Asymptotic safety and Conformal Standard Model

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Scale invariance in particle physics and cosmology workshop
General principles - assumptions
• No new physics between EW and Planck scale - minimal extensions
• Asymptotic safety, perturbative approach
What is asymptotic safety?

In the quantum field theory the couplings change with energy ("run") due to renormalisation group equations:

\[ k \frac{\partial g_i(k)}{\partial k} = \beta_i (\{g_i(k)\}). \]  

(1)

Asymptotic safety:

- Generalisation of asymptotic freedom
- Theory has a UV fixed interacting point. Weinberg hypothesis: Gravity.
- Consequence: prediction of irrelevant couplings, allowed range for relevant couplings
Matter beta functions with gravitational corrections

For the matter beta functions one can calculate the gravitational corrections:

\[ \beta(g_j) = \beta_{SM}(g_j) + \beta_{grav}(g_j, k), \]  

(2)

where due to universal nature of gravitational interactions the \( \beta_{grav} \) are given by:

\[ \beta_{grav}(g_j, k) = \frac{a_j k^2}{M_P^2 + \xi k^2 g_j}. \]  

(3)

The \( \xi \approx 0.024 \) and depends on cosmological constants and \( M_P \) fixed points. The \( a_j \) are unknown parameters, however they can be calculated. Then, depending on a sign of \( a_j \), we have repelling/attracting fixed point at 0 in the perturbative region of couplings.
With the assumption of asymptotic safety of gravity the Higgs mass (self coupling) was calculated. Obtained the value:

\[ m_H = 126 \pm \text{few GeV} \] two years before the detection.
Higgs Portal Models
Higgs Portal Models

• Sterile complex (real) scalar $\phi$ coupled to Higgs doublet:

$$\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) + (\partial_\mu \phi^* \partial^\mu \phi) - V(H, \phi).$$  \hspace{1cm} (4)

$$V(H, \phi) = -m_1^2 H^\dagger H - m_2^2 \phi^* \phi + \lambda_1 (H^\dagger H)^2$$
$$+ \lambda_2 (\phi^* \phi)^2 + 2\lambda_3 (H^\dagger H) \phi^* \phi.$$  \hspace{1cm} (5)

• The scalar particles are combined from two states:

$$m_1^2 = \lambda_1 v_H^2 + \lambda_3 v_\phi^2, \quad m_2^2 = \lambda_3 v_H^2 + \lambda_2 v_\phi^2.$$  \hspace{1cm} (6)

and the lighter is identified with Higgs particle.
The running of couplings

We run following couplings: $g_1, g_2, g_3$ (Gauge couplings), $y_t$ (top Yukawa coupling), $\lambda_1, \lambda_2, \lambda_3$. The beta functions are $\hat{\beta} = 16\pi^2\beta$:

\[
\begin{align*}
\hat{\beta}_{g_1} &= \frac{41}{6} g_1^3, \\
\hat{\beta}_{g_2} &= -\frac{19}{6} g_2^3, \\
\hat{\beta}_{g_3} &= -7 g_3^3, \\
\hat{\beta}_{y_t} &= y_t \left( \frac{9}{2} y_t^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right), \\
\hat{\beta}_{\lambda_1} &= 24 \lambda_1^2 + 4 \lambda_3^2 - 3 \lambda_1 \left( 3 g_2^2 + g_1^2 - 4 y_t^2 \right) \\
&\quad + \frac{9}{8} g_2^4 + \frac{3}{4} g_2^2 g_1^2 + \frac{3}{8} g_1^4 - 6 y_t^4, \\
\hat{\beta}_{\lambda_2} &= (20 \lambda_2^2 + 8 \lambda_3^2), \\
\hat{\beta}_{\lambda_3} &= \frac{1}{2} \lambda_3 \left[ 24 \lambda_1 + 16 \lambda_2 + 16 \lambda_3 - (9 g_2^2 + 3 g_1^2) + 12 y_t^2 \right].
\end{align*}
\]
Conditions for low energy coupling values and $a_i$ values

We impose two conditions:

- absence of Landau poles
- $\lambda_1(\mu) > 0$, $\lambda_2(\mu) > 0$, $\lambda_3(\mu) > -\sqrt{\lambda_2(\mu)\lambda_1(\mu)}$.

Furthermore we take:

$$a_{g_i} = 1, a_{y_t} = -0.5, a_{\lambda_1} = +3$$

and

$$a_{\lambda_2} = +3, a_{\lambda_3} = +3.$$
The low-energy values at $\mu_0 = 173.34$ are taken as: $g_1(\mu_0) = 0.35940$, $g_2(\mu_0) = 0.64754$, $g_3(\mu_0) = 1.1888$ and $y_t(\mu_0) = 0.95113$. 

**Figure 1:** The running of $g_1, g_2, g_3, y_t$
We get: \( \lambda_1 = 0.1537, \lambda_3 = \lambda_2 = 0 \). Can we do something about it?
Include right handed neutrinos coupled to $\phi$ with the coupling $y_M$:

$$\mathcal{L} \ni \frac{1}{2} Y_{ji}^M \phi N^j\alpha N^i\alpha,$$  \hspace{1cm} (10)

where $Y_{ij}^M = y_M \delta_{ij}$. To resolve the baryogenesis problem, via resonant leptogenesis, the right handed neutrinos have to be unstable:

$$M_N = y_M v_\phi / \sqrt{2} > m_2.$$  \hspace{1cm} (11)

Furthermore the CSM can resolve the SM problems like: hierarchy problem and has dark matter candidate, minoron with mass: $v^2 / M_P$, inflation.
Add this new coupling:

\[
\begin{align*}
\hat{\beta}_{y_M} &= \frac{5}{2} y_M^3, \\
\hat{\beta}_{\lambda_2} &= (20 \lambda_2^2 + 8 \lambda_3^2 + 6 \lambda_2 y_M^2 - 3 y_M^4), \\
\hat{\beta}_{\lambda_3} &= \frac{1}{2} \lambda_3 [24 \lambda_1 + 16 \lambda_2 + 16 \lambda_3 \\
&\quad - (9 g_2^2 + 3 g_1^2) + 6 y_M^2 + 12 y_t^2],
\end{align*}
\]

we take \( a_{y_M} = -1 \).
Predictions
Coefficient: $a_{\lambda_3} = +3$, $a_{\lambda_2} = +3$

Still: $\lambda_3 = 0$. So SM and $\phi$ decouple, but:

Figure 2: $\lambda_2$ dependence on $y_M$
Can we have $\lambda_3 \neq 0$? Change $\xi$.

(a) $\xi = 0.024$

(b) $\xi = 1$
Can we have $\lambda_3 \neq 0$? Add new fields

\[ \begin{align*}
\hat{\beta}_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_2^2 - 3\lambda_1 (3g_2^2 + g_1^2 - 4y_t^2) \\
&\quad + \frac{9}{8}g_2^4 + \frac{3}{4}g_2^2g_1^2 + \frac{3}{8}g_1^4 - 6y_t^4, \\
\hat{\beta}_{y_M} &= \frac{5}{2} y_M^3, \\
\hat{\beta}_{\lambda_2} &= (20\lambda_2^2 + 8\lambda_3^2 + 6\lambda_2 y_M^2 - 3y_M^4), \\
\hat{\beta}_{\lambda_3} &= \frac{1}{2} \lambda_3 \left[ 24\lambda_1 + 16\lambda_2 + 16\lambda_3 \\
&\quad - (9g_2^2 + 3g_1^2) + 6y_M^2 + 12y_t^2 \right].
\end{align*} \]  

(13)

we have: $\lambda_1, \lambda_2 \neq 0$, maybe we can make $\lambda_3 \neq 0$. 


Can we have $\lambda_3 \neq 0$? Take $a_{\lambda_3} < 0$

(c) $\lambda_1$ dependence on $\lambda_3, y_M$

(d) $\lambda_2$ dependence on $\lambda_3, y_M$
Conditions on $m_2$

One can parametrize the discrepancies from SM as:

$$\tan \beta = \frac{\lambda_0 - \lambda_1}{\lambda_3} \frac{v_H}{v_\phi}. \quad (14)$$

Conditions:

- $|\tan \beta| < 0.35$.
- Un-stability condition for the second particle: $m_2 > 2m_1$. 
Mass restrictions for $a_{\lambda_2} = +3$, $a_{\lambda_3} = -3$

Take the tree level relations:

$$m_1^2 = \lambda_1 v_H^2 + \lambda_3 v_\phi^2,$$
(15)

$$m_2^2 = \lambda_2 v_\phi^2 + \lambda_3 v_H^2,$$
(16)

and: $m_H = 136\text{GeV}$, $v_H = 246\text{ GeV}$. Then there are only two sets of parameters satisfying the imposed conditions

$$y_M = 0.84, m_2 = 275, v_\phi = 538, M_N = 319,$$
(17)

and

$$y_M = 0.85, m_2 = 296, v_\phi = 574, M_N = 345.$$  
(18)
It constrains the second scalar mass as:

\[ 272 \text{ GeV} < m_2 < 328 \text{ GeV} \quad \text{and} \quad y_M > 0.71. \] (19)

The neutrinos masses:

\[ M_N = 342^{+41}_{-41} \text{ GeV} \] (20)

and they satisfy instability condition.
Higgs portal case: $y_M = 0.0$

For $y_M = 0.0$:

$$m_2 = 160^{+103}_{-100} \text{ GeV}, \quad (21)$$

so classically it is stable.
Comparison with experimental data

- The excess of events with four charged leptons at $E \sim 325$ GeV seen by the CDF [9] and CMS [8] Collaborations can be identified with a detection of a new ‘sterile’ scalar particle proposed by the Conformal Standard Model [7].
- The hypothetical heavy boson mass is measured to be around 272 GeV (in the 270 – 320 GeV range), according to [10, 11].
Thank you for your attention

Talk based on article: arxiv.org/abs/1810.08461
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Non-standard possibilities are:

- Asymptotic safety, \( \lim_{\mu \to \infty} g \neq 0, \forall i \beta_i(g^*) = 0 \). Theory has a UV fixed point. Example: Weinberg hypothesis: Gravity.

- Oscillating \( g \). Theory has a limit cycle. Quantum mechanics: \(-g/r^2\) potential [12].
We consider an effective theory valid below $\Lambda$. We split the bare parameters mass and self-coupling into renormalised parameters and counter-terms:

$$m^2_B(\Lambda) = m^2_R - f_{\text{quad}}(\Lambda, \mu, \lambda_R)\Lambda^2 + m^2_R g \left( \lambda_R, \log \left( \frac{\Lambda}{\mu} \right) \right), \quad (22)$$

where $g \left( \lambda_R, \log \left( \frac{\Lambda}{\mu} \right) \right)$ is some function. Assume that the quadratic divergences depends only on bare couplings:

$$f_{\text{quad}}(\Lambda, \mu, \lambda_R) = f_{\text{quad}}(\lambda_B(\Lambda)). \quad (23)$$

So if $f_{\text{quad}}(\lambda_B) = 0$ at certain scale, then the hierarchy problem is solved.
Conformal Standard Model and Softly Broken Conformal Symmetry

For CSM the $\hat{f}_i^{\text{quad}} = 16\pi^2 f_i^{\text{quad}}$ are:

\[
\hat{f}_1^{\text{quad}}(\lambda, g, y) = 6\lambda_1 + 2\lambda_3 + \frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 6y_t^2, \quad (24)
\]

\[
\hat{f}_2^{\text{quad}}(\lambda, g, y) = 4\lambda_2 + 4\lambda_3 - 3y_M^2. \quad (25)
\]

For the Conformal Standard Model $\Lambda \lesssim M_P$ is sufficient, while the Standard Model requires $\Lambda \gg M_P$. 
Lagrangian in the Jordan frame [6]:

\[
\mathcal{L} = D_\mu H^\dagger D^\mu H + \partial_\mu \phi \partial^\mu \phi^* - \frac{\left( M_P^2 + \xi_1 H^\dagger H + \xi_2 |\phi|^2 \right)}{2} \, R - V_J(H, \phi),
\]

(26)

with \( \xi_i > 0 \). We get the standard result that:

\[
n_s \simeq 1 - \frac{2}{N} \simeq 0.97,
\]

(27)

and:

\[
r \simeq \frac{12}{N^2} \simeq 0.0033,
\]

(28)

however it requires that \( \xi_1, \xi_2 \sim \mathcal{O}(10^4) \).
Coefficients $a_{\lambda_2} = a_{\lambda_3} = -3$, set of allowed couplings $\lambda_2, \lambda_3, y_M$

Figure 3: Maximal (left) and minimal (right) $y_M(\lambda_3, \lambda_2)$, $a_{\lambda_2} = -3, a_{\lambda_3} = -3$
Coefficient: $a_{\lambda_2} = a_{\lambda_3} = -3$, allowed $\lambda_1$

Figure 4: Plot of $\lambda_1(\lambda_2, \lambda_3, y_M)$
1. We checked that the analysed parameters satisfy the Softly Broken Conformal Symmetry requirements at $M_P$ with couplings going to zero (but nowhere else).

2. We analyzed the running of the $\beta$ functions for $m_2$ and $m_1$, where we took $a_{m_i} = -1$. It gives no new bounds on $m_2$ and lambda-couplings.
Bibliography


