

HIGGS PAIR PRODUCTION : NLO QCD AND UNCERTAINTIES

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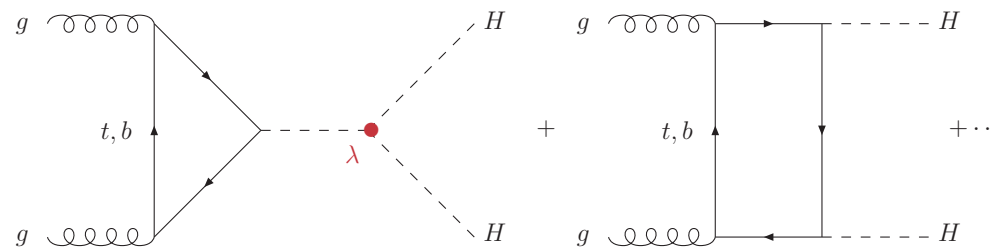
- I Introduction
- II Calculation
- III Conclusions

in collaboration with J. Baglio, F. Campanario, S. Glaus, M. Mühlleit-
ner, J. Streicher

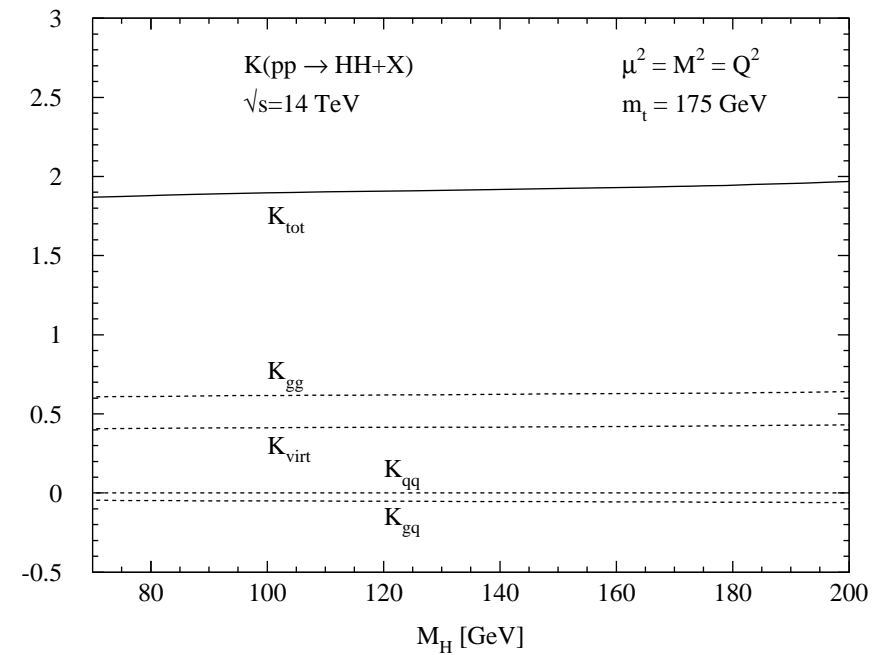
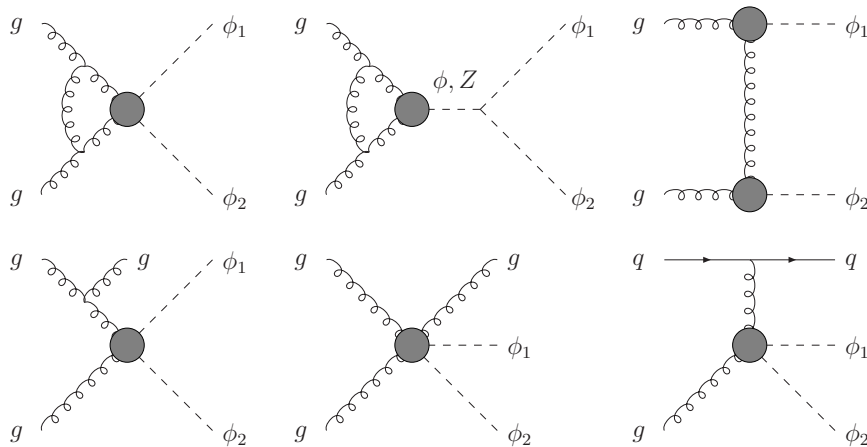
I INTRODUCTION

$gg \rightarrow HH$

SM



- third generation dominant $\rightarrow t, b$
- 2-loop QCD corrections: $\sim 90 - 100\%$
 $[M_H^2 \ll 4m_t^2, \quad \mu = M_{HH}]$



Dawson, Dittmaier, S.

- 2-loop QCD corrections:

$$\sigma = \sigma_0 + \frac{\sigma_1}{m_t^2} + \dots + \frac{\sigma_4}{m_t^8}$$

Grigo, Hoff, Melnikov, Steinhauser

[refinement: full LO at diff. level]

- NLO mass effects @ NLO in real corrections: $\sim -10\%$

Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

→ sizeable virtual mass effects

- NNLO QCD corrections: $\sim 20\%$

$$[M_H^2 \ll 4m_t^2]$$

de Florian, Mazzitelli

Grigo, Melnikov, Steinhauser

- soft gluon resummation: $\sim 10\%$

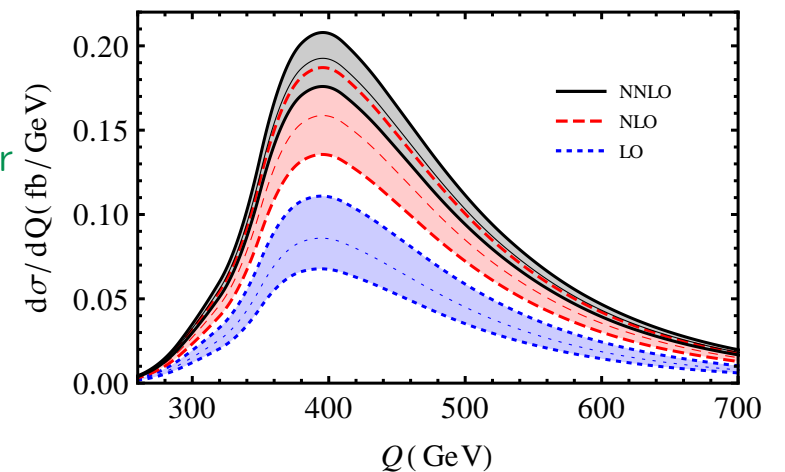
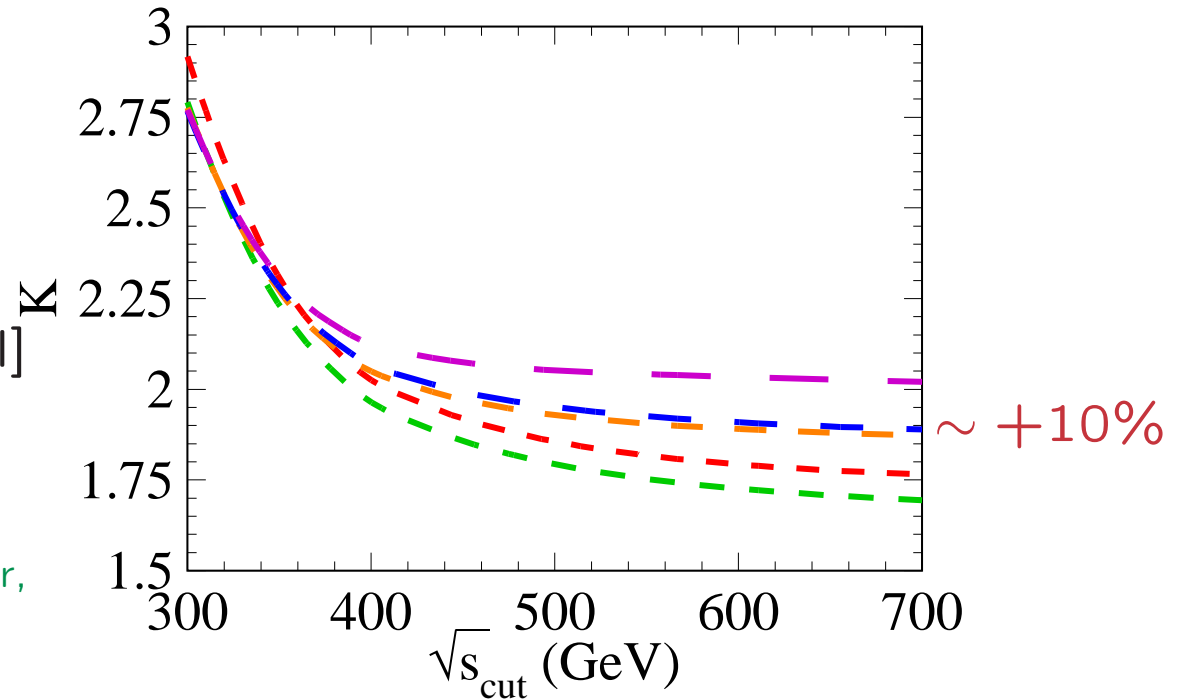
$$[M_H^2 \ll 4m_t^2]$$

Shao, Li, Li, Wang
de Florian, Mazzitelli

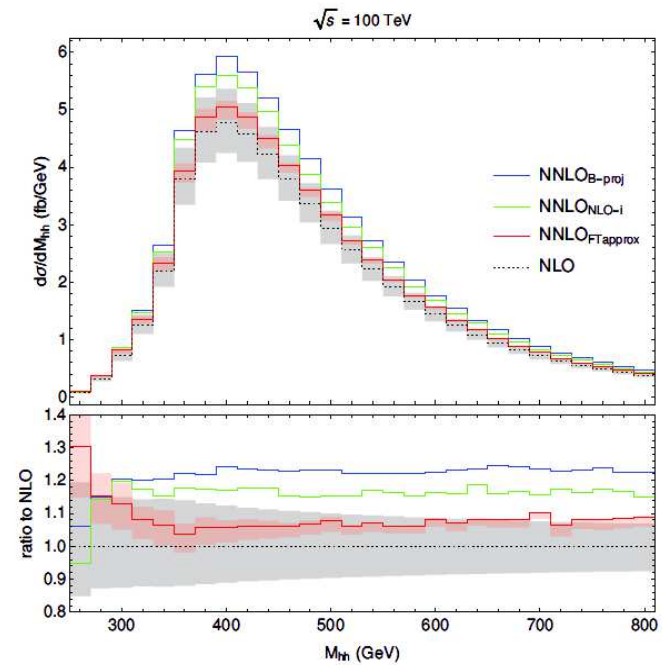
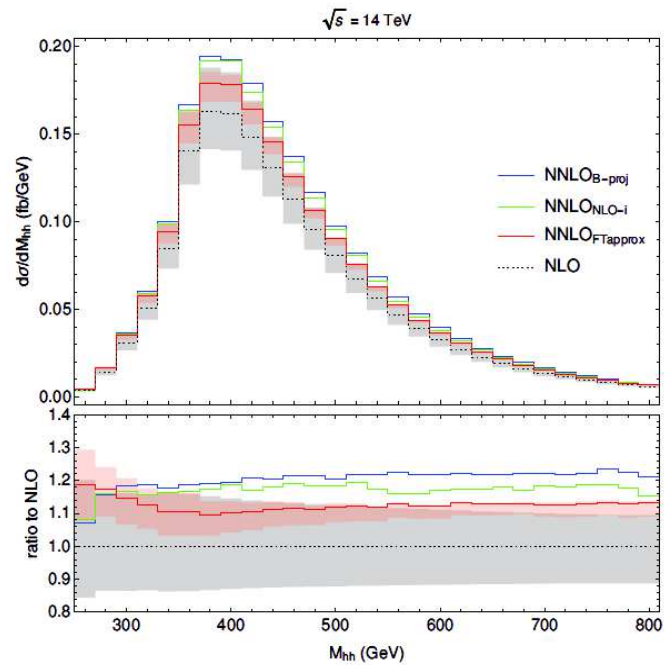
- NLO: small quark mass expansion

$$[Q^2 \gg m_t^2]$$

Davies, Mishima, Steinhauser, Wellmann



- NNLO Monte Carlo: inclusion of full top-mass effects @ NLO



Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli

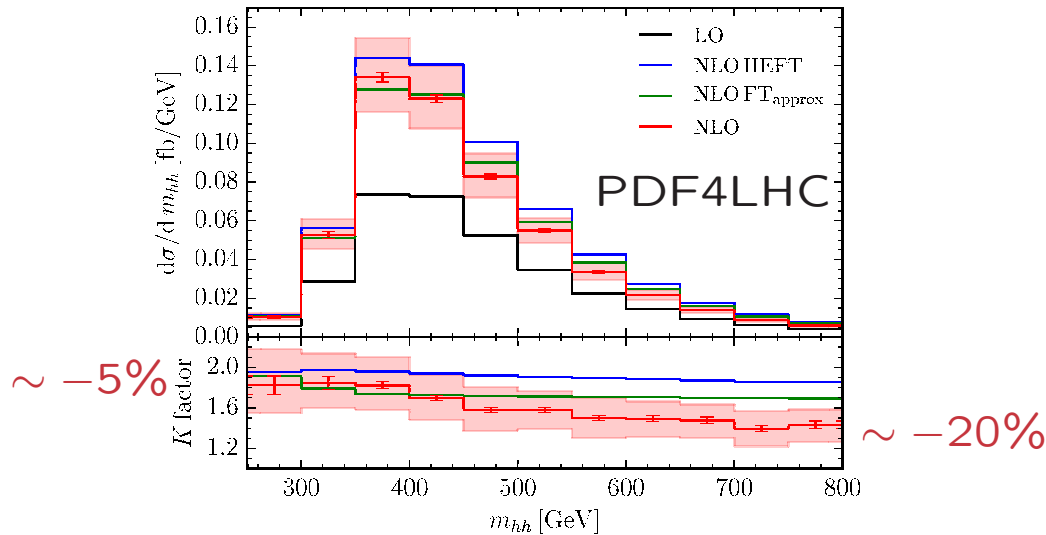
⇒ 20% effects beyond NLO

- NLO: matching to parton showers

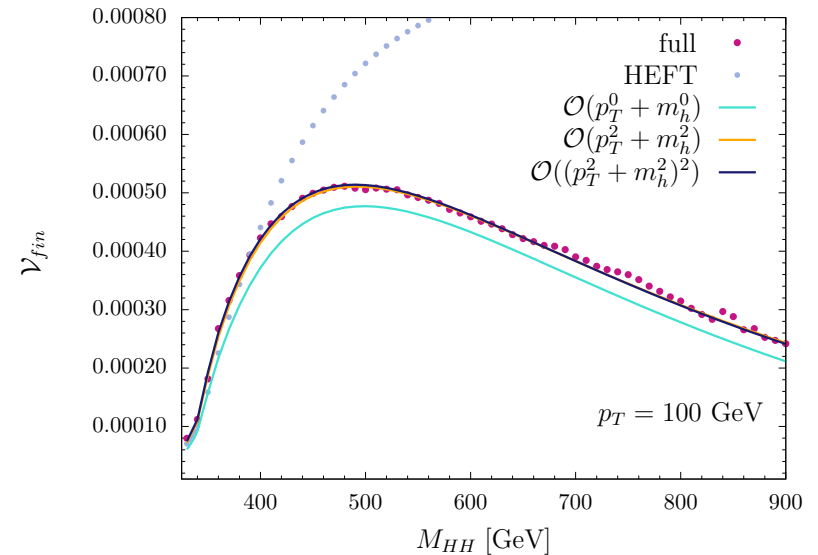
Heinrich, Jones, Kerner, Luisoni, Vryonidou

Full NLO calculation: top only

Numerical integration, sector decomposition, tensor reduction, contour deformation



Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke



Boncianni, Degrassi, Giardino, Gröber

- 14 TeV: ($m_t = 173 \text{ GeV}$) $\sigma_{NLO} = 32.91(10)_{-12.8\%}^{+13.8\%} \text{ fb}$
 $\sigma_{NLO}^{HTL} = 38.75_{-15\%}^{+18\%} \text{ fb}$ (\leftarrow HPAIR)

\Rightarrow -15% mass effects on top of LO

- new expansion/extrapolation methods:

- (i) $1/m_t^2$ expansion + conformal mapping + Padé approximants

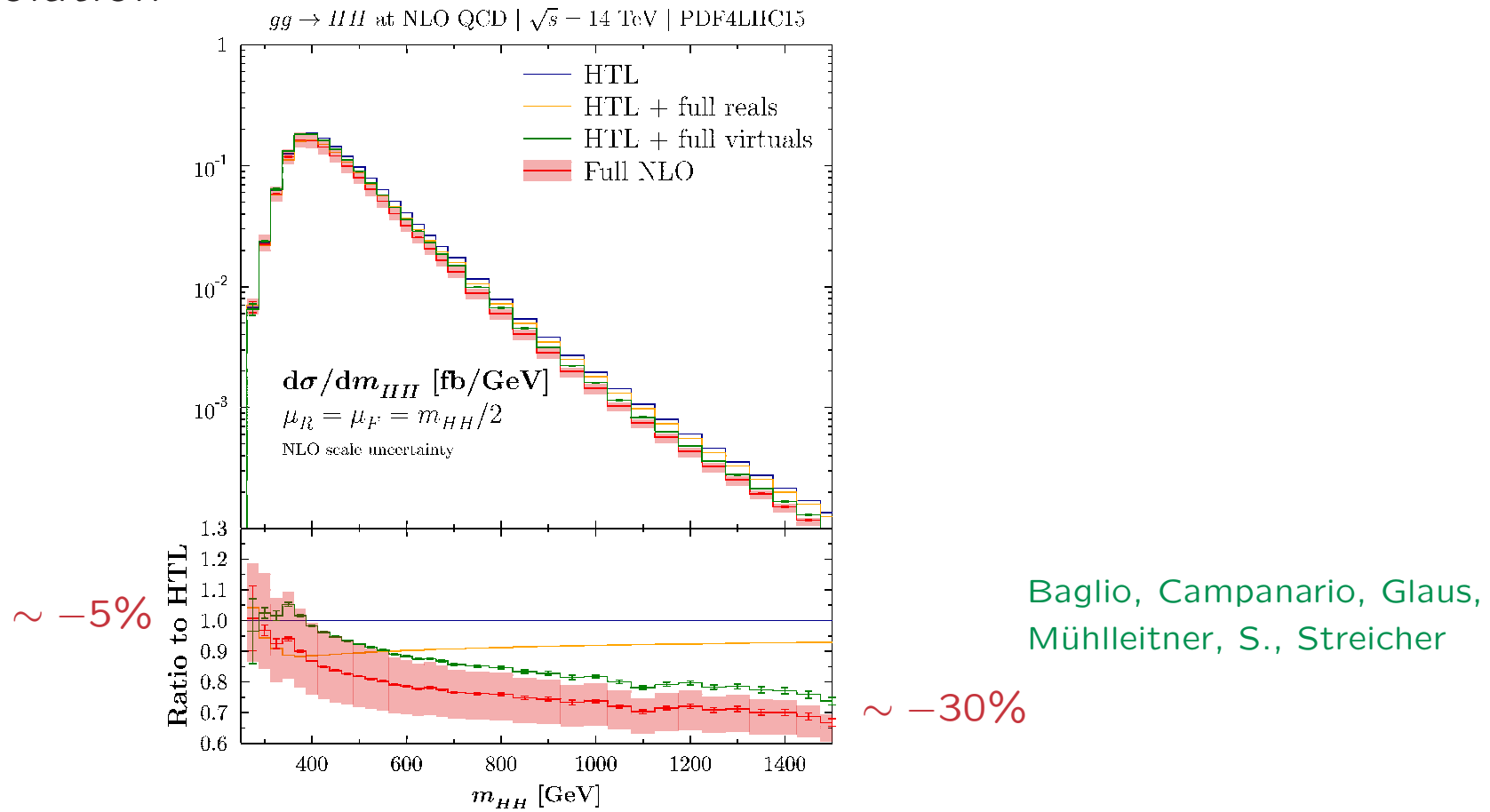
Gröber, Maier, Rauh

- (ii) p_T^2 expansion

Boncianni, Degrassi, Giardino, Gröber

Full NLO calculation: top only

Numerical integration, IR subtraction, no tensor reduction, Richardson extrapolation



- 14 TeV: ($m_t = 172.5$ GeV) $\sigma_{NLO} = 32.78(7)_{-12.5\%}^{+13.5\%}$ fb
 $\sigma_{NLO}^{HTL} = 38.66_{-15\%}^{+18\%}$ fb (← HPAIR)

⇒ -15% mass effects on top of LO

II CALCULATION

$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\sigma_{\text{LO}} = \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s)$$

$$\Delta\sigma_{\text{virt}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \quad C$$

$$\Delta\sigma_{gg} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\ \left. + d_{gg}(z) + 6[1 + z^4 + (1 - z)^4] \left(\frac{\log(1 - z)}{1 - z} \right)_+ \right\}$$

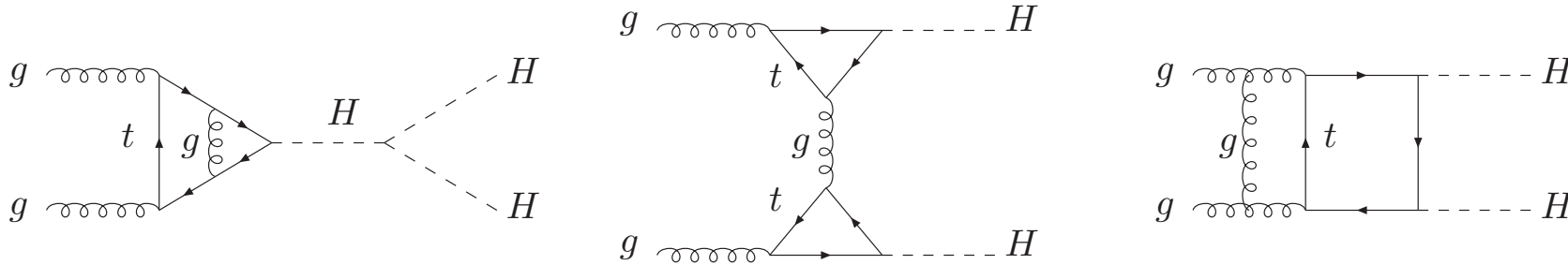
$$\Delta\sigma_{gq} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s(1 - z)^2} + d_{gq}(z) \right\}$$

$$\Delta\sigma_{q\bar{q}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \quad d_{q\bar{q}}(z)$$

$$C \rightarrow \pi^2 + \frac{11}{2} + C_{\Delta\Delta}, \quad d_{gg} \rightarrow -\frac{11}{2}(1 - z)^3, \quad d_{gq} \rightarrow \frac{2}{3}z^2 - (1 - z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27}(1 - z)^3$$

(i) virtual corrections

47 gen. box diags, 8 triangle diags (\leftarrow single Higgs), 1PR ($\leftarrow H \rightarrow Z\gamma$)



- full diagram w/o tensor reduction \rightarrow 6-dim. Feynman integral (2 FF)
- UV-singularities: end-point subtractions

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \frac{f(x) - f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- IR-sing.: IR-subtraction (based on struc. of integr. and rel. to HTL)
- thresholds: $Q^2 \geq 0, 4m_t^2 \rightarrow$ IBP \rightarrow reduction of power of denominator [$m_t^2 \rightarrow m_t^2(1 - ih)$]

$$\int_0^1 dx \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2b} - \frac{f(1)}{2b(a+b)^2} + \int_0^1 dx \frac{f'(x)}{2b(a+bx)^2}$$

- renormalization: α_s : $\overline{\text{MS}}$, 5 flavours
 m_t : on-shell
- PS-integration \rightarrow 7-dim. integrals for $d\sigma/dQ^2$
- subtraction of HTL \rightarrow IR-finite mass effects [adding back HTL results \leftarrow HPAIR]
- extrapolation to NWA ($h \rightarrow 0$): Richardson extrapolation

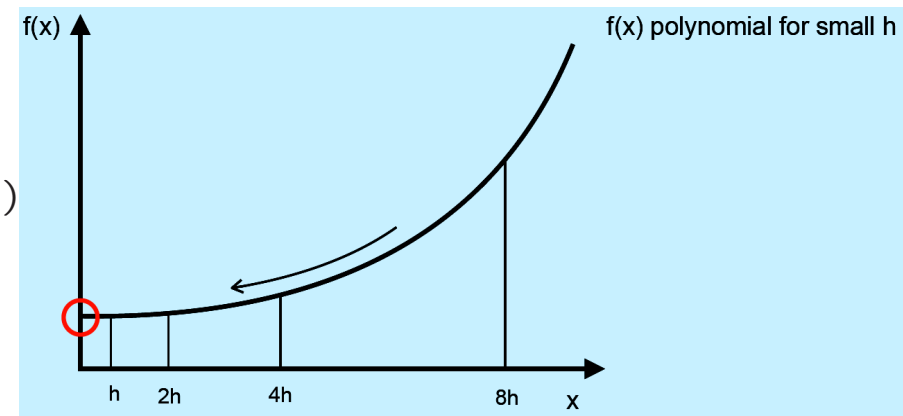
$$M_2 = 2f(h) - f(2h) = f(0) + \mathcal{O}(h^2)$$

$$M_4 = \{8f(h) - 6f(2h) + f(4h)\}/3 = f(0) + \mathcal{O}(h^3)$$

$$M_8 = \{64f(h) - 56f(2h) + 14f(4h) - f(8h)\}/21 = f(0) + \mathcal{O}(h^4)$$

etc.

$[h \geq 0.05]$



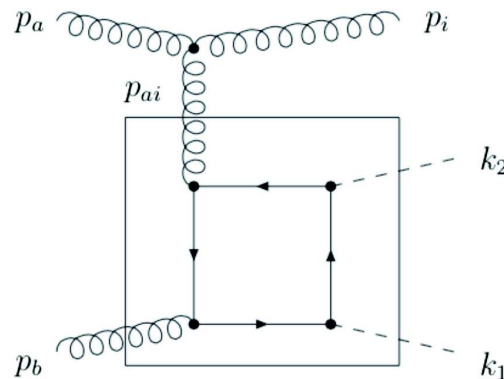
(ii) real corrections

- full matrix elements generated with FeynArts and FormCalc
- matrix elements in HTL involving full LO sub-matrix elements subtracted \rightarrow IR-, COLL-finite [adding back HTL results \leftarrow HPAIR]

$$\sum \overline{|\mathcal{M}_{gg}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{24\pi^2 \alpha_s}{Q^4 \pi} \left\{ \frac{s^4 + t^4 + u^4 + Q^8}{stu} - 4 \frac{\epsilon}{1-\epsilon} Q^2 \right\}$$

$$\sum \overline{|\mathcal{M}_{gq}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{32\pi^2 \alpha_s}{3Q^4 \pi} \left\{ \frac{s^2 + u^2}{-t} + \epsilon \frac{(s+u)^2}{t} \right\}$$

$$\sum \overline{|\mathcal{M}_{q\bar{q}}|^2} = \sum \overline{|\tilde{\mathcal{M}}_{LO}|^2} \frac{256\pi^2 \alpha_s}{9Q^4 \pi} (1-\epsilon) \left\{ \frac{t^2 + u^2}{s} - \epsilon \frac{(t+u)^2}{s} \right\}$$

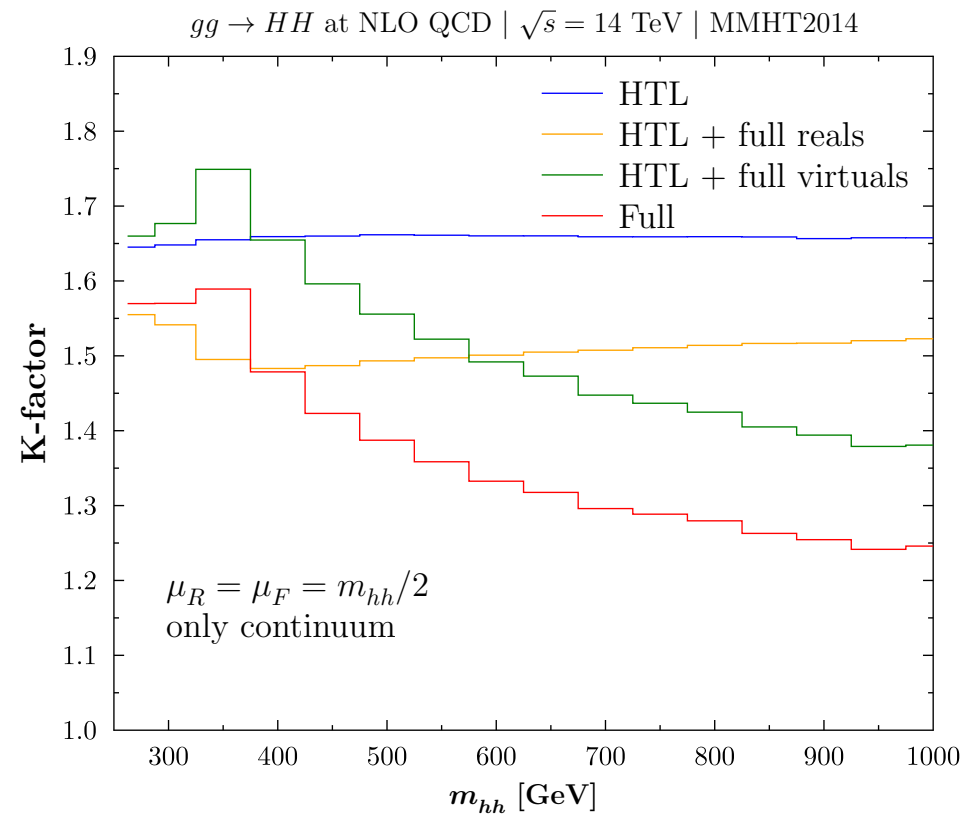
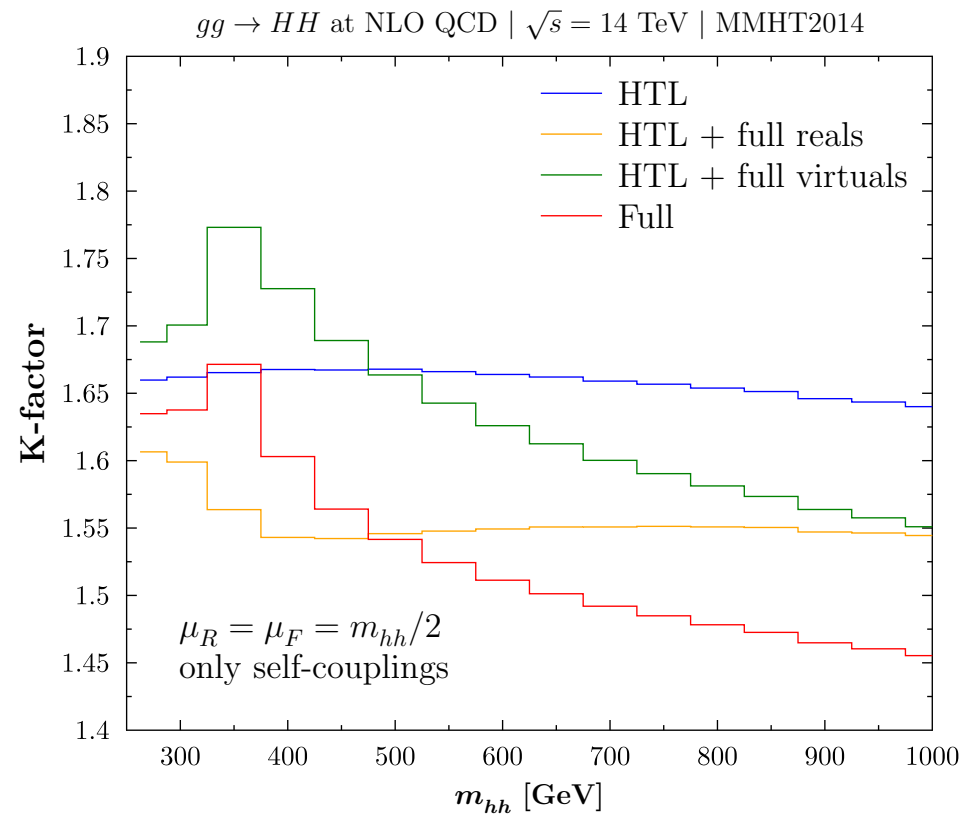


(iii) [results](#)

	PDF4LHC15	MMHT2014
σ_{LO}	19.80 fb	23.75 fb
σ_{NLO}^{HTL}	38.66 fb	39.34 fb
σ_{NLO}	32.78(7) fb	33.33(7) fb

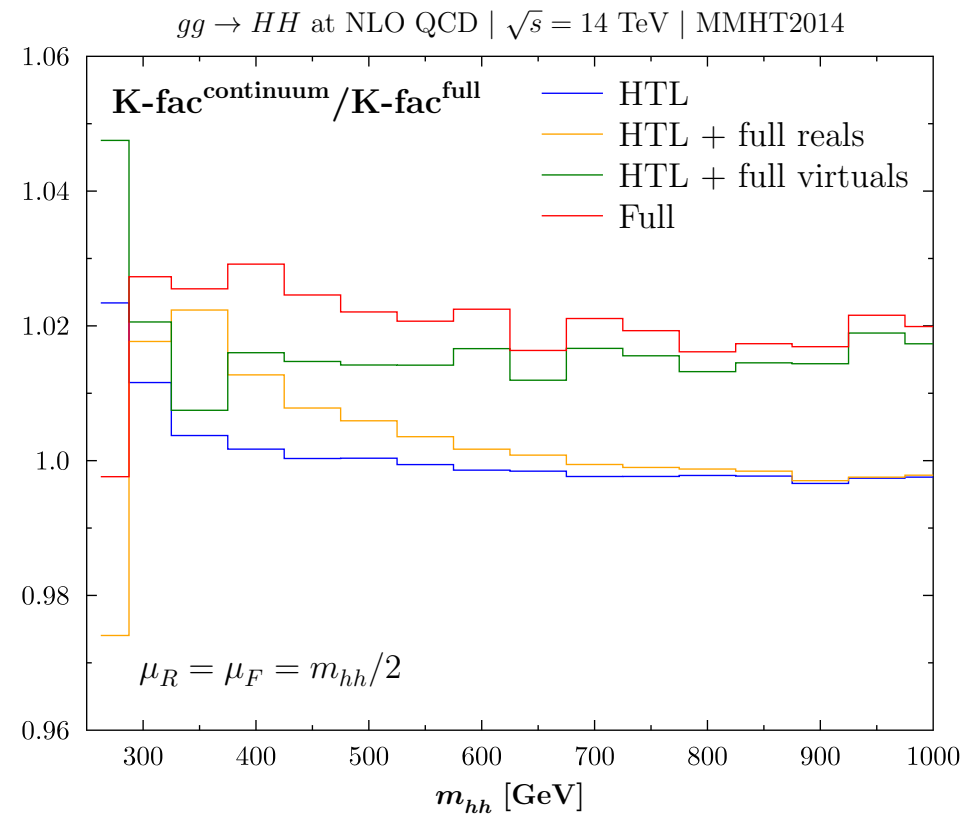
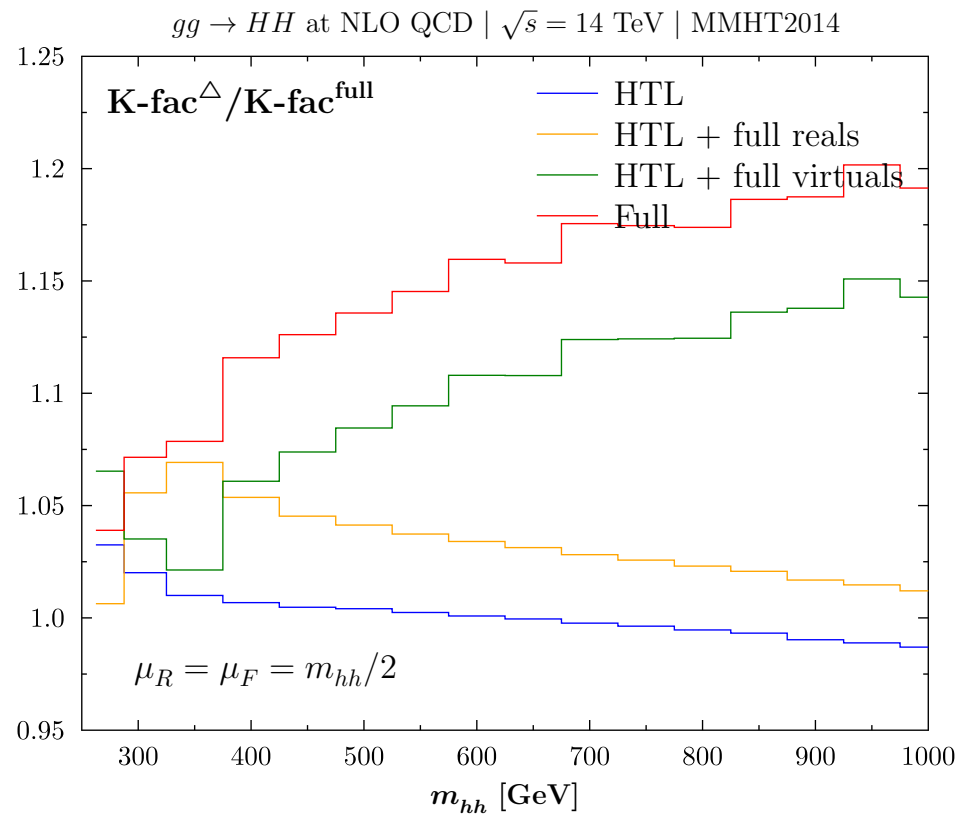
(iv) individual corrections

- take K-factors for triangle (\leftarrow single Higgs) and box diagrams separately: total K-factor approximated by single Higgs K-factor?



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(v) uncertainties due to m_t

- transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$)

→ modification of mass CT

- use $m_t, \overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=300 \text{ GeV}} = 0.031(1)_{-22\%}^{+10\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=400 \text{ GeV}} = 0.1609(4)_{-7\%}^{+7\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=600 \text{ GeV}} = 0.03204(9)_{-26\%}^{+0\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=1200 \text{ GeV}} = 0.000435(4)_{-30\%}^{+0\%} \text{ fb/GeV}$$

- **preliminary** interpolation:

$$\sigma(gg \rightarrow HH) = 32.78_{-25\%}^{+2\%} \text{ fb} \quad (\text{very preliminary})$$

(v) uncertainties due to m_t for single Higgs

- transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$)

→ modification of mass CT

- use $m_t, \overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\sigma(gg \rightarrow H)|_{M_H=125 \text{ GeV}} = 42.17^{+0.4\%}_{-0.5\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=300 \text{ GeV}} = 9.85^{+7.5\%}_{-0.3\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=400 \text{ GeV}} = 9.43^{+0.1\%}_{-0.9\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=600 \text{ GeV}} = 1.97^{+0.0\%}_{-15.9\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=900 \text{ GeV}} = 0.230^{+0.0\%}_{-22.3\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=1200 \text{ GeV}} = 0.0402^{+0.0\%}_{-26.0\%} \text{ pb}$$

III CONCLUSIONS

- Higgs pair production at full NLO for variable top/Higgs masses [top loops]
- top mass effects on top of LO up to 20–30%
- factorization/renormalization scale uncertainties $\sim 15\%$
- uncertainties due to scale/scheme choice of m_t sizeable $\lesssim 30\%$

BACKUP SLIDES

- pole mass \leftrightarrow $\overline{\text{MS}}$ mass:

$$\overline{m}_t(M_t) = \frac{M_t}{1 + \frac{4\alpha_s(M_t)}{3\pi} + 10.9 \left(\frac{\alpha_s(M_t)}{\pi}\right)^2}$$

$$\overline{m}_t(\mu) = \overline{m}_t(M_t) \frac{c[\alpha_s(\mu)/\pi]}{c[\alpha_s(M_t)/\pi]}$$

$$c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}} [1 + 1.398x + 1.793x^2 - 0.6834x^3]$$

$$M_t = 172.5 \text{ GeV}$$

$$\overline{m}_t(\overline{m}_t) = 163.015161017019 \text{ GeV}$$

M_HH	mt (M_HH/4)	mt (M_HH/2)	mt (M_HH)
125	189.209370262526	176.772460597358	166.501914700149
260	176.139964023672	165.972836934324	156.889554725476
275	175.247098219568	165.224863654266	156.188624671063
300	173.888433241807	164.084218616097	155.118481503625
350	171.556916171559	162.101622772544	153.272150436136
375	170.543285547792	161.158290295641	152.465560631846
400	169.611142167793	160.289697463114	151.721739637882
500	166.501914700149	157.384965182267	149.226383426185
600	164.084218616097	155.118481503625	147.270941230420
700	162.101622772544	153.272150436136	145.672596390682
800	160.289697463114	151.721739637882	144.326704798025
900	158.737886290123	150.390138497802	143.168060367441
1000	157.384965182267	149.226383426185	142.153427561240
1100	156.188624671063	148.195135247933	141.252743160739
1200	155.118481503625	147.270941230420	140.444302478362
1300	154.152026867353	146.434896300904	139.711950260189
1400	153.272150436136	145.672596390682	139.043354388391
1500	152.465560631846	144.972828986822	138.428898934501

- triangles for $M_H^2 \gg 4M_t^2$:

$$C \rightarrow \frac{C_A - C_F}{12} \left[\log \frac{M_H^2}{m_t^2} - i\pi \right]^2 - C_F \left[\log \frac{M_H^2}{m_t^2} - i\pi \right] + 3C_F \log \frac{\mu_t^2}{m_t^2}$$

