Conformal symmetry as an exact symmetry with Higgs mechanism

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LHC gave a Higgs mass close to the critical value. This suggests that scale- or conformal invariance may play a role in future theories.

The theory discussed here is still in its infancy. Depends on extensive calculations that have not yet been done.

Theory asserts that local conformal symmetry may be a very powerful constraint on a pseudo-perturbative theory of gravity with matter, which will fix all interaction strengths, masses, Yukawa forces etc., in terms of the Planck units, with the matter algebra as the only adjustable freedom.

Main idea:

conformal invariance in the BEH mode

Merit:

Theory should predict all couplings of the Standard Model,
as soon as we know the complete algebra.

Conformal invariance is usually considered to be broken by

- interactions
- mass terms
- renormalization effects (contribution of loop diagrams to β functions)
- other anomalies

Can all these effects be attributed to spontaneous breakdown of the symmetry / as in BEH mechanism?

Why would this be important?

Gravity Research Foundation 2015 Awards for Essays on Gravitation: "Local Conformal Symmetry: the Missing Symmetry Component for Space and Time" (January 2016.)

A symmetry connecting different scales in physics would be extremely welcome!

It would need to be an exact symmetry, broken as in Brout-Englert-Higgs-Kibble.

Compare scale symmetry with translation, rotation, Lorentz invariance ...
Use the symmetry instead of particle accelerators . . .

Note that translation invariance comes with *locality*. Locality might boost scale invariance into a more powerful *conformal* invariance.

Theme of the Essay: Laws of Nature at one scale give no clues about other scales. Danger of requiring infinite sequences of Laws of Nature, a new one at every scale.

What we need is a more concise - efficient - frame of Laws.

Actually, the Einstein-Hilbert (EH) action already contains the required ingredient: The entire metric tensor is a dynamical field variable. if we write

$$g_{\mu\nu} = \phi^2(x)\hat{g}_{\mu\nu}(x) \ ,$$

$\hat{g}_{\mu\nu}$ obeys "gauge constraint" such as $\det(\hat{g}_{\mu\nu}) = -1$.

$$\langle|\,\phi(x)\,|\rangle = 1 \ .$$

Einstein-Hilbert action:

$$\mathcal{L}_{\rm tot} = \sqrt{-g}\left(\frac{1}{16\pi G_N}(R-2\Lambda) + \mathcal{L}^{\rm matter}\right)$$

Substitute $g_{\mu\nu} = \phi^2(x)\,\hat{g}_{\mu\nu}(x)$:

$$\mathcal{L}_{\rm EH} = \sqrt{-\hat{g}}\left(\frac{1}{16\pi G_N}\left(\phi^2\hat{R} - 2\,\phi^4\Lambda + 6\,\hat{g}^{\,\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right)\right)$$

and, rescaling $\phi = \tilde{\kappa}\eta$, with $\tilde{\kappa} = \sqrt{\frac{4\pi G_N}{3}}$:

$$\begin{aligned}\mathcal{L}^{\rm matter} \quad \rightarrow \quad & -\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \bar{\psi}\,\hat{\gamma}^\mu\hat{D}_\mu\,\psi - \tfrac{1}{2}\hat{g}^{\mu\nu}D_\mu\varphi\,D_\nu\varphi - \tfrac{1}{2}\tilde{\kappa}^2 m_\varphi^2\eta^2\varphi^2 \\ & -\tfrac{1}{12}\hat{R}\varphi^2 - V_4(\varphi) - \tilde{\kappa}\eta V_3(\varphi) \\ & -\bar{\psi}(y_i\varphi_i + iy_i^5\gamma^5\varphi_i + \tilde{\kappa}m_d\eta)\psi\end{aligned}$$

$$\mathcal{L}_{\rm EH} \quad \rightarrow \quad + \tfrac{1}{12}\hat{R}\eta^2 - \tfrac{1}{6}\tilde{\kappa}^2\Lambda\eta^4 + \tfrac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\eta\,\partial_\nu\eta$$

All couplings are dimensionless

Since we substituted $g_{\mu\nu} = \tilde{\kappa}^2 \eta^2 \hat{g}_{\mu\nu}$ everywhere, our Lagrangian $\mathcal{L}$ has the exact – but trivial – "conformal invariance":

$$\hat{g}_{\mu\nu} \to \Omega^2(x)\hat{g}_{\mu\nu} \ , \quad \eta \to \Omega^{-1}\eta \ , \quad \varphi \to \Omega^{-1}\varphi \ , \quad \text{etc.}$$

All this is just notation. No changes in the physics. Theory is still non-renormalizable.

But now, we can treat the 'dilaton' $\eta(x)$ just like any other scalar field.

Since all SM interactions happen to be even in η, the anomalous sign in the propagator disappears completely if we replace $\eta \to i\,\eta$.

Now, compare with electro-weak theory before and after 1970:

How could we have discovered the way to modify the physics?

Around the 1960 's, one could have reasoned as follows:

Start with Fermi's 4-fermion interaction :

$$\mathcal{L}_{\rm int} = \tfrac{1}{2}\,\tilde{G}_F\,J_\mu J_\mu \qquad \text{where} \qquad J_\mu = \bar\psi\gamma_\mu(1+\gamma_5)\psi$$

(scaling away $\sqrt{2}$, and suppressing isospin indices).

Add auxiliary fields $A_\mu(x)$ without kinetic terms, to write

$$\mathcal{L}_{\rm int} \quad = \quad -\tfrac{1}{2}A_\mu^2 + g\,A_\mu J_\mu \ ,$$

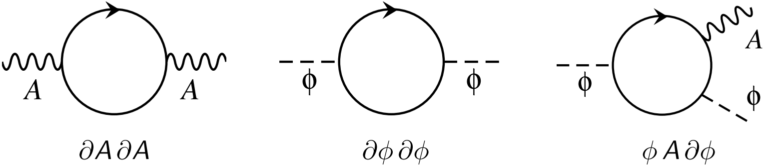
and, solving for A_μ :

$$A_\mu = gJ_\mu \ , \qquad \mathcal{L}_{\rm int} \ \to \ \tfrac{1}{2}g^2\,J_\mu J_\mu \ \to \ \tilde{G}_F = g^2 \ .$$

Fermionic mass terms violate the local gauge symmetry, so that divergences also violate local gauge symmetries. Unless we also introduce auxiliary *schizon* "Higgs" field $\phi_i(x)$, also without kinetic terms, but with self-interaction to generate $\langle|\,\phi(x)\,|\rangle = F \neq 0$.

Now, investigate all divergences. Diagrams with external A_μ and ϕ lines are divergent but have no counter parts in $\mathcal{L}$.

Diagrams with external A_μ and φ_i lines:

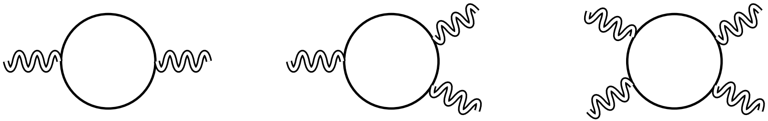


Local symmetry invariances dictate that these take the form of Yang-Mills interactions, $G_{\mu\nu} G_{\mu\nu}$, $(D_\mu \phi)^2$, etc. All renormalizable terms with A_μ and φ, must be added to the original Lagrangian $\mathcal{L}$.

Note that A_μ^2 may have come from $(D_\mu \varphi_i)^2$ if $\langle \varphi \rangle = F$.

This argument naturally derives the Standard Model, a renormalizable Yang-Mills theory, from the original $J_\mu\, J_\mu$ interaction, as soon as we have all terms with the right symmetries and dimensionality.

Similarly, in our conformally invariant gravity theory, one expects divergent diagrams such as:



$\hat{g}\,\partial^4\,\hat{g}$ $\qquad$ $\hat{g}\,\hat{g}\,\partial^4\hat{g}$ $\qquad$ $(\partial^4)(\hat{g}^4)$

Assuming the underlying theory to have exact (but trivial) local conformal invariance, the required counter terms, in terms of $\hat{g}_{\mu\nu}$, should be expected to be conformally invariant as well.

What happens when we try to apply the philosophy that would have been successful in the electro-weak case?

"Add all terms in the Lagrangian that share the original symmetries. All divergences can then be absorbed by counter terms . . . "

Only one term that has the required symmetries, is missing. The Weyl tensor $C_{\mu\nu\alpha\beta}$ is defined by

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta} - \tfrac{1}{2}\left(R_{\mu\alpha}\hat{g}_{\nu\beta} - (\mu \leftrightarrow \nu) - (\alpha \leftrightarrow \beta) + \left({}^{\mu}_{\nu} \leftrightarrow {}^{\alpha}_{\beta}\right)\right) \\ &+ \tfrac{1}{6} R \left(\hat{g}_{\mu\alpha}\hat{g}_{\nu\beta} - (\mu \leftrightarrow \nu)\right) \ , \end{aligned}$$

such that it is traceless ($\hat{g}^{\mu\alpha} C_{\mu\nu\alpha\beta} = \cdots = 0$). For that reason,

$$CC \equiv C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 2R_{\mu\nu} R^{\mu\nu} + \tfrac{1}{3} R^2$$

is a conformally invariant term in the Lagrangian.

$$G \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2$$

is a total derivative and therefore topologically invariant. This leaves CC as a term that has to be added, and R^2 as a term that must be forbidden.

Add to $\mathcal{L}_{\mathrm{EH}}$ a term

$$-\tfrac{1}{2}\lambda\, CC\,, \qquad \text{with} \qquad \lambda = 1/M^2\,.$$

Gives much more convergence than we had before, since the propagator now has a $1/k^4$ behavior at large k.

$\Rightarrow$ renormalizable theory of gravity?

For 2 reasons, the answer is no .

Reason # 1:

In the BEH mode ($\langle|\,\eta\,|\rangle = 1/\tilde{\kappa}$), the graviton propagator is

$$\frac{1}{(k^2 - i\varepsilon)(k^2 + M^2 - i\varepsilon)} = \frac{1}{k^2 - i\varepsilon} - \frac{1}{k^2 + M^2 - i\varepsilon}\ .$$

(omitting indices). The second term is the propagator of

a particle with spin 2, mass M and negative metric

In ordinary theories, such as the SM, this is illegal.

Reason # 2 :

Anomalies.

In the Standard Model, the procedure may be jeopardised by anomalies. If a symmetry only holds for massless particles, and only in 4 space-time dimensions – such as chiral symmetries – then the regularised diagrams may disobey the symmetry: the Adler-Bell-Jackiw anomaly.

Such theories must be cured. Usually: choose the algebra and the representations carefully, to achieve *anomaly cancellation.*

For conformal symmetry, these anomalies are even more severe. Every particle type, scalar, spinor, vector and tensor, contributes to the conformal anomalies. One consequence: all parameters are *running* parameters. Following the Callan-Symanzik β functions.

These all signal scaling violation, and hence must *all cancel out to vanish.*

Apart from the β functions, explicit calculations yield expressions for the counter terms that generate extra R^2 terms, of the kind we had declared forbidden.

Also in pure gravity, with *CC* term, anomalous R^2 terms appear (Duff, Stelle, 1977). This must imply that pure gravity is forbidden. *We must have matter, to cancel this out.*

Yet whether this works at all is not yet known.

Our theory generates as many equations as there are independent couplings in the theory, since *All couplings carry their own β functions.*

They must all be at their fixed points.

Calculations: The β functions. Calculate their fixed points stepwise. In logical order:
First, the conformal anomalies due to pure gravity, with only the CC propagator.
Hopefully, this yields constraints for the matter algebra. Then:

α = coefficient for CC term. RR term must have coefficient 0.

g = Yang-Mills coupling constants.

$$\beta(g^2) = \beta_1 g^4 + \beta_{21} g^6 + \beta_{22} \lambda^2 g^4 \dots$$

y = Yukawa interactions for the spin ½ fields

$$\beta(y) = \beta_{31} y\, g^2 + \beta_{32} y^3 + \dots$$

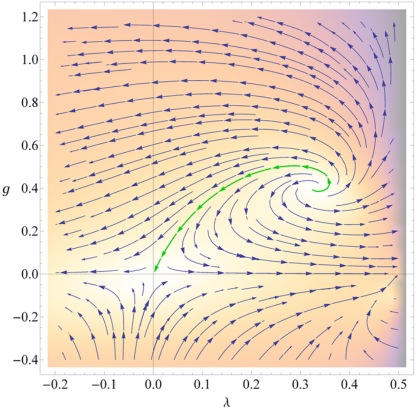
λ = scalar self couplings and cosmological constant.

$$\beta(\lambda) = \beta_{41} \lambda^2 + \beta_{42} \lambda g^2 + \beta_{43} \lambda y^2 + \beta_{44} y^4 \ .$$

Choose the algebra such that $\beta_1 = \mathcal{O}(1/N^2)\ , \quad \beta_2 = \mathcal{O}(1)$ .. Then, fixed point may be found at

$$g^2 = \mathcal{O}(1/N^2)\ , \quad y = \mathcal{O}(g)\ , \quad \lambda = \mathcal{O}(g^2)\ .$$

Connection to the *asymptotic safety* idea (Weinberg)



At the non-trivial fixed point, this theory is also controlled by effective operators with dimension 4.

The Landau ghost in this theory is equivalent to our spin-2 ghost. What we added is the possibility to do perturbation expansions.

But beware, we also have to go to the conformal fixed point!

A major difficulty: the negative metric heavy spin 2 particle. Does it not ruin the theory?

It is worrying. In the past, introduction of negative metric fields has been tried in the Standard Model. But it seems to lead, obviously, to breakdown of unitarity. See however P. Mannheim's attempts to cure such theories.

Alternatively, this issue may be related to the *firewall transformation* in black holes, where very heavy particles are replaced by their gravitational footprints. This reduces the particle spectrum at high energies. Perhaps a negative metric particle is a (poor man's) substitute for a reduction of the highest energy particle states.

One may also try to involve *fundamental quantum effects* at extremely high energies, where one may have that some particles seek the highest energy states rather than the lowest ones. But all this may well be just phantasy. The proposal here is to do the calculations and see where they get us.

Conclusion

Explicit calculations have not yet been carried out. We do not know whether this approach will bring us anywhere.

Interesting aspect:

All physical coupling parameters, including masses (the dimensionless numbers $\tilde{\kappa}m$) and the mass M of the heavy spin 2 excitation of the graviton, should be calculable.

Mixed blessing: shouldn't then all couplings reside in the same domain – the Planck length?
How should we face the hierarchy problem? Is there any reason to expect natural, large ratios of couplings? Why should $\tilde{\kappa}^2\Lambda$ be of order 10^{-122}?

One freedom we (seem to) have: *choose the algebra of the matter fields.*

THANK YOU