Scale invariant theories of gravity and the meaning of the Planck mass

Roberto Percacci[1]

[1]SISSA, Trieste, Italy

Scale invariance in particle physics and cosmology
CERN, January 31, 2019

Outline

General view

Central question in the physics of fundamental interactions is status of symmetries.

Useful to think of gravity as a gauge theory of the linear group in the Higgs phase.

Planck mass is the characteristic scale associated to a Higgs phenomenon – similar to EW scale

Superconductivity

Macroscopic properties of superconductor follow from simple assumption that e.m. $U(1)$ is in Higgs phase.

There is a complex scalar field $\phi = \rho e^{i\varphi}$ with nonzero VEV. IR physics only depends on its phase φ, which behaves like a Goldstone boson. It transforms under $U(1)$ as

$$\varphi(x) \to \varphi(x) + \alpha(x)$$

and has a covariant derivative $D_\mu\varphi = \partial_\mu\varphi - A_\mu$.

The main feature of the superconducting state is

$$D_\mu\varphi = 0$$

Higgsless Higgs mechanism

Goldstone bosons $\sigma \in G/H$ coupled to G-YM fields A_μ.

$$\mathcal{L} = -\frac{f^2}{2} D\sigma^2 \qquad \text{where} \qquad D\sigma = \partial\sigma + A^a K_a(\sigma)$$

$\mathcal{L}(G) = \mathcal{L}(H) \oplus \mathcal{P}$ $A = A|_{\mathcal{L}(H)} + A|_{\mathcal{P}}$

In unitary gauge $\sigma = \sigma_0$

$D\sigma_0 = A^a|_{\mathcal{P}} K_a(\sigma_0)$

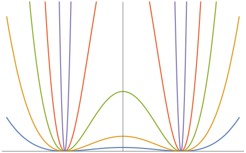
$$\mathcal{L} = -\frac{1}{2} f^2 \sum_{a \in \mathcal{P}} A^a A^a$$

Higgsful Higgs mechanism

Higgs field $\phi \in V$, and σ parametrize orbit of G in V

$W = \frac{\lambda}{4}(\rho^2 - \rho_0^2)^2$, with $\rho = |\phi|$.

$\lim_{\lambda\to\infty} W$ with ρ_0=const



Low Energy EFT

- for $p \ll m_\rho$, $\rho = \rho_0$
- for $p \ll m_A$, $A|_{\mathcal{P}} = 0$ or $D\sigma = 0$

Example: for $p \ll 100\text{GeV}$ Higgsless SM

T. Appelquist, C.W. Bernard, Phys.Rev.D22:200,1980.

A.C. Longhitano, Phys.Rev.D22:1166,1980.

Gravity

In GR the connection is identified with the Levi-Civita connection.

Unique connection that is metric and has vanishing torsion.

This is a necessary consequence of the system being in Higgs phase.

What are torsion and non-metricity?

In metric formalism (coordinate frames)

$$\Theta_\mu{}^\rho{}_\nu = \Gamma_\mu{}^\rho{}_\nu - \Gamma_\nu{}^\rho{}_\mu$$
$$Q_{\lambda\mu\nu} = -\nabla_\lambda g_{\mu\nu}$$

In vierbein formalism (orthonormal frames)

$$\Theta_\mu{}^a{}_\nu = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega_\mu{}^a{}_b\, e^b{}_\nu - \omega_\nu{}^a{}_b\, e^b{}_\mu$$
$$Q_{\lambda ab} = \omega_{\lambda ab} + \omega_{\lambda ba}$$

$\Longrightarrow$ use arbitrary frames

R. Floreanini, R.P Class.Quant.Grav. 7 (1990) 975, ibid. 7 (1990) 1805

R.P. Nucl. Phys. B 353, 271, (1991).

$GL(4)$-Gravity with independent connection

Local bases $\{\partial_\mu\}$, $\{dx^\mu\}$
and $\theta_a = \theta_a{}^\mu \partial_\mu$, $\theta^a = \theta^a{}_\mu dx^\mu$
gauge transformations=linear changes of basis

- metric γ_{ab}, signature $-,+,+,+$,
- frame field $\theta^a{}_\mu$, $\det\theta \neq 0$,
- linear connection, $A_\mu{}^a{}_b$

$\gamma \in GL(4)/O(1,3)$ and $\theta \in GL(4)$ are Goldstone bosons

Composite structures

- $g_{\mu\nu} = \theta^{a}{}_{\mu}\, \theta^{b}{}_{\nu}\, \gamma_{ab}$
- $\Gamma_{\lambda}{}^{\mu}{}_{\nu} = \theta^{-1}{}_{a}{}^{\mu} A_{\lambda}{}^{a}{}_{b} \theta^{b}{}_{\nu} + \theta_{a}^{-1\,\mu} \partial_{\lambda} \theta^{a}{}_{\nu}$

invariant under $GL(4)$

Torsion and Nonmetricity

- $\Theta_\mu{}^a{}_\nu = \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^a{}_b\, \theta^b{}_\nu - A_\nu{}^a{}_b\, \theta^b{}_\mu$
- $Q_{\lambda ab} = -\partial_\lambda \gamma_{ab} + A_\lambda{}^c{}_a\, \gamma_{cb} + A_\lambda{}^c{}_b\, \gamma_{ac}$

are the covariant derivatives of the Goldstone bosons

Transformations under $GL(4)$

$$\begin{aligned} \theta &\mapsto \Lambda^{-1}\theta \\ \gamma &\mapsto \Lambda^{T}\gamma\Lambda \end{aligned}$$

Two “unitary gauges” for $GL(4)$:

- $\theta^a_\mu = \delta^a_\mu$ metric formulation - breaks $GL(4)$ to $\{\mathbf{1}\}$
- $\gamma_{ab} = \eta_{ab}$ vierbein formulation - residual $O(1,3)$

not enough freedom to fix both

Weyl

transformations

$$\Lambda^{a}{}_{b}(x) = \delta^{a}_{b}\Omega(x)$$

form Weyl group and gauge field

$$A_{\mu}{}^{a}{}_{b} = b_{\mu}\delta^{a}_{b}$$

is Weyl gauge field (Ghilencea, Hill, Ross)

Gravitational Higgs mechanism v.I

$$S_G(\theta,\gamma,A) \;=\; \int d^4x\sqrt{|g|}\left[\Theta_{...}\Theta^{...}+Q_{...}Q^{...}+\Theta_{...}Q^{...}\right]$$

expanding around flat background: $A=0$, $\theta=\mathbf{1}$, $\gamma=\eta$

$$\begin{aligned} \Theta_\mu{}^a{}_\nu &= A_\mu{}^a{}_\nu - A_\nu{}^a{}_\mu \\ Q_{\mu ab} &= A_{\mu ab} + A_{\mu ba} \end{aligned}$$

kinetic term of Goldstone bosons becomes

$$S_G = \int d^4x\, \sqrt{|g|}\, A_{...}A^{...}$$

Levi–Civita Connection

given θ, γ, there is a unique $\bar{A}$ such that $\bar{\Theta} = 0$, $\bar{Q} = 0$

$$\bar{A} = \frac{1}{2}\left(E_{cab} + E_{abc} - E_{bac}\right) + \frac{1}{2}\left(C_{abc} + C_{bac} - C_{cab}\right)$$

where

$$E_{cab} = \theta^{-1}{}_c{}^{\lambda}\, \partial_\lambda \gamma_{ab}$$

$$C_{abc} = \gamma_{ad}\, \theta^d{}_\lambda \left(\theta^{-1}{}_b{}^{\mu}\, \partial_\mu \theta^{-1}{}_c{}^{\lambda} - \theta^{-1}{}_c{}^{\mu}\, \partial_\mu \theta^{-1}{}_b{}^{\lambda}\right)$$

Gravitational Higgs mechanism v.II

Any connection A can be split uniquely in

$$A = \bar{A} + \Phi$$

then $S(A, \gamma, \theta) = S(\bar{A}(\theta, \gamma) + \Phi, \theta, \gamma) = S'(\Phi, \theta, \gamma)$

$$\begin{aligned} \Theta_{\mu}{}^{a}{}_{\nu} &= \Phi_{\mu}{}^{a}{}_{\nu} - \Phi_{\nu}{}^{a}{}_{\mu} \\ Q_{\mu ab} &= \Phi_{\mu ab} + \Phi_{\mu ba} \end{aligned}$$

(in any gauge) therefore

$$S_G = \int d^4x \, \sqrt{|g|} \, \Phi_{\dots} \Phi^{\cdots}$$

The Palatini term

$$S_P(A,\gamma,\theta) = \frac{m_P^2}{2}\int d^4x\,\sqrt{|g|}\,F(A)_{ab}{}^{ab}$$

since

$$F(A) = F(\bar{A}) + \bar{\nabla}\Phi + \Phi^2$$

Palatini term also gives

$$\begin{aligned} S_P(A,\gamma,\theta) \;&=\; \frac{m_P^2}{2}\int d^4x\,\sqrt{|g|}\,F(\bar{A})_{ab}{}^{ab} \\ &\quad + \frac{m_P^2}{2}\int d^4x\,\sqrt{|g|}\,\left(\Phi_a{}^{ac}\Phi_{bc}{}^{b} - \Phi_{bac}\Phi^{acb}\right) \end{aligned}$$

(a degenerate quadratic form)

Macroscopic gravity

If mass matrix is nondegenerate,
assuming all masses are $O(m)$
at $p \ll m$

$$\Phi = 0 \iff \Theta = Q = 0 \iff A = \bar{A}(\theta, \gamma)$$

independent of the detailed form of the action.

Natural to assume that $m = m_P$

To summarize

GR looks like a low energy EFT of a theory where Higgs phenomenon occurs at Planck scale.

Central question for quantum theory of spacetime:

- why is there a nondegenerate metric?

Extension in the UV

In Landau-Ginzburg theory or SM the Goldstone bosons are embedded in a linear realization

In gravity no new field needed

Simplest linear realization would keep the same fields: metric/vierbein and connection (Dilaton $\rightarrow$ scalar)

In Landau-Ginzburg theory or SM VEV dictated by potential

Can write potential in bi-metric formulations of gravity.

$$V(\mathrm{tr}(\tilde{g}g^{-1}), \mathrm{tr}(\tilde{g}g^{-1}\tilde{g}g^{-1}))$$

R. Floreanini, R.P. Phys. Rev. D 46, 1566 (1992)

GraviGUT

Unify gravity and gauge interactions e.g. by

$$SO(1,3) \times SO(10) \subset SO(3,11)$$

Hint from Nature:

$$\mathbf{2_C} \times \mathbf{16_C} = \mathbf{64_R}$$

R.P. Phys. Lett. B 144, 37 (1984), Nucl. Phys. B 353, 271, (1991).

F. Nesti, R.P., Phys. Rev. D 81, 025010 (2010)

K. Krasnov, R.P., CQG arXiv:1712.03061 [hep-th]

GraviGUT III

$$A = \begin{bmatrix} A^{(4)} & H \\ H^T & A^{(10)} \end{bmatrix}$$

kinetic term of θ gives mass to $A^{(4)}$, H
$A^{(10)}$ remains massless

From here on choose metric gauge

General action of MAG

$$
\begin{aligned}
S \;=\; -\frac{1}{2}\int d^4x\,\sqrt{|g|}\,\Big[& F^{\mu\nu\rho\sigma}\left(c_1 F_{\mu\nu\rho\sigma} + c_2 F_{\mu\nu\sigma\rho} + c_3 F_{\rho\sigma\mu\nu} + c_4 F_{\mu\rho\nu\sigma}\right.\\
& \left. + c_5 F_{\mu\sigma\nu\rho} + c_6 F_{\mu\sigma\rho\nu}\right) + L^{\mu\nu}\left(c_7 L_{\mu\nu} + c_8 L_{\nu\mu}\right) + K^{\mu\nu}\left(c_9 K_{\mu\nu} + c_{10} K_{\nu\mu}\right)\\
& + K^{\mu\nu}\left(c_{11} L_{\mu\nu} + c_{12} L_{\nu\mu}\right) + F^{\mu\nu}\left(c_{13} F_{\mu\nu} + c_{14} L_{\mu\nu} + c_{15} K_{\mu\nu}\right) + c_{16} L^2\\
& + T^{\mu\rho\nu}\left(a_1 T_{\mu\rho\nu} + a_2 T_{\mu\nu\rho}\right) + a_3 T^\mu T_\mu\\
& + Q^{\rho\mu\nu}\left(a_4 Q_{\rho\mu\nu} + a_5 Q_{\nu\mu\rho}\right) + a_6 U^\mu U_\mu + a_7 V^\mu V_\mu + a_8 U^\mu V_\mu\\
& + a_9 T^{\mu\rho\nu} Q_{\mu\rho\nu} + a_{10} T^\mu U_\mu + a_{11} T^\mu V_\mu - m_P^2 L\Big]
\end{aligned}
$$

where

$$
\begin{aligned}
F_{\mu\nu} \;&:=\; F_{\mu\nu\lambda}{}^{\lambda}\,, \quad K_{\mu\nu} := F_{\lambda\mu\nu}{}^{\lambda}\,, \quad L_{\mu\nu} := F_{\lambda\mu}{}^{\lambda}{}_{\nu}\,, \quad L := L^{\mu}{}_{\mu} = -K^{\mu}{}_{\mu}\\
T_\mu \;&:=\; T_\lambda{}^{\lambda}{}_{\mu}\,, \qquad U_\mu := Q_{\mu\lambda}{}^{\lambda}\,, \qquad V_\mu := Q_\lambda{}^{\lambda}{}_{\mu}\,,
\end{aligned}
$$

Spin$^{\text{parity}}$ states

A_{abc}

	ts	hs	ha	ta
TTT	$3^-, 1_1^-$	$2_1^-, 1_2^-$	$2_2^-, 1_3^-$	0^-
$TTL + TLT + LTT$	$2_1^+, 0_1^+$	-	-	1_3^+
$\frac{3}{2}LTT$	-	$2_2^+, 0_2^+$	$1_2^+,$	-
$TTL + TLT - \frac{1}{2}LTT$	-	1_1^+	$2_3^+, 0_3^+$	-
$TLL + LTL + LLT$	1_4^-	1_5^-	1_6^-	-
LLL	0_4^+	-	-	-

h_{ab} symmetric

TT	$2_4^+, 0_5^+$
TL	1_7^-
LL	0_6^+

Systematic analysis of states in case $Q = 0$

E. Sezgin, P. van Nieuwenhuizen, Phys. Rev. D21, 3269 (1980)

Y.C. Lin, M.P. Hobson and A.N. Lasenby, arXiv:1812.02675

Analysis of general case under way

R.P. and E.Sezgin, to appear

Generically expect many massive states including ghosts.

Scale invariance

Only for $m_P = a_1 = a_2 = \ldots = a_{11} = 0$.

Free parameters c_1, $c_2 \ldots c_{16}$

Quantum scale invariance

May be realized at a (free or interacting) fixed point.

Literature on asymptotic safety of gravity almost entirely focused on metric formalism.

Using $F = R + \nabla\Phi + \Phi^2$ we can rewrite schematically

$$
\begin{aligned}
S(g,A) &= S'(g,\Phi) \\
\int d^4x\sqrt{|g|}F^2 &= \int d^4x\sqrt{|g|}\big[R^2 + R\nabla\phi + R\Phi^2 \\
&\qquad +(\nabla\Phi)^2 + \nabla\Phi\Phi^2 + \Phi^4\big]
\end{aligned}
$$

HDG and AS

Perturbative calculation

I.Avramidi, A.O. Barvinsky Phys. Lett. 159B (1985) 269-274

Using FRG at one loop

A. Codello and R. P. Phys.Rev.Lett. 97 (2006) 221301

M.Niedermaier Nucl.Phys. B833 (2010) 226-270

N.Ohta and R.P. Class.Quant.Grav. 31 (2014) 015024

same FP but with $\tilde{G}_{N*} \neq 0$

Using FRG beyond one loop

D.Benedetti, P.Machado, F.Saueressig Mod.Phys.Lett. A24 (2009) 2233-2241

find FP with $f_{2*} \neq 0$, $f_{0*} \neq 0$. Not settled

AS of gravity with independent connection

Few calculations

D. Benedetti and S. Speziale, JHEP 116 (2011) 107

J.E. Daum and M. Reuter, Phys. Lett. B710 (2012) 215

J.E. Daum and M. Reuter, Ann. Phys. 334 (2013) 351

U. Harst and M. Reuter, Ann. Phys. 354 (2015) 637

C. Pagani and R.P. Class. Quant. Grav. 32 (2015) no.19, 195019

but none of these takes into account the F^2 terms

A special model

$$
\begin{aligned}
S \;=\; & \int d^4x\,\sqrt{|g|}\,\Big[F^{\mu\nu\rho\sigma}\left(g_1 F_{\mu\nu\rho\sigma} + g_2 F_{\mu\rho\nu\sigma} + g_3 F_{\rho\sigma\mu\nu}\right) \\
& + L^{\mu\nu}\left(g_4 L_{\mu\nu} + g_5 L_{\nu\mu}\right) + g_6 L^2 \\
& + T^{\mu\rho\nu}\rho^2\left(b_1 T_{\mu\rho\nu} + b_2 T_{\mu\nu\rho}\right) + b_3\rho^2 T^\mu T_\mu + g_0\rho^2 L \\
& + \frac{1}{2} b_0 \partial_\mu\rho\partial^\mu\rho + 2b_1\rho\partial_\mu\rho T^\mu + V(\rho)\Big]
\end{aligned}
$$

RG flow of VEV in UV limit

$$\langle\rho\rangle \sim k$$

but what about g_i?

Special case $F_{\mu\nu ab}F^{\mu\nu ab}$ is AF

J. Donoghue Phys.Rev. D96 (2017) no.4, 044003

Quantum scale invariance in MAG possible, even likely

The ghost issue

- Ghosts are an artifact of a finite truncation
- Ghosts are an artifact of the expansion around flat space

A.Bonanno, M. Reuter Phys.Rev. D87 (2013) no.8, 084019

- There is no pole in the ghost propagator due to scale invariance at high energy

A. Salam and J. Strathdee, Phys.Rev. D18 (1978) 4480

R. Floreanini and R.P. Phys. Rev. D52 (1995) 896

$$m_{\rm phys} = m(k = m_{\rm phys})$$

Summary

- Higgs phenomenon generates mass for the gravitational connection near the Planck scale
- at low scales the connection is frozen to the Levi-Civita and only the Goldstone boson(s) remain dynamical
- there may be asymptotically safe UV completion of theory in terms of the same field variables
- enhanced predictivity (Eichhorn's talk)
- consistent with quantum scale invariance at high energy, there may be no propagating states at Planck scale
- in the symmetric phase there is no metric - not clear that it makes sense to talk of a spacetime
- dynamical origin of the VEV related to origin of spacetime
- there is room for unification of all interactions