Quantum scale invariance and Weyl gravity

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Based on: arXiv: 1812.08613, 1712.06024, 1809.09174 with Hyun Min Lee

PRD 96 (2017), with Z. Lalak, P. Olszewski

Outline:

I. Flat space:

- Quantum corrections in scale-invariant regularization $\lambda\phi^4$ & SM
- spontaneous breaking (SB) $\Rightarrow$ flat direction (dilaton)

$\rightarrow$ classical hierarchy of: higgs vev $\ll$ dilaton vev's $\Rightarrow$ quantum stable?

II. Curved space:

- $M_{\rm Planck} \sim$ dilaton vev $\Rightarrow$ Weyl gravity/gauge symmetry
- Stueckelberg: dilaton "eaten" by Weyl gauge field ω_μ $(m_\omega \sim M_{\rm Planck})$

$\rightarrow$ SB of Weyl quadratic gravity $\Rightarrow$ Einstein gravity $+$massive ω_μ

- Weyl geometry $\Rightarrow$ Riemann geometry (below m_ω)
- Adding matter (Higgs) $\Rightarrow$ EWSB.

- "New physics" beyond SM: new symmetry?

- SUSY @ TeV: hierarchy problem [and other problems] solved [in theory....]

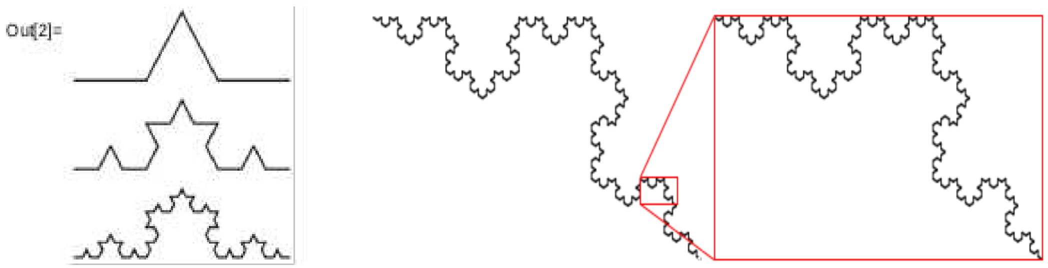
- scale invariance (SI); - SM with $m_h = 0$ has classical scale invariance. [Bardeen 1995]

$$x \to \rho\, x, \quad \phi \to \rho^{-1}\, \phi, \quad [\phi] = 1, \quad \text{SI forbids} \quad \int d^4x \; m^2 \phi^2 + \cdots$$

- no dimensionful couplings; All scales from vev's! (including M_{Planck}!)
- Classical SI: models: "Higgs portal": $\lambda \phi^2 \sigma^2$. Quantum level?

- Nature: discrete SI: self-similarity across different scales [fractals]

In[2]:= Column[Table[Graphics[KochCurve[n]], {n, 1, 3}]] snowflake/Koch curve



• Scale-invariant regularization (SIR): [Englert 1976, Itzykson/Zuber, Shaposhnikov 2008]

DR: $d=4-2\epsilon$: $L = \frac{1}{2}(\partial_\mu\phi)^2 - \lambda_\phi\,\mu^{2\epsilon}\,\phi^4$, explicit $\not{\text{SI}}$ by UV regulators, to avoid.

replace $\mu \to$ field σ, spontaneous $\langle\sigma\rangle\neq 0 \Rightarrow$ Goldstone (dilaton): $\sigma = \langle\sigma\rangle\; e^\tau$, $\tau \to \tau - \ln\rho$

[nothing new, recall M_{string} moduli dep]

• Action: $d=4$, $$S = \int d^4x \left[\underbrace{(\partial_\mu\phi)^2 - V(\phi)}_{\text{visible}}\right] + \int \underbrace{d^4y\, L_h(\sigma,\partial\sigma)}_{\text{hidden}}$$

- spectrum extended by σ! potential: $\lambda_\sigma\,\sigma^4$ but Poincaré symmetry demands $\lambda_\sigma = 0$ [Fubini 1976]

- each sector SI (shift symmetry) $\to$ enhanced shift symmetry: $S_h \times S_v \Rightarrow \lambda_m\phi^2\sigma^2$: λ_m naturally small

[Volkas, Kobakhidze, Foot 2013]

$d=4-2\epsilon$: $\mu = z\,\sigma^{2/(d-2)}$, $V \to \tilde V = \left[z\,\sigma^{2/(d-2)}\right]^{4-d} V(\phi)$, ($z$: dim-less), $\sigma = \langle\sigma\rangle + \tilde\sigma$

$$\tilde V = \left(z\langle\sigma\rangle^{1/(1-\epsilon)}\right)^{2\epsilon}\left[1 + 2\,\epsilon\left(\frac{\tilde\sigma}{\langle\sigma\rangle} - \frac{\tilde\sigma^2}{2\langle\sigma\rangle^2} + \ldots\right) + \epsilon^2\left(\frac{2\tilde\sigma}{\langle\sigma\rangle} + \ldots.\right) + \mathcal{O}(\epsilon^3)\right] V(\phi),$$

$\Rightarrow$ SIR=DR+dilaton with ∞-many ϵ-couplings. $\langle\sigma\rangle$:'new physics'. Q: if $\langle\phi\rangle \ll \langle\sigma\rangle$: is it quantum stable?

[$\sigma\sigma \to \sigma\sigma$ at 3 loops! $(\partial_\mu \ln\sigma)^4$]

• One-loop SI potential: $$L = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \overbrace{\frac{1}{4!}\lambda\,\phi^4}^{=V(\phi)}, \qquad V \to \tilde V = z^{2\epsilon}\,\sigma^{2\epsilon/(1-\epsilon)}\,V(\phi).$$

$$V_1 = -\frac{i}{2}\int\frac{d^dp}{(2\pi)^d}\ \mathrm{tr}\,\ln\left[p^2 - \tilde V_{\alpha\beta} + i\varepsilon\right] = \sum_{s=\phi,\sigma}\frac{\tilde M_s^4}{4\kappa}\left[\frac{-1}{\epsilon} + \ln\frac{\tilde M_s^2}{c_0}\right]\mu(\sigma)^{2\epsilon},$$

$$\tilde M_\phi^4 = M_\phi^4 + \epsilon\ldots, \quad \tilde M_\sigma^4 \sim \epsilon^2, \quad \kappa = (4\pi)^2$$

One-loop correction: $$U_1 = \frac{V_{\phi\phi}^2}{4\kappa}\left[\overline{\ln}\frac{V_{\phi\phi}}{(z\,\sigma)^2} - \frac{1}{2}\right] = \underbrace{\frac{V_{\phi\phi}^2}{4\kappa}\left[\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} - \frac{1}{2}\right]}_{CW} + \underbrace{\frac{V_{\phi\phi}^2}{4\kappa}\left[\frac{-\tilde\sigma}{\langle\sigma\rangle} + \frac{\tilde\sigma^2}{2\langle\sigma\rangle^2} + \ldots\right]}_{\to 0;\ \text{small yet maintains SI}}$$

$\sigma = \langle\sigma\rangle + \tilde\sigma$, $\tilde\sigma$: fluctuations

$V_{\phi\phi} = \frac{\lambda}{2}\,\phi^2$

$\Rightarrow$ U_1 scale invariant due to dilaton ($\ln\sigma$). No new poles, same beta function $\beta_\lambda^{(1)}$ (as for no dilaton)

$\lambda^B = \lambda Z_\lambda Z_\phi^{-2}$, $d\lambda^B/d(\ln z) = 0$, $\beta_\lambda^{(1)} = d\lambda/(d\ln z) = 3\lambda^2/\kappa$. Callan-Symanzik: $d(U_1 + V)/d(\ln z) = 0$

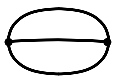
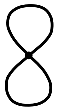
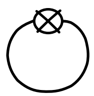
$\Rightarrow$ Quantum SI with $\beta_\lambda \neq 0$ [$\beta_\lambda = 0$ is sufficient but not necessary for QSI: different spectrum/symmetry]

$\Rightarrow$ dilatation current: $\partial_\mu D^\mu = 0$ [Englert et al 1979, Shaposhnikov et al; Tamarit 2014]

• Two-loop SI potential: using: $\tilde V(\phi,\sigma) = z^{2\epsilon}\,\sigma^{2\epsilon/(1-\epsilon)}\,V(\phi) \sim \sigma^{2\epsilon}\,V(\phi)$ [background field method]

$$\Rightarrow \tilde V(\phi+\delta_\phi,\sigma+\delta_\sigma) = \tilde V(\phi,\sigma)+\tilde V_\alpha\delta_\alpha+\frac{1}{2}\,\tilde V_{\alpha\beta}\,\delta_\alpha\delta_\beta+\frac{1}{3!}\,\tilde V_{\alpha\beta\gamma}\,\delta_\alpha\delta_\beta\delta_\gamma+\frac{1}{4!}\,\tilde V_{\alpha\beta\gamma\rho}\,\delta_\alpha\delta_\beta\delta_\gamma\delta_\rho+\cdots \quad \alpha,\beta=\phi,\sigma.$$

$$\tilde V_{\alpha\beta\ldots} = \partial_\alpha\partial_\beta..\tilde V$$

$V_2 = \frac{i}{12}$  $+\frac{i}{8}$  $+\frac{i}{2}$  $= \frac{i}{12}\,\tilde V_{\alpha\beta\gamma}\tilde V_{\alpha'\beta'\gamma'}\int\frac{d^dp}{(2\pi)^d}\frac{d^dq}{(2\pi)^d}\,(\tilde D_p)_{\alpha\alpha'}\,(\tilde D_q)_{\beta\beta'}\,(\tilde D_{p+q})_{\gamma\gamma'}+\cdots$

$$= (z\sigma)^{2\epsilon}\,\frac{\lambda^3\,\phi^4}{32\kappa^2}\Big\{-\frac{3}{\epsilon^2}+\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\Big\};$$

$$(\tilde D_p)_{\alpha\beta} = (D_p)_{\alpha\beta}+\epsilon\,(\ldots)_{\alpha\beta}+\epsilon^2\,(\ldots)_{\alpha\beta}$$

same poles, ϵ-shifts to propagators, vertices: $\tilde V_{\alpha\beta\gamma\ldots} = V_{\alpha\beta\gamma\ldots}+\epsilon\,(\ldots)_{\alpha\beta\gamma\ldots}+\epsilon^2\,(\ldots)_{\alpha\beta\gamma\ldots}$

[Lalak, Olszewski,DG]
[1712.06024]

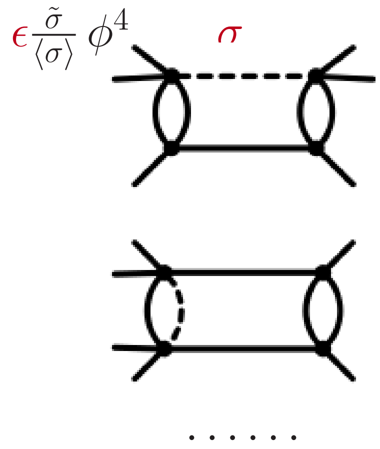
Two-loop correction:

$$U_2 = \frac{\lambda}{4!}\,\phi^4\,\Big\{\frac{3\lambda^2}{4\kappa^2}\Big(4+A_0-4\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}+3\,\overline{\ln}^2\frac{V_{\phi\phi}}{(z\sigma)^2}\Big)+\frac{5\lambda^2}{\kappa^2}\frac{\phi^2}{\sigma^2}+\frac{7\lambda^2}{24\kappa^2}\frac{\phi^4}{\sigma^4}\Big\}, \qquad V_{\phi\phi}\equiv\frac{\lambda}{2}\phi^2.$$

$V^{(2)}$: 'old result' [$\mu\to z\sigma$] $V^{(2,n)}$: new, finite; no z, (!) if $\phi\sim\sigma$ [Callan-Symanzik verified]

- Three-loop SI potential: Counterterms!

[Monin 2015]



$$\delta L_3 = \frac{1}{2}\,\delta_\phi^{(3)}\,(\partial_\mu \phi)^2 - \mu^{2\epsilon}\Big\{ \frac{1}{4!}\,\delta_\lambda^{(3)}\,\lambda\,\phi^4 + \frac{1}{6}\,\delta_{\lambda_6}^{(3)}\,\lambda_6\,\frac{\phi^6}{\sigma^2} + \frac{1}{8}\,\delta_{\lambda_8}^{(3)}\,\lambda_8\,\frac{\phi^8}{\sigma^4}\Big\}$$

$$\delta_{\lambda_6}^{(3)} = \frac{3}{2}\frac{\lambda^4}{\lambda_6\,\kappa^3\,\epsilon}, \qquad \delta_{\lambda_8}^{(3)} = \frac{275}{864}\frac{\lambda^4}{\lambda_8\,\kappa^3\,\epsilon.} \;\Rightarrow\; \gamma_\phi^{(3)},\, \beta_{\lambda_6},\, \beta_{\lambda_8} = ...$$

Integrate CS: Three-loop term: $U_3 = \Delta V + V^{(3)} + V^{(3,n)}$. $\quad V^{(3)}$='old'$[\,\mu \to z\sigma\,]$; new: $V^{(3,n)}$ and ΔV

$$\Delta V = \frac{\lambda_6\,\phi^6}{\sigma^2} + \frac{\lambda_8\,\phi^8}{\sigma^4}, \qquad V^{(3)} = \frac{\lambda^4\,\phi^4}{\kappa^3}\Big\{\mathcal{Q} + \Big(\frac{97}{128} + \frac{9}{64}A_0 + \frac{\zeta[3]}{4}\Big)\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2} - \frac{31}{96}\overline{\ln}^2\frac{V_{\phi\phi}}{(z\sigma)^2} + \frac{9}{64}\overline{\ln}^3\frac{V_{\phi\phi}}{(z\sigma)^2}\Big\}$$

$$V^{(3,n)} = \frac{\lambda^3\,\phi^4}{2\,\kappa^3}\Big\{\Big(27\lambda - \frac{\lambda_6}{2}\Big)\frac{\phi^2}{8\,\sigma^2} + \Big(\frac{401\,\lambda}{72} - \lambda_8\Big)\frac{\phi^4}{16\,\sigma^4}\Big\}\,\overline{\ln}\frac{V_{\phi\phi}}{(z\sigma)^2}. \qquad V_{\phi\phi} \equiv \frac{\lambda}{2}\phi^2.$$

[DG 1712.06024]

$\Rightarrow$ SI effective operators always suppressed by $\sigma \Rightarrow$. At large σ enhanced symmetry $S_v \times S_h$ restored $\Rightarrow$

$\Rightarrow$ No c-terms $\quad \lambda^2\,\phi^2\sigma^2 = \lambda^2\,\langle\sigma\rangle^2\,\phi^2 + \cdots$ No tunning of Higgs selfcoupling λ for large σ.

• SM+dilaton: one-loop SI potential [Shaposhnikov et al 2008; DG, Lalak, Olszewski]

$$V = \frac{\lambda_\phi}{3!}(H^\dagger H)^2 + \frac{\lambda_m}{2}(H^\dagger H)\,\sigma^2 + \frac{\lambda_\sigma}{4!}\sigma^4 + \cdots$$

$$= \frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_m}{4}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4!}\lambda_\sigma\,\sigma^4 + \cdots; \qquad (\lambda_m < 0),$$

$$\to \frac{\lambda_\phi}{4}\left(\phi^2 + \frac{3\lambda_m}{\lambda_\phi}\sigma^2\right)^2 + \text{loops.}$$

min $\langle\sigma\rangle \neq 0$:

1) $9\lambda_m^2 = \lambda_\phi\lambda_\sigma$ + loops

2) $\dfrac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = \dfrac{-3\lambda_m}{\lambda_\phi}(1 + \text{loops})$

$\not{S}\!I$: tuning (i) or $V_{\text{min}} = 0 \Rightarrow$ EWSB; $m_{\tilde\phi} \approx \lambda_\phi\langle\phi\rangle^2 = (-3)\lambda_m\langle\sigma\rangle^2 \ll \langle\sigma\rangle^2$ if $|\lambda_m| \ll \lambda_\phi$ one classical tuning

• SM+dilaton: one-loop SI potential [Shaposhnikov et al 2008; DG, Lalak, Olszewski]

$$V = \frac{\lambda_\phi}{3!}(H^\dagger H)^2 + \frac{\lambda_m}{2}(H^\dagger H)\,\sigma^2 + \frac{\lambda_\sigma}{4!}\sigma^4 + \frac{4\,\lambda_6}{3}\frac{(H^\dagger H)^3}{\sigma^2} + \cdots$$

$$= \frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_m}{4}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4!}\lambda_\sigma\,\sigma^4 + \frac{\lambda_6}{6}\frac{\phi^6}{\sigma^2}\cdots;\ (\lambda_m<0),$$

$$\to \frac{\lambda_\phi}{4}\Big(\phi^2 + \frac{3\lambda_m}{\lambda_\phi}\sigma^2\Big)^2 + \text{loops.}$$

min $\langle\sigma\rangle\neq 0$:

1) $9\lambda_m^2 = \lambda_\phi\lambda_\sigma + \text{loops}$

2) $\frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = \frac{-3\lambda_m}{\lambda_\phi}(1+\text{loops})$

S̸I : tuning (i) or $V_{\text{min}}=0 \Rightarrow$ EWSB; $m_{\tilde\phi} \approx \lambda_\phi\langle\phi\rangle^2 = (-3)\lambda_m\langle\sigma\rangle^2 \ll \langle\sigma\rangle^2$ if $|\lambda_m| \ll \lambda_\phi$ one classical tuning

$$V^{(1)} \equiv \sum_{j=\phi,\sigma;G,t,W,Z} \frac{n_j\,m_j^4(\phi,\sigma)}{4\,\kappa}\ln\frac{m_j^2(\phi,\sigma)}{c_j\,(z\sigma)^2},$$

'old CW' $[\mu \to z\,\sigma]$

$$V^{(1,n)} \equiv \frac{1}{48\kappa}\Big[(-16\lambda_m\lambda_\phi - 18\lambda_m^2 + \lambda_\phi\lambda_\sigma)\,\phi^4 - \lambda_m(48\lambda_m + 25\lambda_\sigma)\phi^2\sigma^2 - 7\lambda_\sigma^2\sigma^4$$

$$+ (\lambda_\phi\lambda_m + 6\lambda_6\lambda_\sigma)\frac{\phi^6}{\sigma^2} + 8\,(4\lambda_\phi - 2\lambda_m)\,\lambda_6\frac{\phi^8}{\sigma^4} + (192\lambda_6 + 2\lambda_\phi)\lambda_6\frac{\phi^{10}}{\sigma^6} + 40\lambda_6^2\frac{\phi^{12}}{\sigma^8}\Big],$$

large σ : $S_v \times S_h$

$$\Delta V = \frac{\lambda_p}{p}\frac{\phi^p}{\sigma^{p-4}},\ p = 6, 8, 10, 12.$$

$\to 0$ if $\phi \ll \sigma$ and $\lambda_m \to 0$.

no $\lambda_\phi^2\phi^2\sigma^2 \sim \lambda_\phi\,\phi^2\langle\sigma\rangle^2$

One-loop corrected potential $U = V + V^{(1)} + V^{(1,n)}$ gives:

$$\Delta m^2_{\tilde\phi} = \frac{-\lambda_m}{\lambda_\phi}\frac{\langle\sigma\rangle^2}{16\kappa}\Big\{ 27\Big[g^4\Big(\ln\frac{g^2}{4}+\frac{1}{3}\Big)+2\,g_2^4\Big(\ln\frac{g_2^2}{4}+\frac{1}{3}\Big)-16\,h_t^4\Big(\ln\frac{h_t^2}{2}-\frac{1}{3}\Big)\Big] + 4\,\lambda_\phi^2\Big[5\ln\frac{\lambda_\phi^2}{12}-8+\ln 27\Big]\Big\} = \lambda_m\lambda_\phi\langle\sigma\rangle^2+\cdots \ll \langle\sigma\rangle^2, \text{but no } \lambda_\phi^2\,\langle\sigma\rangle^2.$$

- no tuning of λ_m beyond classical one: $\beta_{\lambda_m} \propto \lambda_m$, so λ_m stays ultraweak, natural.

- classical hierarchy of vev's: $\langle\phi\rangle \ll \langle\sigma\rangle$ protected by quantum scale inv $S_v \times S_h$, (all orders, spont $\not{S}$I)

- All scales from fields' vev's $\Rightarrow$ result is not a DR artefact! More scalars?

• Curved spacetime - Einstein gravity breaks $S_v \times S_h$ by $\big(\xi_1\phi^2+\xi_2\sigma^2\big)R \Rightarrow \beta_{\lambda_m} = \xi_1(\ldots)+\xi_2(\ldots)+\lambda_m(\ldots)$

- what happens to the flat direction (dilaton)?

- hierarchy of vev's (Higgs vs dilaton)?

• Einstein Gravity $\rightarrow$ Weyl gauge symmetry: $g'_{\mu\nu}(x) = e^{2\alpha(x)}\, g_{\mu\nu}(x), \quad \phi'_j(x) = e^{-\alpha(x)}\phi_j(x),$ (a)

Wanted: L_E by spont breaking. An invariant of (a) $\phi_j : \phi, \sigma$

$$L_E = -\frac{1}{2}\sqrt{g} M_p^2\, R, \quad \rightarrow \quad L_1 = -\sqrt{g}\,\frac{\xi_j}{2}\Big[\frac{1}{6}\,\phi_j^2\, R + g^{\mu\nu}\,\partial_\mu\phi_j\,\partial_\nu\phi_j\Big] - V(\phi_j)$$

1) ghost (or $G_N < 0$), 2) Fake conf symmetry?[Jackiw, Pi 2015] 3) Go to Einstein frame: # dof violated! Avoid.

['something' is missing here....]

$\Rightarrow$ Add: $\omega'_\mu = \omega_\mu - \frac{2}{q}\,\partial_\mu\alpha(x)$ (b)

$$L_2 = \frac{\sqrt{g}}{2}(1+\xi_j)\; g^{\mu\nu}\,\tilde{D}_\mu\phi_j\,\tilde{D}_\nu\phi_j, \quad \text{invariant}, \qquad \tilde{D}_\mu\phi \equiv (\partial_\mu - \frac{q}{2}\,\omega_\mu)\phi$$

$$L_{12} \equiv L_1 + L_2 = \sqrt{g}\,\Big\{-\frac{\xi_j}{12}\,\phi_j^2\,R + \frac{g^{\mu\nu}}{2}(\partial_\mu\phi_j)\,(\partial_\nu\phi_j) - \frac{q}{4}\,g^{\mu\nu}\,\omega_\mu\,\partial_\nu K + \frac{q^2}{8}\,K\,\omega_\mu\,\omega^\mu - V(\phi_j)\Big\},$$

$$K \equiv (1+\xi_j)\,\phi_j^2.$$

$$L_G = -\frac{\sqrt{g}}{4}\,F_{\mu\nu}\,F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu,$$

$\Rightarrow L = L_{12} + L_G$: Weyl gravity, gauge invariant under (a), (b) for any ξ_j, no ghost, $G_N > 0$. Dirac 1973

• Riemann $\to$ Weyl geometry: Riemann: $D_\mu g_{\alpha\beta} = 0$, with $\Gamma^\rho_{\mu\nu}$

Weyl: $\tilde\nabla_\mu\, g_{\alpha\beta} = -q\,\omega_\mu\, g_{\alpha\beta}, \;\Rightarrow\; (\tilde\nabla_\mu + q\,\omega_\mu)\, g_{\alpha\beta} = D_\mu\, g_{\alpha\beta} = 0,$ $\tilde\nabla_\mu$: Weyl-covariant deriv with $\tilde\Gamma^\rho_{\mu\nu}$

$$\tilde\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + \frac{q}{2}\Big[\delta^\rho_\mu\,\omega_\nu + \delta^\rho_\nu\,\omega_\mu - g_{\mu\nu}\,\omega^\rho\Big],$$ no torsion; invariant (a), (b)

$$\Rightarrow\;\tilde R = R - 3\,q\,D_\mu\omega^\mu - \frac{3}{2}q^2\,\omega^\mu\omega_\mu. \;\Rightarrow\; \tilde R \to \tilde R' = e^{-2\alpha(x)}\tilde R.$$ (like ϕ^2!)

Simplest Lagrangian: $L_{1w} = \dfrac{-\sqrt{g}}{12}\,\xi_j\,\phi_j^2\,\tilde R$: invariant! $L_{2w} = \sqrt{g}\left[\dfrac{g^{\mu\nu}}{2}\,\tilde D_\mu\phi_j\,\tilde D_\nu\phi_j - V(\phi_j)\right]$; invariant!

$$\Rightarrow L_{1w} + L_{2w} + L_G = L_{12} + L_G + \text{tot deriv} = L,$$ (L_G and $F_{\mu\nu}$ unchanged)

[Smolin 1979; Cheng 1988; H. M. Lee and D.G. 2018]

• L is not most general Weyl gravity: missing: $\sqrt{g}\,\tilde R^2$ and $\sqrt{g}\,\tilde C_{\mu\nu\rho\sigma}\,\tilde C^{\mu\nu\rho\sigma}$, each invariant under (a),(b)

• If $\omega_\mu \to 0$: Weyl geometry $\to$ Riemann geometry: $\tilde\Gamma^\rho_{\mu\nu} \to \Gamma^\rho_{\mu\nu}$, $\tilde R \to R$ and Weyl tensor $\tilde C_{\mu\nu\rho\sigma} \to C_{\mu\nu\rho\sigma}$

$\Rightarrow$ Let us first turn off matter ϕ_j

note: $\tilde X$ denotes the expression of X in Weyl-geometry.

• No matter: Weyl quadratic gravity $\to$ Einstein gravity + massive ω_μ [DG 1812.08613]

$$\mathcal{L}_1 = \sqrt{g}\,\frac{\xi_0}{4!}\,\tilde{R}^2, \quad \xi_0 > 0, \qquad \mathcal{L}_1 + \mathcal{L}_2: \text{ Weyl quadratic action}$$

$$\mathcal{L}_2 = \frac{\sqrt{g}}{\zeta}\,\tilde{C}_{\mu\nu\rho\sigma}\,\tilde{C}^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\zeta}\Big[C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma} + \frac{3}{2}q^2\,F_{\mu\nu}F^{\mu\nu}\Big]$$

+ Gauss-Bonnet.

So:

$$\mathcal{L}_1 = \sqrt{g}\,\frac{\xi_0}{4!}\Big[-2\,\phi_0^2\,\tilde{R} - \phi_0^4\Big] \qquad \text{eom: } \phi_0^2 = -\tilde{R}; \quad \text{dilaton: } \ln\phi_0 \to \ln\phi_0 - \ln\Omega$$

$$= \sqrt{g}\,\Big[-\frac{\xi_0}{12}\,\phi_0^2\,R - \frac{q}{4}g^{\mu\nu}\omega_\mu\,\partial_\nu K + \frac{q^2}{8}\,g^{\mu\nu}\,K\,\omega_\mu\,\omega_\nu - \frac{\xi_0}{4!}\,\phi_0^4\Big] + \text{total deriv}, \quad K = \xi_0\,\phi_0^2$$

- $\mathcal{L}_1$ has no k.t. for ω_μ: integrating ω_μ $\quad\Rightarrow\quad \mathcal{L}_{\text{eff}} = \sqrt{g}\,(-\xi_0/6)\Big[1/6\,\phi_0^2\,R + g^{\mu\nu}\,\partial_\mu\phi_0\partial_\nu\phi_0\Big] - \xi_0/4!\,\phi_0^4.$

- Include $\mathcal{L}_2$ or only $\delta\mathcal{L}_2 = \frac{-1}{4}\,\sqrt{g}\,F_{\mu\nu}F^{\mu\nu}$:

- Consider $\mathcal{L} = \mathcal{L}_1 + \delta\mathcal{L}_2$: then:

$$\partial^\alpha(F_{\alpha\mu}\,\sqrt{g}) + \frac{1}{2}\sqrt{g}\,\xi_0\,\phi_0\,\Big[\partial_\mu - \frac{q}{2}\,\omega_\mu\Big]\,\phi_0 = 0$$

conserved current

Canonical Einstein term $\rightarrow$ Einstein gauge ("frame") $\hat{g}_{\mu\nu} = \Omega\, g_{\mu\nu},\ \ \phi_0' = \frac{1}{\sqrt{\Omega}}\,\phi_0,\ \ \Omega \equiv \frac{\xi_0\,\phi_0^2}{6\,M^2},$

$M \sim \langle \phi_0 \rangle$

Then:
$$\mathcal{L}_1 = \sqrt{\hat{g}}\,\Big[\frac{-1}{2}\,M^2\,\hat{R} + \frac{3\,M^2}{4\,\Omega^2}\,\hat{g}^{\mu\nu}\,(\partial_\mu\Omega)(\partial_\nu\Omega) + \frac{\hat{g}^{\mu\nu}}{\Omega}\Big(\frac{-q}{4}\omega_\mu\,\partial_\nu K + \frac{q^2}{8} K\,\omega_\mu\,\omega_\nu - \frac{\xi_0}{4!}\frac{\phi_0^4}{\Omega}\Big)\Big].$$

$K = \xi_0\,\phi_0^2$

Also $\omega_\mu' = \omega_\mu - \frac{1}{q}\,\partial_\mu \ln \Omega,$

$$\Rightarrow \mathcal{L}_1 + \delta\mathcal{L}_2 = \sqrt{\hat{g}}\,\Big[-\frac{1}{2}\,M^2\,\hat{R} - \frac{3\,M^4}{2\,\xi_0}\Big] + \sqrt{\hat{g}}\,\Big[-\frac{1}{4}\,\hat{g}^{\mu\rho}\,\hat{g}^{\nu\sigma}\,F_{\mu\nu}'\,F_{\rho\sigma}' + \frac{3\,q^2}{4}\,M^2\,\hat{g}^{\mu\nu}\omega_\mu'\,w_\nu'\Big]$$

Weyl quadratic action Einstein Proca [D.G. 1812.08613]

$m_{\omega'}^2 = (3/2) q^2\,M^2.$

- Stueckelberg: ϕ_0 eaten by ω_μ: massive; $\#$ dof conserved (=3): ϕ_0+massless $\omega_\mu \rightarrow$no ϕ_0+massive ω_μ
- Below $m_\omega \sim M_p$, Weyl geometry $\rightarrow$ Riemann geometry ($\omega_\mu = 0$): Avoids criticisms of Weyl gravity!

$\Rightarrow$ SB: Weyl quadratic gravity (no matter)= Einstein action $(+\Lambda)$ + massive Weyl ω_μ action.

$\Rightarrow$ The same Stueckelberg mechanism also works in the linear Weyl gravity case, with L of page 10 and 11.

• Adding matter: Weyl gravity → EW breaking: Add ϕ_1 higgs field

In Weyl-geometry language:

$$\mathcal{L} = \sqrt{g}\,\Big[\,\frac{\xi_0}{4!}\,\tilde{R}^2 - \frac{\sqrt{g}}{4}\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}\,\xi_1\phi_1^2\,\tilde{R} + \frac{1}{2}\,g^{\mu\nu}\,\tilde{D}_\mu\phi_1\tilde{D}_\nu\phi_1 - \frac{\lambda_1}{4!}\phi_1^4\Big] \qquad \tilde{D}_\mu\phi_1 \equiv \partial_\mu - \frac{q}{2}\,\omega_\mu$$

$$= \sqrt{g}\,\Big[\,\frac{\xi_0}{4!}\,(-2\phi_0^2\,\tilde{R} - \phi_0^4) - \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}\,\xi_1\phi_1^2\,\tilde{R} + \frac{1}{2}\,g^{\mu\nu}\,\tilde{D}_\mu\phi_1\tilde{D}_\nu\phi_1 - \frac{\lambda_1}{4!}\phi_1^4\Big],$$

In Riemannian language:

$$\mathcal{L} = \sqrt{g}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}(\xi_0\phi_0^2 + \xi_1\phi_1^2)\,R - \frac{q}{4}\omega^\mu\,\partial_\mu\mathcal{K} + \frac{q^2}{8}\,\mathcal{K}\,\omega^\mu\,\omega_\mu + \frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 - \mathcal{V}\Big]$$

$$\mathcal{V} = \frac{\xi_0}{4!}\,\phi_0^4 + \frac{\lambda_1}{4!}\,\phi_1^4, \qquad \mathcal{K} = \xi_0\phi_0^2 + (1+\xi_1)\phi_1^2,$$

Einstein gauge: only one kinetic term left!

$$\hat{g}_{\mu\nu} = \Omega\,g_{\mu\nu}, \;\; \Omega = \frac{1}{6M^2}\,(\xi_0\phi_0^2 + \xi_1\phi_1^2), \quad \mathcal{Z} = \xi_0\,\frac{\phi_0^2}{\phi_1^2} + \xi_1, \quad \mathcal{L}\Big|_{k.t.} = \frac{3}{4}\,\frac{\sqrt{M^4\,\hat{g}}}{1+\mathcal{Z}}\,\partial^\mu \ln\mathcal{Z}\;\partial_\mu\ln\mathcal{Z}$$

Use: $\phi_0 = \frac{\rho \sin\theta}{\sqrt{\xi_0}}, \quad \phi_1 = \frac{\rho\cos\theta}{\sqrt{1+\xi_1}}, \quad \Rightarrow \quad \mathcal{K} = \rho^2, \qquad \mathcal{L}\Big\vert_{k.t.} = \frac{\sqrt{g}}{2} M^2 f(\theta)(\partial^\mu \cot\theta)(\partial_\mu \cot\theta)$

Then $\omega'_\mu = \omega_\mu - \frac{1}{q}\partial_\mu \ln\rho^2$, dilaton ρ "eaten" by ω'_μ. $\qquad \frac{\tilde{h}}{M} = \frac{\cot\theta}{\sqrt{1+\xi_1}}\big(1 + c_1 \cot^2\theta + \dots\big) \ll 1.$

$$\mathcal{L} = \sqrt{\hat{g}}\Big[-\frac{M^2}{2}\hat{R} + \frac{\hat{g}^{\mu\nu}}{2}(\partial_\mu\tilde{h})(\partial_\nu\tilde{h}) - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{m^2(\tilde{h})}{2}\omega'_\mu\omega'^\mu - \hat{\mathcal{V}}(\tilde{h})\Big], \quad \text{with}$$

$$m^2(\tilde{h}) = \frac{3\,q^2 M^2}{2}\Big[1 + \frac{\tilde{h}^2}{6\,M^2} + \mathcal{O}\Big(\frac{\tilde{h}^4}{M^4}\Big)\Big], \quad \hat{\mathcal{V}}(\tilde{h}) = \frac{3\,M^4}{2\,\xi_0} - \frac{\xi_1 M^2}{2\,\xi_0}\tilde{h}^2 + \Big(\frac{\lambda_1}{4!} + \frac{\xi_1(3\xi_1 - 2)}{72\,\xi_0}\Big)\tilde{h}^4 + \mathcal{O}\Big(\frac{\tilde{h}^6}{M^6}\Big)$$

$\Rightarrow$ Einstein action $+\ \Lambda\ +$ Higgs $+$ massive ω'_μ; # dof (=4) is conserved (Jordan: 2 scalars+massless ω_μ).

$\Rightarrow$ EWSB breaking; $m^2_{\text{higgs}} = \xi_1/\xi_0\, M^2$. Quantum stable? The limit $\xi_1 \to 0 \Rightarrow S_v \times S_h \Rightarrow \beta_{\xi_1} \propto \xi_1$!

not $(\xi + 1/6)$!

$\Rightarrow$ renormalizable Weyl quadratic gravity? $\tilde{R}^2 + \tilde{C}^2_{\mu\nu\rho\sigma}$? (quadratic gravity renorm [K. Stelle 1977]).

- Conclusions

I. Flat space:

- Quantum SI: V in $\lambda\phi^4$ (3loop) & SM (1 loop) spontaneous breaking (SB)
- classical hierarchy of: higgs vev $\ll$ dilaton vev $\Rightarrow$ quantum stable
- SI effective operators, suppressed by dilaton
- all scales from fields' vevs, result not a DR artefact

II. Curved space:

- $M_{\rm Planck} \sim$ dilaton vev $\Rightarrow$ Weyl gravity/gauge symmetry
- Stueckelberg: dilaton "eaten" by Weyl gauge field ω_μ

$\rightarrow$ SB: Weyl quadratic gravity (no matter) $=$ Einstein gravity $+$ massive ω_μ

- Weyl geometry $\Rightarrow$ Riemann geometry (near $m_\omega \sim M_{\textsf{Planck}}$)
- Adding matter (higgs) $\xi_1 \phi_1^2 \tilde R$: $\rightarrow$ EWSB; m_h quantum stable?
- renormalizability of Weyl quadratic gravity $\tilde R^2 + \tilde C^2_{\mu\nu\rho\sigma}$?

BACKUP SLIDES

- Two-loop: No new poles. Two-loop $\beta_\lambda^{(2)}$, anom dims $\gamma_\phi^{(2)}$ unchanged. Usual counterterm:

$$\delta L_2 = \frac{1}{2}(\partial_\mu\phi)^2\,\delta_\phi^{(2)} - \mu(\sigma)^{2\epsilon}\frac{1}{4!}\,\lambda\,\phi^4\,\delta_\lambda^{(2)}, \qquad \delta_\lambda^{(2)} = \frac{\lambda^2}{\kappa^2}\Big(\frac{9}{4\,\epsilon^2} - \frac{3}{2\,\epsilon}\Big), \qquad \delta_\phi^{(2)} = \frac{-\lambda^2}{24\,\kappa^2\,\epsilon}.$$

$$\beta_\lambda^{(2)} = -\frac{17}{3\,\kappa^2}\,\lambda^3, \qquad \text{unchanged (as if no dilaton \& } \mu = \text{const).} \qquad \gamma_\sigma^{(2)} = 0.$$

Callan-Symanzik (consistency check):

$$\frac{\partial\,V^{(2)}}{\partial\ln z} + \Big[\beta_\lambda^{(2)}\frac{\partial}{\partial\lambda} + \gamma_\phi^{(2)}\phi\frac{\partial}{\partial\phi} + \gamma_\sigma^{(2)}\sigma\frac{\partial}{\partial\sigma}\Big]V + \beta_\lambda^{(1)}\frac{\partial V^{(1)}}{\partial\lambda} = O(\lambda^4), \quad \text{“usual” CS}$$

$$\frac{\partial\,V^{(2,\mathrm{n})}}{\partial\ln z} = O(\lambda^4), \qquad \beta^{(k)},\ \gamma^{(k)},\ V^{(k)}\text{: k-loop correction.}$$

- $\text{if } V \to V + \frac{\lambda_m}{2}\,\phi^2\sigma^2, \quad V^{(2,n)} \supset \frac{\lambda\,\lambda_m}{96\kappa^2\,\epsilon}\Big\{7\,(2\lambda_m - \lambda)\frac{\phi^6}{\sigma^2} - \frac{3\lambda_m}{2}\frac{\phi^8}{\sigma^4}\Big\} \;\Rightarrow\; \beta_\lambda \to \beta_\lambda + \frac{\lambda_m}{\kappa^2}\Big[12\lambda_m^2 - 7\lambda_m\lambda - 40\lambda^2\Big]$

$$\beta_{\lambda_m} \propto \lambda_m^2; \quad \lambda_m \to 0\text{: } S_v \times S_h.$$

• Two-loop SI potential: Taylor expand about $\sigma = \langle\sigma\rangle + \tilde{\sigma}$:

$$U = \frac{\lambda}{4!}\phi^4\left\{1+\frac{3\lambda}{2\kappa}\Big(\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2}-\frac{1}{2}\Big)+\frac{3\lambda^2}{4\kappa^2}\Big(4+A_0-4\,\overline{\ln}\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2}+3\,\overline{\ln}^2\frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2}\Big)+\mathcal{O}\Big(\frac{1}{\langle\sigma\rangle}\Big)\right.$$

- This is the "usual" CW result with $\mu = \langle\sigma\rangle$, broken SI, no dilaton present. [Cheng, I. Jack, T. Jones, S. Martin]
- new terms comparable/larger than standard two-loop terms

$$\frac{\phi^n}{\sigma^n} \sim 1., \qquad n = 1, 2; \qquad\qquad \frac{\phi}{\sigma} = \frac{\phi}{\langle\sigma\rangle}\Big(1 - \frac{\tilde{\sigma}}{\langle\sigma\rangle} + \frac{\tilde{\sigma}^2}{\langle\sigma\rangle^2} + \cdots\Big)$$

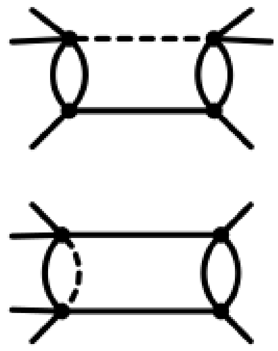
- Non-polynomial terms: - vanish for $\phi \ll \sigma$, only $\log\sigma$ is left.
  - respect symmetries of the theory.
  - finite counterterms, cannot be "seen" in a scheme that breaks this symmetry.
  - if not forbidden (by a symmetry), operators generated at quantum level.
  - non-renormalizability.

• Three-loop SI potential: Counterterms! [Monin 2015]

$$\delta L_3 = \frac{1}{2}\,\delta_\phi^{(3)}\,(\partial_\mu\phi)^2 - \mu^{2\epsilon}\Big\{\frac{1}{4!}\,\delta_\lambda^{(3)}\,\lambda\,\phi^4 + \frac{1}{6}\,\delta_{\lambda_6}^{(3)}\,\lambda_6\,\frac{\phi^6}{\sigma^2} + \frac{1}{8}\,\delta_{\lambda_8}^{(3)}\,\lambda_8\,\frac{\phi^8}{\sigma^4}\Big\}$$

$$\delta_\phi^{(3)} = -\frac{\lambda^3}{4\kappa^3}\Big(\frac{1}{6\epsilon^2} - \frac{1}{12\epsilon}\Big), \quad \delta_{\lambda_6}^{(3)} = \frac{3}{2}\frac{\lambda^4}{\lambda_6\,\kappa^3\,\epsilon}, \quad \delta_{\lambda_8}^{(3)} = \frac{275}{864}\frac{\lambda^4}{\lambda_8\,\kappa^3\,\epsilon.}$$

$\epsilon\frac{\tilde\sigma}{\langle\sigma\rangle}\,\phi^4$



. . . . . .

So $\gamma_\phi^{(3)} = \lambda^3/(16\kappa^3)$. With $Z_X = 1 + \delta_X$ and $\lambda_6^B = \mu^{2\epsilon}(\sigma)\lambda_6 Z_{\lambda_6} Z_\phi^{-3} Z_\sigma$ and $(d/d\ln z)\,\lambda_6^B = 0$,

$$\Rightarrow\ \beta_{\lambda_6} \ = \ \frac{\lambda^2\,\lambda_6}{2\kappa^2} + \frac{\lambda^3}{\kappa^3}\Big(9\lambda - \frac{3}{8}\lambda_6\Big), \qquad \text{(similar } \beta_{\lambda_8}\text{).}$$

Callan-Symanzik: $V^{(3)}$ ["usual" with $\mu\to\sigma$]$+V^{(3,n)}$ [new]:

$$\frac{\partial V^{(3)}}{\partial\ln z} + \beta_\lambda^{(1)}\frac{\partial V^{(2)}}{\partial\lambda} + \beta_\lambda^{(2)}\frac{\partial V^{(1)}}{\partial\lambda} + \beta_\lambda^{(3)}\frac{\partial V}{\partial\lambda} + \gamma_\phi^{(2)}\,\frac{\partial V^{(1)}}{\partial\ln\phi} + \gamma_\phi^{(3)}\,\frac{\partial V}{\partial\ln\phi} = \mathcal{O}(\lambda_j^5).$$

$$\frac{\partial V^{(3,n)}}{\partial\ln z} + \beta_{\lambda_j}^{(1)}\frac{\partial V^{(2,n)}}{\partial\lambda_j} + \beta_{\lambda_j}^{(3,n)}\frac{\partial V}{\partial\lambda_j} = \mathcal{O}(\lambda_j^5), \qquad \lambda_j = \lambda, \lambda_6, \lambda_8.$$

• Dilatation current D_μ: $$D^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} (x^\nu \partial_\nu \phi_j + d_\phi) - x^\mu \mathcal{L}, \qquad d_\phi = (d-2)/2 \quad \text{(scalars)},$$

- in $d = 4 - 2\epsilon$, potential $\tilde{V}$: $$\partial_\mu D^\mu = (d_\phi + 1)(\partial_\mu \phi_j) \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} + d_\phi \phi_j \frac{\partial \mathcal{L}}{\partial \phi_j} - d\mathcal{L}$$
$$= d\,\tilde{V} - \frac{d-2}{2} \phi_j \frac{\partial \tilde{V}}{\partial \phi_j}, \quad \phi_j = \phi, \sigma; \quad \text{(onshell, canonical k.t.)}$$

• in SI theory: $\tilde{V}$ homogeneous in d dim's: $\tilde{V}(\rho\,\phi_j) = \rho^{2d/(d-2)}\,\tilde{V}(\phi_j) \Rightarrow \partial_\mu D^\mu = 0.$

• in "usual" reg: μ =const, no σ: $\tilde{V} = \mu^{2\epsilon} V(\phi)$, and V is scale inv in $d = 4$:

$$\partial_\mu D^\mu = 2\,\epsilon\,\mu^{2\epsilon} V \sim 2\,\epsilon\,\mu^{2\epsilon} \left[\lambda + \frac{\beta_\lambda}{\epsilon} + \cdots\right] \frac{\partial V}{\partial \lambda} \propto \beta_\lambda \frac{\partial V}{\partial \lambda}, \quad [=0 \text{ only if } \beta_\lambda = 0]. \quad \text{(scale anomaly)}$$

- different field content! [Shaposhnikov et al, Tamarit 2013]

"For scale invariance, though, the situation is hopeless; any cutoff procedure necessarily involves a large mass and a large mass necessarily breaks scale invariance in a large way. This argument does not show that the occurence of anomalies is inevitable [.....]" (S. Coleman: Aspects of Symmetry, p.82, 1985).

• SM + dilaton SI potential: one-loop Beta functions:

$$\beta_{\lambda_\phi} = \frac{1}{\kappa}\left[3\left(\frac{9}{4}g_2^4+\frac{3}{4}g_1^4+\frac{3}{2}g_1^2g_2^2-12h_t^4\right)-4\lambda_\phi\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)+4\lambda_\phi^2+3\lambda_m^2+96\lambda_m\lambda_6\right]$$

$$\beta_{\lambda_m} = \frac{2\lambda_m}{\kappa}\left[\lambda_\phi+2\lambda_m+\frac{1}{2}\lambda_\sigma-\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]\propto\lambda_m, \quad \Rightarrow \lambda_m \text{ stays small (f.p.).}$$

• for new couplings

$$\beta_{\lambda_6} = \frac{3\lambda_6}{\kappa}\left[6\lambda_\phi-8\lambda_m+\lambda_\sigma-2\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$$\beta_{\lambda_8} = \frac{2}{\kappa}\left[2\lambda_6\left(28\lambda_6+\lambda_m\right)-4\lambda_8\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$$\beta_{\lambda_{10}} = 10\left[4\lambda_6^2-\lambda_{10}\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$$\beta_{\lambda_{12}} = 2\left[3\lambda_6^2-6\lambda_{12}\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2-3h_t^2\right)\right]$$

$\Rightarrow$ if one sets $\lambda_{6,8,10,..}=0$ at tree level, then $\beta_{\lambda_{6,8,10,12}}=0$ at one-loop, but emerge at 2-loops.