Scale invariance and strong dynamics as the origin of inflation and the Planck mass

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In collaboration with J. Kubo, M. Lindner and K. Schmitz

31/01/2019 Scale invariance in particle physics and cosmology@CERN

Scales

- Planck scale
- GUT scale (??)
- Electroweak scale
- QCD scale
- . . .

Scales

- **Planck scale**
- GUT scale (??)
- Electroweak scale
- QCD scale
- . . .

Origin of Planck Scale

$$\int \mathrm{d}^4x\sqrt{-g}\,\frac{M_{\mathrm{Pl}}^2}{2}R$$

Origin of Planck Scale

$$\int \mathrm{d}^4x\sqrt{-g}\, S^\dagger S R$$

Origin of Planck Scale

$$\int \mathrm{d}^4x\sqrt{-g}\, S^\dagger S R$$

Scale invariant

What is the dynamics of scalar?

Quantum dynamics generates a scale.

- How to generate a scale from a dimensionless theory?

- Dimensional transmutation:

  - The Coleman-Weinberg mechanism. (perturbation)

    Salvio's talk

  - Strong dynamics (non-perturbative)

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- Dimensional transmutation:
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    Salvio's talk
  - Strong dynamics (non-perturbative)

Let's call "<u>Scalegenesis</u>"

Contents

- Model:
  - Scalegenesis in scale invariant scalar-gauge theory
- The model has two possible inflatons.
- Predictions

Scale invariant scalar-gauge theory

- SU(Nc)×scale inv. scalar gauge theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big(- \hat{\beta}(S^\dagger S)\, R + \hat{\gamma}R^2 - \frac{1}{2}\mathrm{Tr}\, F^2 + g^{\mu\nu}[D_\mu S]^\dagger D_\nu S - \hat{\lambda}(S^\dagger S)^2 + \hat{\kappa}W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}\Big)$$

- Due to the strong dynamics, the scalar condensate takes place: $\langle S^\dagger S\rangle$

Dynamical scale symmetry breaking!

$$\hat{\beta}(S^\dagger S)R \Longrightarrow \hat{\beta}\langle S^\dagger S\rangle R$$

Non-perturbative dynamics is difficult…

- We cannot use the perturbation theory…
- How to describe scalegenesis from the strong dynamics of the theory?
- We attempt to formulate an effective theory!
- What we want to see is that the scale invariant theory dynamically generates a scale.

Effective theory

- Scale invariant scalar-tensor theory

cf. J. Kubo and **MY**, Phys.Rev. D93 (2016) no.7, 075016

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big(-\beta(S^\dagger S)\,R + \gamma R^2 + g^{\mu\nu}[\partial_\mu S]^\dagger \partial_\nu S - \lambda(S^\dagger S)^2 + \kappa W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}\Big)$$

- Ignore scale symmetry (hard) breaking by quantum anomaly.
- We want to see that the scale invariant theory generates a scale.
- We do not consider quantum gravity effects.

Effective potential

- Using the mean field approximation,

$$V_{\rm eff}(f,\bar{S},R) = (2f\lambda+\beta R)\bar{S}^\dagger\bar{S} - \lambda f^2 + \underline{\frac{N_c}{32\pi^2}(2\lambda f+\beta R)^2\log\left(\frac{2\lambda f+\beta R}{\Lambda_B^2}\right)}$$

Loop effect

- Mean-field introduced: $f = \langle S^\dagger S\rangle$
- Vacuum: $\langle f\rangle = f_0 = \frac{\Lambda_B^2}{2\lambda}\exp\left(\frac{8\pi^2}{N_c\lambda}-\frac{1}{2}\right)$

$$\langle\bar{S}\rangle = 0$$
$$\langle R\rangle = 0$$

- The Planck scale:

$$M_{\rm Pl}^2 = 2\beta f_0$$

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Effective action in Jordan frame

- Effective action:

$$\Gamma_{\rm eff} = \int \mathrm{d}^4x\sqrt{-g}\Big(\frac{Z^{-1}}{2}g^{\mu\nu}\partial_\mu f\partial_\nu f - V_{\rm eff}(f,R) + \gamma R^2 + \kappa W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}\Big)$$

Canonical normalisation and polynomial expansion of *R*

Effective Lagrangian in Jordan frame

$$\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} = \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi) - \frac{M_{\rm Pl}^2}{2}B(\chi)R + G(\chi)R^2 + \cancel{\kappa W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta} + \mathcal{O}(R^3)}$$

$$= \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi) - \frac{M_{\rm Pl}^2}{2}\underbrace{\left[B(\chi) - \frac{4G(\chi)}{M_{\rm Pl}^2}\psi\right]}_{=\Omega^2}R - G(\chi)\psi$$

Go to Einstein frame

- (Local) Weyl transformation $g_{\mu\nu} = \Omega^2 g^J_{\mu\nu}$

$$\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} = \frac{1}{2}e^{-\Phi(\phi)}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\chi,\phi) - \frac{M_{\rm Pl}^2}{2}R$$

  - Functions

$$\phi = \sqrt{\frac{3}{2}}M_{\rm Pl}\ln\Omega^2 \qquad \Phi(\phi) = \frac{\sqrt{2}}{\sqrt{3}M_{\rm Pl}}\phi$$

$$V(\chi,\phi) = e^{-2\Phi(\phi)}\left[U(\chi) + \frac{M_{\rm Pl}^2}{16G(\chi)}\left(B(\chi) - e^{\Phi(\phi)}\right)^2\right]$$

There are two dynamical scalar fields.

- χ (dilaton)
  - The pseudo-NG boson of dynamical scale symmetry breaking.
- ϕ (scalaron)
  - A gravitational scalar degree of freedom that originates from the R^2 term.

We choose a specific inflationary trajectory.

- Local extremum in the scalaron direction at $\phi = \phi_*$

$$\left.\frac{\partial V}{\partial \phi}\right|_{(\chi, \phi_*(\chi))} = 0 \longrightarrow \text{We obtain } \phi_*(\chi)$$

- Trajectory: $\mathcal{C}(\sigma) = \{\chi(\sigma), \phi(\sigma)\} = \{\sigma, \phi_*(\sigma)\}$

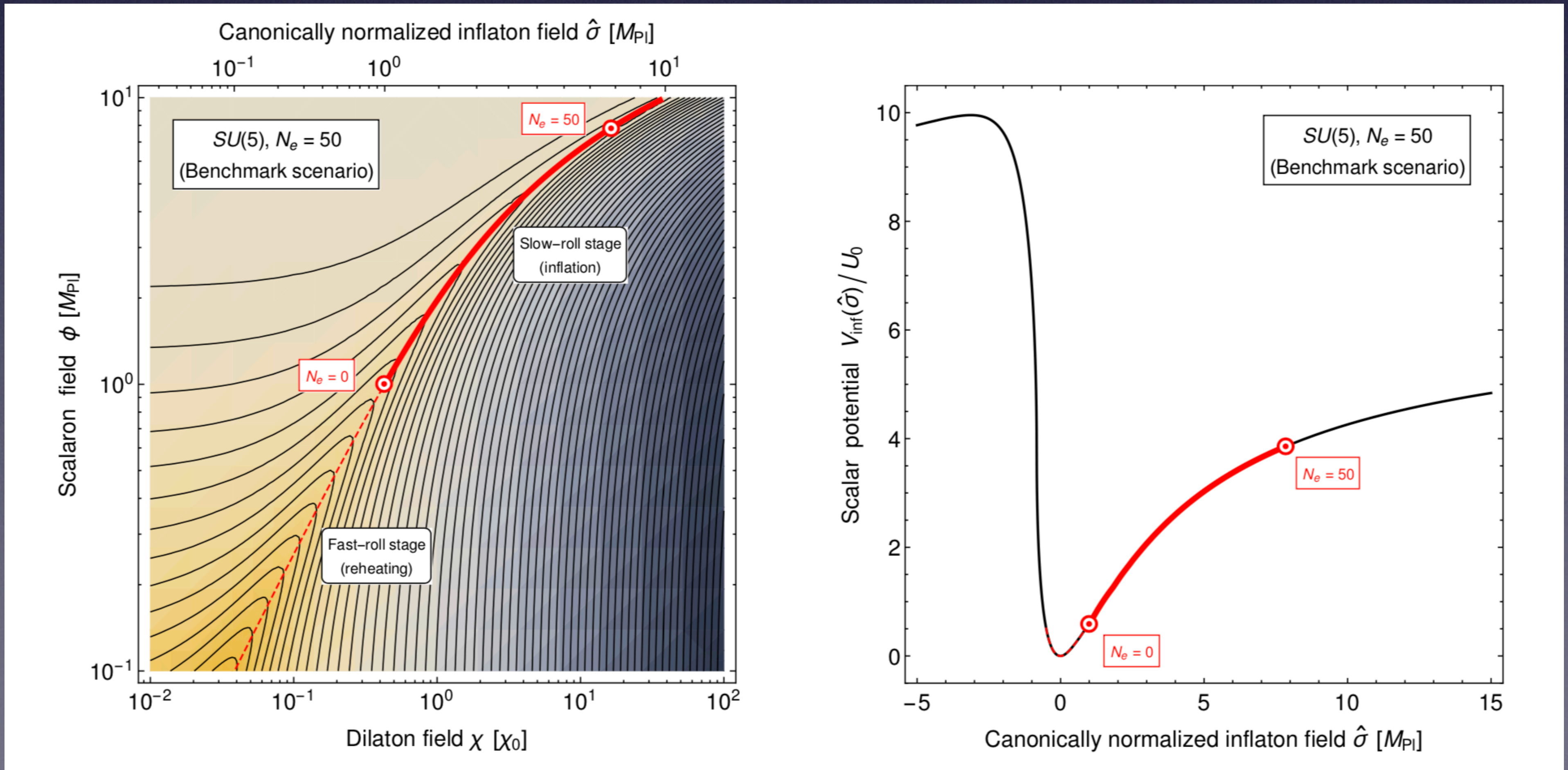
- Inflation is described by an effective single-field σ.

- Stability condition: $m_\phi^2(\sigma) = \left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\mathcal{C}} > 0$

We choose a specific inflationary trajectory.

- Effective Lagrangian

$$\left.\frac{\mathcal{L}_{\rm eff}^{E}}{\sqrt{-g}}\right|_{\mathcal{C}} = -\frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\sigma}\partial_\nu\hat{\sigma} - V_{\rm inf}(\hat{\sigma})$$



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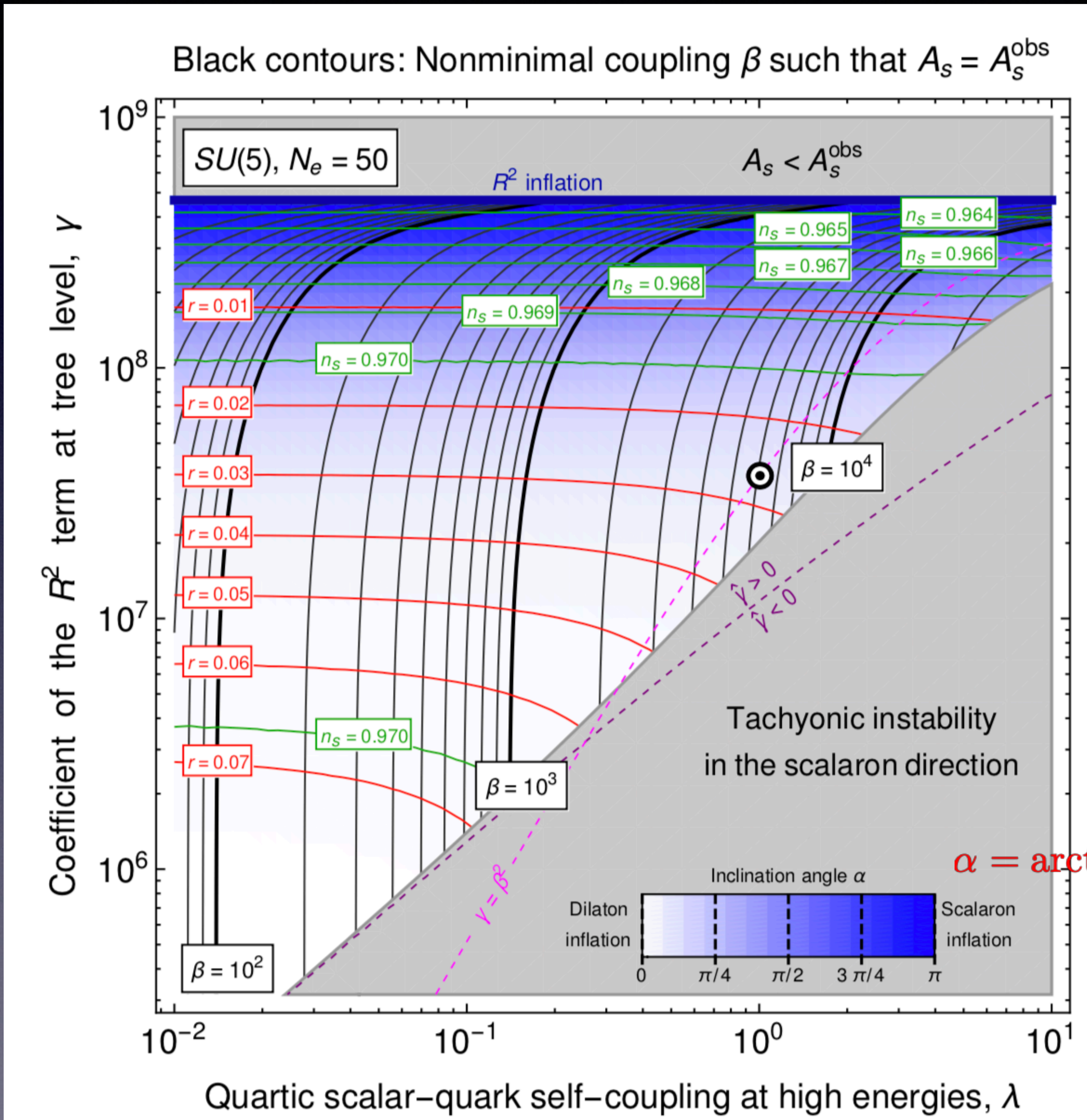
Parameter space



Benchmark

$$N_c = 5, \quad N_e = 50$$

From the PLANCK

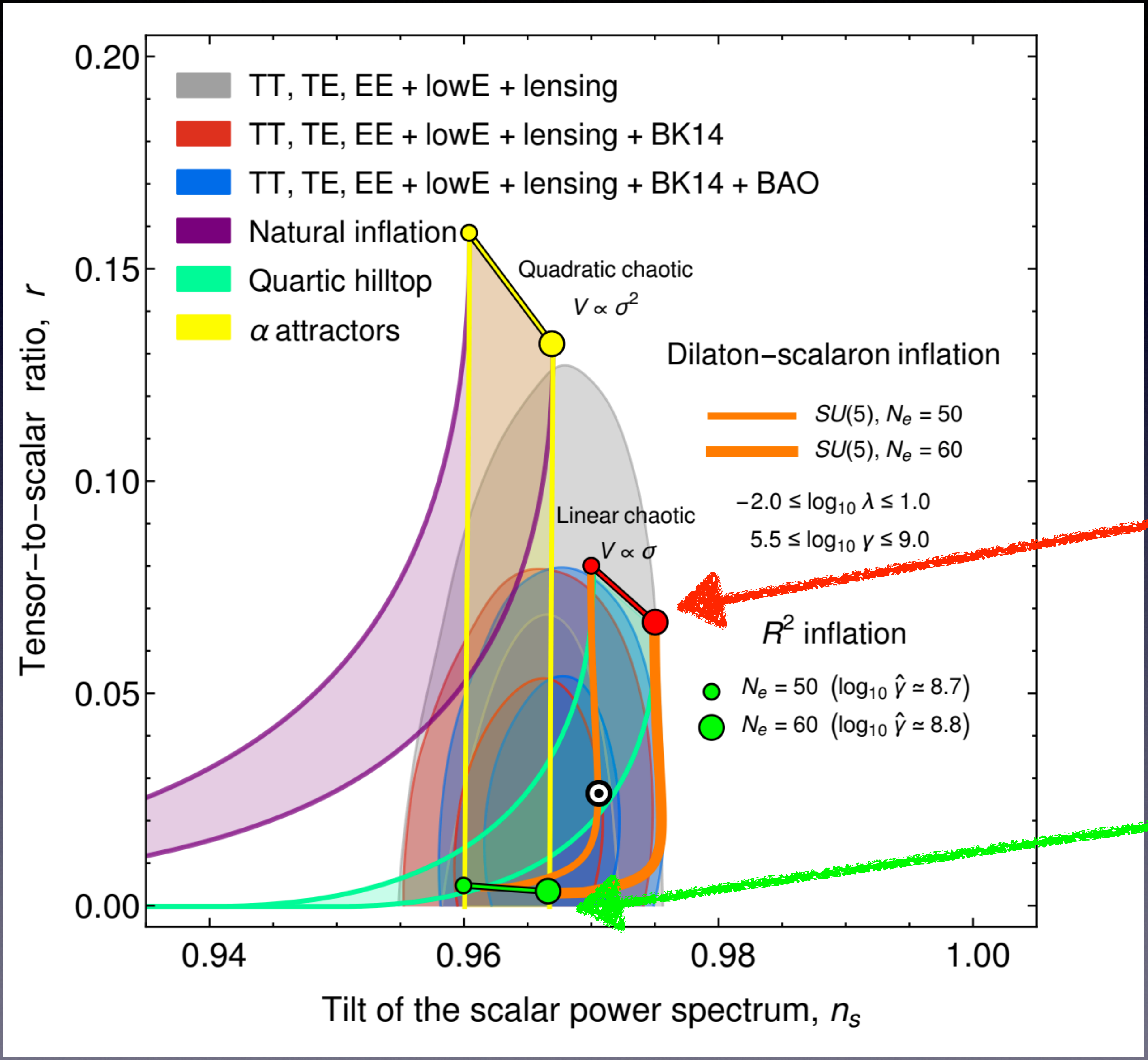
[arXiv:1807.06211]

$$A_s^{\rm obs} \simeq 2.1 \times 10^{-9}$$

$$\alpha = \arctan\left(\frac{\chi_0}{M_{\rm Pl}} \frac{{\rm d}\phi_*}{{\rm d}\chi}\right)$$

The angle between the dilaton axis and the inflationary trajectory.

On $n_s - r$ Plane



Linear chaotic

$$n_s \simeq 1 - \frac{3}{2N_e} \qquad r \simeq \frac{12}{N_e^2}$$

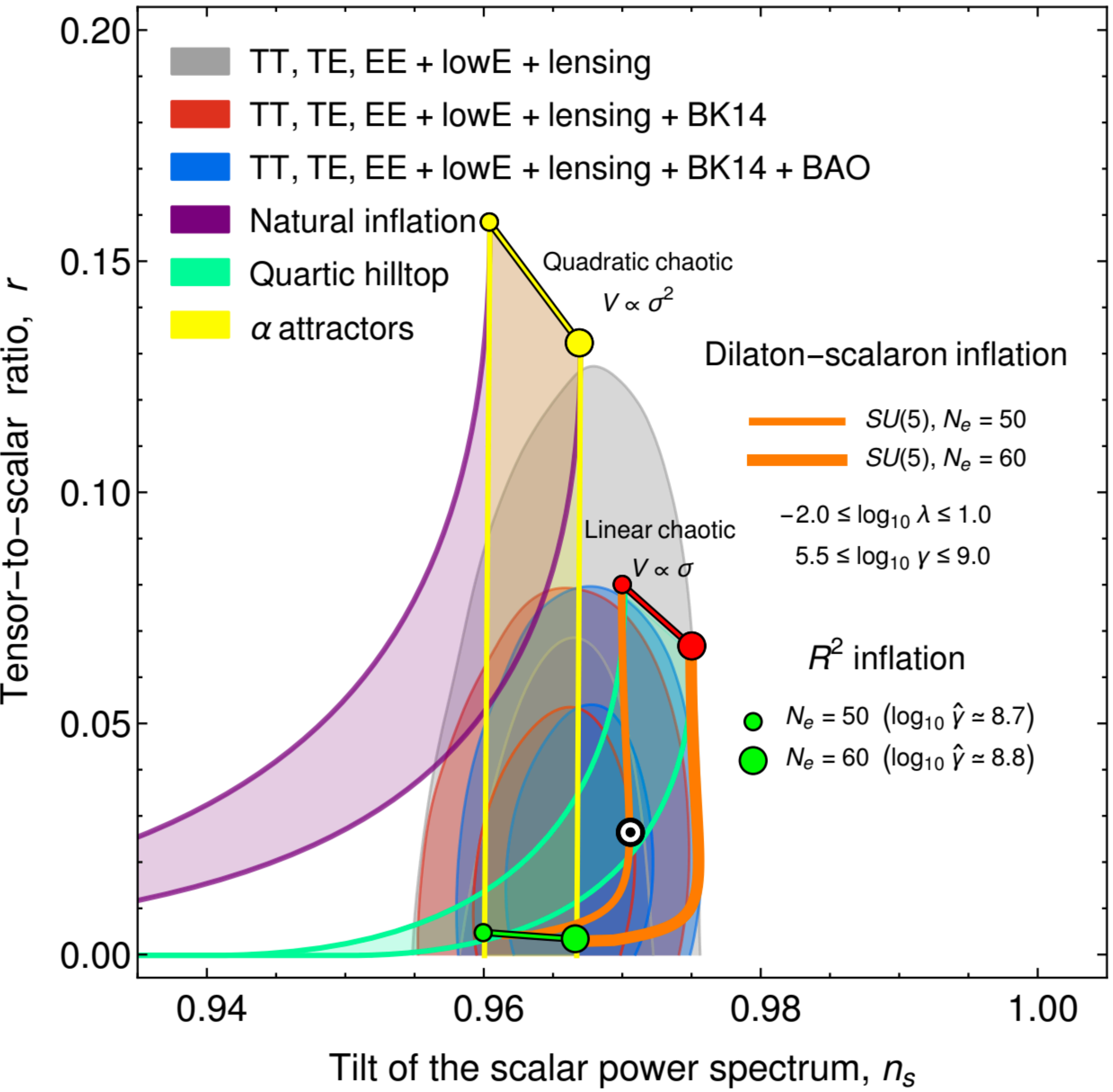
R^2 inflation

$$n_s \simeq 1 - \frac{2}{N_e} \qquad r \simeq \frac{4}{N_e}$$

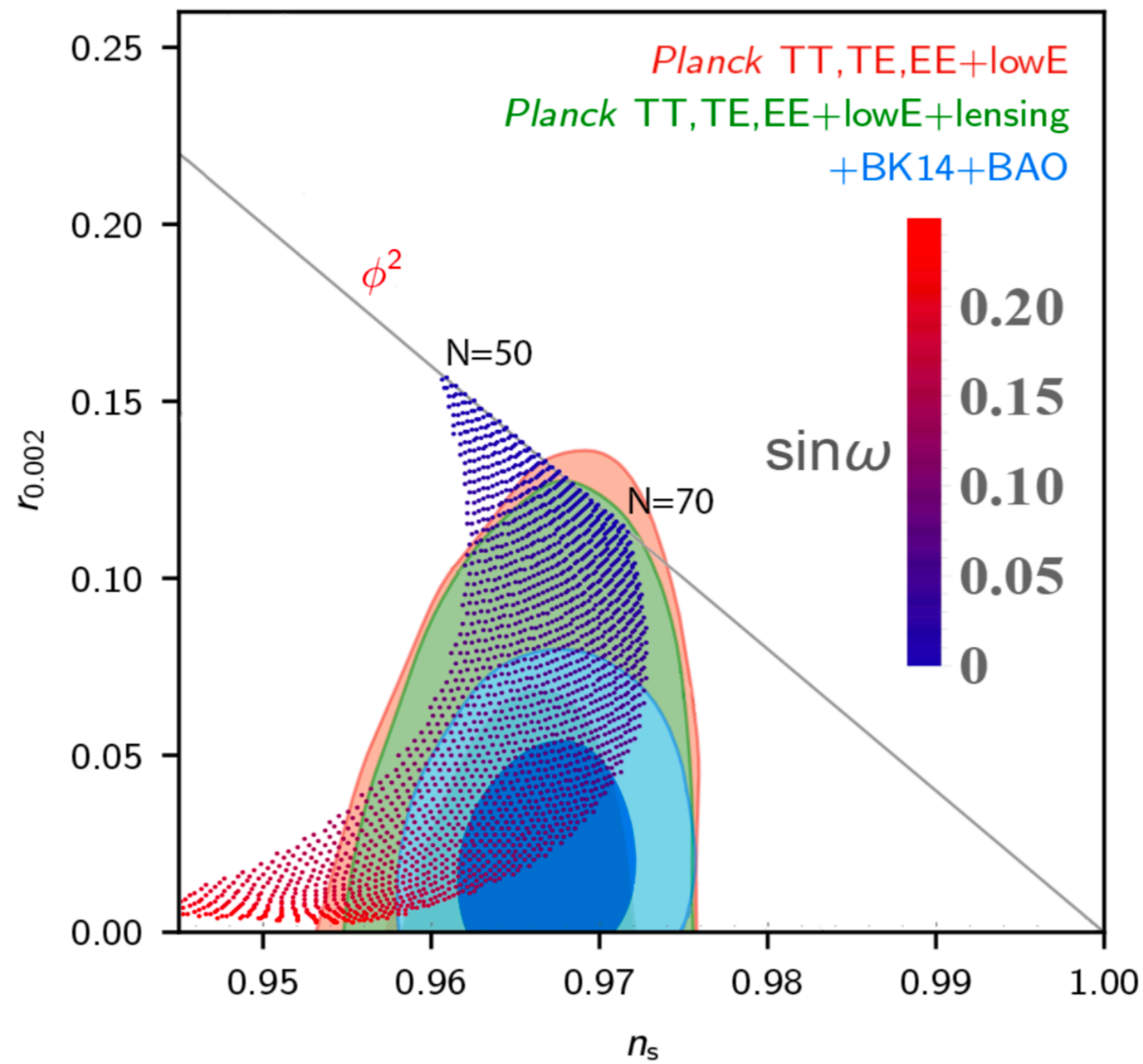
Distinguishable

A. Karam et. al. [arXiv:1810.12884]

Our work
(Non-perturbative)



Coleman-Weinberg mechanism
(Perturbative)



$$\mathcal{L}^J = \sqrt{-\bar{g}}\left[\frac{\xi\phi^2}{2}\bar{R} + \frac{\alpha}{2}\bar{R}^2 - \frac{1}{2}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi - \frac{\lambda_\phi}{4}\phi^4\right]$$

Summary

- Planck-Scalegenesis by scalar condensate.

$$M_{\rm Pl}^2 \propto \langle S^\dagger S \rangle$$

- Inflation by dilaton and scalaron.
  - For large gravitational couplings, R^2 inflation.
  - For small gravitational couplings, linear chaotic inflation.
- The model could be tested by PLANCK.

Prospects

- Quantum gravity…
- Dynamics of the scale invariant scalar-gauge theory above the Planck scale.
    - Ghost problem?

Appendix

Potentials

$$\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} = \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi) - \frac{M_{\rm Pl}^2}{2}B(\chi)R + G(\chi)R^2 + \kappa W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}$$

$$B(\chi) = \left(1+\frac{\chi}{\chi_0}\right)^2\left[1+2\,\ell\ln\left(1+\frac{\chi}{\chi_0}\right)^2\right],$$

$$G(\chi) = \hat{\gamma} - \frac{N_c\beta^2}{32\pi^2}\ln\left(1+\frac{\chi}{\chi_0}\right)^2,$$

$$U(\chi) = U_0\left(1+\frac{\chi}{\chi_0}\right)^4\left[2\ln\left(1+\frac{\chi}{\chi_0}\right)^2 - 1\right] + U_0\,.$$

$$\hat{\gamma} = \gamma - \frac{\beta^2}{2\lambda}\left(\frac{1}{2}+\ell\right) \qquad \ell = \frac{N_c\lambda}{16\pi^2}$$

Trajectory

- Condition $$\left.\frac{\partial V}{\partial \phi}\right|_{(\chi,\phi_*(\chi))} = 0$$

$$\phi_*(\chi) = \sqrt{\frac{3}{2}} M_{\rm Pl} \ln[(1+4A(\chi))B(\chi)]$$

$$B(\chi) = \left(1+\frac{\chi}{\chi_0}\right)^2 \left[1 + 2\,\ell \ln\left(1+\frac{\chi}{\chi_0}\right)^2\right],$$

$$G(\chi) = \hat{\gamma} - \frac{N_c \beta^2}{32\pi^2} \ln\left(1+\frac{\chi}{\chi_0}\right)^2,$$

$$U(\chi) = U_0 \left(1+\frac{\chi}{\chi_0}\right)^4 \left[2\ln\left(1+\frac{\chi}{\chi_0}\right)^2 - 1\right] + U_0\,.$$

$$A(\chi) = \frac{4G(\chi)\,U(\chi)}{B^2(\chi)\,M_{\rm Pl}^4}$$

Effective Lagrangian

- On the trajectory

$$\left.\frac{\mathcal{L}_{\text{eff}}^{E}}{\sqrt{-g}}\right|_{\mathcal{C}} = -\frac{1}{2}M_{\text{Pl}}^{2}R + \frac{1}{2}N_{\sigma}^{2}(\sigma)\, g^{\mu\nu}\partial_{\mu}\sigma\partial_{\nu}\sigma - V_{\text{inf}}(\sigma)$$

$$N_{\sigma}(\sigma) = \frac{1}{(1+4\,A(\sigma))\,B(\sigma)}\left[(1+4\,A(\sigma))\,B(\sigma) + \frac{3}{2}\,M_{\text{Pl}}^{2}\left((1+4\,A(\sigma))\,B'(\sigma) + 4\,A'(\sigma)\,B(\sigma)\right)^{2}\right]^{1/2}.$$

$$V_{\text{inf}}(\sigma) = \frac{U(\sigma)}{(1+4\,A(\sigma))\,B^{2}(\sigma)}.$$

In the limits

- R^2 inflation (large γ)

$$B(\chi) \approx 1, \qquad G(\chi) \approx \hat{\gamma}, \qquad U(\chi) = 0.$$

$$V(\chi, \phi) \approx \frac{M_{\rm Pl}^2}{16\hat{\gamma}}\left(1 - e^{-\Phi(\phi)}\right)^2 \qquad n_s \simeq 1 - \frac{2}{N_e} \qquad r \simeq \frac{12}{N_e^2}$$

- Linear-chaotic inflation (small γ)

$$V(\hat{\sigma}) \approx U_0\left[e^{-\hat{\sigma}/\sigma_0} - 1 + \frac{\hat{\sigma}}{\sigma_0}\right] \qquad n_s \simeq 1 - \frac{3}{2N_e} \quad r \simeq \frac{4}{N_e}$$

Breaking effects

- Perturbative way (Coleman-Weinberg mechanism)
  - Effects of quantum scale anomaly (**hard breaking**)

$$V_{\mathrm{eff}}(\phi) = \frac{\lambda_H}{4}\phi^4 + \sum_\alpha \frac{N_\alpha M_\alpha^4}{64\pi^2}\left(\log\left(\frac{M_\alpha^2}{\mu^2}\right) - C_\alpha\right)$$

$$\Rightarrow v_h^2\phi^2 + \cdots$$ In broken phase

- Non-Perturbative way
  - The mass term (**soft breaking**) is dynamically generated.

$$V_{\mathrm{eff}}(\phi) = M^2\phi^2 + \cdots$$

$$\Rightarrow M^2 v_h \phi + \cdots$$ In broken phase