ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Gravity, Scale Invariance and the Hierarchy Problem

A. Shkerin

In collaboration with M. Shaposhnikov

M. Shaposhnikov, AS, Phys. Lett. B783, 253 (2018), 1803.08907
M. Shaposhnikov, AS, JHEP 1810, 024 (2018), 1804.06376

Drowning by numbers

The fact is that

$$\frac{G_F \hbar^2}{G_N c^2} \sim 10^{33}$$

where G_F - Fermi constant, G_N - Newton constant

Quantum complications:

G.F. Giudice, (2008) 155, 0801.2562

Let M_X be some heavy mass scale. Then, one expects

$$\delta m^2_{H,X} \sim M_X^2$$

Even if one assumes that there are no heavy thresholds beyond the EW scale, then, naively,

$$\delta m^2_{H,grav.} \sim M_P^2$$

EFT approach and beyond

The Effective Field Theory paradigm:

Low energy description of Nature, provided by the SM, can be affected by an unknown UV physics only through a finite set of parameters.

This "**Naturalness principle**" is questioned now in light of the absence of signatures of new physics at the LHC.

G.F. Giudice, PoS EPS-HEP2013 (2013) 163, 1307.7879

What if one goes beyond the EFT approach? Many examples are known:

- Multiple point criticality principle — D. L. Bennett, H. B. Nielsen'94; C. D. Froggatt, H. B. Nielsen'96
- Asymptotic safety of gravity — S. Weinberg'09; M. Shaposhnikov, C. Wetterich'09
- EW vacuum decay — V. Branchina, E. Messina, M. Sher'14; F. Bezrukov, M. Shaposhnikov'14

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It is tempting to write something like

$$v \sim M_P e^{-B} \qquad (*)$$

where *v* is the Higgs vev and $B \approx 37$ in order to match with experiment.

Eq. (*) can be viewed as resulting from a saddle-point approximation of some functional integral. At this point euclidean classical configurations come into play.

Outline of the idea

Let us endow Eq. (*) with the physical meaning.

Consider a theory containing the real scalar field φ , the metric $g_{\mu\nu}$, and, possibly, other fields whose presence we ignore for the moment. Let M_P be the only classical scale in the theory. Then,

The vacuum expectation value of φ is $\langle\varphi\rangle \sim \int \mathscr{D}\varphi \mathscr{D} g_{\mu\nu} \varphi(0) e^{-S}$.

Assume that in the large-field limit the scalar degree of freedom is reorganised according to

$$\varphi \to M_P e^{\bar{\varphi}/M_P} \,, \quad \varphi \gtrsim M_P$$

Then, the expression in the path integral in this limit becomes

$$\varphi(0) e^{-S} \to M_P e^{-S'} \,, \quad S' = -\bar{\varphi}(0)/M_P + S$$

Assume that the saddle-point approximation (SPA) can be applied to S' . Then,

$$\langle\varphi\rangle = M_P e^{-B+B_0}$$

where B - the value of S' evaluated on a saddle, B_0 - the euclidean action evaluated on a vacuum solution.

For this to work, it is necessary to find

- appropriate saddle points of S' ,
- semiclassical parameter that would justify the SPA,
- physical argumentation that would justify the change of the scalar field variable.

Framework

Conjectures:

- Scale Invariance: The idea of reducing an amount of dimensionful parameters as a way towards the fundamental theory seems fruitful. Besides, SI can protect the Higgs mass against large radiative corrections.
- No degrees of freedom beyond the EW scale: Experimental data?
- Dynamical gravity: We believe gravity plays a crucial role in the effect we look for.

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Structure of the instanton:

$$B = B_{HE} + B_{LE}$$

The core of the instanton probes UV physics.

The tail is sensitive to low energies.

Several options are possible:

- $B \approx B_{HE}$
- Both B_{HE} and B_{LE} contribute significantly to B

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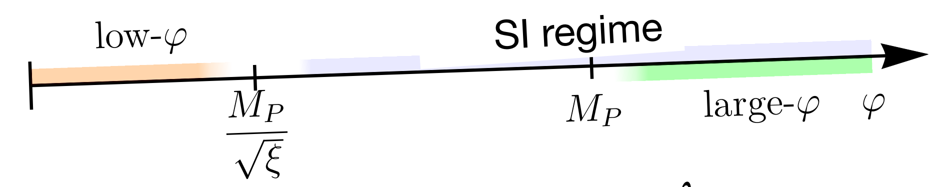
Disclaimer: We do not argue that the models we choose to test the instanton mechanism can indeed be embedded into the UV complete theory of gravity.

Example 1: Higgs inflation scenario

Lagrangian of the model: (inspired by Higgs inflation)



$$\frac{\mathscr{L}_J}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + V$$

$$V = \frac{\lambda}{4}\varphi^4$$

Some fields redefinition:

to get rid of the non-minimal coupling

to make the kinetic term canonical

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \qquad \Omega^2 = \frac{M_P^2 + \xi\varphi^2}{M_P^2}\,, \qquad \varphi = M_P e^{\bar{\varphi}/M_P}$$

``Einstein frame" Lagrangian:

$$\frac{\mathscr{L}_E}{\sqrt{\tilde{g}}} = -\frac{1}{2}M_P^2\tilde{R} + \frac{1}{2a}(\tilde{\partial}\bar{\varphi})^2 + V\Omega^{-4}$$

SI regime

$$a \longrightarrow a_{SI} = \frac{1}{1/\xi + 6}$$

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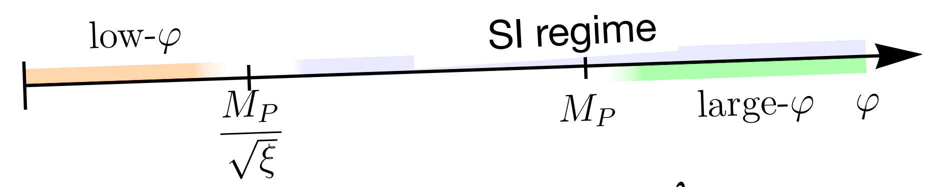

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SI regime

$$a \longrightarrow a_{SI} = \frac{1}{1/\xi + 6}$$

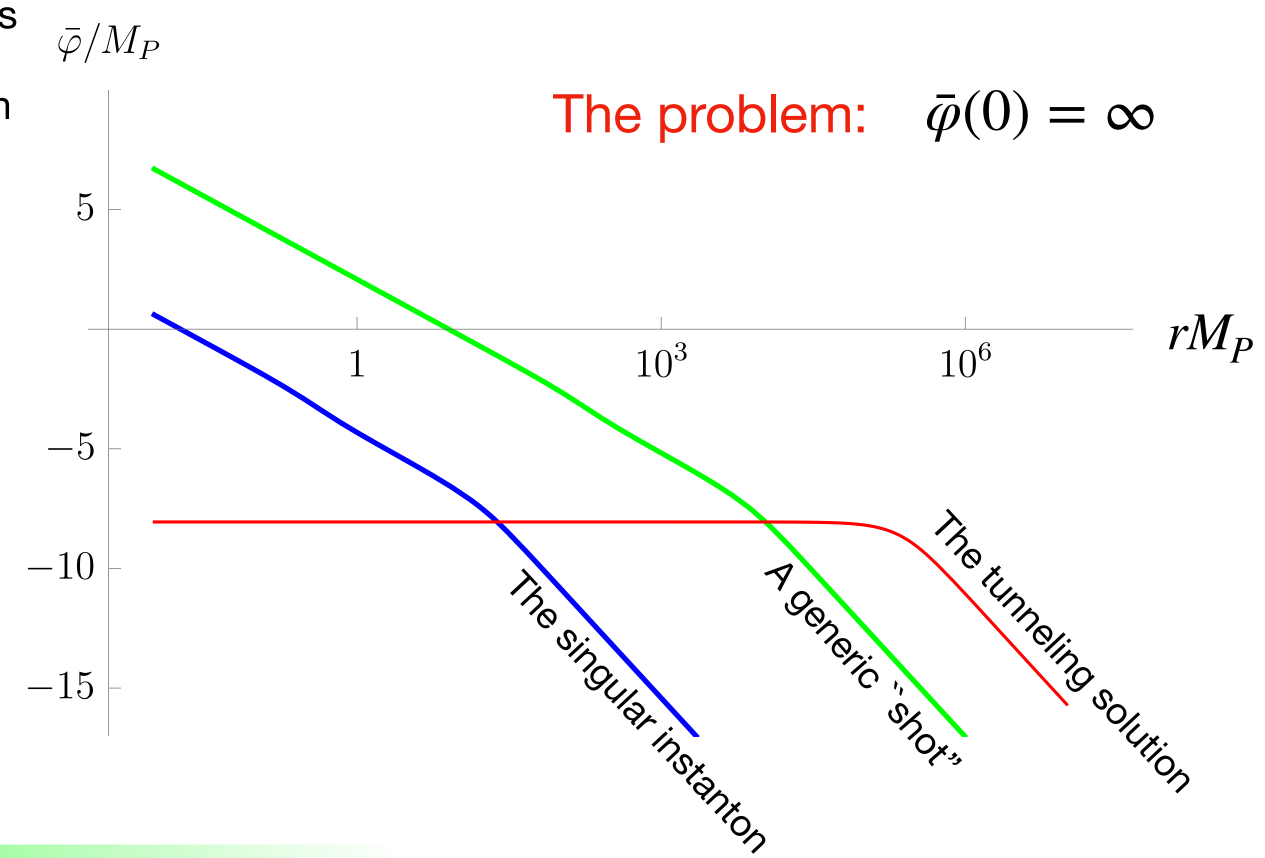
Action to vary: $S' = -\bar{\varphi}(0)/M_P + S$

the source provides an additional boundary condition

Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

EoM for $\bar{\varphi}$ in the SI regime: $\frac{r^3\bar{\varphi}'}{f a_{SI}} = -\frac{1}{M_P}$

Short-distance asymptotics of the instanton: $\bar{\varphi}' \sim M_P r^{-1}$



Example 1: Higgs inflation scenario

To cure the problem, let us modify the Lagrangian:

some operator with higher degree of the derivative of φ . For example, take $n = 2$

$$\frac{\mathcal{L}_J}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + V + \delta_n \frac{(\partial\varphi)^{2n}}{(M_P\Omega)^{4n-4}}$$

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$$\frac{\delta\mathcal{L}_E}{\sqrt{\tilde{g}}} \xrightarrow{\text{SI regime}} \delta_2 \frac{(\tilde{\partial}\bar{\varphi})^4}{\xi^2 M_P^4}$$

$$a \xrightarrow{\text{SI regime}} a_{SI} = \frac{1}{1/\xi + 6}$$

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Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

EoM for $\bar{\varphi}$ in the SI regime: $\frac{4\delta}{M_P^4}\frac{\rho^3\bar{\varphi}'^3}{f^3} + \frac{r^3\bar{\varphi}'}{fa_{SI}} = -\frac{1}{M_P}$

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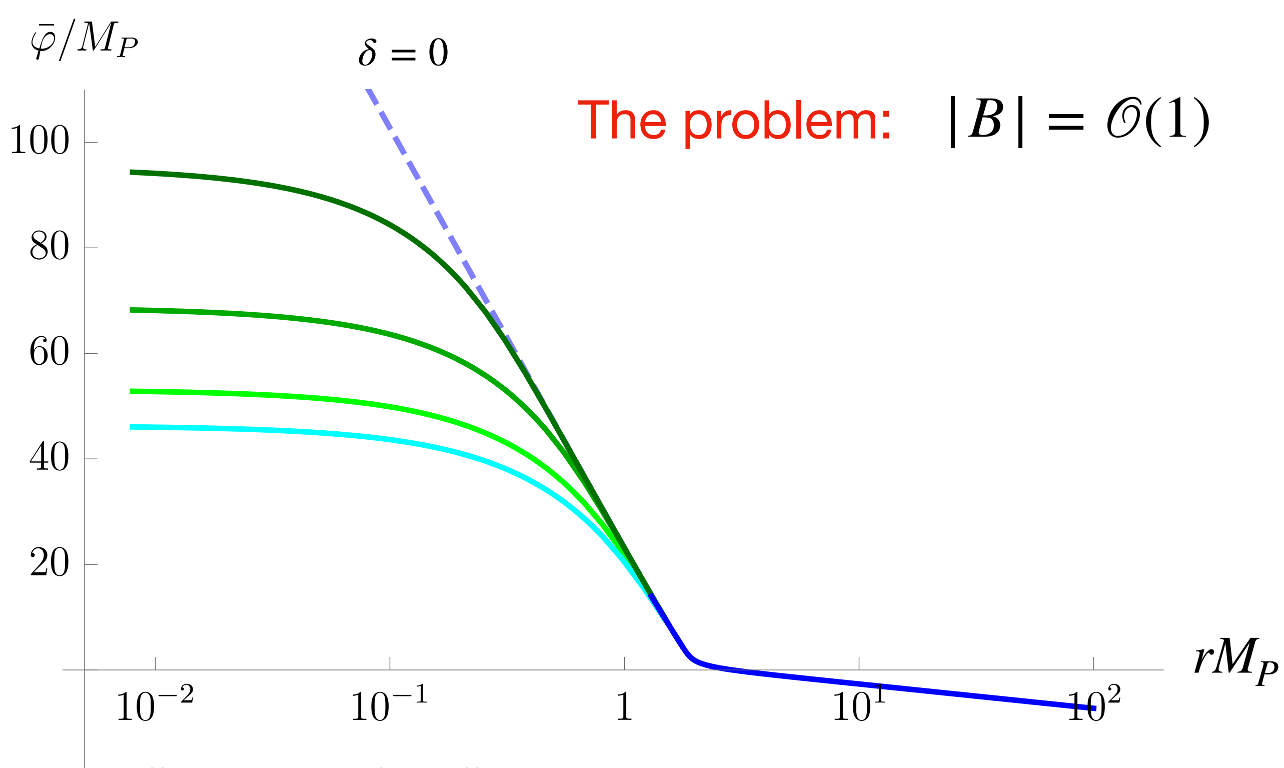
Short-distance asymptotics of the instanton: $\bar{\varphi}' \sim M_P^2\delta^{-1/6}$



Different colors for different δ

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SI regime

$$\frac{\delta\mathcal{L}_E}{\sqrt{\tilde{g}}} \longrightarrow \delta_2 \frac{(\tilde{\partial}\bar{\varphi})^4}{\xi^2 M_P^4}$$

large-φ regime

$$a' \longrightarrow a_{HE} \gg a_{SI}$$

Action to vary: $S' = -\bar{\varphi}(0)/M_P + S$

Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

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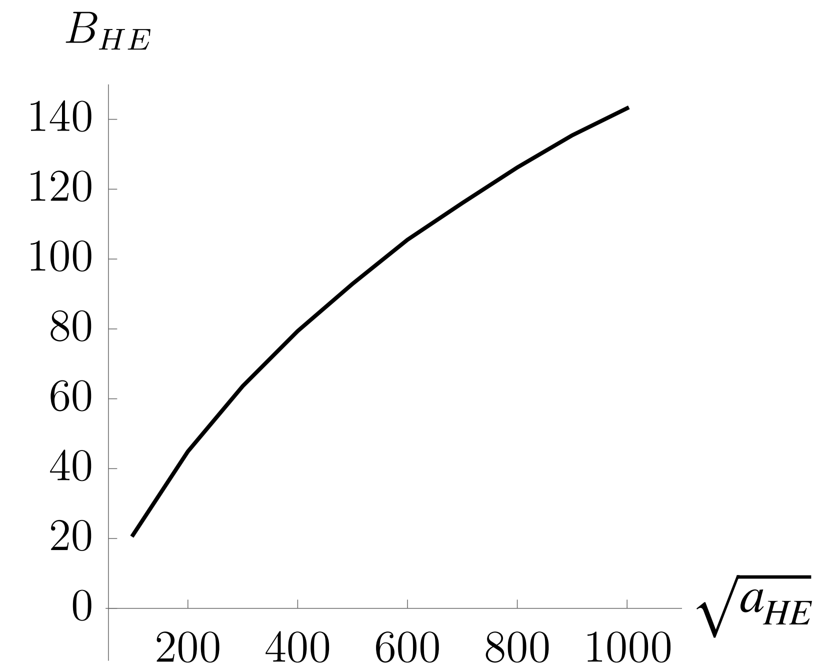

Here $\xi \sim 10^3$, $\delta \sim 10^{-8}$

The result: $B_{HE} \sim \sqrt{a_{HE}}$ $\quad B \approx B_{HE}$ $\quad \langle\varphi\rangle \sim M_P e^{-B}$

Discussion

Points to notice:

- There is nothing special about $B \approx 37$
- The instanton is not sensitive to the physics at low energies
- The mechanism is not sensitive to the particular model, but:
- Approximate Weyl invariance at high energies seems to be necessary

Example 2: No scale scenario

The Higgs-Dilaton Lagrangian: (inspired by the fact that all scales emerge dynamically)

$$\frac{\mathcal{L}_J}{\sqrt{g}} = -\frac{1}{2}(\xi_\chi \chi^2 + \xi_h h^2)R + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial h)^2 + V$$

the vev of the dilaton gives the Planck mass

Some fields redefinition…

``polar coordinates”

$$\chi = \frac{M_P \cos\theta}{\sqrt{1+6\xi_\chi}} \cdot e^{\rho/M_P}\,, \qquad h = \frac{M_P \sin\theta}{\sqrt{1+6\xi_h}} \cdot e^{\rho/M_P}$$

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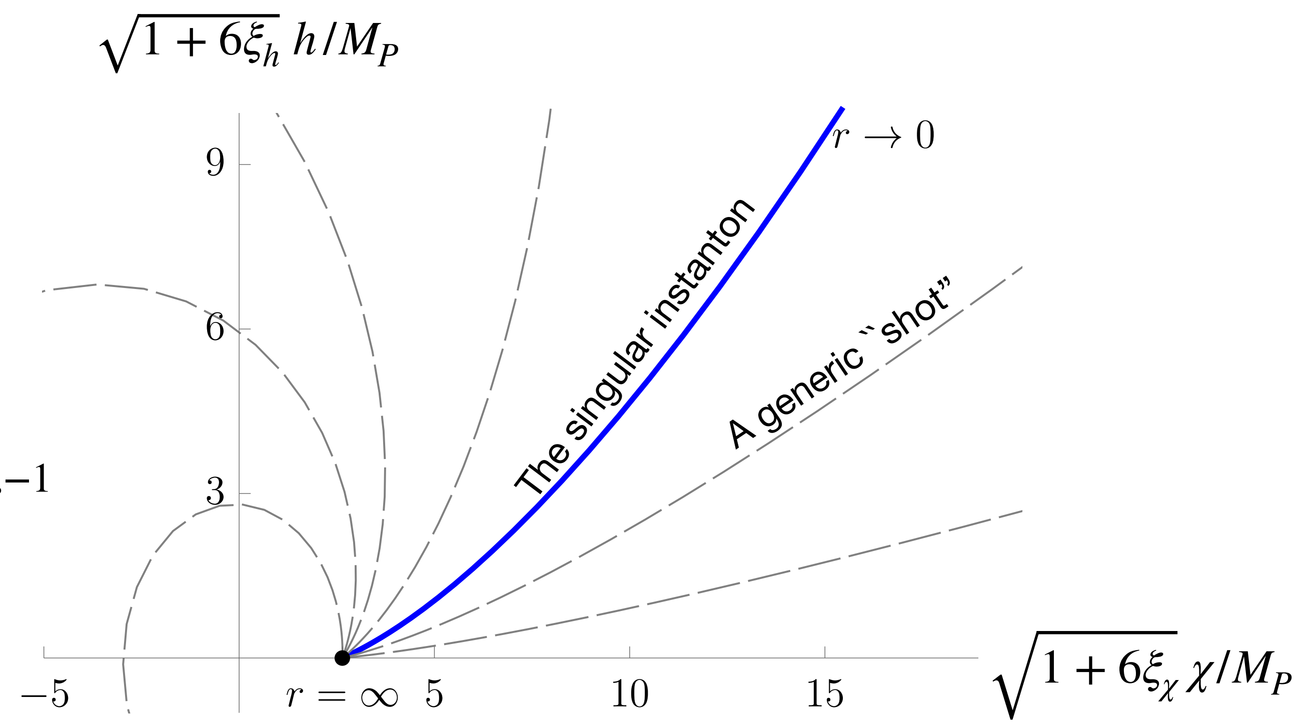
Action to vary: $S' = -\rho(0)/M_P + S$

Metric ansatz: $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

EoM for ρ: $\frac{r^3\rho'}{fa(\theta)} = -\frac{1}{M_P}$

Short-distance asymptotics of the instanton: $\rho' \sim -M_P r^{-1}$, $\theta \to \pi/2$



The same problems as in example 1.
The same treatment.

Example 2: No scale scenario

The modified Lagrangian: $(\varphi^1, \varphi^2) = (\chi, h)$

$$\frac{\mathscr{L}_J}{\sqrt{g}} = -\frac{1}{2}(\xi_\chi \xi^2 + \xi_h h^2)R + \frac{1}{2}\gamma^{(2)}_{ij} g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j + \frac{1}{2}\gamma^{(4)}_{ijkl} g^{\mu\nu}\partial_\mu\varphi^i\partial_\nu\varphi^j g^{\rho\sigma}\partial_\rho\varphi^k\partial_\sigma\varphi^l + V$$

Some fields redefinition…

$$\chi = \frac{M_P \cos\theta}{\sqrt{1+6\xi_\chi}} \cdot e^{\rho/M_P}, \qquad h = \frac{M_P \sin\theta}{\sqrt{1+6\xi_h}} \cdot e^{\rho/M_P}$$

``Einstein frame" Lagrangian:

just an example

$$\frac{\mathscr{L}_E}{\sqrt{\tilde{g}}} = -\frac{1}{2}M_P^2\tilde{R} + \frac{1}{2a'(\theta)}(\tilde{\partial}\rho)^2 + \frac{b(\theta)}{2}(\tilde{\partial}\theta)^2 + \delta\frac{(\tilde{\partial}\rho)^4}{M_P^4} + V\Omega^{-4}$$

$$a' \xrightarrow{\theta \to \pi/2} a_{HE}$$

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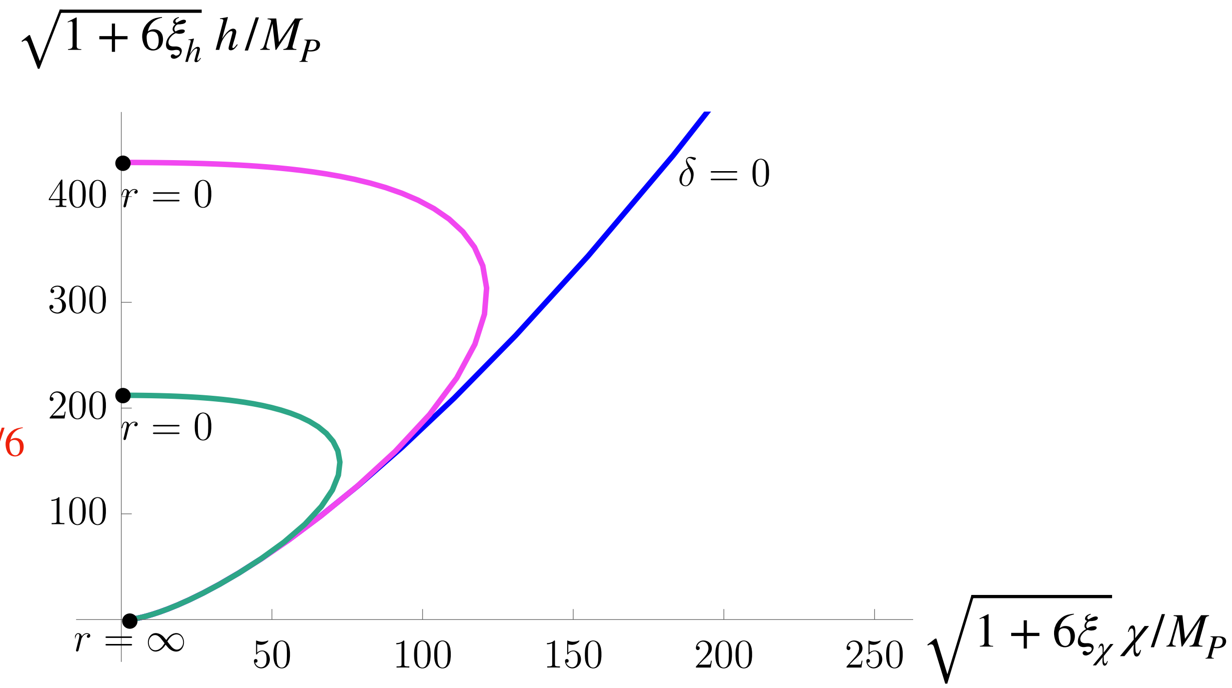
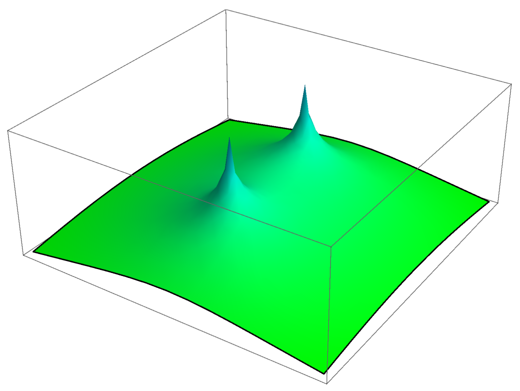
As before, $B_{HE} \sim \sqrt{a_{HE}}$ $\quad B \approx B_{HE}$



Different colors for different δ

Outlook

- More understanding of the singular instanton is needed (what about fluctuations above it?),
- More understanding of the conditions for the successful implementation of the mechanism is needed (what is special about gravity?),
- The option where both low-energy and high-energy parts of B are important is also possible,
- 2-point scalar correlation function via instanton?
- Majorana masses via instanton?



Thank you!