Inertial Symmetry Breaking, Dilaton and Weyl Photon

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"Scale Invariance in Particle Physics and Cosmology"
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Inertial Symmetry Breaking

Inertial Spontaneous Symmetry Breaking and Quantum Scale Invariance, By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross, **Phys.Rev. D98, (2018), 116012**; arXiv:**1801.07676**

Inertial Symmetry Breaking,
By C. T. Hill, Mar. 2018; arXiv:**1803.06994**

Inflation in a scale invariant universe
Pedro G. Ferreira, Christopher T. Hill, Johannes Noller, Graham G. Ross. Feb 16, 2018.
Phys.Rev. D97 (2018) no.12, 123516

No fifth force in a scale invariant universe
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross. **Phys.Rev. D95 (2017) no.6, 064038**

Weyl Current, Scale-Invariant Inflation and Planck Scale Generation
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross. **Phys.Rev. D95 (2017) no.4, 043507.**

Scale-Independent Inflation and Hierarchy Generation
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross. **Phys.Lett. B763 (2016) 174-178.B**

Some foundational and related earlier works

Hermann Weyl, Raum, Zeit, Materie (Space, Time, Matter), "Lectures on General Relativity," (in German) Berlin, Springer (1921).

C. Wetterich, Nucl Phys. B302 (1988) 668
M. Shaposhnikov, and D. Zenhausen Phys Lett B671 162; B671, 187;
D. Blas, M. Shaposhnikov, and D. Zenhausen, Phys Rev D84 (144001) (2011);
J. Garcia, J. Rubio, M Shaposhnikov and D. Zenhausen, Phs. Rev D 84 (23504) (2011);
F.L.Bezrukov and M. Shaposhnikov, Phys. Lett B659 (2008) 703
D. M Ghilencea, arXiv:1812.08613 [hep-th].
D. M. Ghilencea and H Lee, arXiv:1809.09174 [hep-th]

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A conventional scalar-gravity action:

(notation g = (1,-1,-1,-1))

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \Lambda + \frac{1}{2}M_P^2 R\right)$$

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A *global* scale invariant scalar-gravity action:

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4 - \frac{1}{12}\alpha\phi^2 R\right)$$

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$\Updownarrow$ "Weyl Equivalent"

A scale invariant scalar-gravity action:

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Do a Weyl Field Redefinition:

$$\phi(x) \to \phi_0 \exp(\sigma(x)/f), \qquad g_{\mu\nu}(x) \to \exp(-2\sigma(x)/f)g_{\mu\nu}(x)$$

$$\int \sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{\lambda}{4}\phi^4 - \tfrac{1}{12}\alpha\phi^2 R\right)$$

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$$= \int \sqrt{-g}\left(\tfrac{1}{2f^2}\phi_0^2(1-\alpha)g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \tfrac{\lambda}{4}\phi_0^4 - \tfrac{1}{12}\alpha\phi_0^2 R(g)\right)$$

$$\int \sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{\lambda}{4}\phi^4 - \tfrac{1}{12}\alpha\phi^2 R\right)$$

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choose $f^2 = \phi_0^2(1-\alpha) \quad \Lambda = \frac{\lambda}{4}\phi_0^4 \qquad M_{Planck}^2 = -\frac{1}{6}\alpha\phi_0^2$

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \Lambda + \frac{1}{2}M_P^2 R\right)$$

A conventional scalar-gravity action:

(notation g = (1,-1,-1,-1))

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \Lambda + \frac{1}{2}M_P^2 R\right)$$

How do we see that
the dynamics is equivalent?

A scale invariant scalar-gravity action:

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4 - \frac{1}{12}\alpha\phi^2 R\right)$$

A scale invariant scalar-gravity action:

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 - \frac{1}{12} \alpha \phi^2 R \right)$$

Transplanckian expanding universe?

Note that this situation contrasts what happens in conventional Einstein gravity with a fixed M_P and a $\lambda\phi^4/4$ potential. Inflation is possible for super Plankian values of ϕ which slow-roll to $\phi = 0$. Hence, while normal Einstein gravity causes ϕ to relax to zero, the scale-invariant gravity theory leads to constant nonzero $\phi = \phi_0$ which generates M_P and eternal inflation.

Why does ϕ stop?

Compute in the scale invariant frame

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4 - \frac{1}{12}\alpha\phi^2 R\right)$$

Einstein Equations

$$\frac{1}{6}\alpha\phi^2 G_{\alpha\beta} = \left(\frac{3-\alpha}{3}\right)\partial_\alpha\phi\partial_\beta\phi - g_{\alpha\beta}\left(\frac{3-2\alpha}{6}\right)\partial^\mu\phi\partial_\mu\phi$$
$$+\frac{1}{3}\alpha\left(g_{\alpha\beta}\phi D^2\phi - \phi D_\beta D_\alpha\phi\right) + g_{\alpha\beta}V(\phi)$$

Trace $$-\frac{1}{6}\alpha\phi^2 R = (\alpha-1)\partial^\mu\phi\partial_\mu\phi + \alpha\phi D^2\phi + 4V(\phi)$$

Klein-Gordon $$0 = \phi D^2\phi + \phi\frac{\delta}{\delta\phi}V(\phi) + \frac{1}{6}\alpha\phi^2 R$$

Combine trace + KG to eliminate R:

$$0 = (1-\alpha)\phi D^2\phi + (1-\alpha)\partial^\mu\phi\partial_\mu\phi + \phi\frac{\partial}{\partial\phi}V(\phi) - 4V(\phi)$$

Combine trace + KG to eliminate R:

$$0 = (1-\alpha)\phi D^2\phi + (1-\alpha)\partial^\mu\phi\partial_\mu\phi + \phi\frac{\partial}{\partial\phi}V(\phi) - 4V(\phi)$$

$$\Rightarrow \quad D^\mu K_\mu = 4V(\phi) - \phi\frac{\partial}{\partial\phi}V(\phi)$$

$$K_\mu = (1-\alpha)\phi\partial_\mu\phi$$ Weyl Current

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$$K_\mu = (1-\alpha)\phi\partial_\mu\phi$$ Weyl Current

If $V(\phi) = \frac{\lambda}{4}\phi^4$ then K_μ is conserved

This is the Noether Current

"Theorem:"
The current for any Weyl Invariant action with two scalar derivatives can be written as

$$K_\mu = \partial_\mu K$$

K is the "kernel"

$K = (1-\alpha)\phi^2/2$ in the present model

$$D^\mu K_\mu = D^2 K = 0$$

Consider arbitrary chaotic initial conditions,
K is "lumpy" with arbitrary space-time
derivatives: a "disordered phase."

But K redshifts like a massless field
in a vanishing potential
if the Weyl current is conserved

$$D^\mu K_\mu = D^2 K = 0$$

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e.g. a FRW metric:

$$g_{\mu\nu} = [1, -a^2(t), -a^2(t), -a^2(t)] \qquad H = \frac{\dot{a}}{a}$$

$$G_{00} = -3\frac{\dot{a}^2}{a^2} \qquad R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$

$$D^\mu K_\mu = 0 \quad \Rightarrow \quad K(t) = c_1 + c_2 \int_{t_0}^{t} \frac{dt'}{a^3(t')}$$

K $\Rightarrow$ CONSTANT

This is why ϕ stops

The Kernel is the "order parameter"
of Weyl scale symmetry breaking
disorder - order transition

general initial conditions, $K(t)$ will evolve to a constant

$$K = K(t \to \infty) \quad \to \quad \frac{1}{2}(1-\alpha)\phi_0^2$$ Constant

Dilaton decay constant $f^2 = \phi_0^2(1-\alpha)$ = 2K

"Inertial" spontaneous scale generation

Multi-Scalar Theories

N fields:

$$S = \int \sqrt{-g} \left(\sum_i^N \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - V(\phi_i) - \sum_i^N \tfrac{1}{12} \alpha_i \phi_i^2 R \right)$$

$K_\mu = \partial_\mu K$ The kernel is $K = \frac{1}{2} \sum_i^N (1 - \alpha_i) \phi_i^2$

$K = K(t \to \infty)$ Constant

Transition to an "ordered phase"

Multi-Scalar Theories

$$S = \int \sqrt{-g} \left(\sum_i^N \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - V(\phi_i) - \sum_i^N \tfrac{1}{12} \alpha_i \phi_i^2 R \right)$$

Define a set of $N+1$ fields, $(\phi_i', \sigma(x))$,

$$\phi_i(x) = \exp(\sigma(x)/f)\phi_i'(x) \ , \ 1 \ldots N$$

Impose a constraint

$$\overline{K} = \tfrac{1}{2} \sum^N (1 - \alpha_i)\phi_i'^2$$

Hence $(\phi_i', \sigma(x))$ with $\overline{K}$ describes N fields.

Define the new metric, $g_{\mu\nu}(x) = \exp(-2\sigma(x)f) g'_{\mu\nu}(x)$

Multi-Scalar Theories

$$S => \int \sqrt{-g'} \left(\sum_i^N \tfrac{1}{2} g'^{\mu\nu} \partial_\mu \phi_i' \partial_\nu \phi_i' - V(\phi_i') - \sum_i^N \tfrac{1}{12} \alpha_i \phi_i'^2 R(g') \right)$$
$$+ \int \sqrt{-g'} \left(\sum_i^N (1-\alpha_i)(\phi_i' \partial_\mu \phi_i') \left(\frac{\partial^\mu \sigma(x)}{f} \right) + \sum_i^N \tfrac{1}{2} (1-\alpha_i) \phi_i'^2 \left(\frac{\partial_\rho \sigma(x) \partial^\rho \sigma(x)}{f^2} \right) \right)$$

Multi-Scalar Theories

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$$+\int \sqrt{-g'}\left(\sum_i^N (1-\alpha_i)(\phi_i'\partial_\mu\phi_i')\left(\tfrac{\partial^\mu\sigma(x)}{f}\right) + \sum_i^N \tfrac{1}{2}(1-\alpha_i)\phi_i'^2\left(\tfrac{\partial_\rho\sigma(x)\partial^\rho\sigma(x)}{f^2}\right)\right)$$

Choose the decay constant: $f^2 = 2\overline{K}$

$$= \int \sqrt{-g'}\left(\sum_i^N \tfrac{1}{2} g'^{\mu\nu}\partial_\mu\phi_i'\partial_\nu\phi_i' - V(\phi_i') - \sum_i^N \tfrac{1}{12}\alpha_i\phi_i'^2 R(g')\right)$$
$$+\int \sqrt{-g'}\left(\tfrac{1}{2}(\partial_\rho\sigma(x)\partial^\rho\sigma(x))\right)$$

Multi-Scalar Theories

Canonical normalization $f^2 = 2\overline{K}$

$$\int \sqrt{-g'}\left(\sum_i^N \tfrac{1}{2} g'^{\mu\nu}\partial_\mu\phi_i'\partial_\nu\phi_i' - V(\phi_i') - \sum_i^N \tfrac{1}{12}\alpha_i\phi_i'^2 R(g')\right)$$
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Decoupling of the dilaton.

Multi-Scalar Theories

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$$+\int \sqrt{-g'}\left(\tfrac{1}{2}(\partial_\rho\sigma(x)\partial^\rho\sigma(x))\right)$$

Decoupling of the dilaton.

In this representation the Weyl transformation is

$\sigma(x) \to \sigma(x) + \epsilon f.$ The current is now $K_\mu = f\partial_\mu\sigma.$

The decay constant of an NGB
is the **order parameter** of SSB

Two Scalars, globally Weyl symmetric

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\upsilon}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\upsilon}\partial_\mu\chi\partial_\nu\chi - W(\phi,\chi) - \frac{1}{12}\alpha_1\phi^2 R - \frac{1}{12}\alpha_2\chi^2 R\right)$$

$$W(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{\xi}{4}\chi^4 + \frac{\delta}{2}\phi^2\chi^2$$

Effective Planck Mass $M^2_{Planck} = \frac{1}{6}(-\alpha_1\phi^2 - \alpha_2\chi^2)$

Two Scalar K-current

$$K_\mu = (1-\alpha_1)\phi\partial^\mu\phi + (1-\alpha_2)\chi\partial^\mu\chi$$

Two Scalar Potential

$$W(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{\xi}{4}\chi^4 + \frac{\delta}{2}\phi^2\chi^2$$

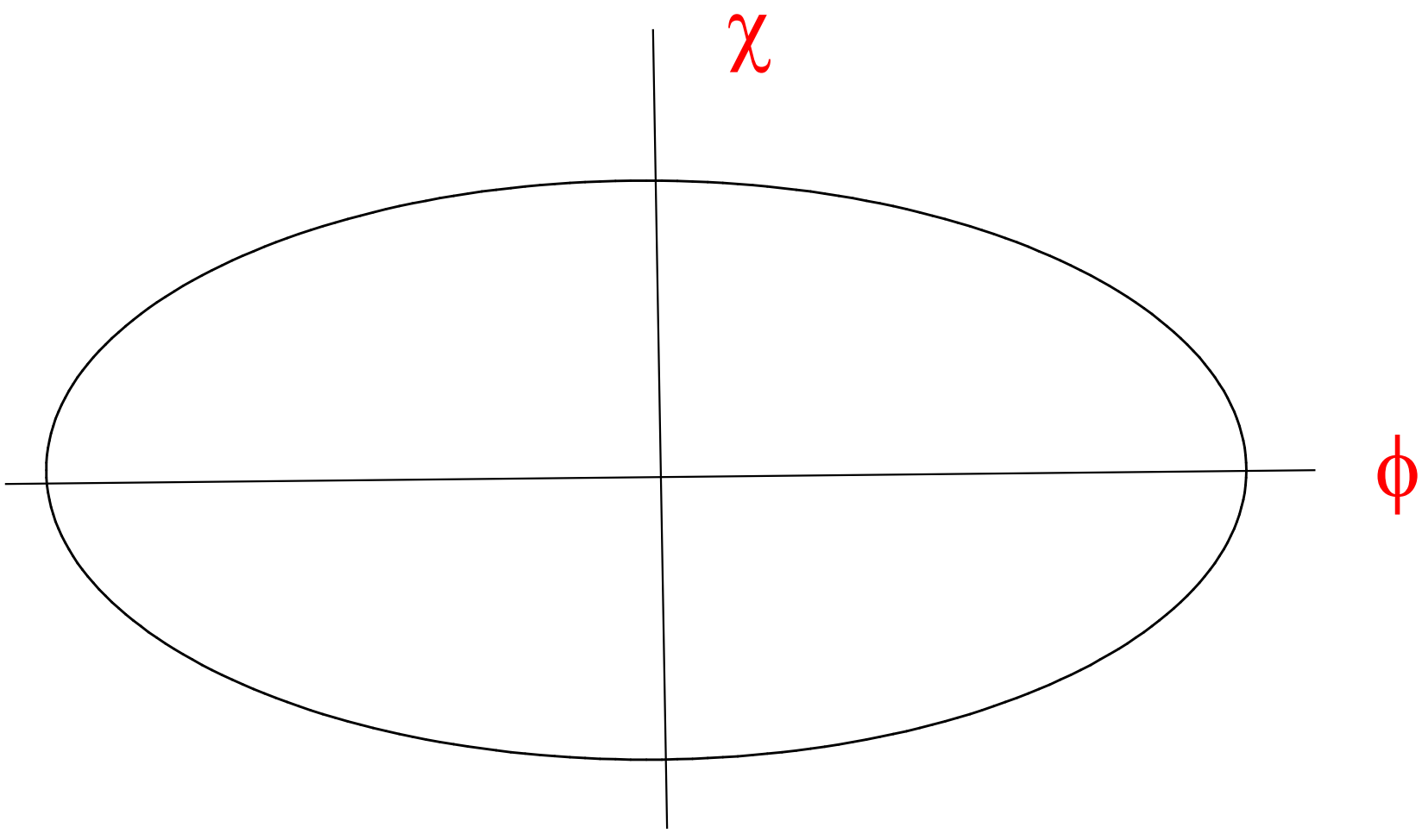
Current divergence

$$D_\mu K^\mu = 4W - \phi\frac{\delta W}{\delta\phi} - \chi\frac{\delta W}{\delta\chi} = 0$$

Kernel

$$K = \frac{1}{2}\left[(1-\alpha_1)\phi^2 + (1-\alpha_2)\chi^2\right]$$

$$K = \frac{1}{2}\left[(1-\alpha_1)\phi^2 + (1-\alpha_2)\chi^2\right] \qquad M_P^2 = -\tfrac{1}{6}\left(\alpha_1\phi^2 + \alpha_2\chi^2\right)$$

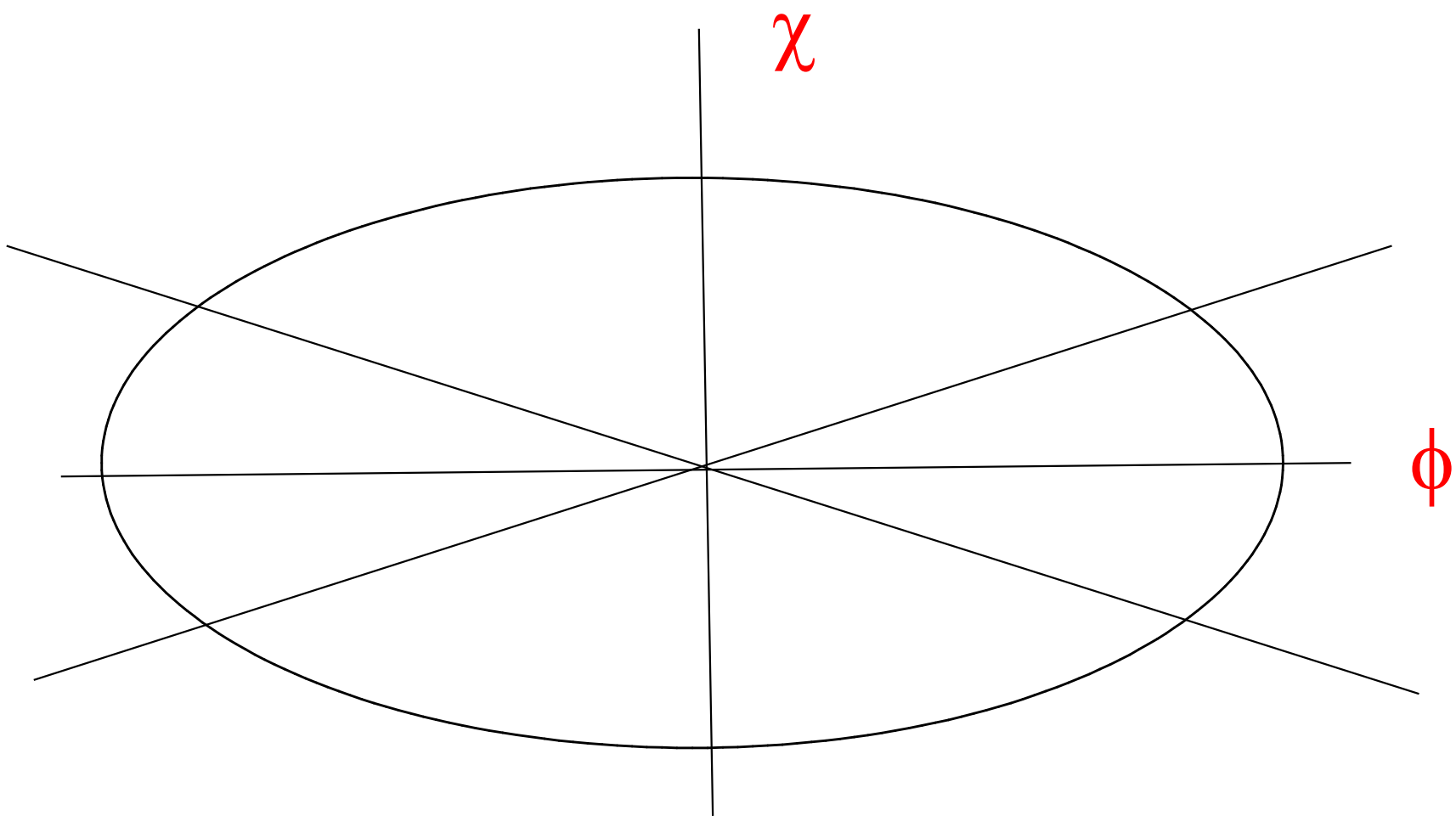


$$K = \frac{1}{2}\left[(1-\alpha_1)\phi^2 + (1-\alpha_2)\chi^2\right]$$

$$M_P^2 = -\tfrac{1}{6}\left(\alpha_1\phi^2 + \alpha_2\chi^2\right)$$

$$V(\phi,\chi) = \tfrac{\lambda}{4}\left(\chi^2 - \epsilon\phi^2\right)^2$$

Flat Directions: $\chi = \sqrt{\epsilon}\,\phi$



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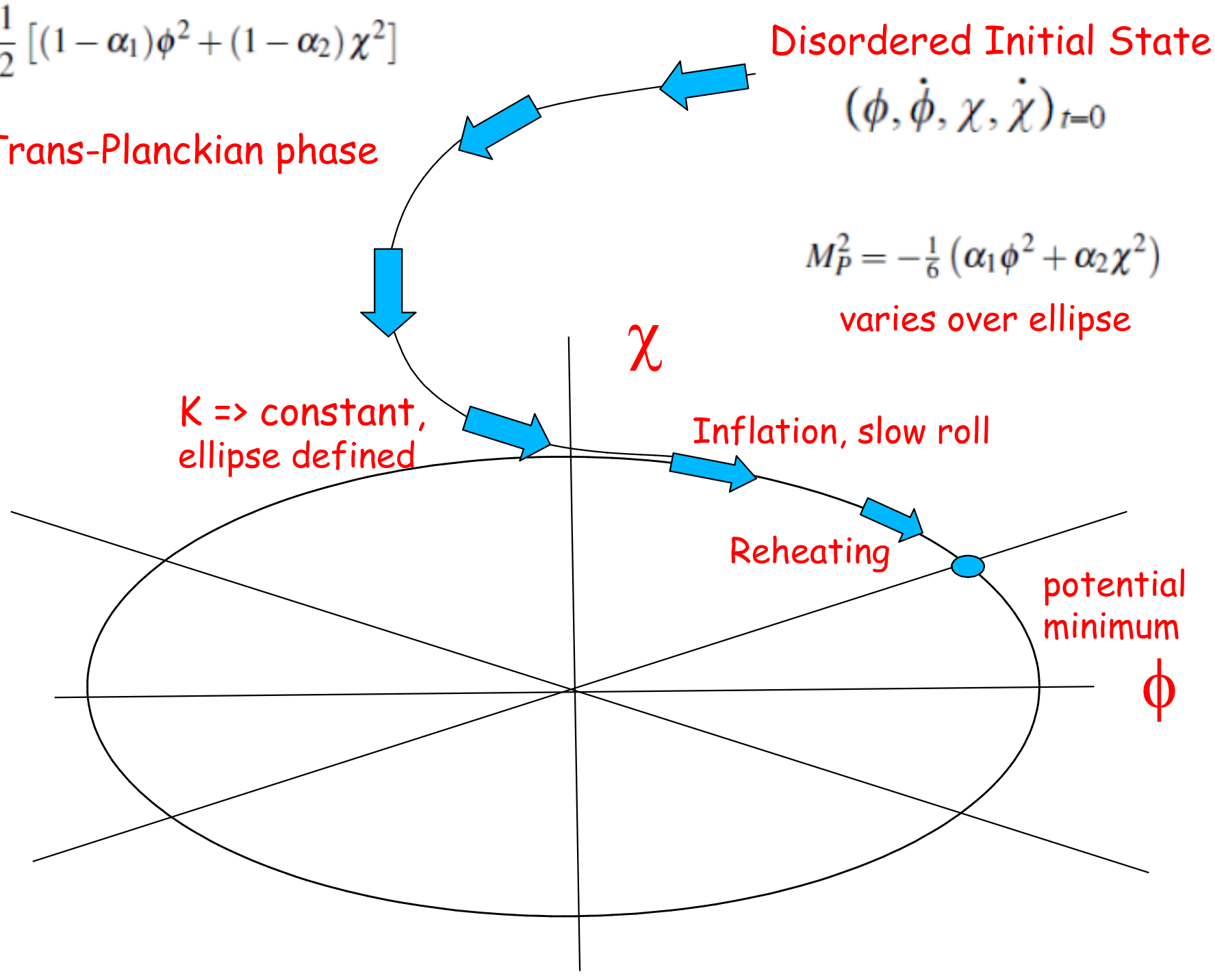
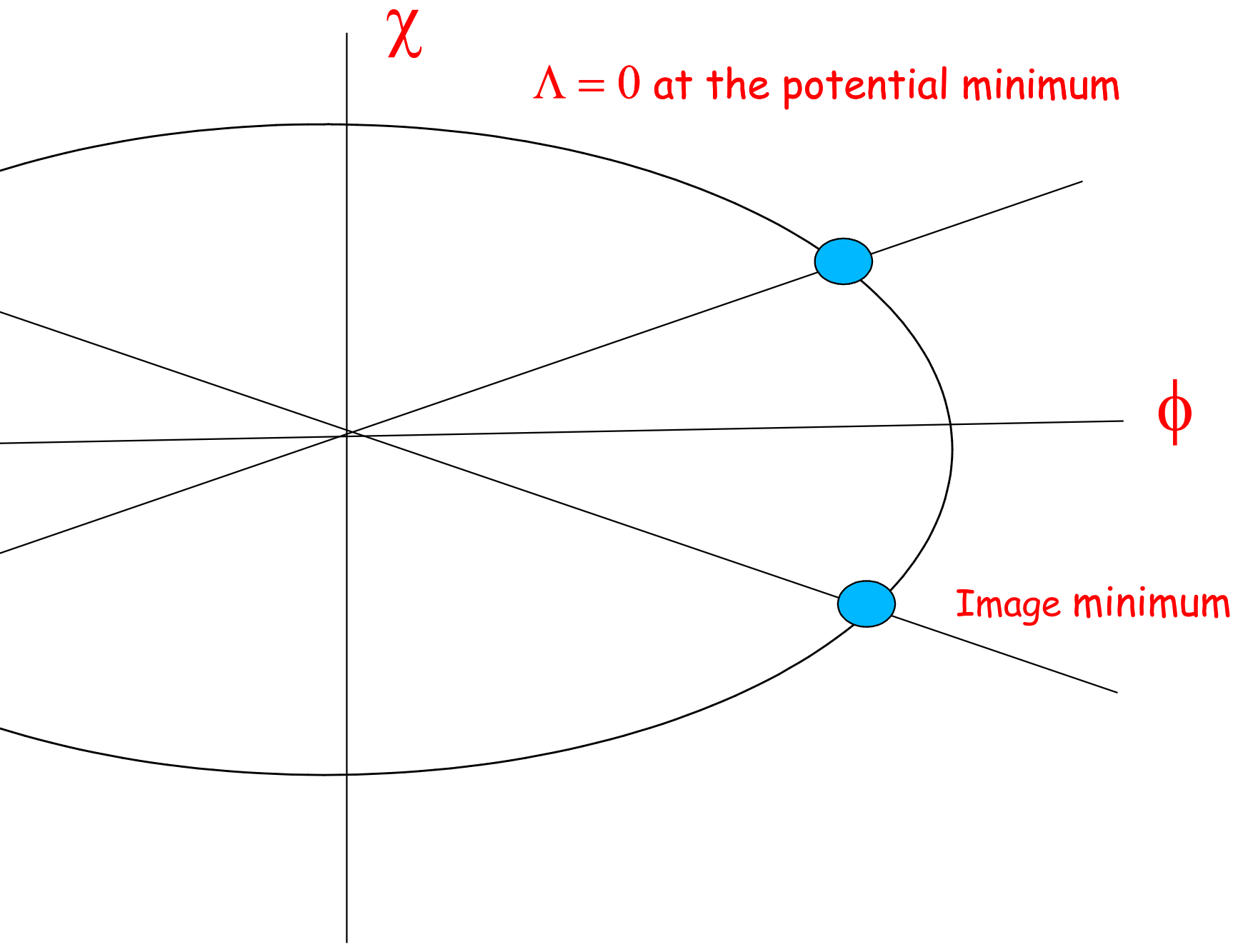
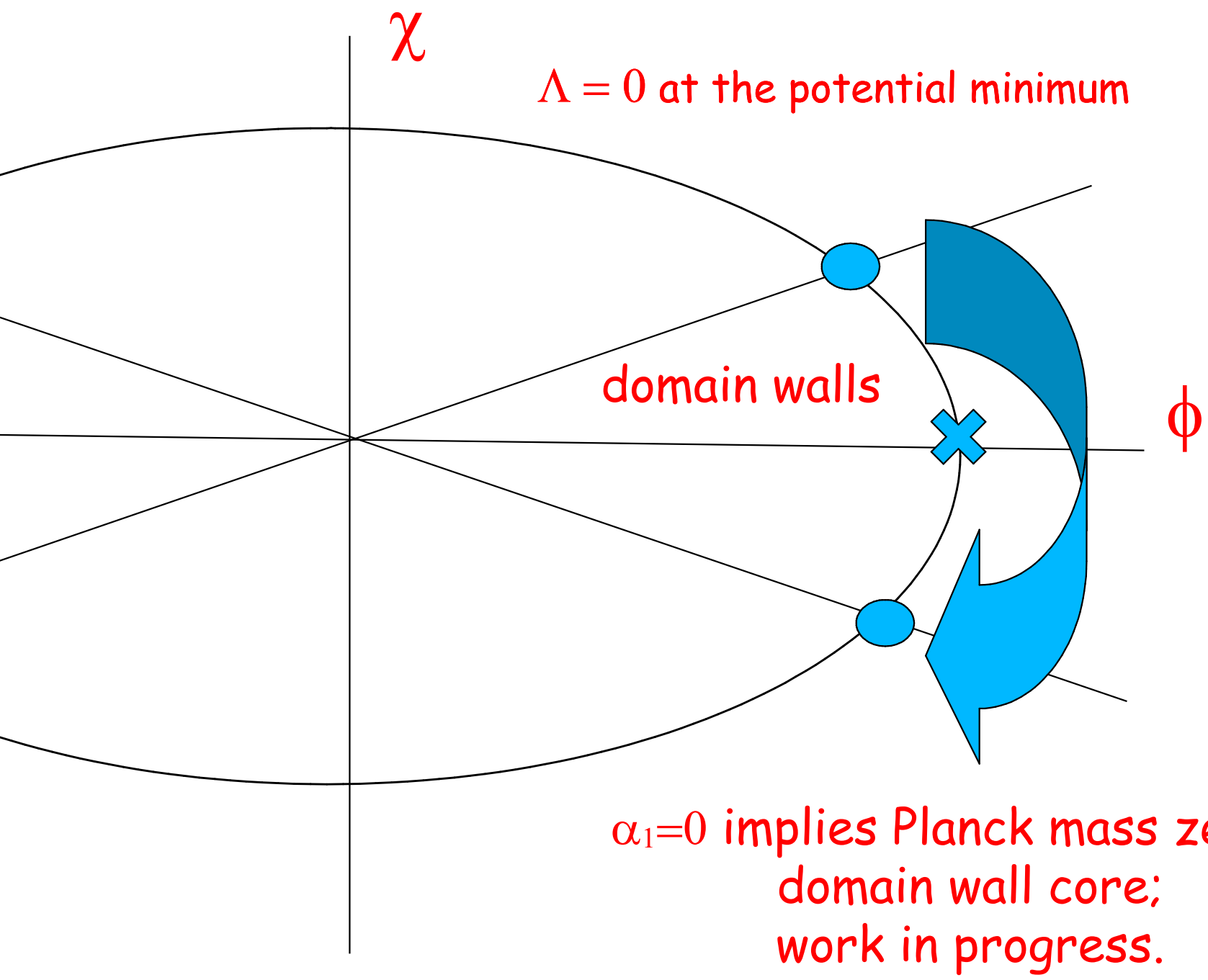



χ
$\Lambda = 0$ at the potential minimum
ϕ
Image minimum



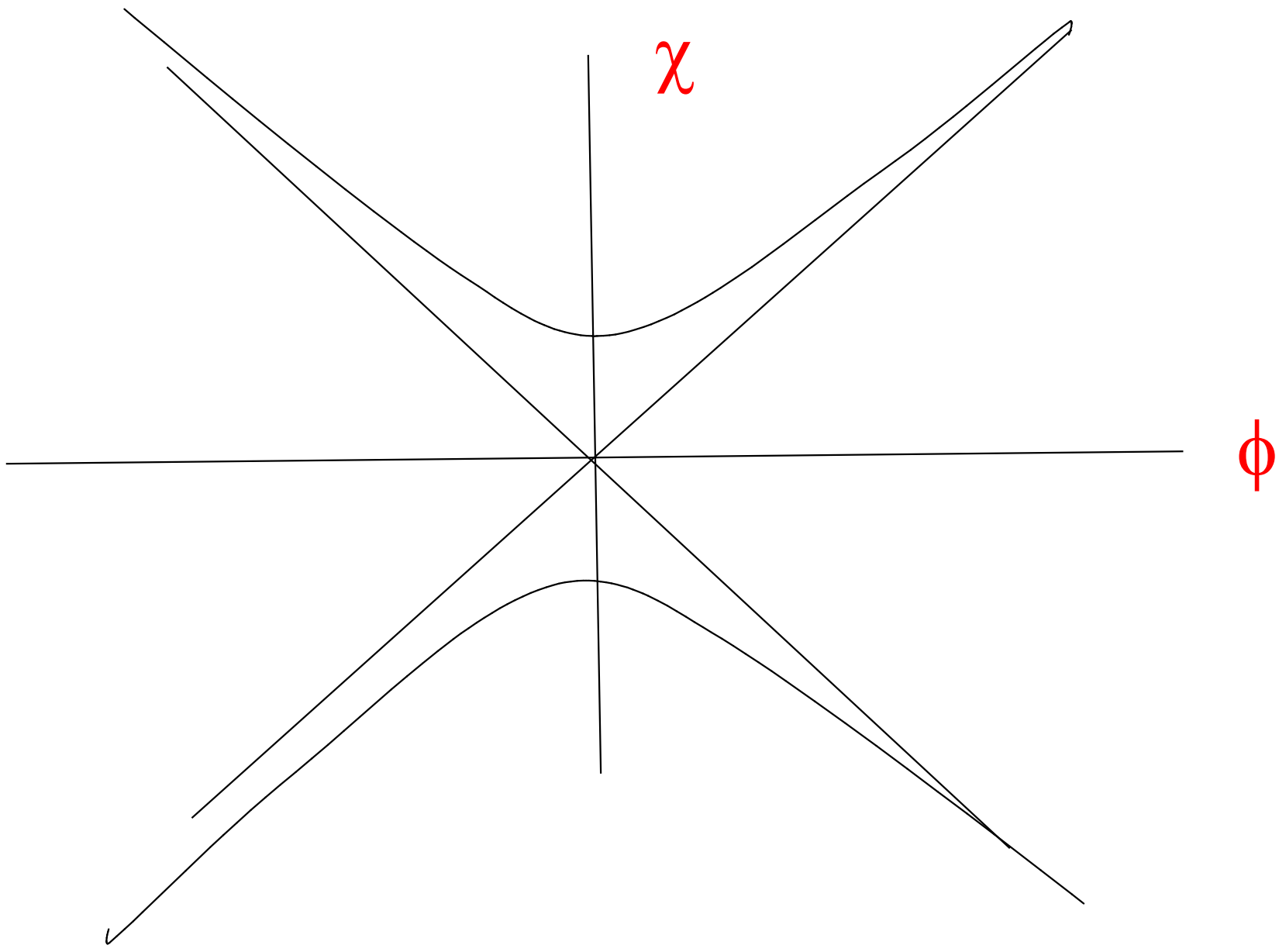
$\alpha_1=0$ implies Planck mass zero in domain wall core; work in progress.

$$K = \frac{1}{2}\left[(1-\alpha_1)\phi^2 + (1-\alpha_2)\chi^2\right]$$

Hyperbolic locus

$\alpha_1 = -1 \qquad \alpha_2 = 3$

$\Rightarrow \quad 2K = \phi^2 - \chi^2$



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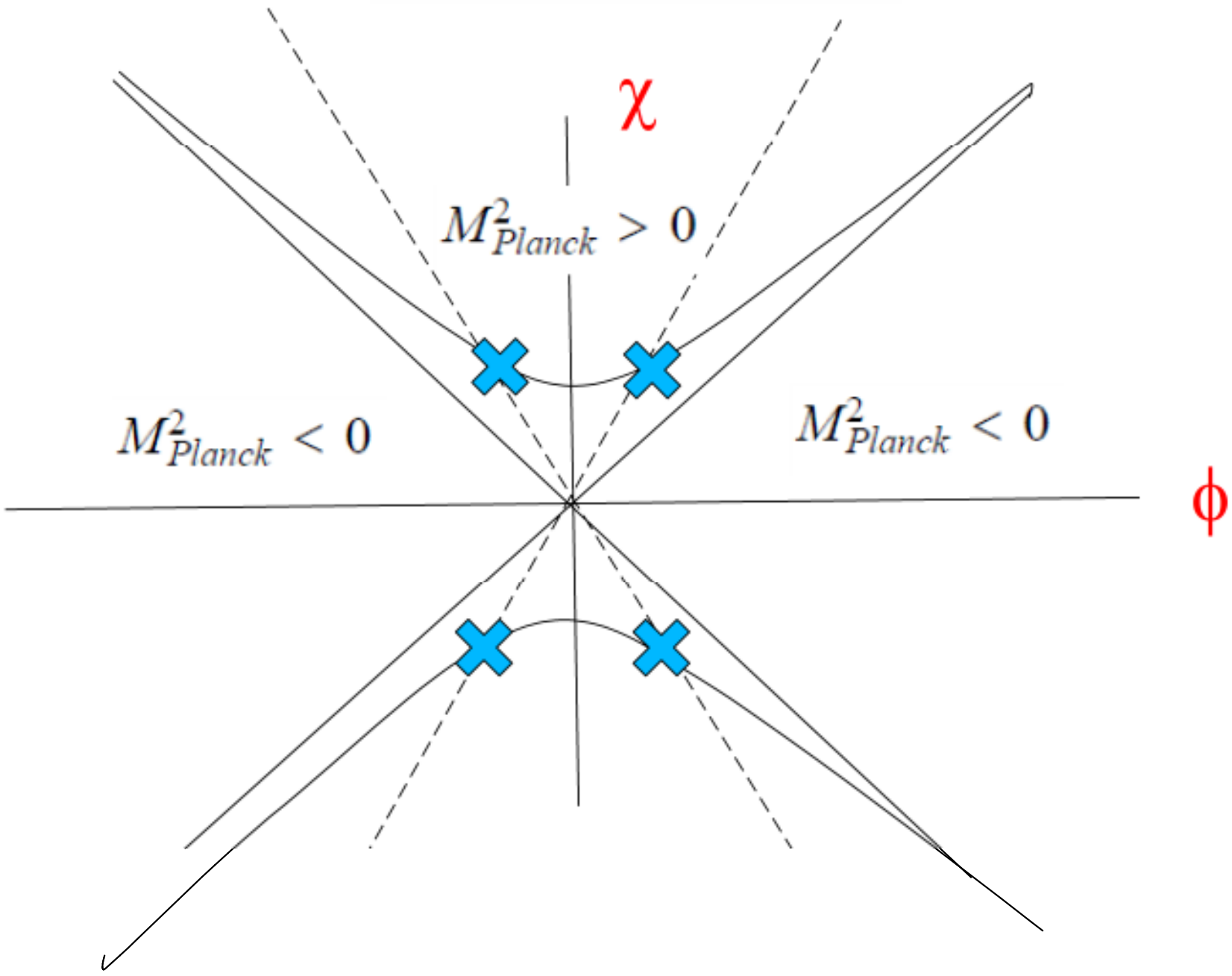
Hyperbolic locus

$$M^2_{Planck} = \tfrac{1}{6}(\phi^2 - 3\chi^2) \qquad \phi = \pm\sqrt{3}\,\chi$$



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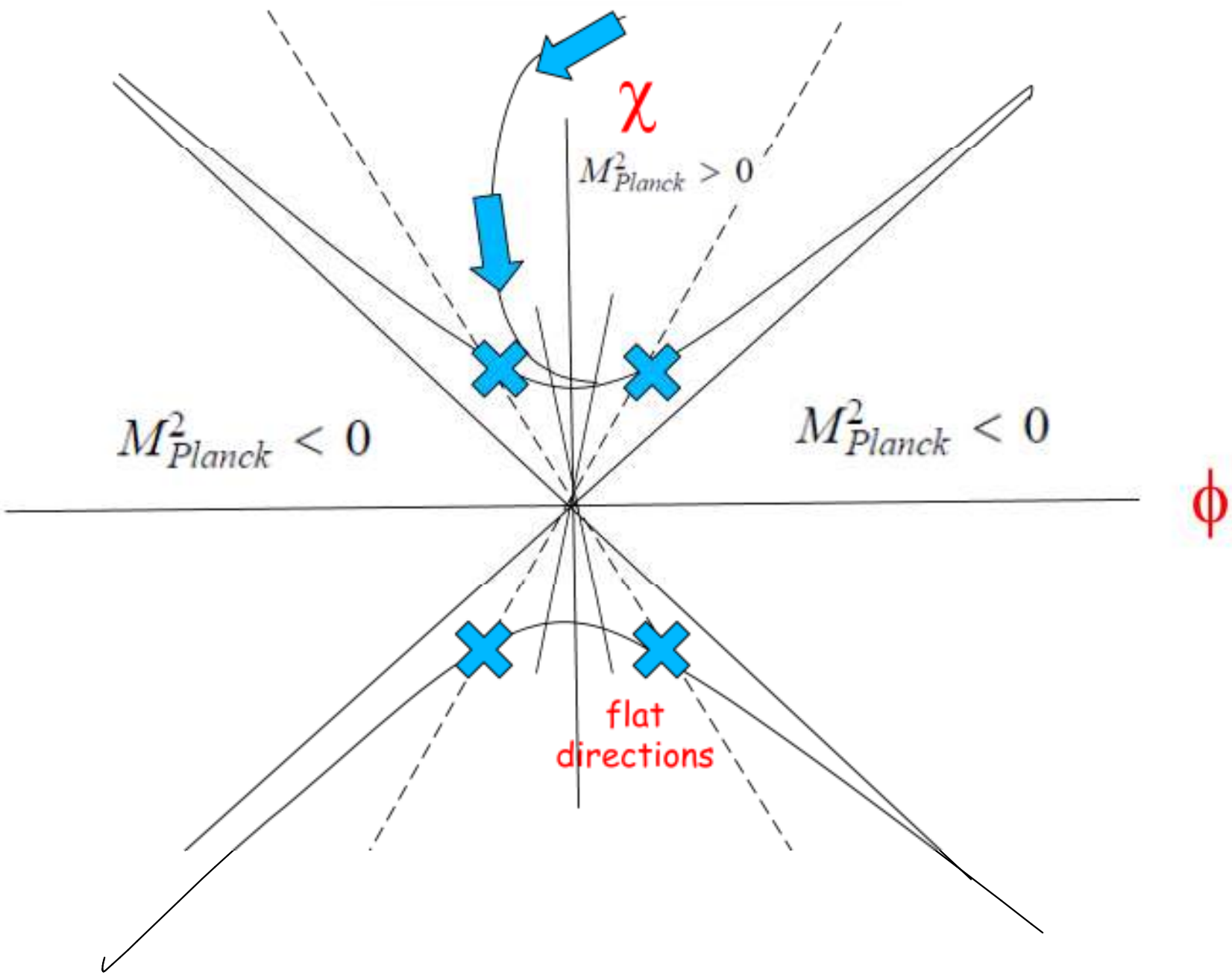
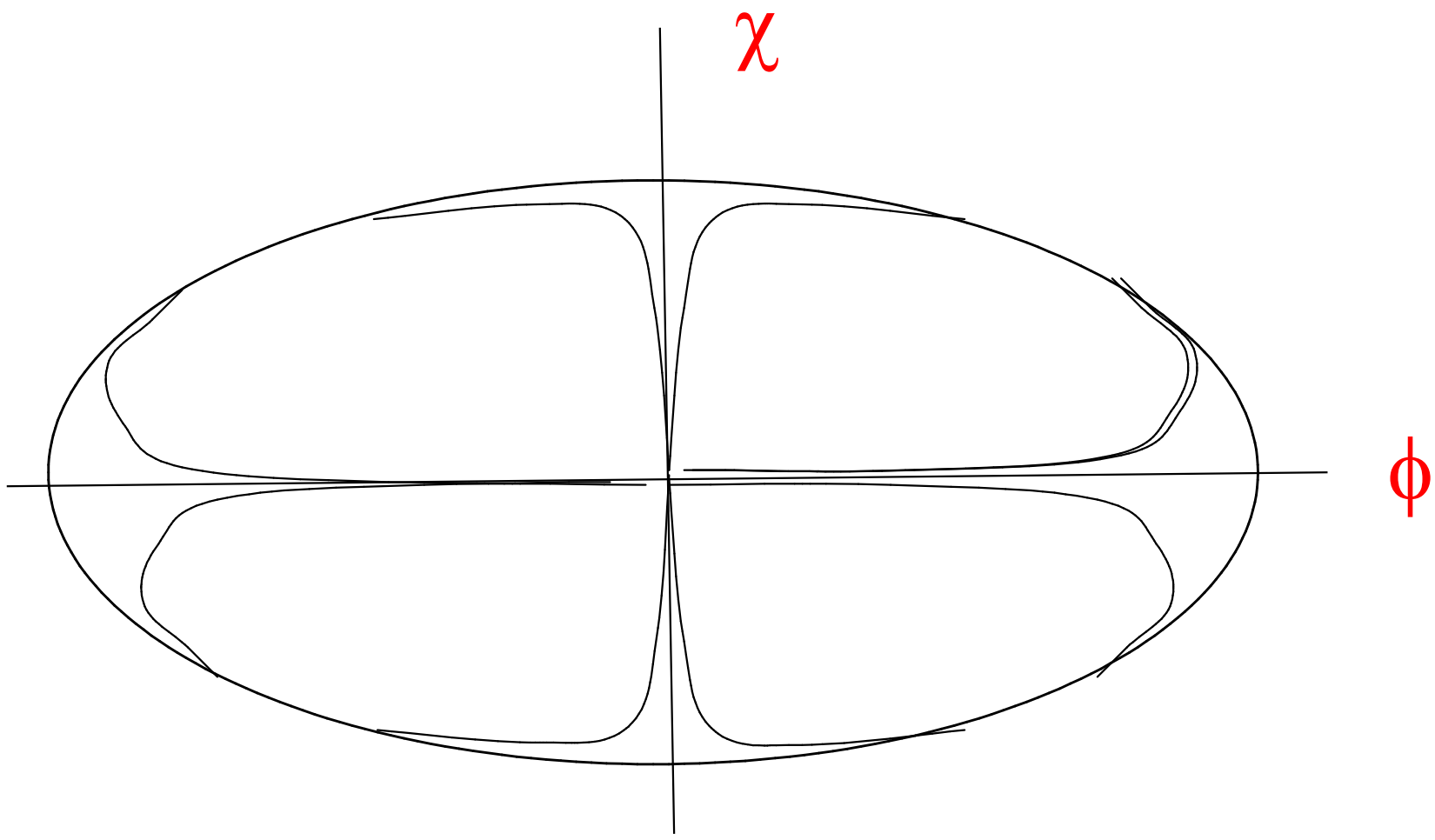


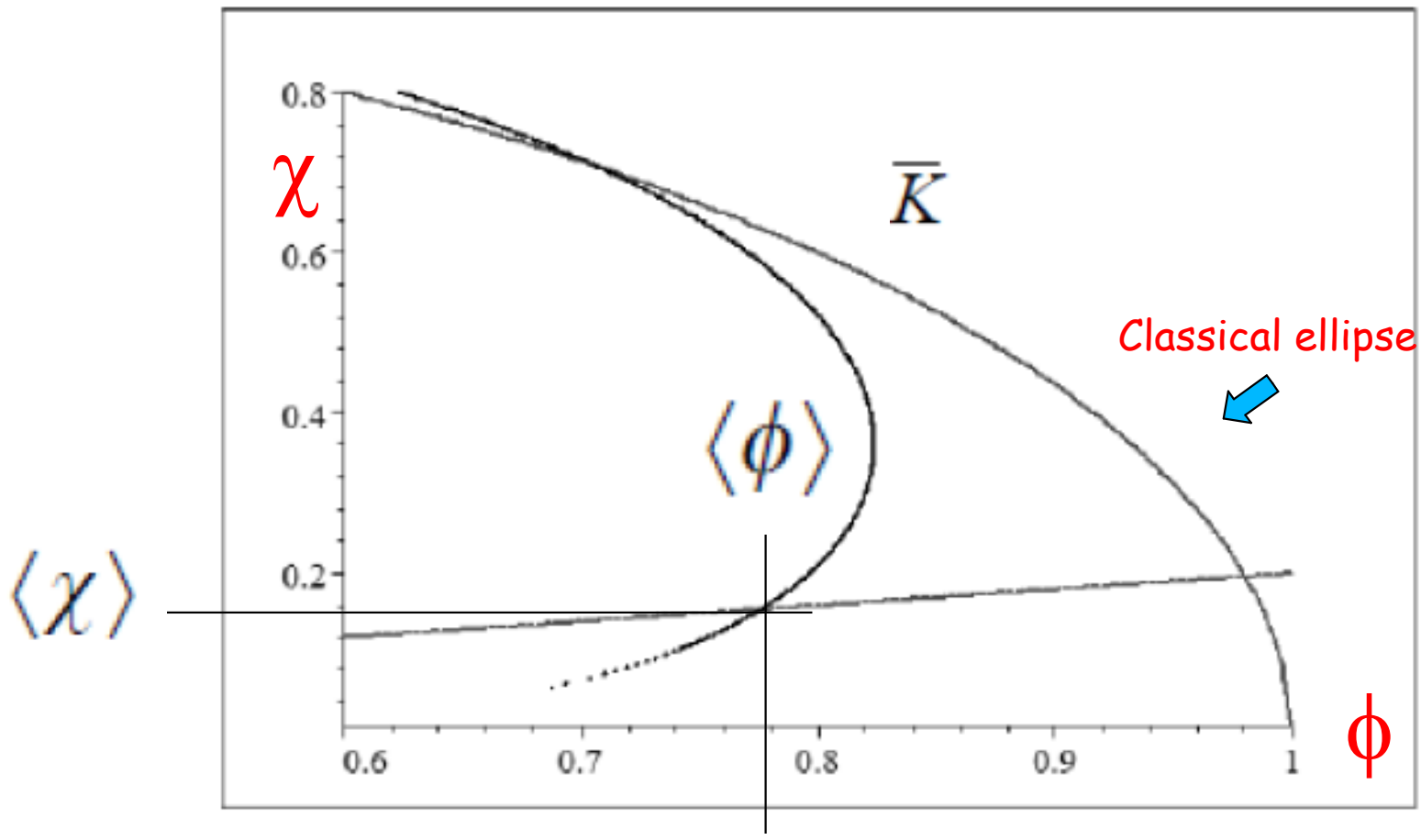
$$K = \frac{1}{2}\left[(1-\alpha_1)\phi^2 + (1-\alpha_2)\chi^2\right] \qquad M_P^2 = -\tfrac{1}{6}\left(\alpha_1\phi^2 + \alpha_2\chi^2\right)$$

RG running of α's deforms ellipse



Quantum deformation of ellipse and heirarchies

$$M_P^2 = \frac{1}{6}\left((1-\alpha_{10})\left(\frac{\phi}{\chi}\right)^{\gamma_1}\right)\phi^2 \qquad \chi\rangle << \langle\phi\rangle << \overline{K}$$



$$\phi \sim (M_P)^{2/(2+\gamma_1)}\chi^{\gamma_1} \sim (10^{19})^{2/3}(100)^{1/3} \sim 2\times 10^{13}\ \text{GeV}$$

Quick Summary of Inertial SSB

K initially is "disordered"
by local fluctuations in $\phi_i(x)$ $\partial\phi_i(x)$ etc.

Conserved Weyl Current $D_\mu K^\mu = 0,$

Current structure $K_\mu = \partial_\mu K$

General expansion $K \rightarrow \overline{K} = \frac{1}{2}\sum_i^N (1-\alpha_i)\phi_i'^2$

Dilaton decay constant $f = \sqrt{2\overline{K}}$

Inertial Breaking Does Not Involve a Potential

Maintaining
Local Weyl Invariance

Weyl Photon

Introduce the Weyl photon via the covariant derivative

$\widetilde{D}_\mu \phi = \partial_\mu \phi - qA_\mu \phi$ where q is the Weyl charge of ϕ
Under a local WT we have:

$$g_{\mu\nu}(x) \to \exp(-2\epsilon(x))g_{\mu\nu}(x) \qquad g^{\mu\nu}(x) \to \exp(2\epsilon(x))g^{\mu\nu}(x)$$
$$\sqrt{-g} \to \sqrt{-g}\exp(-4\epsilon(x))$$
$$\phi(x) \to \exp(\epsilon(x))\phi(x)$$
$$A_\mu \to A_\mu + \partial_\mu \epsilon(x)$$

It is important to realize that the metric now is nontrivial under $\widetilde{D}_\mu$
since $\widetilde{D}_\rho g_{\mu\nu}(x) = D_\rho g_{\mu\nu}(x) + 2A_\rho g_{\mu\nu}(x) = 2A_\rho g_{\mu\nu}(x)$.
The covariant metric has Weyl charge +2; contravariant is charge -2.

Weyl Photon

Consider the action

FHR, 1801.07676, 1803.06994
Ghilencea & Lee, 1809.09174
Ghilencea, 1812.08613

$$S = \int \sqrt{-g}\left(\tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\alpha\phi^2 R\right)$$

This is locally Weyl invariant since I multiplied everything by α. (that is, this is really $\alpha\left[\int \sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\phi^2 R\right)\right]$ in disguise, where $[\ldots]$ is locally invariant;

Weyl Photon

$$S = \int \sqrt{-g}\left(\tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\alpha\phi^2 R\right)$$

Do a Weyl Local Gauge Transformation:

$$\phi(x) \to \hat{\phi}(x)\exp(\sigma(x)), \quad g_{\mu\nu}(x) \to \exp(-2\sigma(x))g_{\mu\nu}(x)$$

use $R(e^{-2\sigma}g) = e^{2\sigma}R(g) + 6e^{3\sigma}D\partial e^{-\sigma}$

$$\to \int \sqrt{-g}\left(\tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi} - \tfrac{1}{12}\alpha\hat{\phi}^2 R\right)$$

Local Weyl Gauge Invariance
Noether Current -> 0

Weyl Photon

$$S = \int \sqrt{-g}\left(\tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\alpha\phi^2 R\right)$$

field redefinition:

$$\phi(x) \to f\exp(\sigma(x)/f), \qquad g_{\mu\nu}(x) \to \exp(-2\sigma(x)/f)g_{\mu\nu}(x)$$

$$\int \sqrt{-g}\left(\tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\alpha\phi^2 R\right)$$

$$\to \int \sqrt{-g}\left(\tfrac{1}{2}\alpha f^2 g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \tfrac{1}{12}\alpha f^2 e^{-2\sigma}R(e^{-2\sigma}g)\right)$$

$$\to \int \sqrt{-g}\left(\tfrac{1}{2}\alpha f^2 g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \tfrac{1}{12}\alpha f^2 R(g) - \tfrac{1}{2}\alpha f^2 e^{\sigma}D\partial e^{-\sigma}\right)$$

$$S \to \int \sqrt{-g}\left(-\tfrac{1}{12}\alpha f^2 R\right)$$

Scalar Field is Eliminated

Weyl Photon

$$S = \int \sqrt{-g}\left(\tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\alpha\phi^2 R\right)$$

KT is not canonically normalized

To fix this, AND maintain local invariance, we write:

$$S \Rightarrow \int \sqrt{-g}\left(\tfrac{1}{2}(1-\alpha)g^{\mu\nu}\widetilde{D}_\mu\phi\widetilde{D}_\nu\phi + \tfrac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{12}\alpha\phi^2 R\right)$$

$$S = \int \sqrt{-g}\left(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (1-\alpha)g^{\mu\nu}A_\mu\phi\partial_\nu\phi\right.$$

$$\left. + (1-\alpha)g^{\mu\nu}A_\mu A_\nu\phi^2 - \tfrac{1}{12}\alpha\phi^2 R\right)$$

Locally Weyl Invariant Theory

Weyl Photon

Do a Weyl Field Redefinition:

$$\phi(x) \to \phi_0 \exp(\sigma(x)/f) \qquad g_{\mu\nu}(x) \to \exp(-2\sigma(x)/f) g_{\mu\nu}(x)$$

$$S \Rightarrow \int \sqrt{-g}\,\Big(\tfrac{1}{2f^2}(1-\alpha)\phi_0^2 g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \tfrac{1}{f^2}(1-\alpha)\phi_0^2 g^{\mu\nu}A_\mu f\partial_\nu\sigma + \tfrac{1}{2}(1-\alpha)\phi_0^2 g^{\mu\nu}A_\mu A_\nu - \tfrac{1}{12}\alpha\phi_0^2 R\Big)$$

WT field redefinition cancelled the original $\frac{1}{2}\alpha g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ piece.

What is left is a perfect square:

$$S \Rightarrow \int \sqrt{-g}\,\Big(\tfrac{1}{2}(1-\alpha)\phi_0^2\, g^{\mu\nu}(A_\mu - \partial_\mu\sigma/f)(A_\nu - \partial_\nu\sigma/f) - \tfrac{1}{12}\alpha\phi_0^2 R\Big)$$

Weyl Photon

Now define Stueckelberg field $B_\mu = A_\mu - \partial_\mu \sigma / f$.

$$-\tfrac{1}{12}\alpha\phi_0^2 = \tfrac{1}{2}M_{Planck}^2 \quad \text{and} \quad \overline{K} = \tfrac{1}{2}(1-\alpha)\phi_0^2 = \tfrac{1}{2}\phi_0^2 + 6M_{Planck}^2$$

$$S \to \int \sqrt{-g}\left(\overline{K} g^{\mu\nu} B_\mu B_\nu + \tfrac{1}{2}M_{Planck}^2 R\right)$$

Include a gauge invariant kinetic term for B_μ with

$$F_{\mu\nu} = -\tfrac{1}{q}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$S \to \int \sqrt{-g}\left(-\tfrac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \overline{K} g^{\mu\nu} B_\mu B_\nu + \tfrac{1}{2}M_{Planck}^2 R\right)$$

Weyl Photon

Canonically normalize

$$S \to \int \sqrt{-g}\left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 g^{\mu\nu}B_\mu B_\nu + \tfrac{1}{2}M^2_{Planck}R\right)$$

$$m^2 = 2g^2\overline{K}$$

This is a massive spin one field of mass $m = g\sqrt{2K}$.

Weyl Photon in multiscalar theory

$$S = \int \sqrt{-g} \left(\sum_i \alpha_i \tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - \sum \alpha_i \tfrac{1}{12} \phi_i^2 R - V(\phi_i) \right)$$

locally invariant under the Weyl Transformation:

$$\phi(x) \to \hat{\phi}_i \exp(\sigma(x)) \qquad g_{\mu\nu}(x) \to \exp(-2\sigma(x)) g_{\mu\nu}(x)$$

$$\Rightarrow \sum \int \sqrt{-g} \left(+\tfrac{1}{2} \alpha_i g^{\mu\nu} \partial_\mu \hat{\phi}_i \partial_\nu \hat{\phi}_i - \tfrac{1}{12} \alpha_i \hat{\phi}_i^2 R \right)$$

Local Weyl Gauge Invariance
for N scalar Fields !!!

Weyl Photon in multiscalar theory

Again the KT's aren't canonically normalized, so we add

$$\int \sqrt{-g}\left(\sum(1-\alpha_i)\tfrac{1}{2}g^{\mu\nu}\widetilde{D}_\mu\phi_i\widetilde{D}_\nu\phi_i+\sum\alpha_i\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i-\sum\alpha_i\tfrac{1}{12}\phi_i^2R\right)$$

$$=\int\sqrt{-g}\,(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i-\sum(1-\alpha_i)g^{\mu\nu}A_\mu(\phi_i\partial_\nu\phi_i)$$
$$+\tfrac{1}{2}\sum(1-\alpha_i)g^{\mu\nu}A_\mu A_\nu(\phi_i\phi_i)-\sum\alpha_i\tfrac{1}{12}\phi_i^2R$$

Do a Weyl Field Redefinition:

$$\phi(x)\to\hat{\phi}_i\exp(\sigma(x)/f),\qquad g_{\mu\nu}(x)\to\exp(-2\sigma(x)/f)g_{\mu\nu}(x)$$

Impose a constraint $\quad \overline{K}=\frac{1}{2}\sum^{N}(1-\alpha_i)\phi_i'^2$

Weyl Photon in multiscalar theory

$$S => \int \sqrt{-g}\,(\sum(1-\alpha_i)\tfrac{1}{2}g^{\mu\nu}\widetilde{D}_\mu\hat{\phi}_i\widetilde{D}_\nu\hat{\phi}_i + \sum\alpha_i\tfrac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \sum_i \tfrac{1}{12}\alpha_i\hat{\phi}_i^2 R)$$

We see upon expanding:

$$S => \int \sqrt{-g}\,(\sum\tfrac{1}{2}(1-\alpha_i)g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i + \sum\tfrac{1}{2}\alpha_i g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \sum_i \tfrac{1}{12}\alpha_i\hat{\phi}_i^2 R$$
$$-\sum(1-\alpha_i)(\hat{\phi}_i\partial_\mu\hat{\phi}_i)B^\mu + \tfrac{1}{2}\sum(1-\alpha_i)\left(\hat{\phi}_i^2\right)(B_\mu^2))$$

and the term $\sum(1-\alpha_i)(\hat{\phi}_i\partial_\mu\hat{\phi}_i)B^\nu = B^\mu\partial_\mu\overline{K} = 0.$

$$\tfrac{1}{2}\sum(1-\alpha_i)\left(\hat{\phi}_i^2\right)(B_\mu^2) = \overline{K}(B_\mu^2)$$

Weyl Photon in multiscalar theory

Include a gauge invariant kinetic term for B_μ

$$S \to \int \sqrt{-g}\,(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 B_\mu^2$$

$$+\sum \tfrac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \sum \tfrac{1}{12}\alpha_i\hat{\phi}_i^2 R - V(\hat{\phi}_i))$$

$$m^2 = 2g^2\overline{K}$$

Weyl Photon in multiscalar theory

Dilaton is eaten; Weyl Photon acquires mass

$$m^2 = 2g^2\overline{K}$$

The $\hat{\phi}_i$ are decoupled from the B^μ !!!

g+g -> B + B Pair Produced by Gravitons

B^μ is stable.

Dark matter? Note $m^2 >> 12g^2 M^2_{Planck}$.

For all of this to work
Weyl symmetry must be exact:

$$D_\mu K^\mu = 0$$

(Failure is not an option)

Does this conflict with Quantum Mechanics?

The Weyl current (trace) anomaly underlies Coleman-Weinberg Symmetry Breaking

Coleman & Weinberg PR D7 (1973) 1888

Coleman Weinberg Symmetry Breaking

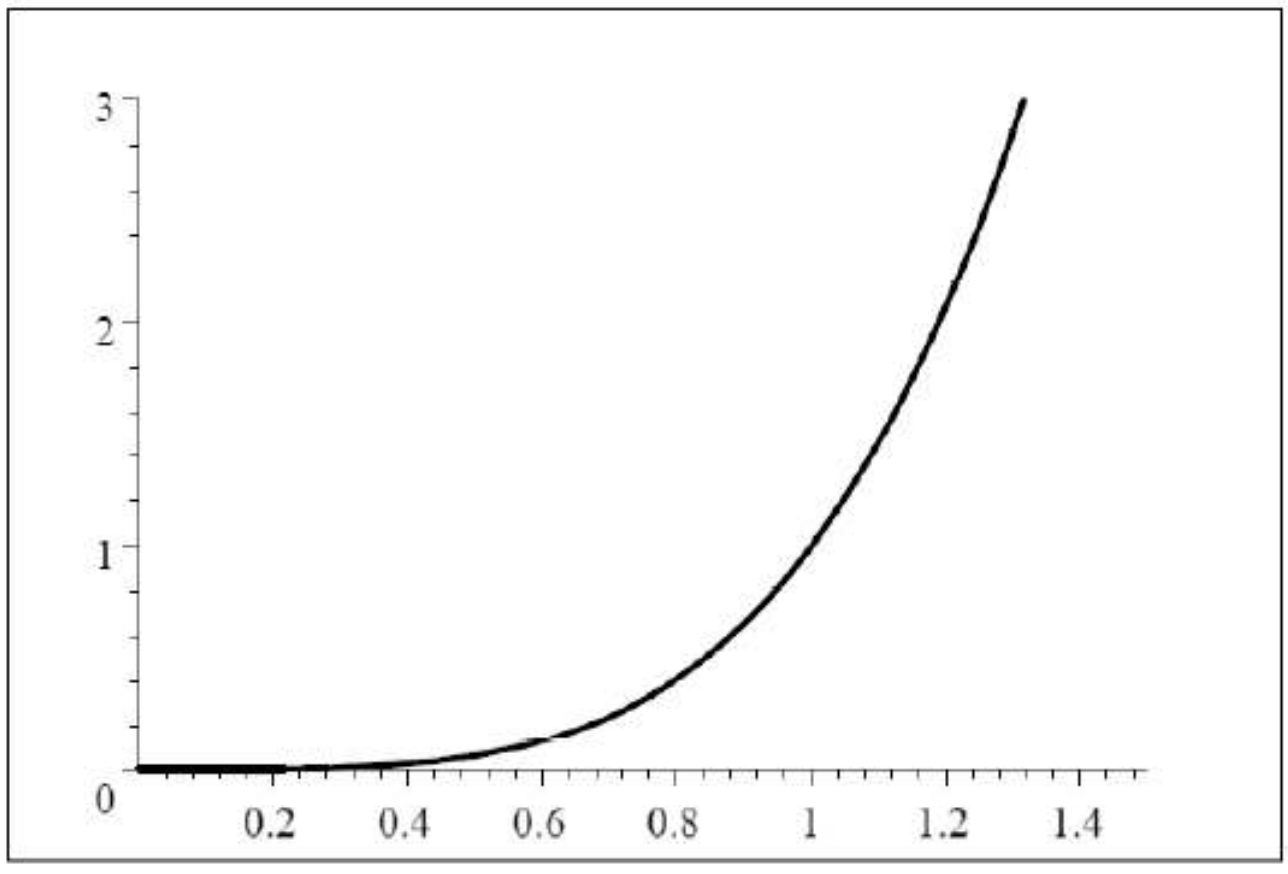


FIG. 1: Classical $\sim \lambda v^4$ potential.

Coleman Weinberg Symmetry Breaking

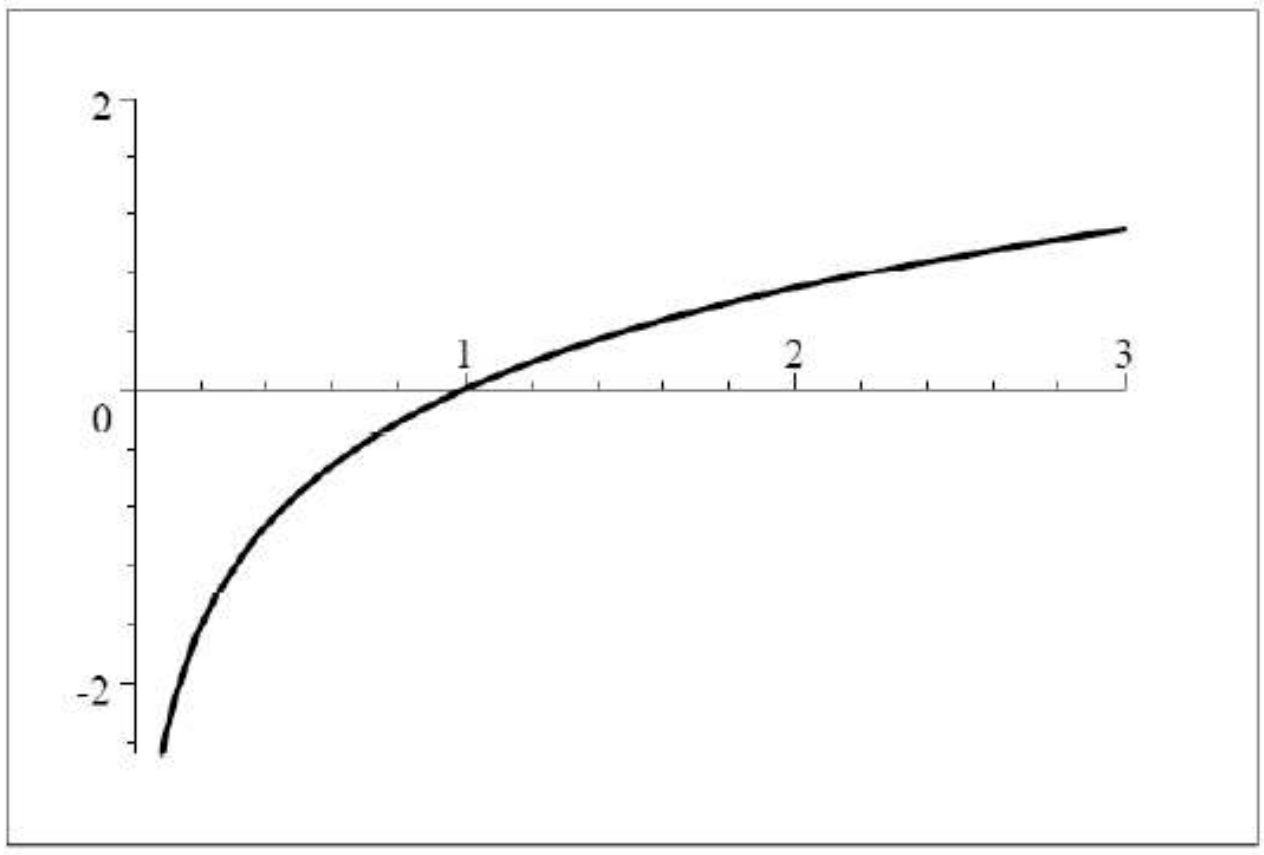


FIG. 2: Typical RG trajectory $\lambda \sim \beta \ln(v/M)$

Coleman Weinberg Symmetry Breaking

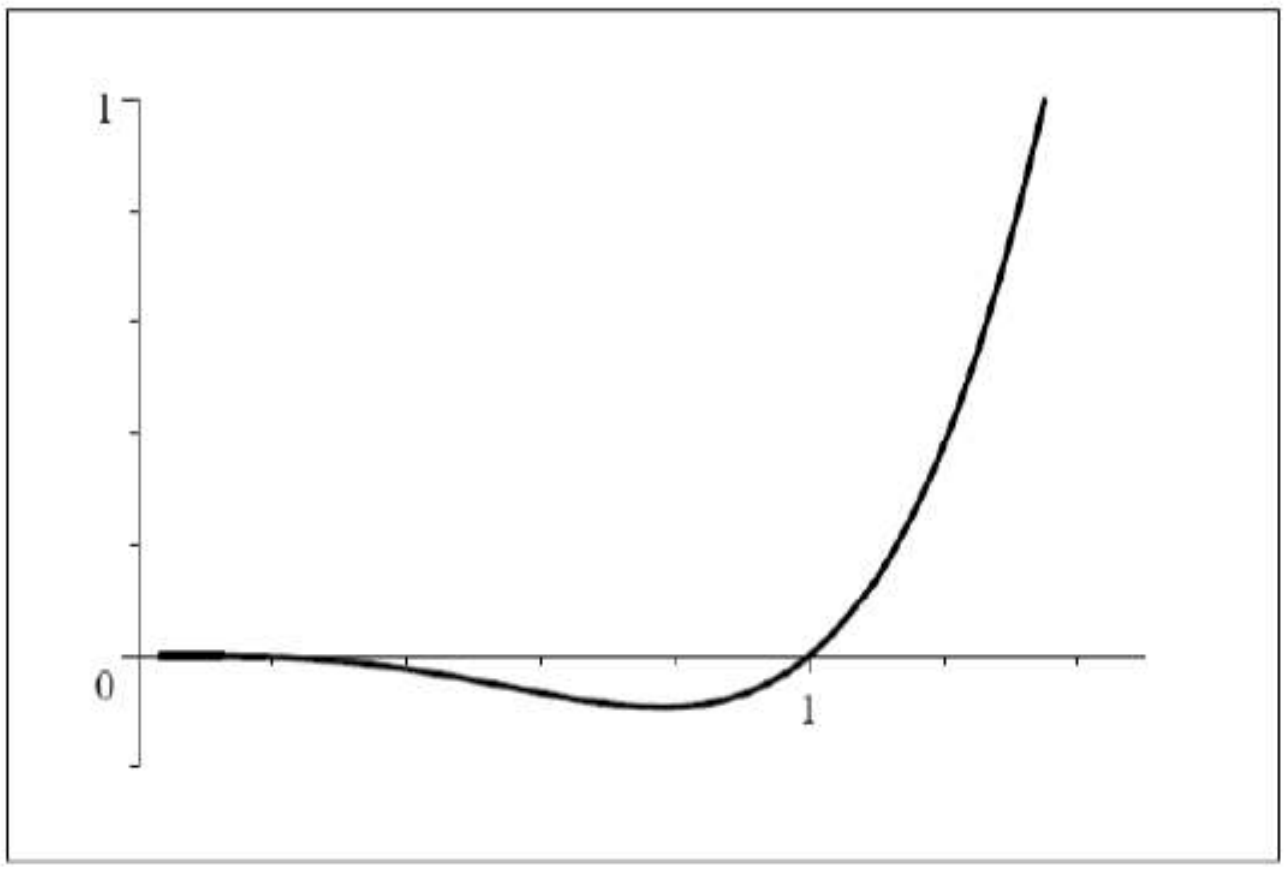


FIG. 3: Resulting CW potential, $\sim \beta v^4 \ln(v/M)$

Conventional Trace Anmaly

$$D^{\mu}K_{\mu} = 4V(\phi) - \phi\frac{\partial}{\partial\phi}V(\phi)$$

Consider $V(\phi) = \frac{\lambda}{4}\phi^4$ $\qquad \frac{d\lambda(\mu)}{d\ln\mu} = \beta(\lambda)$

$$D^{\mu}K_{\mu} = 4V(\phi) - \phi\frac{\partial}{\partial\phi}V(\phi) = \frac{\beta}{\lambda}V(\phi)$$

Renormalization Group Derivation of the Coleman-Weinberg Potential:

$$\phi \frac{\delta}{\delta \phi} V(\phi) - 4V(\phi) = \frac{\beta}{\lambda} V(\phi)$$

This can be viewed as a defining equation for the CW potential !!!

$$V(\phi) = \frac{1}{4} \lambda_r \phi_c^4$$

$$\frac{d\lambda_r}{d\ln(\mu)} = \frac{9\lambda^2}{8\pi^2}$$

$$V(\phi) = \frac{9\lambda^2 \phi_c^4}{64\pi^2} \ln\left(\frac{\phi_c^2}{M^2}\right)$$

Leading log CW Potential

External mass M enters from Renormalization

Coleman & Weinberg PR D7 (1973) pg. 1892:

. That is to say, we define the renormalized coupling constant by

M is external to the action

$$\left.\frac{d^4V}{d\varphi_c{}^4}\right|_M = \lambda \ , \tag{3.7}$$

where M is some number with the dimensions of a mass. We emphasize that M is completely arbitrary, just as is the corresponding quantity in the momentum-space analysis; different choices for M will lead to different definitions of the coupling constant, different parametrizations of the theory, but any nonzero M is as good as any other. Al-

Injection of external renormalization scale leads to Scale Anomaly:

Coleman-Weinberg Potential (one loop):

$\lambda\phi^4/4$ theory. $$V(\phi) = \frac{\beta_\lambda}{4}\phi^4 \ln(\phi/M)$$

$$D^\mu K_\mu = 4V(\phi) - \phi\frac{\partial}{\partial\phi}V(\phi)$$

$$\Rightarrow \quad D^\mu K_\mu = -\frac{1}{4}\beta_\lambda \phi^4 .$$

Scale Anomaly due to Weyl noninvariant M

Weyl Invariant Renormalization: Replace M by a Weyl covariant element of the action.

Coleman & Weinberg PR D7 (1973) pg. 1892:

That is to say, we define the renormalized coupling constant by

$$\left.\frac{d^4V}{d\varphi_c{}^4}\right|_{\chi} = \lambda \,, \tag{3.7}$$

where χ is some VEV with the dimensions of a mass. We emphasize that χ is completely arbitrary, just as is the corresponding quantity in the momentum-space analysis; different choices for χ will lead to different definitions of the coupling constant, different parametrizations of the theory, but any nonzero χ is as good as any other. Al-

$M \Rightarrow \chi$

Two Scalar Fields

$$D^\mu K_\mu = 4V - \phi \frac{\partial}{\partial \phi} V - \chi \frac{\partial}{\partial \chi} V$$

Weyl Invariant Renormalization: $V(\phi, \chi) = \frac{\lambda}{4}\phi^4$

$$V(\phi, \chi) = \frac{\beta_\lambda}{4}\phi^4 \ln\left(\frac{\phi}{M}\right) \quad \Rightarrow \quad V(\phi, \chi) = \frac{\beta_\lambda}{4}\phi^4 \ln\left(\frac{\phi}{\chi}\right)$$

Ratio of scalars in the log in effective action does not modify K current.

Divergence of the K-current $\quad D^\mu K_\mu = 0 \quad$ No trace anomaly.

Second field can be gravity itself

Weyl Invariant Renormalization: $V(\phi) = \frac{\lambda}{4}\phi^4$

$$V(\phi) = \frac{\beta_\lambda}{4}\phi^4 \ln\left(\frac{\phi}{M}\right) \quad \Rightarrow \quad V(\phi, R) = \frac{\beta_\lambda}{8}\phi^4 \ln\left(\frac{\phi^2}{R}\right)$$

R in the log significantly modifies the current.
Messy!!!
Rather, use a local Weyl invariant:

$$V(\phi, R) = \frac{\beta_\lambda}{8}\phi^4 \ln\left(\frac{\phi^2}{R + 6\phi^{-1} g^{\mu\nu} D_\mu(\partial_\nu \phi)}\right)$$

K-current and K are unmodified.

Divergence $D^\mu K_\mu = 0$ No trace anomaly.

Beta-function governs the potential; No trace anomaly if renormalization is Weyl Invariant

(1) Yes we have renormalization group!
"running of couplings" in Weyl invariant ratios

(2) But, there is No Trace Anomaly!

(3) Current and K are unmodified if arguments of logs (subtractions) are locally invariant forms.

(4) Analogous to Wess-Zumino-Witten term to cancel chiral anomalies. A deeper origin?

Speculation: A Ghostly Connection?

This construction is inspired by the way in which R^2 and W^2 make gravity renormalizeable by introducing k^4 terms.

see also:

Ghost condensation and a consistent infrared modification of gravity
Nima Arkani-Hamed, Hsin-Chia Cheng, Markus A. Luty, Shinji Mukohyama
JHEP 0405 (2004) 074 e-Print: hep-th/0312099

Speculation: A Ghostly Connection?

$$L_0 = \frac{1}{2}\partial\phi\partial\phi - \frac{1}{4}\lambda\phi^4 + \frac{1}{2}\partial\chi\partial\chi - \ldots \text{ (other potential terms)}$$

$$- \frac{1}{12}(\alpha_1\phi^2 + \alpha_2\chi^2)R$$

Couple χ to an auxilliary field Θ :

$$L_{auxilliary} = \partial\Theta\partial\phi - \frac{1}{2}\Theta^2\chi^2$$

Modified K current

$$K_\mu = \frac{1}{2}\partial_\mu(1-\alpha_1)\phi^2 + \frac{1}{2}\partial_\mu(1-\alpha_2)\chi^2 + \partial_\mu(\Theta\phi)$$

$$K = \frac{1}{2}(1-\alpha_1)\phi^2 + \frac{1}{2}(1-\alpha_2)\chi^2 + \Theta\phi$$

Speculation: A Ghostly Connection?

Auxilliary field equation of motion:

$$\partial^2\phi + \Theta\chi^2 = 0 \qquad \rightarrow \qquad \Theta = \frac{-\partial^2\phi}{\chi^2}$$

$$K => \frac{1}{2}(1-\alpha_1)\phi^2 + \frac{1}{2}(1-\alpha_2)\chi^2 - \frac{1}{2}\frac{\phi\partial^2\phi}{\chi^2}$$

So, K => (constant) but we also have $\partial^2\phi$ => 0 by redshifting so it appears we still get the ellipse, but there could be some weird pathologies where χ^2 => 0.

Eliminate auxilliary field: $L_{auxilliary} = \partial\Theta\partial\phi - \frac{1}{2}\Theta^2\chi^2$ $\qquad \Theta = \frac{-\partial^2\phi}{\chi^2}$

Integate by parts:

$$\int L_{auxilliary} \rightarrow \int \frac{\partial^2\phi\partial^2\phi}{\chi^2} - \frac{1}{2}\left(\frac{-\partial^2\phi}{\chi^2}\right)^2\chi^2 = \int \frac{\partial^2\phi\partial^2\phi}{2\chi^2}$$

Speculation: A Ghostly Connection?

Full action:

$$\int L_{auxilliary} + L_0 = \int \tfrac{1}{2}\partial\phi\partial\phi - \tfrac{1}{4}\lambda\phi^4 + \tfrac{1}{2}\partial\chi\partial\chi + \frac{\partial^2\phi\partial^2\phi}{2\chi^2} - \tfrac{1}{4}\lambda\phi^4 + \ldots$$

Coleman-Weinberg: add sources for ϕ, χ,
shift, $\phi \to \phi_c + \hat{\phi}$ $\quad \chi \to \chi_c + \hat{\chi}$
sources cancel tadpoles,

leading order in $\hbar$ expansion:

$$L_0 = \tfrac{1}{2}\partial\phi\partial\phi + \frac{\partial^2\phi\partial^2\phi}{2\chi^2} - \tfrac{1}{4}\lambda\phi^4 \to \tfrac{1}{2}\partial\hat{\phi}\partial\hat{\phi} + \frac{\partial^2\hat{\phi}\partial^2\hat{\phi}}{2\chi_c^2} - \tfrac{1}{4}\lambda\phi_c^4 - \tfrac{3}{2}\lambda\phi_c^2\hat{\phi}^2$$

Speculation: A Ghostly Connection?

$\tilde{\phi}$ propagator: $-i\left(k^2 + \frac{k^2k^2}{\chi_c^2} - 3\lambda\phi_c^2\right)^{-1} = \frac{-i}{k^2-m^2} - \frac{-i}{k^2-\tilde{M}^2}$

poles: $k^2k^2 - (M^2 + m^2)k^2 + M^2m^2 = 0$

assume $M^2 >> m^2$ $\quad \chi_c^2 >> \lambda\phi_c^2$

$M^2 \approx -\chi_c^2 + O\left(\frac{1}{\chi_c^2}\right)$ $\qquad m^2 \approx 3\lambda\phi_c^2 + O\left(\frac{1}{\chi_c^2}\right)$

Compute CW potential: $V = \left(3\lambda\phi_c^2\right)^2 \int \frac{d^4k}{(2\pi)^4}\left(\frac{-i}{k^2-m^2} - \frac{-i}{k^2+M^2}\right)^2$

The resulting integral is finite.

$$V = \frac{1}{8\pi^2}\left(3\lambda\phi_c^2\right)^2\left(\ln\left(\frac{3\lambda\phi_c^2}{\chi_c^2}\right) - 1\right)$$

Speculation: A Ghostly Connection?
A Finite, Weyl Invariant CW Potential

$$V = \frac{1}{8\pi^2}\left(3\lambda\phi_c^2\right)^2\left(\ln\left(\frac{3\lambda\phi_c^2}{\chi_c^2}\right) - 2\right)$$

Inertial ghost condensation stability?

$$K_\mu = \tfrac{1}{2}\partial_\mu(1-\alpha_1)\phi^2 + \tfrac{1}{2}\partial_\mu(1-\alpha_2)\chi^2 + \partial_\mu(\Theta\phi)$$

$$K = \tfrac{1}{2}(1-\alpha_1)\phi^2 + \tfrac{1}{2}(1-\alpha_2)\chi^2 + \Theta\phi$$

$$\partial^2\phi + \Theta\chi^2 = 0 \qquad \rightarrow \qquad \Theta = \frac{-\partial^2\phi}{\chi^2}$$

$$K => \tfrac{1}{2}(1-\alpha_1)\phi^2 + \tfrac{1}{2}(1-\alpha_2)\chi^2 - \tfrac{1}{2}\frac{\phi\partial^2\phi}{\chi^2}$$

Conclusions

The Planck Mass and all other mass scales are dynamical if there is a hidden exact Weyl invariance. Only ratios of masses are physical.

Multi-scalar models can dynamically generate the Planck Mass at the same time as inflation occurs. The two phenomena are unified.

The dilaton decouples with exact Weyl symmetry; there is no 5th force.

Can introduce Weyl photon and maintain a local Weyl gauge invariance .

Weyl photon eats the dilaton and becomes massive;
Weyl photon is stable and interacts only theorugh gravity
A dark matter candidate !

Conclusions

Through renormalization **without external mass scales** Weyl symmetry of the overall quantum theory is maintained. This is the quantum implementation of the idea that there exist no fundamental mass scales.

There remains RG running, but running only in locally Weyl invariant quantities like ϕ/χ ; hence no scale anomalies occur. The Weyl current and its kernel are then unmodified and exactly conserved.

Ghosts may provide an explicit realization of ϕ/χ running.
Are ghosts physical
(with a nontrivial stabilization)? Inertial stabilization? Ghosts seem to arise naturally in GR. Inertial stability?
Finite Quantum Field Theories.