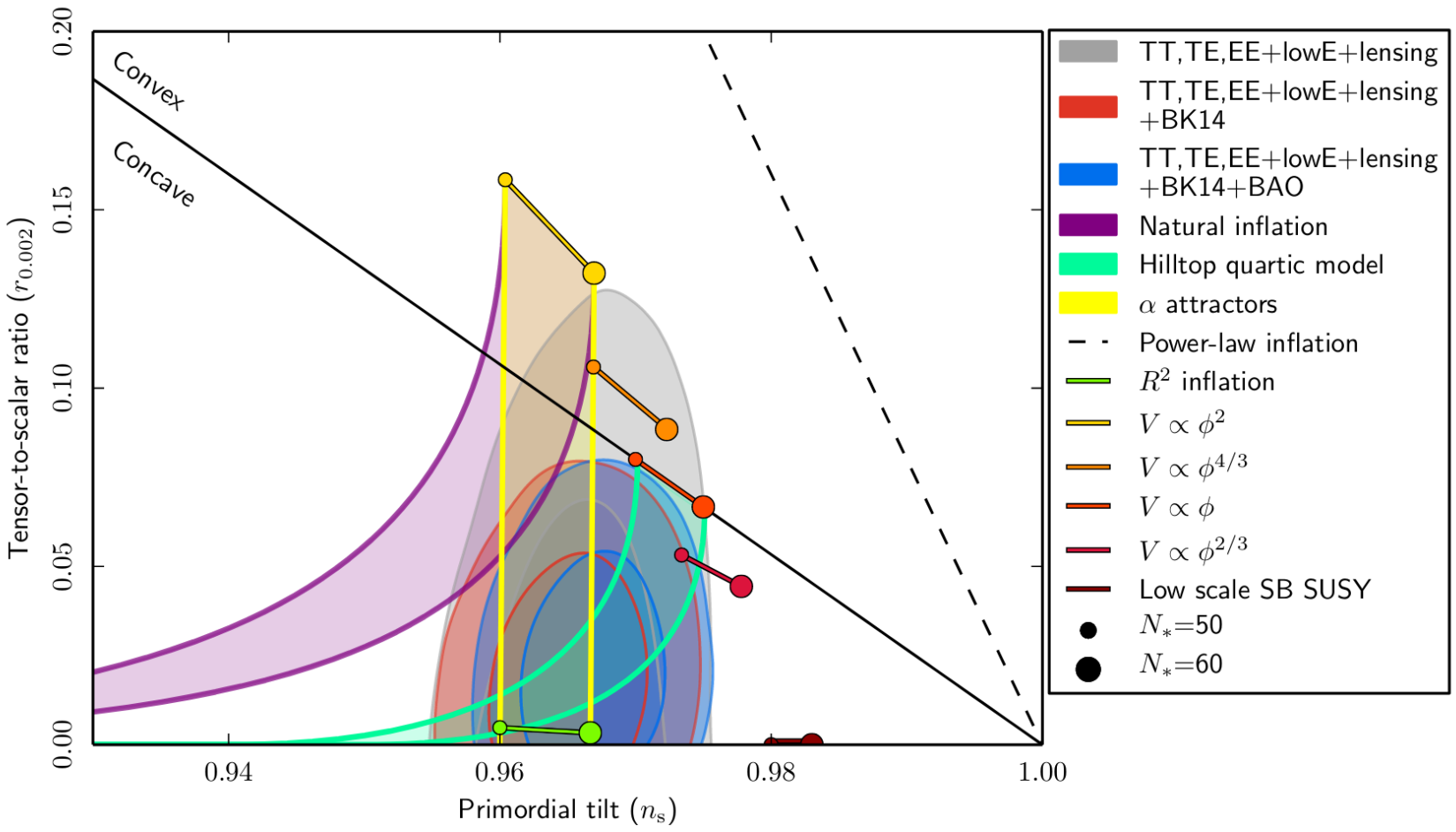
Experimental observations

- Dark Matter
- Baryon asymmetry of the Universe
- Inflation (nearly scale invariant spectrum of initial density perturbations)

Laboratory also asks for SM extensions

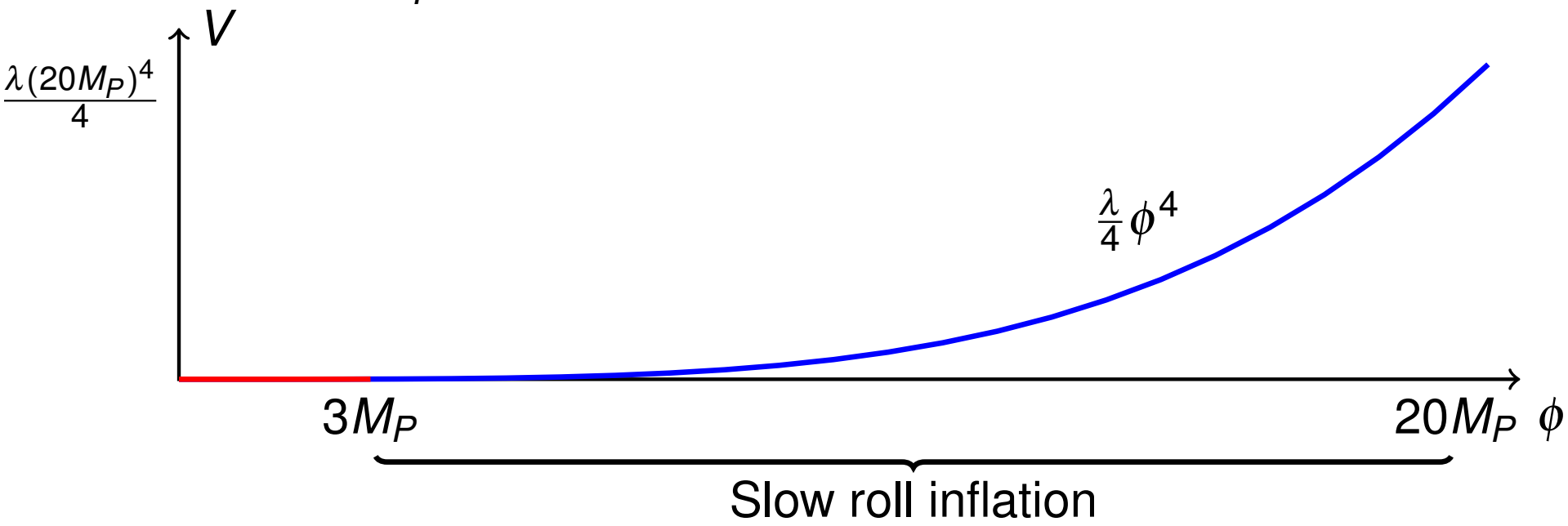
- Neutrino oscillations

CMB observations favour flat potentials



Chaotic inflation–a scalar field

$$\mathscr{H}^2 \simeq \frac{1}{3M_P^2}\left(V(\phi)+\dot{\phi}^2/2\right), \qquad \ddot{\phi}+3\mathscr{H}\dot{\phi}+V'(\phi)=0$$



$\delta T/T \sim 10^{-5}$ normalization

quartic coupling: $\lambda \sim 10^{-13}$ (or mass: $m \sim 10^{13}$ GeV)

Can not be the SM Higgs field?

Non-minimal coupling to gravity solves the problem

Quite an old idea

For a scalar field coupling to the Ricci curvature is possible (actually *required* by renormalization)

- [A.Zee'78, L.Smolin'79, B.Spokoiny'84]
- [D.Salopek J.Bond J.Bardeen'89]

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$

- h is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4\times 10^{18}$GeV
- SM higgs vev $v \ll M_P/\sqrt{\xi}$ – can be neglected in the early Universe
- At $h \gg M_P/\sqrt{\xi}$ *all* masses are proportional to h – scale invariant spectrum!

[FB, Shaposhnikov'08]

Conformal transformation – nice way to calculate

It is possible to get rid of the non-minimal coupling by the conformal transformation (change of variables)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \,, \qquad \Omega^2 \equiv 1 + \frac{\xi h^2}{M_P^2}$$

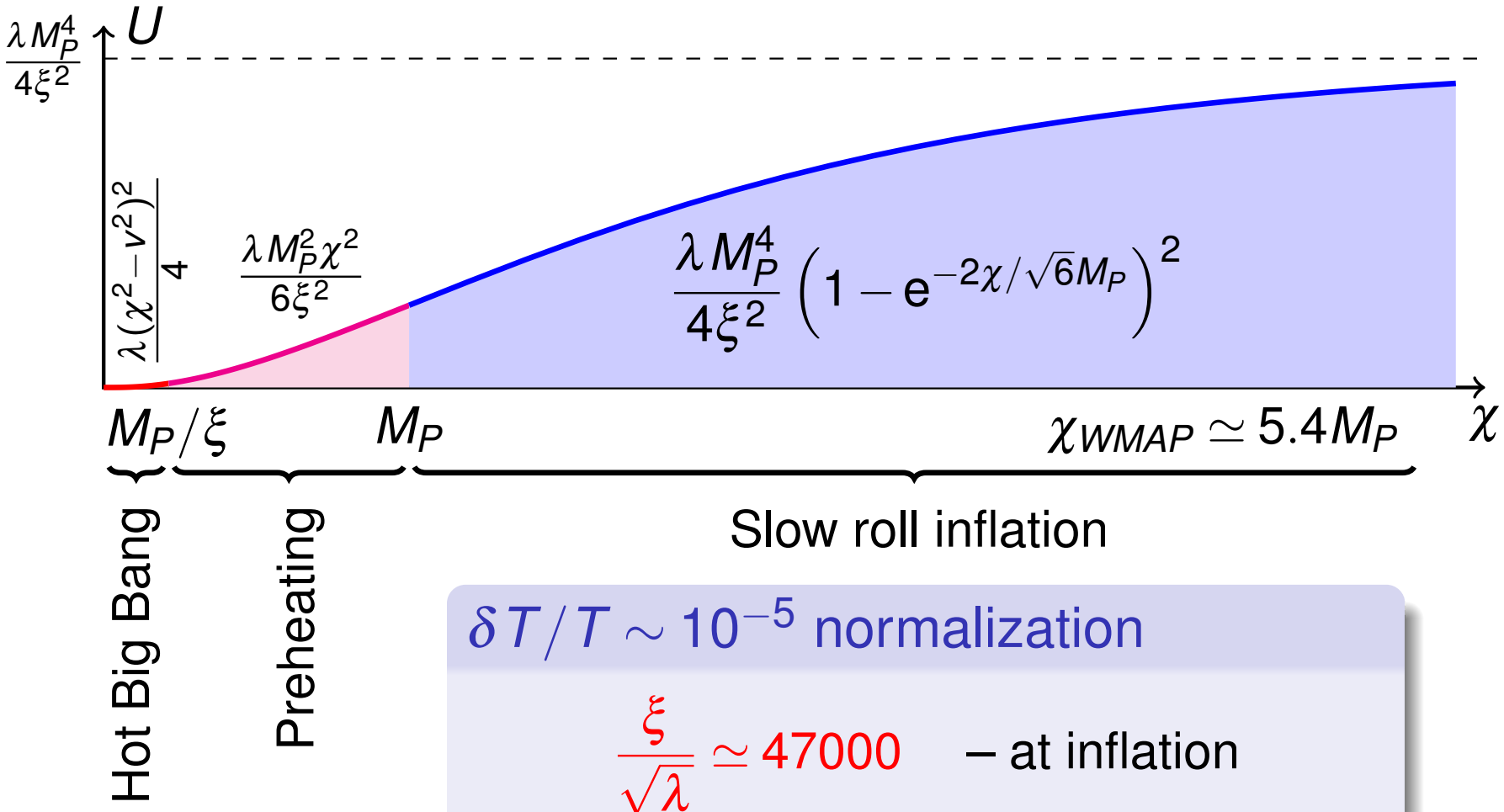
Redefinition of the Higgs field to get canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ \Omega^2 \simeq \exp\left(\frac{2\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\xi \end{cases}$$

Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{\lambda}{4}\frac{h(\chi)^4}{\Omega(\chi)^4} \right\}$$

Potential – different stages of the Universe



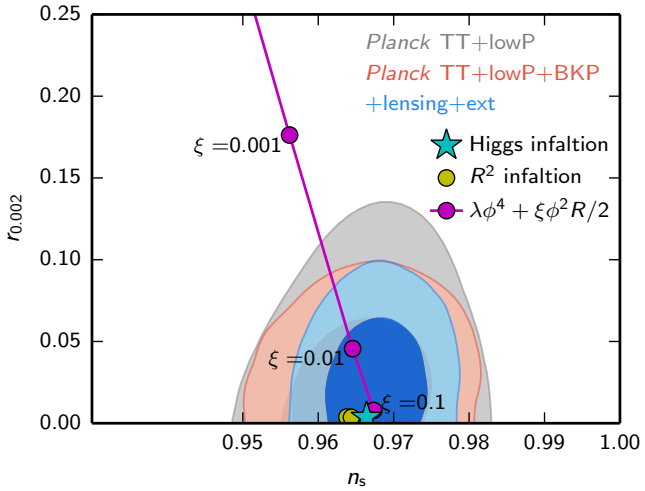
$\delta T/T \sim 10^{-5}$ normalization

$$\frac{\xi}{\sqrt{\lambda}} \simeq 47000 \quad \text{– at inflation}$$

Small λ is traded for large ξ

CMB parameters are predicted

Exactly like preferred by CMB



For large ξ Higgs inflation

spectral index $\quad n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$

tensor/scalar ratio $\quad r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$

$$\delta T/T \sim 10^{-5} \quad \Longrightarrow \quad \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

Note: for very near critical top quark/Higgs masses results change and allow for larger r

Consistency

Up to now we neglected the quantum effects, assuming they do not spoil the story.

Is this really the case?

Cut off scale today

Let us work in the Einstein frame for simplicty

Change of variables: $\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(\xi+6\xi^2)h^2}}{M_P^2+\xi h^2}$ leads to the higher order terms in the potential (expanded in a power law series)

$$V(\chi) = \lambda\frac{h^4}{4\Omega^4} \simeq \lambda\frac{h^4}{4} \simeq \lambda\frac{\chi^4}{4} + \#\frac{\chi^6}{(M_P/\xi)^2} + \cdots$$

Unitarity is violated at tree level

in scattering processes (eg. $2 \to 4$) with energy above the "cut-off"

$$E > \Lambda_0 \sim \frac{M_P}{\xi}$$

Hubble scale at inflation is $H \sim \lambda^{1/2}\frac{M_P}{\xi}$ – not much smaller than the today cut-off Λ_0 :(

[Burgess, Lee, Trott'09, Barbon, Espinosa'09, Hertzberg'10]

"Cut off" is background dependent!

Classical background Quantum perturbations

$$\chi(x,t) \quad = \quad \bar{\chi}(t) \quad + \quad \delta\chi(x,t)$$

leads to background dependent suppression of operators of dim $n > 4$

$$\frac{\mathscr{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}}$$

Example

Potential in the inflationary region $\chi > M_P$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

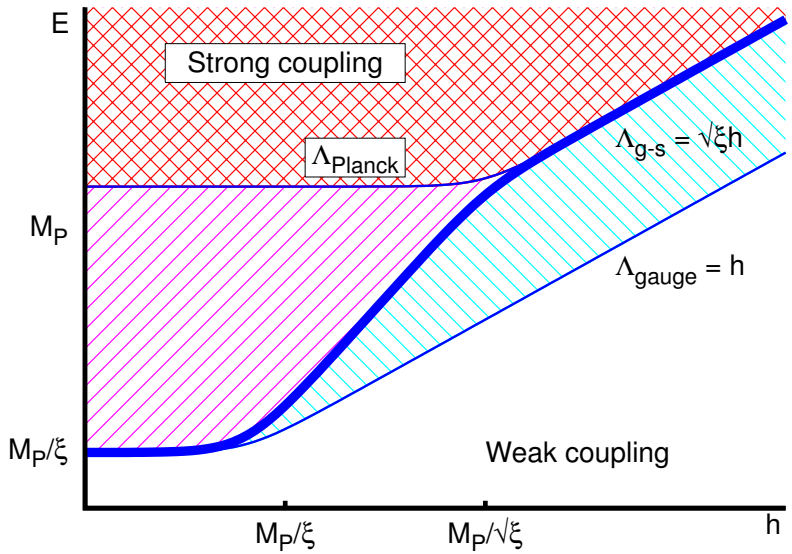
leads to operators of the form: $\frac{\mathscr{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta\chi)^n}{M_P^n}$

Leading at high n to the "cut-off"

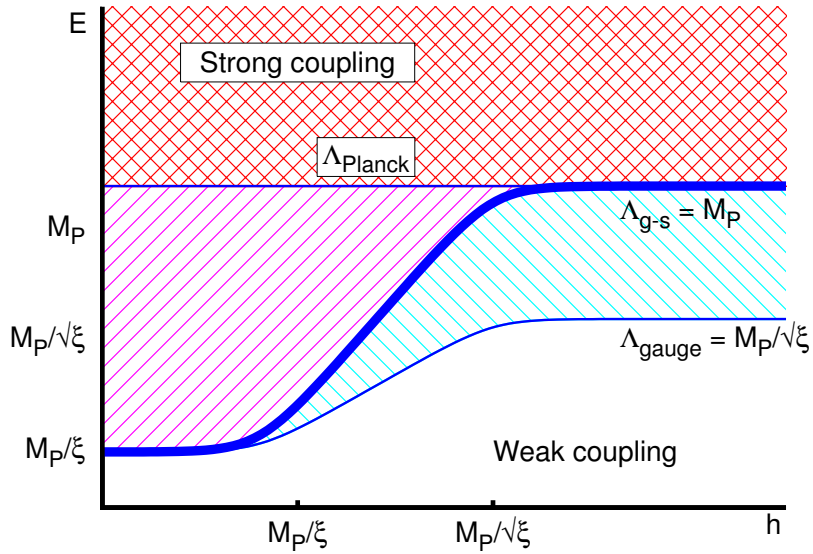
$$\Lambda \sim M_P$$

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Relevant scales

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Problems during reheating – Kohei Kamada's talk yesterday

[FB, Magnin, Shaposhnikov,, Sibiryakov'11, FB, Gorbunov, Shaposhnikov'11]

- Below M_P/ξ
  - Renormalizable ϕ^4-like Standard Model
- Above $M_P/\sqrt{\xi}$
  - Non-renormalizable, but the potential nicely arranges

$$e^{-\chi/M} + e^{-2\chi/M} + e^{-3\chi/M} + \cdots$$

  Higher terms are irrelevant
- Can we go "through" the M_P/ξ scale?

Shift symmetric UV completion allows to have effective theory during inflation

$$\mathscr{L} = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum u_n e^{-n\cdot\chi/M}\right)$$

Effective action (from quantum corrections of loops of $\delta\chi$)

$$\mathscr{L}_{\text{eff}} = f^{(1)}(\chi)\frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial \chi)^4}{M^4} + \cdots$$

All the divergences are absorbed in u_n and in $f^{(n)} \sim \sum f_l e^{-n\chi/M}$

Required (asymptotically at large fields)

Shift symmetry (Einstein frame)	Scale (Jordan frame)
$\chi \mapsto \chi + \text{const}$	$h \to \Omega h$
	$x \to \Omega^{-1} x$

Should be true in quantum case!

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How to get scale invariant quantum theory

Regularization breaks scale invariance

Classically fully scale invariant action

$$\mathscr{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \phi)^2 - \lambda_0 (h^2 - \alpha_0 \phi^2)^2$$

c.f. previous talk by J.Rubio

In the dimensional regularization the renormalized parameters λ, α

$$\lambda_0 = \mu^{4-n}\left(\lambda + \sum \frac{C_k(\lambda,\alpha)}{(n-4)^k}\right)$$

$$\alpha_0 = \left(\alpha + \sum \frac{D_k(\lambda,\alpha)}{(n-4)^k}\right)$$

Scale μ explicitly breaks scale invariance on quantum level

How to get scale invariant quantum theory

Scale invariant regularization at the cost of renormalizability

Generate scale μ out of the fields

Replace

$$\mu^{4-n} \to (\phi^2)^{\frac{4-n}{n-2}} F(h/\phi)$$

- The renormalized action is explicitly scale invariant
- At the same time, it does not have the same form, as the original – it has arbitrary powers of the fields (when expanded around some $\phi = \bar{\phi} + \delta\phi$). non-renormalizable

[Shaposhnikov, Zenhausern'09], also
[Englert, Truffin, Gastmans'76, Wetterich'88]

Just asymptotically scale invariant is also possible

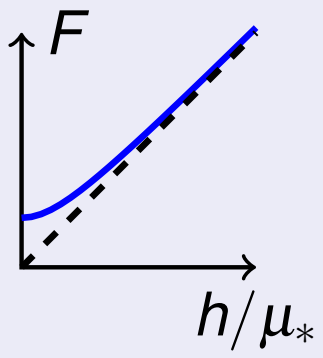
- Actually, no need to introduce ϕ
- Start from original *asymptotically* scale invariant theory

Generate scale μ out of the fields

Replace

$$\mu^{4-n} \to (\mu^2)^{\frac{4-n}{n-2}} F(h/\mu_*)$$

with some μ_* and F of the form:



[Shaposhnikov, Shimada'18]

Quantum scale invariance at the cost of renormalizability

- Not loosing too much – the theory with gravity was not renormalizable in any case
- However – there are infinitely many counterterms

Difference between any two choices of $F(h)$ can be absorbed in redefinition of tree level (counterterm) potential

Popular choices:

	choice I	choice II
Jordan frame	$M_P^2 + \xi h^2$	M_P^2
Einstein frame	M_P^2	$\dfrac{M_P^4}{M_P^2 + \xi h^2}$

[Hamada, Kawai, Nakanishi,, Oda'17, Mooij, Shaposhnikov, Voumard'18]

Further assumptions

1. Assume something about subtraction rules

   strong: Only add infinite part of counterterms ("$\overline{\text{MS}}$")

   weaker: Add only required operators, but with some finite coefficients

2. Try to embed this in something with better UV behaviour ("UV-complete")

1. Adding only required counterterms to the action

- Generic – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the *required* counterterms at each order in loop expansion

$$\mathscr{L} = \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t\partial\!\!\!/\psi_t + \frac{y_t}{\sqrt{2}}F(\chi)\bar{\psi}_t\psi_t$$

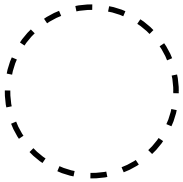
$$F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\}$$

Doing quantum calculations we should add

$$\mathscr{L} + \mathscr{L}_{\text{1-loop}} + \delta\mathscr{L}_{\text{1-loop c.t.}} + \cdots$$

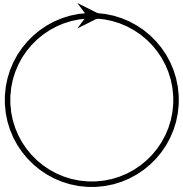
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$

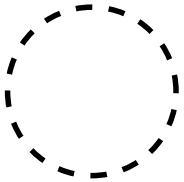


$$= -\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{y_t^2 F^2}{2\mu^2} + \frac{3}{2}\right)F^4$$

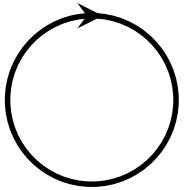
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$\delta\mathscr{L}_{\mathrm{ct}} = \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1a}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$\delta\mathscr{L}_{\mathrm{ct}} = -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1b}\right)F^4$$

Small χ : $F'^4 F^4 \sim \chi \sim F^4$

Large χ : $F'^4 F^4 \sim \mathrm{e}^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

$\delta\lambda_{1b}$ – just λ redefinition, while $\delta\lambda_{1a}$ is not!

Modified "evolution" of $\lambda(\mu)$

For RG we should in principle write infinite series

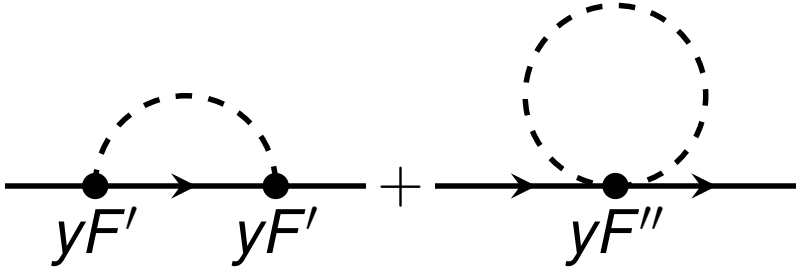
$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, \lambda_1, a\ldots)$$
$$\frac{d\lambda_1}{d\ln\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \ldots)$$
$$\ldots$$

- Assuming δ_i are small and have the same hierarchy, as the loop expansion, we truncate this to just first equation.
- Neglect change of $\delta\lambda_1$ between $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right],$$

Counterterms: Top Yukawa coupling

Calculating propagation of the top quark in the background χ



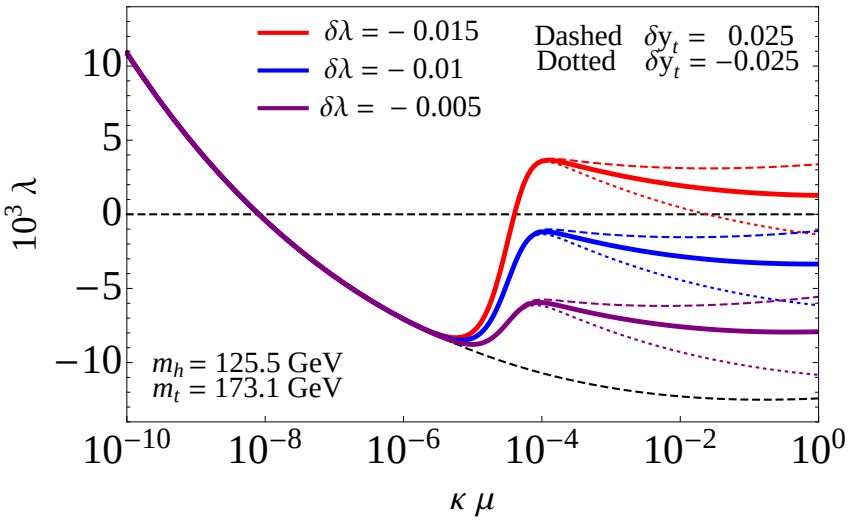
$$\begin{aligned}\delta\mathscr{L}_{\text{ct}} &\sim \left(\#\frac{y_t^3}{\bar{\varepsilon}} + \delta y_{t1}\right) F'^2 F \bar{\psi}\psi \\ &+ \left(\#\frac{y_t\lambda}{\bar{\varepsilon}} + \delta y_{t2}\right) F''(F^4)''\bar{\psi}\psi\end{aligned}$$

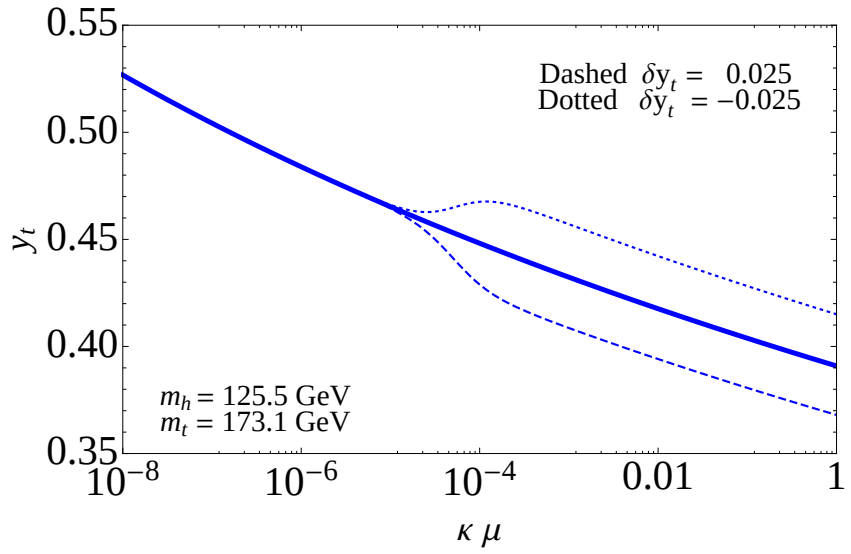
$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1\right]$$

Threshold effects at M_P/ξ summarized by two new arbitrary constants $\delta\lambda$, δy_t

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$





Large ξ – return to tree level predictions

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

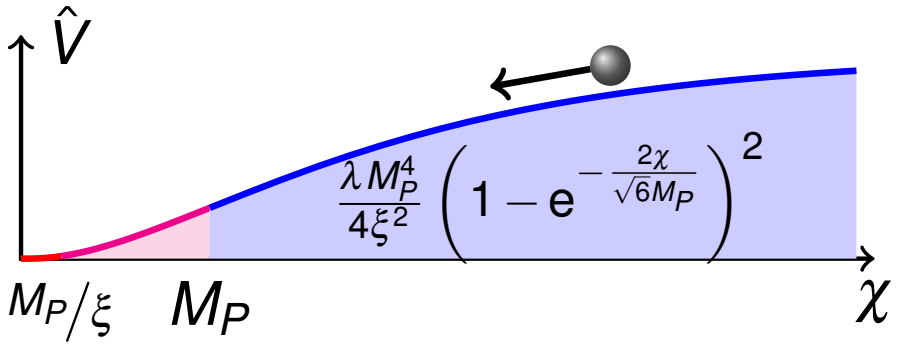
- If $\xi \sim 5 \times 10^4\sqrt{\lambda} \gg 1$
  - logarithmic RG running of λ is neglidgible
  - threshold "jumps" at $\mu \sim M_P/\xi$ are below inflationary scale – irrelevant for inflationary observables.
- All this story is not needed – we are in general attractor class of inflationary models

spectral index

$$n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$$

tensor/scalar ratio

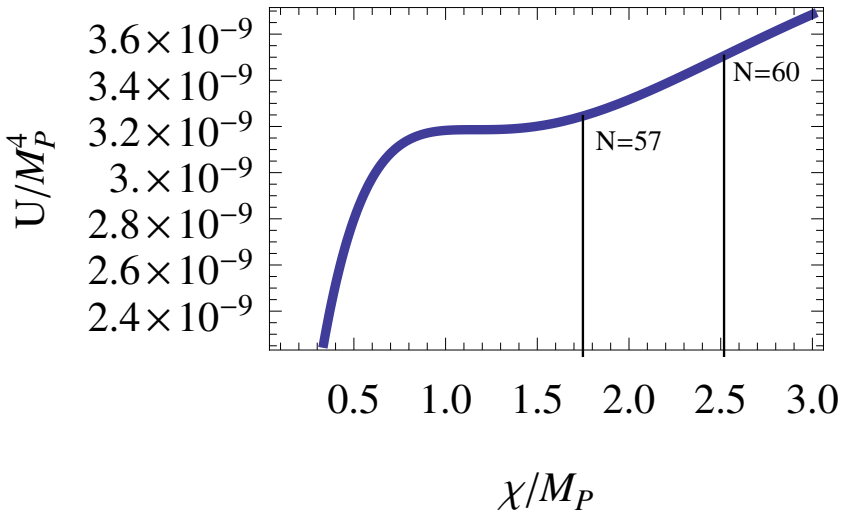
$$r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$$

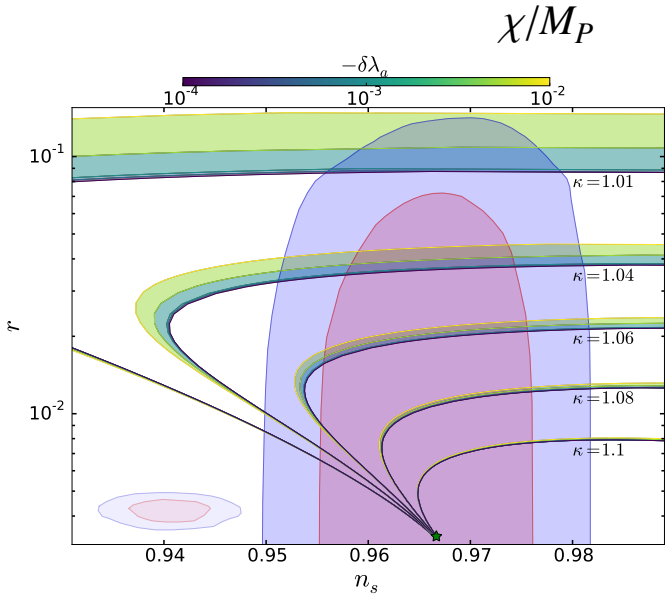


Small ξ – critical HI

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

- Small $\xi \lesssim 10$ – λ vs. $\delta\lambda$ significant, may give interesting "features" in the potential ("critical inflation", large r)
- However – tend to get both inflation *and* $\delta\lambda$ "jumps" in the same scale around M_P/ξ
- Loop corrections change result – harder to control





Bezrukov, Pauly, Rubio'17

2) UV completions

Embed into something with better behaviour

- R^2-higgs inflation
  [Ema'17, Gorbunov, Tokareva'19]

$$S = \int d^4x \sqrt{-g} \left\{ -\left(\frac{M_P^2}{2} + \frac{\xi h^2}{2} \right) R + \frac{M_P}{12M^2} R^2 - \frac{(\partial h)^2}{2} - \frac{\lambda h^4}{4} \right\}$$

K.Kohei's talk yesterday

  - R^2 is complete up to M_P
  - If scalaron is below the problematic scale the whole theory is weakly coupled up to M_P

$$M \lesssim \frac{M_P}{\xi}$$

- More generic additional scalar below M_P/ξ
  [Giudice, Lee'11]

Relation between low energy and inflationary dynamics change

- Basic HI
  - Low energy: λ, and high order operators controlled by ξ
  - Inflation: perturbations fixed by $\sqrt{\xi}/\lambda$
- R^2-Higgs
  - Low energy: λ, and high order operators controlled by ξ, M
  - Inflation: perturbations fixed by $(\sqrt{\xi}/\lambda + M_P^2/3M^2)$

No known UV completion without a state with $M < M_P/\xi$

See, however [Calmet, Casadio'14]

Further note on variable choice:

- How do we interpret the gravity action:
  - Metric – $g_{\mu\nu}(x)$ is an independent field, Connection – $\Gamma^{\lambda}_{\mu\nu} \equiv \frac{g^{\lambda\rho}}{2}(g_{\rho\mu,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho})$
  - Palatiny – $g_{\mu\nu}(x)$, $\Gamma^{\lambda}_{\mu\nu}(x)$ are independent fields
- Different *classical* dynamics if $\xi \neq 0$
  Can be seen as different transformation under
  $g_{\mu\nu} \to \Omega(x) g_{\mu\nu}$

Rather different inflationary predictions!

Metric	Palatini
$R \to \Omega^2 R + 6 g^{\mu\nu} \partial_\mu \ln\Omega \partial_\nu \ln\Omega$	$R \to \Omega^2 R$
$\xi \sim 5 \times 10^4 \sqrt{\lambda}$ $r \sim 3.2 \times 10^{-3}$	$\xi \sim 1.5 \times 10^{10} \lambda$ $r \sim 3.5 \times 10^{-14} \lambda^{-1}$

e.g. Rasanen,Wahlman'17; Järv,Racioppi,Tenkanen'17

Conclusions

- Scale invariance is indispensable to make sense of non-minimally coupled inflation
- Much more assumptions needed to relate low and high energy physics
  - intermediate M_P/ξ scale
- Still lacking
  - UV completion without even more scales?
  - Arguments how to be fully predictive without one?

F. L. Bezrukov and M. Shaposhnikov *Phys. Lett.* **B659** (2008) 703–706.

C. P. Burgess, H. M. Lee and M. Trott *JHEP* **09** (2009) 103.

J. L. F. Barbon and J. R. Espinosa *Phys. Rev.* **D79** (2009) 081302.

M. P. Hertzberg *JHEP* **11** (2010) 023.

F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov *JHEP* **1101** (2011) 016, `arXiv:1008.515`.

F. Bezrukov, D. Gorbunov and M. Shaposhnikov *JCAP* **1110** (2011) 001.

M. Shaposhnikov and D. Zenhausern *Phys. Lett.* **B671** (2009) 162–166.

F. Englert, C. Truffin and R. Gastmans *Nucl.Phys.* **B117** (1976) 407–432.

C. Wetterich *Nucl.Phys.* **B302** (1988) 668–696.

M. Shaposhnikov and K. Shimada 2018.

Y. Hamada, H. Kawai, Y. Nakanishi, and K.-y. Oda *Phys.Rev.* **D95** (2017), no. 10, 103524.

S. Mooij, M. Shaposhnikov and T. Voumard 2018.

Y. Ema *arXiv:1701.07665 [astro-ph, physics:hep-ph]*, 2017 `arXiv:1701.07665`.

D. Gorbunov and A. Tokareva *Phys.Lett.* **B788** (2019) 37–41.

G. F. Giudice and H. M. Lee *Phys.Lett.* **B694** (2011) 294–300.

X. Calmet and R. Casadio *Phys.Lett.* **B734** (2014) 17–20.