Leptogenesis and the baryon asymmetry of the universe

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Why going beyond the SM?

Even ignoring:
- (more or less) compelling theoretical motivations (quantum gravity theory, flavour problem, hierarchy problem, naturalness(?),...) and
- Experimental anomalies in $(g-2)_\mu$, B decays, ...

The SM cannot explain:

- **Cosmological Puzzles**
  1. Dark matter
  2. Matter - antimatter asymmetry
  3. Inflation
  4. Accelerating Universe

- **Neutrino masses and mixing**
Bridging neutrino physics and cosmology

**Cosmology (early Universe)**

1. Matter - antimatter asymmetry

\[ \eta_B \approx 6.1 \times 10^{-10} \]

- New stage in early Universe history:

  - \( T_{RH} \)??
  - Inflation
  - Leptogenesis
  - EWSSB
  - 100 GeV
  - BBN
  - 0.1-1 MeV
  - Recombination
  - 0.1-1 eV

**Neutrino Physics**

Leptogenesis complements low energy neutrino experiments testing the seesaw high energy parameters and providing a guidance toward the model underlying the seesaw
Baryon asymmetry of the universe

(Hu, Dodelson, astro-ph/0110414)

\[ \Omega_{B0} h^2 = 0.02237 \pm 0.00015 \]

\[ \eta_{B0} \equiv \frac{n_{B0} - \bar{n}_{B0}}{n_{\gamma0}} \approx \frac{n_{B0}}{n_{\gamma0}} \approx 273.5 \Omega_{B0} h^2 \times 10^{-10} = (6.12 \pm 0.04) \times 10^{-10} \]

- Consistent with (older) BBN determination but more precise and accurate
- Asymmetry coincides with matter abundance since there is no evidence of primordial antimatter...not so far at least (see AMS-02 results and Poulin, Salati, Cholis, Kamionkowski, Silk 1808.08961)

(Planck 2018, 1807.06209)

(68% CL, TT, TE, EE+lowE+lensing)
Neutrino masses \( (m_1' < m_2' < m_3') \)

\[
\begin{align*}
\text{NO: } m_2 &= \sqrt{m_1^2 + m_{\text{sol}}^2}, \quad m_3 = \sqrt{m_1^2 + m_{\text{atm}}^2} \\
\text{IO: } m_2' &= \sqrt{m_1^2 + m_{\text{atm}}^2 - m_{\text{sol}}^2}, \quad m_3' = \sqrt{m_1^2 + m_{\text{atm}}^2}
\end{align*}
\]

\[
m_{\text{sol}} = (8.6 \pm 0.1) \text{ meV}
\]

\[
m_{\text{atm}} = (49.9 \pm 0.3) \text{ meV}
\text{ (\nu fit 2018)}
\]

\[
m_1' \leq 0.07 \text{ eV} \text{ (95\%C.L.)}
\text{ (Planck 2015)}
\]
Neutrino mixing parameters

Pontecorvo-Maki-Nakagawa-Sakata matrix

\[
U_{\alpha i} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
e^{i\rho} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{i\sigma}
\end{pmatrix}
\]

Atmospheric, LB

Reactors, Accel., LB

CP violating phase

Solar, Reactor

bb0ν decay

Asymmetry of the leptonic mixing matrix can be parameterized in

\[a_{31} = 2(\sigma - \rho)\]

\[a_{21} = -2\rho\]

3σ ranges (NO):

\[\theta_{12} = [31.42°, 36.05°]\]

\[\theta_{13} = [8.09°, 8.98°]\]

\[\theta_{23} = [40.3°, 51.5°]\]

\[\delta = [-216°, +14°]\]

\[\rho, \sigma = [-180°, +180°]\]

3σ ranges (IO):

\[\theta_{12} = [31.43°, 36.06°]\]

\[\theta_{13} = [8.14°, 9.01°]\]

\[\theta_{23} = [38°, 53°]\]

\[\delta = [-168°, -6°]\]

\[\rho, \sigma = [-180°, +180°]\]

(νFIT collaboration, January 2018)

\[c_{ij} = \cos \theta_{ij}, \text{and } s_{ij} = \sin \theta_{ij}\]

NO favoured over IO

\[(\Delta x^2)_{(IO-NO)} = 4.14 \Rightarrow \sim 2\sigma\]
Minimally extended SM

\[ \mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_Y^\nu \]

\[ -\mathcal{L}_Y^\nu = \bar{\nu}_L h^\nu \nu_R \phi \Rightarrow -\mathcal{L}_{\text{mass}} = \bar{\nu}_L m_D \nu_R \]

(in a basis where charged lepton mass matrix is diagonal)

diagonalising \( m_D \):

\[ m_D = V_L^\dagger D m_D U_R \]

\[ D_{m_D} \equiv \begin{pmatrix} m_{D1} & 0 & 0 \\ 0 & m_{D2} & 0 \\ 0 & 0 & m_{D3} \end{pmatrix} \]

\[ \Rightarrow \]

neutrino masses:

\[ m_i = m_{Di} \]

leptonic mixing matrix:

\[ U = V_L^\dagger \]

Too many unanswered questions:

- Why neutrinos are much lighter than all other fermions?
- Why large mixing angles?
- Cosmological puzzles?
- Why not a Majorana mass term as well?
**Minimal seesaw mechanism (type I)**

- Dirac + (right-right) Majorana mass terms

(Minkowski '77; Gell-mann, Ramond, Slansky; Yanagida; Mohapatra, Senjanovic '79)

\[-\mathcal{L}_\text{mass}^\nu = \overline{\nu_L} m_D \nu_R + \frac{1}{2} \overline{\nu_R^c} M \nu_R + \text{h.c.}\]

In the see-saw limit \((M \gg m_D)\) the mass spectrum splits into 2 sets:

- 3 light Majorana neutrinos with masses (seesaw formula):

\[
\text{diag}(m_1, m_2, m_3) = -U^\dagger m_D \frac{1}{M} m_D^T U^*
\]

- 3(?) very heavy Majorana neutrinos \(N_1, N_2, N_3\) with masses \(M_3 > M_2 > M_1 \gg m_D\)

**1 generation toy model \((U=1)\):**

- \(m_D \sim m_{\text{top}}\)

- \(M \sim M_{\text{GUT}} \sim 10^{16}\) GeV

\[\Rightarrow m \sim m_{\text{atm}} \sim 50\ \text{meV}\]
Minimal scenario of leptogenesis

(Fukugita, Yanagida ’86)

- Type I seesaw mechanism
- Thermal production of RH neutrinos: $T_{RH} \geq T_{lep} \approx M_i / (2\div10)$
  
  heavy neutrinos decay
  
  $$N_i \xrightarrow{\Gamma} L_i + \phi^+$$
  $$N_i \xrightarrow{\bar{\Gamma}} \bar{L}_i + \phi$$

  total CP asymmetries
  
  $$\epsilon_i \equiv -\frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$
  $$\Rightarrow N_{B-L}^{fin} = \sum_{i=1,2,3} \epsilon_i \times K_i^{fin}$$

- Sphaleron processes in equilibrium
  
  $$\Rightarrow T_{lep} \geq T_{sphalerons} \sim 100 \text{ GeV}$$
  
  (Kuzmin, Rubakov, Shaposhnikov ’85)

$$\eta_{B0}^{lep} = \frac{a_{sph}}{N_{B-L}^{rec}} \frac{N_{B-L}^{fin}}{N_{\gamma}} \approx 0.01 N_{B-L}^{fin}$$

$\Delta B = \Delta L = 3$
Seesaw parameter space

Imposing $\eta_{lep}^B \approx \eta_{CMB}^B \approx 6 \times 10^{-10} \Rightarrow$ can we test seesaw and leptog.? 

Problem: too many parameters

(Casas, Ibarra'01) $m_\nu = -m_D \frac{1}{M} m_T^T \Leftrightarrow \Omega^T \Omega = I$

Orthogonal parameterisation

(in a basis where charged lepton and Majorana mass matrices are diagonal)

light neutrino parameters

heavy neutrino parameters escaping experimental information

Popular solution: “low-scale” leptogenesis, though no signs so far of new physics at the TeV scale or below able to explain $\eta_{B0}$

Insisting with high scale leptogenesis is challenging but there are a few strategies able to reduce the number of parameters in order to obtain testable predictions on low energy neutrino parameters
Vanilla leptogenesis $\Rightarrow$ upper bound on $\nu$ masses

(Buchmüller, PDB, Plümacher '04; Blanchet, PDB '07)

1) Lepton flavor composition is neglected

$$N_i \rightarrow \Gamma \ell_i + \phi^\dagger \quad N_i \rightarrow \tilde{\ell}_i + \phi$$

2) Hierarchical spectrum ($M_2 \geq 2M_1$)

3) Strong lightest RH neutrino wash-out

$$\eta_{B0} \approx 0.01N^{final}_{B-L} \approx 0.01\varepsilon_1\kappa_{1}^{fin}(K_1,m_1)$$

decay parameter: $K_1 \equiv \frac{\Gamma_{N_1}(T=0)}{H(T=M_1)}$

All the asymmetry is generated by the lightest RH neutrino

4) Barring fine-tuned cancellations

(Davidson, Ibarra '02)

$$\varepsilon_1 \leq \varepsilon_1^{max} \approx 10^{-6} \left( \frac{M_1}{10^{10} \text{ GeV}} \right) \frac{m_{\text{atm}}}{m_1 + m_3}$$

$$\eta_B^{max}(m_1, M_1) \geq \eta_{B}^{CMB}$$

$m_1 < 0.12 \text{ eV}$

No dependence on the leptonic mixing matrix $U$: it cancels out
Independence of the initial conditions (strong thermal leptogenesis)

(Buchmüller, PDB, Plümacher '04)

wash-out of a pre-existing asymmetry $N_{B-L}^{P,\text{initial}}$

$$N_{B-L}^{P,\text{final}} = N_{B-L}^{P,\text{initial}} e^{-\frac{3\pi}{8} K_1} \ll N_{B-L}^{f,\text{N}_1}$$

decay parameter:

$$K_1 \equiv \frac{\Gamma_{N_1}}{H(T=M_1)} \sim \frac{m_{\text{sol, atm}}}{m_*} \sim 10^{-3} \text{eV} \sim 10 \div 50$$

equilibrium neutrino mass:

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} \frac{\sqrt{g_*}}{M_{\text{Pl}}} \nu^2 \approx 1.08 \times 10^{-3} \text{eV}.$$
Beyond vanilla Leptogenesis

Vanilla Leptogenesis

Degenerate limit, resonant leptogenesis

Non minimal Leptogenesis:
SUSY, non thermal, in type II, III, inverse seesaw, doublet Higgs model, soft leptogenesis, from RH neutrino mixing (ARS), Dirac lep.,....

Improved Kinetic description
(momentum dependence, quantum kinetic effects, finite temperature effects, ...., density matrix formalism)

Flavour Effects
(heavy neutrino flavour effects, charged lepton flavour effects and their interplay)
Flavor composition of lepton quantum states matters!

\[ |l_1\rangle = \sum_\alpha \langle l_\alpha | l_1 \rangle |l_\alpha\rangle \quad (\alpha = e, \mu, \tau) \]

\[ |\overline{l}_1\rangle = \sum_\alpha \langle l_\alpha | \overline{l}_1 \rangle |\overline{l}_\alpha\rangle \]

\( T \ll 10^{12} \text{ GeV} \Rightarrow \tau\text{-Yukawa interactions are fast enough break the coherent evolution of } |l_1\rangle \text{ and } |\overline{l}_1\rangle \)

\( \Rightarrow \text{incoherent mixture of a } \tau \text{ and of a } \infty+e \text{ components } \Rightarrow 2\text{-flavour regime} \)

\( T \ll 10^9 \text{ GeV} \text{ then also } \infty\text{-Yukawas in equilibrium } \Rightarrow 3\text{-flavour regime} \)

\[ N_{B-L}^{\text{final}} = \varepsilon_1 \kappa_1^{\text{fin}} \]

\[ 2 \text{ Flavour regime (} \tau, e+\mu\text{)} \]

\[ \varepsilon_{1\tau} \kappa_1^{\text{fin}} (K_{1\tau}) + \varepsilon_{1e+\mu} \kappa_1^{\text{fin}} (K_{1e+\mu}) \]

\[ 3 \text{ Flavour regime (} e, \mu, \tau\text{)} \]

\[ \varepsilon_{1\tau} \kappa_1^{\text{fin}} (K_{1\tau}) + \varepsilon_{1\mu} \kappa_1^{\text{fin}} (K_{1\mu}) + \varepsilon_{1e} \kappa_1^{\text{fin}} (K_{1e}) \]
Heavy neutrino lepton flavour effects: 10 scenarios

Heavy neutrino flavored scenario

Typically rising in discrete flavour symmetry models

\[ M_i \]

\( \sim 10^{12} \text{ GeV} \)

\( \sim 10^{9} \text{ GeV} \)

\( N_2 \)-dominated scenario:

- \( N_1 \) produces negligible asymmetry;
- It emerges naturally in SO(10)-inspired models;

Example: ARS leptog, (Drewes et al.1711.02862)
What determines the RH neutrino mass spectrum?

In the flavour basis (both charged lepton mass and Majorana mass matrices are diagonal):

\[
-L^\nu+\ell_{\text{mass}} = \bar{\alpha}_L m_\alpha \alpha_R + \bar{\nu}_L m_{D\alpha I} \nu_{RI} + \frac{1}{2} \nu^c_{RI} M_I \nu_{RI} + \text{h.c.}
\]

diagonalising again \( m_D \) with a bi-unitary transformation:

\[
m_D = V_L^T D_{m_D} U_R
\]

The seesaw formula becomes:

\[
U D_m U^T = V_L^T D_{m_D} U_R \frac{1}{D_{m_D}} U_R^T D_{m_D} V_L^*
\]

\[
D_m \equiv \text{diag}(m_1, m_2, m_3) \quad D_{m_D} \equiv \text{diag}(m_{D1}, m_{D2}, m_{D3}) \quad D_M \equiv \text{diag}(M_1, M_2, M_3)
\]

AN EASY LIMIT (typically realised imposing a flavour symmetry):

- \( U_R = I \) \( \implies \) again \( U = V_L^T \) but neutrino masses:

\[
m_i = \frac{m^{2}_{Di}}{M_I}
\]

If also \( m_{D1} = m_{D2} = m_{D3} = \lambda \) then simply:

\[
M_I = \frac{\lambda^2}{m_i}
\]
A less easy limit: SO(10)-inspired models

\[
UD_m U^T = V_L^d D_{mD} U_R \frac{1}{D_M} U_R^T D_{mD} V_L^*
\]

\[D_m \equiv \text{diag}(m_1, m_2, m_3) \quad D_{mD} \equiv \text{diag}(m_{D1}, m_{D2}, m_{D3}) \quad D_M \equiv \text{diag}(M_1, M_2, M_3)\]

- \[V_L = I \implies M_1 = \frac{m_{D1}^2}{m_{\beta\beta}}; \quad M_2 = \frac{m_{D2}^2}{m_1 m_2 m_3} \left| (m^{-1}_\nu)^{\tau\tau} \right|; \quad M_3 = m_{D3}^2 \left| (m^{-1}_\nu)^{\tau\tau} \right|\]

If also: \[m_{D1} = \alpha_1 m_{up}; \quad m_{D2} = \alpha_2 m_{charm}; \quad m_{D3} = \alpha_3 m_{top}; \quad \alpha_i = O(1)\]

Barring fine-tuned solutions, one obtains a very hierarchical RH neutrino mass spectrum requiring \[N_2\] leptogenesis: DOES IT WORK?

The analytical expressions for the \[M_i\] 's can be nicely extended for a generic \[V_L\]
The $N_2$-dominated scenario

- **Unflavoured case:** asymmetry produced from $N_2$ - RH neutrinos is typically washed-out
  \[ \eta_{B0}^{lep(N_2)} \approx 0.01 \cdot \varepsilon_2 \cdot \kappa^{fin}(K_2) \cdot e^{-\frac{3\pi}{8}K_1} << \eta_{B0}^{CMB} \]

- **Adding flavour effects:** lighest RH neutrino wash-out acts on individual flavour ⇒ much weaker

- With flavor effects the domain of successful $N_2$ dominated leptogenesis greatly enlarges: the probability that $K_1 < 1$ is less than 0.1% but the probability that either $K_{1e}$ or $K_{1\mu}$ or $K_{1\tau}$ is less than 1 is ~30%!

- Existence of the heaviest RH neutrino $N_3$ is necessary for the $\varepsilon_{2a}$'s not to be negligible

- It is the only hierarchical scenario that can realise strong thermal leptogenesis (independence of the initial conditions) if the asymmetry is tauon-dominated and if $m_1 \gtrsim 10$ meV (corresponding to $\Sigma_i m_i \gtrsim 80$meV)
SO(10)-inspired leptogenesis is predictive

(PDB, Riotto 0809.2285;1012.2343;He,Lew,Volkas 0810.1104)

- $I \leq V_L \leq V_{CKM}$
- dependence on $\alpha_1$ and $\alpha_3$ cancels out $\Rightarrow$ only on $\alpha_2 \equiv \frac{m_{D2}}{m_{charm}}$

$\alpha_2 = 5$

$\alpha_2 = 4$

$\alpha_2 = 1$

NORMAL ORDERING

- Lower bound $m_1 \gtrsim 10^{-3}$ eV
- $\Theta_{23}$ preferred in the first octant
- Majorana phases constrained about specific regions

- only marginal allowed regions for INVERTED ORDERING
It is possible to lower $T_{RH}$ to values consistent with the gravitino problem for $m_g \gtrsim 30$ TeV (Kawasaki, Kohri, Moroi, 0804.3745)

Alternatively, for lower gravitino masses, one has to consider non-thermal SO(10)-inspired leptogenesis (Blanchet, Marfatia 1006.2857)
Can SO(10)-inspired leptogenesis be strong?

(PDB, Marzola, Re Fiorentin 1411.5478)
Strong SO(10)-inspired leptogenesis confronting long baseline experiments (PDB, Marco Chianese 1802.07690)

Pre-existing initial asymmetry: \( N^{p,i}_{B-L} = 10^{-3} \)

\[ \alpha_2 = \frac{m_D}{m_{charm}} = 5 \]

\[ \alpha_2 = \frac{m_D}{m_{charm}} = 6 \]

\( \delta = -75^\circ \)
SO(10)-inspired leptogenesis confronting long baseline and absolute neutrino mass experiments
A popular class of SO(10) models


In SO(10) models each SM particles generation + 1 RH neutrino are assigned to a single 16-dim representation. Masses of fermions arise from Yukawa interactions of two 16s with vevs of suitable Higgs fields. Since:

\[16 \otimes 16 = 10_S \oplus \overline{126}_S \oplus 120_A,\]

The Higgs fields of renormalizable SO(10) models can belong to 10-, 126-, 120-dim representations yielding Yukawa part of the Lagrangian

\[\mathcal{L}_Y = 16 \left( Y_{10} Y_{10}^H + Y_{126} Y_{126}^H + Y_{120} Y_{120}^H \right) 16.\]

After SSB of the fermions at \(M_{\text{GUT}} = 2 \times 10^{16}\) GeV one obtains the masses:

- **up-quark mass matrix**
  \[
  M_u = v^u_{10} y_{10} + v^u_{126} y_{126} + v^u_{120} y_{120},
  \]

- **down-quark mass matrix**
  \[
  M_d = v^d_{10} y_{10} + v^d_{126} y_{126} + v^d_{120} y_{120},
  \]

- **neutrino mass matrix**
  \[
  M_D = v^u_{10} y_{10} - 3 v^u_{126} y_{126} + v^D_{120} y_{120},
  \]

- **charged lepton mass matrix**
  \[
  M_l = v^l_{10} y_{10} - 3 v^l_{126} y_{126} + v^l_{120} y_{120},
  \]

- **RH neutrino mass matrix**
  \[
  M_R = v^R_{126} y_{126},
  \]

- **LH neutrino mass matrix**
  \[
  M_L = v^L_{126} y_{126},
  \]

**NOTE:** these models do respect SO(10)-inspired conditions

Simplest case but clearly non-realistic: it predicts no mixing at all (both in quark and lepton Sectors). For realistic models one has to add at least the 126 contribution.
Recent fits within SO(10) models

- Joshipura Patel 2011; Rodejohann, Dueck '13: the obtained quite good fits especially including supersymmetry but no leptogenesis and usually compact Spectrum solutions very fine tuned

- Babu, Bajc, Saad 1612.04329: they find a good fit with NO, hierarchical RH neutrino spectrum but no leptogenesis

- Ohlsson, Pernow 1804.04560: a fit found for NO but minimum $\chi^2=18.4$

- de Anda, King, Perdomo 1710.03229: SO(10) $\times S_4 \times Z_4^R \times Z_4^3$ model: it fits fermion parameters and also find successful leptogenesis respecting the constraints we showed: interesting prediction on neutrinoless double beta decay effective neutrino mass $m_{ee} \sim 11$ meV.

In all recent fits a type II term does not seem to help and best fits are type I dominated
An example of realistic model combining GUT+discrete symmetry: SO(10)-inspired leptogenesis in the “A2Z model”

(S.F.King 2014, PDB, S.F.King 1507.06431)

Figure 1: A to Z of flavour with Pati-Salam, where $A \equiv A_4$ and $Z \equiv Z_5$. The left-handed families form a triplet of $A_4$ and are doublets of $SU(2)_L$. The right-handed families are distinguished by $Z_5$ and are doublets of $SU(2)_R$. The $SU(4)_C$ unifies the quarks and leptons with leptons as the fourth colour, depicted here as white.

CASE A:

\[ m_{\nu_1}^D = m_{\text{up}}, \quad m_{\nu_2}^D = m_{\text{charm}}, \quad m_{\nu_3}^D = m_{\text{top}} \]

CASE B:

\[ m_{\nu_1}^D \approx m_{\text{up}}, \quad m_{\nu_2}^D \approx 3 m_{\text{charm}}, \quad m_{\nu_3}^D \approx \frac{1}{3} m_{\text{top}} \]
They can be obtained from 3 RH neutrino models in the limit $M_3 \rightarrow \infty$

Number of parameters get reduced to 11

Contribution to asymmetry from both 2 RH neutrinos. The contribution from the lightest ($N_1$) typically dominates but the contribution from next-to-lightest ($N_2$) opens new regions that correspond to light sequential dominated neutrino mass models realised in some GUT models. In any case there is still a lower bound

$$M_1 \gtrsim 2 \times 10^{10} \text{ GeV} \implies T_{\text{RH}} \gtrsim 6 \times 10^9 \text{ GeV}$$

Recent 2 RH neutrino model realised in $A_4 \times SU(5)$ SUSY GUT model with interesting link between “leptogenesis phase” and Dirac phase (F. Bjørkeroth, S.F. King 1505.05504)

2 RH neutrino model can be also obtained from 3 RH neutrino models with 1 vanishing Yukawa eigenvalue $\Rightarrow$ potential DM candidate (A. Anisimov, PDB hep-ph/0812.5085)
Total CP asymmetries

(Flanz, Paschos, Sarkar’95; Covi, Roulet, Vissani’96; Buchmüller, Plümacher’98)

It does not depend on \( U \)!
### N$_1$ dominated leptogenesis

\[ Z \equiv \frac{M_1}{T} \]

\[
\frac{dN_{N_1}}{dz} = -D_1 \left( N_{N_1} - N_{eq}^{N_1} \right)
\]

\[
\frac{dN_{B-L}}{dz} = -\varepsilon_1 \frac{dN_{N_1}}{dz} - W_{ID} N_{B-L}
\]

\[ D_1 = \frac{\Gamma_{D,1}}{H z} = K_1 z \left\langle \frac{1}{\gamma} \right\rangle, \quad W_{ID} \propto D_1 \propto K_1 \]

\[ N_{B-L}(z; K_1, z_{in}) = N_{B-L}^{in} e^{-\int_{z_{in}}^{z} dz' W_{ID}(z')} + \varepsilon_1 \kappa_1(z) \]

\[ \kappa_1(z; K_1, z_{in}) = -\int_{z_{in}}^{z} dz' \left[ \frac{dN_{N_1}}{dz'} \right] e^{-\int_{z'}^{z} dz'' W_{ID}(z'')} \]

- **Weak wash-out regime** for $K_1 \lesssim 1$ (out-of-equilibrium picture recovered for $K_1 \to 0$)
- **Strong wash-out regime** for $K_1 \gtrsim 1$
Weak and strong wash-out: comparison
STSO10 solution: on the right track?

(PDB, Marzola '13)

What is the probability that the agreement is due to a coincidence? This sets the statistical significance of the agreement

\( N_{B-L}^p = 0, 0.001, 0.01, 0.1 \)

If the first octant is found then \( p \lesssim 10\% \)

If NO is found then \( p \lesssim 5\% \)

If \( \sin \delta < 0 \) is confirmed then \( p \lesssim 2\% \)

If \( \cos \delta < 0 \) is found then \( p \lesssim 1\% \) and then?
Majorana phases are constrained around definite values.

Sharp prediction on the absolute neutrino mass scale: both on $m_1$ and $m_{ee}$.

Despite one has normal ordering, $m_{ee}$ value might be within exp. Reach

If also these predictions are satisfied exp, then $p \lesssim 0.01\%$ (conservative)

$\alpha_2 = 5$

$\text{NORMAL ORDERING}$

$\text{Majorana phases}$

$m_{ee} \approx 0.8m_1 \approx 15\text{ meV}$
Decrypted SO(10)-inspired models

\( U_R \approx \begin{pmatrix}
1 & -\frac{m_{D1}^*}{m_{D2}} m_{\nu e} \mu & \frac{m_{D1}^*}{m_{D3}} (m^{-1}_{\nu})_{\tau\tau} \\
\frac{m_{D1}}{m_{D2}} m_{\nu e} & 1 & \frac{m_{D2}^*}{m_{D3}} (m^{-1}_{\nu})_{\tau\tau} \\
\frac{m_{D1}}{m_{D3}} m_{\nu e} & \frac{m_{D2}^*}{m_{D3}} (m^{-1}_{\nu})_{\tau\tau} & 1
\end{pmatrix} \, D_\Phi \quad D_\phi \equiv (e^{-i \frac{\Phi_1}{2}}, e^{-i \frac{\Phi_2}{2}}, e^{-i \frac{\Phi_3}{2}})

M_1 \approx \frac{m^2_{D1}}{|m_{\nu e}|} \approx \frac{\alpha^2}{|m_{\nu e}|} \approx \alpha^2 \, 10^5 \text{ GeV} \left( \frac{m_u}{1 \text{ MeV}} \right)^2 \left( \frac{10 \text{ meV}}{|m_{\nu e}|} \right)

\Phi_1 = \text{Arg}[-m^*_{\nu e}].

M_2 \approx \frac{\alpha^2 m^2_c}{m_1 m_2 m_3} \frac{|m_{\nu e}|}{|(m^{-1}_{\nu})_{\tau\tau}|} \approx \alpha^2 \, 10^{11} \text{ GeV} \left( \frac{m_c}{400 \text{ MeV}} \right)^2 \left( \frac{|m_{\nu e}|}{10 \text{ meV}} \right)

\Phi_2 = \text{Arg} \left[ \frac{m_{\nu e}}{(m^{-1}_{\nu})_{\tau\tau}} \right] - 2 (\rho + \sigma)

M_3 \approx \alpha^2 m^2_t |(m^{-1}_{\nu})_{\tau\tau}| \approx \alpha^2 \, 10^{15} \text{ GeV} \left( \frac{m_t}{100 \text{ GeV}} \right)^2 \left( \frac{\text{meV}}{m_1} \right).

\Phi_3 = \text{Arg}[-(m^{-1}_{\nu})_{\tau\tau}].

0\nu\beta\beta \text{ neutrino mass}
Deciphering SO(10)-inspired leptogenesis

Finally, putting all together, one arrives to an expression for the final asymmetry:

\[
\frac{N_{B-L}^{\text{lep,f}}}{16 \pi} \approx \frac{3}{16 \pi} \frac{\alpha^2 m_c^2}{v^2} \frac{|m_{\nu e e}|}{m_1 m_2 m_3} \left( \frac{|(m^{-1}_\nu)_{\mu \tau}|^2}{m_* \sin \alpha_L} \right) \left( \frac{|(m^{-1}_\nu)_{\mu \tau}|^2}{|m_{\nu e e}| \left| (m^{-1}_\nu)_{\tau \tau} \right|} \right) \left( m_1 m_2 m_3 \right) \left( m_* \right) e^{-\frac{3\pi}{8} \frac{|m_{\nu e e}|^2}{m_* |m_{\nu e e}|}}.
\]

Effective SO(10)-inspired leptogenesis phase

\[
\alpha_L = \text{Arg}[m_{\nu e e}] - 2 \text{Arg}[(m^{-1}_\nu)_{\mu \tau}] + \pi - 2(\rho + \sigma).
\]

This analytical expression for the asymmetry fully reproduces all numerical constraints for \(V_L=\mathbb{I}\).

These results can be easily generalised to the case \(V_L \neq \mathbb{I}\): all given expressions are still valid with the replacement: (Akhmedov, Frigerio, Smirnov, 2005)

\[
m_\nu \rightarrow \tilde{m}_\nu \equiv \tilde{V}_L m_\nu \tilde{V}^T_L
\]
An example of realistic model:

SO(10)-inspired leptogenesis in the “A2Z model”

(S.F. King 2014)

Figure 1: A to Z of flavour with Pati-Salam, where $A = A_4$ and $Z = Z_5$. The left-handed families form a triplet of $A_4$ and are doublets of $SU(2)_L$. The right-handed families are distinguished by $Z_5$ and are doublets of $SU(2)_R$. The $SU(4)_C$ unifies the quarks and leptons with leptons as the fourth colour, depicted here as white.

Neutrino sector:

$$Y'_{LR} = \begin{pmatrix} 0 & be^{-i\frac{3\pi}{5}} & 0 \\ ae^{-i\frac{3\pi}{5}} & 4be^{-i\frac{3\pi}{5}} & 0 \\ ae^{-i\frac{3\pi}{5}} & 2be^{-i\frac{3\pi}{5}} & ce^{i\phi} \end{pmatrix}, \quad M'_R = \begin{pmatrix} M'_{11}e^{2i\xi} & 0 & M'_{13}e^{i\xi} \\ 0 & M'_{22}e^{i\xi} & 0 \\ M'_{13}e^{i\xi} & 0 & M'_{33} \end{pmatrix}$$

**CASE A:**

$$m_{\nu_1}^D = m_{\text{up}}, \quad m_{\nu_2}^D = m_{\text{charm}}, \quad m_{\nu_3}^D = m_{\text{top}}$$

**CASE B:**

$$m_{\nu_1}^D \approx m_{\text{up}}, \quad m_{\nu_2}^D \approx 3m_{\text{charm}}, \quad m_{\nu_3}^D \approx \frac{1}{3} m_{\text{top}}$$
There are 2 solutions (only for NO)

(PDB, S.King 1507.06431)

<table>
<thead>
<tr>
<th>CASE</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>$+4\pi/5$</td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{\text{min}}$</td>
<td>5.15</td>
<td>6.1</td>
</tr>
<tr>
<td>$M_1/10^7\text{GeV}$</td>
<td>15</td>
<td>2.7</td>
</tr>
<tr>
<td>$M_2/10^{10}\text{GeV}$</td>
<td>0.483</td>
<td>4.35</td>
</tr>
<tr>
<td>$M_3/10^{12}\text{GeV}$</td>
<td>2.16</td>
<td>1.31</td>
</tr>
<tr>
<td>$</td>
<td>\gamma</td>
<td>$</td>
</tr>
<tr>
<td>$m_1/\text{meV}$</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>$m_2/\text{meV} \ (p_{\Delta m^2_{12}})$</td>
<td>8.93 (-0.22)</td>
<td>8.94 (-0.25)</td>
</tr>
<tr>
<td>$m_3/\text{meV} \ (p_{\Delta m^2_{13}})$</td>
<td>49.7 (+0.17)</td>
<td>49.7 (+0.21)</td>
</tr>
<tr>
<td>$\sum_i m_i/\text{meV}$</td>
<td>61</td>
<td>61</td>
</tr>
<tr>
<td>$m_{ee}/\text{meV}$</td>
<td>1.95</td>
<td>1.95</td>
</tr>
<tr>
<td>$\theta_{12}/^\circ \ (p_{\theta_{12}})$</td>
<td>33.0 (-0.58)</td>
<td>33.0 (-0.66)</td>
</tr>
<tr>
<td>$\theta_{13}/^\circ \ (p_{\theta_{13}})$</td>
<td>8.40 (-0.47)</td>
<td>8.40 (-0.49)</td>
</tr>
<tr>
<td>$\theta_{23}/^\circ \ (p_{\theta_{23}})$</td>
<td>53.3 (+2.1)</td>
<td>54.0 (+2.3)</td>
</tr>
<tr>
<td>$\delta/^\circ$</td>
<td>20.8</td>
<td>23.5</td>
</tr>
<tr>
<td>$\eta_B/10^{-10} \ (p_{\eta_B})$</td>
<td>6.101 (+0.01)</td>
<td>6.101 (+0.01)</td>
</tr>
<tr>
<td>$\varepsilon_{2\tau}$</td>
<td>$-8.1 \times 10^{-6}$</td>
<td>$-1.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>$K_{1\mu}$</td>
<td>0.11</td>
<td>0.58</td>
</tr>
<tr>
<td>$K_{1\tau}$</td>
<td>4341</td>
<td>800</td>
</tr>
<tr>
<td>$K_{2\tau}$</td>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>$K_{2\mu}$</td>
<td>29.2</td>
<td>29.2</td>
</tr>
<tr>
<td>$K_{2e}$</td>
<td>1.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

The spectrum is not so strongly hierarchical: it is in the proximity of crossing level solutions.
There are 2 solutions (only for NO)

CASE A

CASE B

This region will be tested relatively quickly
Quantifying the fine-tuning

(PDB, S. King 2015)

Analytical expression also for the orthogonal matrix:

\[ \Omega \approx \left( \begin{array}{ccc}
\sqrt{m_1 |\tilde{m}_{\nu 11}|} & \sqrt{m_2 m_3 |(\tilde{m}_{\nu})_{33}|} & U^*_{\mu 1} - U^*_{\tau 1} (\tilde{m}_{\nu})_{23} \\
-\sqrt{m_2 |\tilde{m}_{\nu 11}|} & \sqrt{m_1 m_3 |(\tilde{m}_{\nu})_{33}|} & U^*_{\mu 2} - U^*_{\tau 2} (\tilde{m}_{\nu})_{23} \\
-\sqrt{m_3 |\tilde{m}_{\nu 11}|} & \sqrt{m_1 m_2 |(\tilde{m}_{\nu})_{33}|} & U^*_{\mu 3} - U^*_{\tau 3} (\tilde{m}_{\nu})_{23}
\end{array} \right) \frac{U_{31}^*}{\sqrt{m_1 |(\tilde{m}_{\nu})_{33}|}} \frac{U_{32}^*}{\sqrt{m_2 |(\tilde{m}_{\nu})_{33}|}} \frac{U_{33}^*}{\sqrt{m_3 |(\tilde{m}_{\nu})_{33}|}} D_\Phi, \]

\[ \Omega^{(\text{CASEA})} \approx \left( \begin{array}{ccc}
-4.40016 - 15.9889 i & 0.0930875 - 0.894045 i & -16.0396 + 4.38107 i \\
-15.9446 + 3.40333 i & -1.15394 + 0.0537137 i & 3.40494 + 15.9553 i \\
-3.69174 + 4.35811 i & 0.709793 + 0.204576 i & 4.37787 + 3.64191 i
\end{array} \right) \]

\[ \Omega^{(\text{CASEB})} \approx \left( \begin{array}{ccc}
-1.77835 - 6.85986 i & 0.108413 - 0.897431 i & -6.97828 + 1.73423 i \\
-6.87598 + 1.34103 i & -1.15331 + 0.0386159 i & 1.34278 + 6.90018 i \\
-1.64314 + 1.81259 i & 0.710523 + 0.199612 i & 1.85785 + 1.52677 i
\end{array} \right) \]

- Fine tuned cancellations in the see-saw formula at the level of \(|\Omega_{ij}|^{-2}\) this seems to be quite a recurrent issue in fits.....
A popular class of SO(10) models


In SO(10) models each SM particles generation + 1 RH neutrino are assigned to a single 16-dim representation. Masses of fermions arise from Yukawa interactions of two 16s with vevs of suitable Higgs fields. Since:

\[16 \otimes 16 = 10_S + \overline{126}_S + 120_A;\]

The Higgs fields of renormalizable SO(10) models can belong to 10-, 126-, 120-dim representations yielding Yukawa part of the Lagrangian

\[\mathcal{L}_Y = 16 \left( Y_{10}^{10}_H + Y_{126}^{126}_H + Y_{120}^{120}_H \right) 16.\]

After SSB of the fermions at \(M_{\text{GUT}} = 2 \times 10^{16}\) GeV one obtains the masses:

- **up-quark mass matrix**
  \[M_u = v_{10}^{u} Y_{10} + v_{126}^{u} Y_{126} + v_{120}^{u} Y_{120},\]

- **down-quark mass matrix**
  \[M_d = v_{10}^{d} Y_{10} + v_{126}^{d} Y_{126} + v_{120}^{d} Y_{120},\]

- **neutrino mass matrix**
  \[M_D = v_{10}^{\nu} Y_{10} - 3 v_{126}^{\nu} Y_{126} + v_{120}^{\nu} Y_{120},\]

- **charged lepton mass matrix**
  \[M_l = v_{10}^{\ell} Y_{10} - 3 v_{126}^{\ell} Y_{126} + v_{120}^{\ell} Y_{120},\]

- **RH neutrino mass matrix**
  \[M_R = v_{126}^{R} Y_{126},\]

- **LH neutrino mass matrix**
  \[M_L = v_{126}^{L} Y_{126},\]

Simplest case but clearly non-realistic: it predicts no mixing at all (both in quark and lepton Sectors). For realistic models one has to add at least the 126 contribution

NOTE: these models do respect SO(10)-inspired conditions
### Recent fits within SO(10) models

(Joshipura Patel 2011; Rodejohann, Dueck '13)

#### Minimal Model with $10_H + \overline{126}_H$ (MN, MS)

"full" Higgs Content $10_H + \overline{126}_H + 120_H$ (FN, FS)

<table>
<thead>
<tr>
<th>Mod</th>
<th>Comments</th>
<th>$\langle m_\nu \rangle$ [meV]</th>
<th>$\delta^{\ell}_{CP}$ [rad]</th>
<th>$\sin^2 \theta_{23}$</th>
<th>$m_0$ [meV]</th>
<th>$M_3$ [GeV]</th>
<th>$M_2$ [GeV]</th>
<th>$M_1$ [GeV]</th>
<th>$\chi^2_{\min}$</th>
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</thead>
<tbody>
<tr>
<td>MN</td>
<td>no RGE, NH</td>
<td>0.35</td>
<td>0.7</td>
<td>0.406</td>
<td>3.03</td>
<td>$5.5 \times 10^{12}$</td>
<td>$7.2 \times 10^{11}$</td>
<td>$1.5 \times 10^{10}$</td>
<td>1.10</td>
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<tr>
<td>MN</td>
<td>RGE, NH</td>
<td>0.49</td>
<td>6.0</td>
<td>0.346</td>
<td>2.40</td>
<td>$3.6 \times 10^{12}$</td>
<td>$2.0 \times 10^{11}$</td>
<td>$1.2 \times 10^{11}$</td>
<td>23.0</td>
</tr>
<tr>
<td>MS</td>
<td>no RGE, NH</td>
<td>0.38</td>
<td>0.27</td>
<td>0.387</td>
<td>2.58</td>
<td>$3.9 \times 10^{12}$</td>
<td>$7.2 \times 10^{11}$</td>
<td>$1.6 \times 10^{10}$</td>
<td>9.41</td>
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<tr>
<td>MS</td>
<td>RGE, NH</td>
<td>0.44</td>
<td>2.8</td>
<td>0.410</td>
<td>6.83</td>
<td>$1.1 \times 10^{12}$</td>
<td>$5.7 \times 10^{10}$</td>
<td>$1.5 \times 10^{10}$</td>
<td>3.29</td>
</tr>
<tr>
<td>FN</td>
<td>no RGE, NH</td>
<td>4.96</td>
<td>1.7</td>
<td>0.410</td>
<td>8.8</td>
<td>$1.9 \times 10^{13}$</td>
<td>$2.8 \times 10^{12}$</td>
<td>$2.2 \times 10^{10}$</td>
<td>$6.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>FN</td>
<td>RGE, NH</td>
<td>2.87</td>
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<td>0.410</td>
<td>1.54</td>
<td>$9.9 \times 10^{14}$</td>
<td>$7.3 \times 10^{13}$</td>
<td>$1.2 \times 10^{13}$</td>
<td>11.2</td>
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<tr>
<td>FS</td>
<td>no RGE, NH</td>
<td>0.75</td>
<td>0.5</td>
<td>0.410</td>
<td>1.16</td>
<td>$1.5 \times 10^{13}$</td>
<td>$5.3 \times 10^{11}$</td>
<td>$5.7 \times 10^{10}$</td>
<td>$9.0 \times 10^{-10}$</td>
</tr>
<tr>
<td>FS</td>
<td>RGE, NH</td>
<td>0.78</td>
<td>5.4</td>
<td>0.410</td>
<td>3.17</td>
<td>$4.2 \times 10^{13}$</td>
<td>$4.9 \times 10^{11}$</td>
<td>$4.9 \times 10^{11}$</td>
<td>$6.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>FN</td>
<td>no RGE, IH</td>
<td>35.37</td>
<td>5.4</td>
<td>0.590</td>
<td>35.85</td>
<td>$2.2 \times 10^{13}$</td>
<td>$4.9 \times 10^{12}$</td>
<td>$9.2 \times 10^{11}$</td>
<td>$2.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>FN</td>
<td>RGE, IH</td>
<td>35.52</td>
<td>4.7</td>
<td>0.590</td>
<td>30.24</td>
<td>$1.1 \times 10^{13}$</td>
<td>$3.5 \times 10^{12}$</td>
<td>$5.5 \times 10^{11}$</td>
<td>13.3</td>
</tr>
<tr>
<td>FS</td>
<td>no RGE, IH</td>
<td>44.21</td>
<td>0.3</td>
<td>0.590</td>
<td>6.27</td>
<td>$1.2 \times 10^{13}$</td>
<td>$4.2 \times 10^{11}$</td>
<td>$3.5 \times 10^{10}$</td>
<td>$3.9 \times 10^{-8}$</td>
</tr>
<tr>
<td>FS</td>
<td>RGE, IH</td>
<td>24.22</td>
<td>3.6</td>
<td>0.590</td>
<td>11.97</td>
<td>$1.2 \times 10^{13}$</td>
<td>$3.1 \times 10^{11}$</td>
<td>$2.0 \times 10^{3}$</td>
<td>0.602</td>
</tr>
</tbody>
</table>

Recently Fong, Meloni, Meroni, Nardi (1412.4776) have included leptogenesis for the non-SUSY case obtaining successful leptogenesis: but such a compact RN neutrino spectrum implies huge fine-tuning. Too simplistic models? What solution: non renormalizable terms? Type II seesaw term? SUSY seems to improve the fits and also give $1$ hier. solution.
SUSY SO(10)-inspired leptogenesis

(PDB, Re Fiorentin, Marzola, 1512.06739)

\[
tan \beta = 5
\]

\[
tan \beta = 50
\]

It is possible to lower \(T_{RH}\) to values consistent with the gravitino problem for \(m_g \gtrsim 30\) TeV

(Kawasaki, Kohri, Moroi, 0804.3745)

Alternatively, for lower gravitino masses, one has to consider non-thermal SO(10)-inspired leptogenesis

(Blanchet, Marfatia 1006.2857)
The Dark Matter of the Universe

(Hu, Dodelson, astro-ph/0110414)

(Planck 2015, 1502.10589)

\[
\Omega_{CDM,0}h^2 = 0.1188 \pm 0.0010 \sim 5\Omega_{B,0}h^2
\]
Unifying Leptogenesis and Dark Matter

(PDB, NOW 2006; Anisimov, PDB, 0812.5085; PDB, P. Ludl, S. Palomarez-Ruiz 1606.06238)

1 RH neutrino has vanishing Yukawa couplings

\[ m_D \sim \begin{pmatrix} m_{De1} & m_{De2} & 0 \\ m_{D\mu 1} & m_{D\mu 2} & 0 \\ m_{Dr 1} & m_{Dr 2} & 0 \end{pmatrix} \]

\[ \Rightarrow \text{2 RH neutrino seesaw model} \]

\[ \Rightarrow \text{2 RH neutrino seesaw model} \]

...but couples to one “source” RH neutrino \( N_S \) via Higgs portal-type interactions

\[ \mathcal{L} = \frac{\lambda_{IJ}}{\Lambda} \phi^\dagger \phi \bar{N}_I^c N_J \]

These are responsible both for production via RH neutrino mixing (non-adiabatic conversions) and for decays \( \Rightarrow \) decays are unavoidable \( \Rightarrow \) it predicts some contribution to high energy neutrino flux potentially testable with neutrino telescopes (IceCube, Km\(^3\),......)

Correct DM abundance and life time for \( \Lambda/\lambda \sim 10^{25} \) GeV

Interference of \( N_S \) with the third RH neutrino \( \Rightarrow (1-100) \) TeV leptogenesis
Nicely predicted a signal at IceCube
(Anisimov, PDB, 0812.5085; PDB, P. Ludl, S. Palomarez-Ruiz 1606.06238)

- DM neutrinos unavoidably decay today into $A+$leptons ($A=H,Z,W$) through the same mixing that produced them in the very early Universe
- Potentially testable high energy neutrino contribution

Energy neutrino flux

Flavour composition at the detector

Hard component

Neutrino events at IceCube: 2 examples

$M_{DM}=300$ TeV

$M_{DM}=8$ PeV
High energy scale leptogenesis is the most attractive scenario of baryogenesis if absence of new physics at TeV scale of below will persist.

N$_2$-dominated scenario provides is naturally realised in SO(10)-inspired models and also to satisfy STRONG THERMAL LEPTOGENESIS.

STRONG SO(10) thermal solution has strong predictive power and current data are in line. Deviation of neutrino masses from the hierarchical limits is expected; despite normal ordering, m$_{ee}$ ~ 15meV might be still within reach; Despite NO neutrinoless double beta decay signal also within reach.

Study of realistic models incorporating leptogenesis started but there is still not a satisfactory model able to fit everything.

SUSY SO(10)-inspired models can be still reconciled with gravitino problem and improve of quark+lepton sectors parameters;

A unified scenario of DM and resonant leptogenesis can be tested with IceCube high energy neutrino data.
Leptogenesis in the “A2Z model”

The only sizeable CP asymmetry is the tauon asymmetry but $K_{1\tau} \gg 1$!

Flavour coupling (mainly due to the hypercharge Higgs asymmetry) is then crucial to produce the correct asymmetry:

$(\text{Antusch, PDB, Jones, King 2011})$

\[ \eta_B \simeq \sum_{\alpha=e,\mu,\tau} \eta^{(\alpha)}_B, \quad \eta^{(\tau)}_B \simeq 0.01 \varepsilon_{2\tau} \kappa(K_{2\tau}) e^{-\frac{3\pi}{8}K_{1\tau}} \]

\[ \eta^{(e)}_B \simeq -0.01 \varepsilon_{2\tau} \kappa(K_{2\tau}) \frac{K_{2e}}{K_{2e} + K_{2\mu}} C^{(2)}_{\tau\perp\tau} e^{-\frac{3\pi}{8}K_{1e}} \]

\[ \eta^{(\mu)}_B \simeq -\left( \frac{K_{2\mu}}{K_{2e} + K_{2\mu}} C^{(2)}_{\tau\perp\tau} - \frac{K_{1\mu}}{K_{1\tau}} C^{(3)}_{\mu\tau} \right) e^{-\frac{3\pi}{8}K_{1\mu}} . \]
Density matrix and CTP formalism to describe the transition regimes

(De Simone, Riotto '06; Beneke, Gabrecht, Fidler, Herranen, Schwaller '10)

\[
\frac{dY_{\alpha\beta}}{dz} = \frac{1}{s_H(z)} \left[ (\gamma_D + \gamma_{\Delta L=1}) \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \epsilon_{\alpha\beta} - \frac{1}{2Y_\epsilon}\{Y_D + Y_{\Delta L=1}, Y\}_{\alpha\beta} \right] - \left[ \sigma_2 \text{Re}(\Lambda) + \sigma_1 |\text{Im}(\Lambda)| \right] Y_{\alpha\beta}
\]
Neutrino mass bounds and role of PMNS phases

(Abada et al. '07; Blanchet, PDB, Raffelt; Blanchet, PDB '08)

PMNS phases off

Imposing the validity of the Boltzmann equations

One-flavour transition

Two-flavour transition

$M_1$ (GeV)

$m_1$ (eV)

$10^8$

$10^9$

$10^{12}$
Affleck-Dine Baryogenesis  

(Affleck, Dine ’85)

In the Supersymmetric SM there are many “flat directions” in the space of a field composed of squarks and/or sleptons

\[ V(\phi) = \sum_{i} \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} \sum_{A} \left( \sum_{ij} \phi_i^*(t_A)_{ij} \phi_j \right)^2 \]

\begin{align*}
\text{F term} & \quad \text{D term}
\end{align*}

A flat direction can be parametrized in terms of a complex field (AD field) that carries a baryon number that is violated dynamically during inflation

\[ \frac{n_B}{s} \sim 10^{-10} \left( \frac{m_{3/2}}{m_\Phi} \right) \left( \frac{m_\Phi}{\text{TeV}} \right)^{-1/2} \left( \frac{M}{M_P} \right)^{3/2} \left( \frac{T_R}{10 \text{ GeV}} \right) \]

The final asymmetry is \( T_{RH} \) and the observed one can be reproduced for low values \( T_{RH} \sim 10 \text{ GeV} \).
Gravitational Baryogenesis (Davoudiasl, Kribs, Kitano, Murayama, Steinhardt '04)

It works efficiently and asymmetries even much larger than the observed one are generated for $T_{RH} \gg 100$ GeV.

The key ingredient is a CP-violating interaction between the derivative of the Ricci scalar curvature $R$ and the baryon number current $J^\mu$:

It is natural to have this operator in quantum gravity and in supergravity.
Total CP asymmetries

(FLanz, Paschos, Sarkar'95; Covi, Roulet, Vissani'96; Buchmüller, Plümacher'98)

\[ \varepsilon_i \simeq \frac{1}{8\pi v^2 (m_D^{\dagger} m_D)_{ii}} \sum_{j \neq i} \text{Im} \left[ (m_D^{\dagger} m_D)^2_{ij} \right] \times \left[ f_V \left( \frac{M_j^2}{M_i^2} \right) + f_S \left( \frac{M_j^2}{M_i^2} \right) \right] \]

It does not depend on U!
Additional contribution to CP violation:

\[
\varepsilon_{1\alpha} = P_{1\alpha}^0 \varepsilon_1 + \frac{\Delta P_{1\alpha}}{2}
\]

depends on \( U \)!

1) \( \Gamma \neq \bar{\Gamma} \)

\[
N_1 \rightarrow \begin{cases} N_{\ell_1} \\ \bar{N}_{\ell_1}' \end{cases} \rightarrow \begin{cases} l_1 \\ \bar{l}_1' \end{cases} \quad \Rightarrow \quad P_{1\alpha}^0 \varepsilon_1
\]

2) \( |\bar{l}_1\rangle \neq CP|l_1\rangle \)

\[
N_1 \rightarrow \begin{cases} e+\mu \\ \bar{e}+\bar{\mu} \end{cases} \quad \Rightarrow \quad \frac{\Delta P_{1\alpha}}{2}
\]

(Nardi, Racker, Roulet '06)
A lower bound on neutrino masses (IO)

$N_{B-L}^{P,i} = 0.001, 0.01, 0.1$  
$\max[|\Omega_{21}^2|] = 2$  
INVERTED ORDERING

$m_1 \geq 3 \text{ meV} \Rightarrow \sum_i m_i \geq 100 \text{ meV} \text{ (not necessarily deviation from HL)}$
About the crossing levels the $N_1$ CP asymmetry is enhanced

The correct BAU can be attained for a fine tuned choice of parameters: many realistic models have made use of these solutions

(e.g. Ji, Mohapatra, Nasri ’10; Buccella, Falcone, Nardi, ‘12; Altarelli, Meloni ‘14, Feng, Meloni, Meroni, Nardi ‘15; Addazi, Bianchi, Ricciardi 1510.00243)
A possible GUT origin

\[
\frac{1}{\Lambda_{\text{eff}}} = \frac{h\mu}{M_{\text{GUT}}^2}
\]

\[\Lambda_{\text{eff}} \gg M_{\text{GUT}}\]

(Anisimov, PDB, 2010, unpublished)
Total CP asymmetries

(Flanz, Paschos, Sarkar'95; Covi, Roulet, Vissani'96; Buchmüller, Plümacher'98)

\[ \varepsilon_i \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{ii}} \sum_{j \neq i} \text{Im} \left[ (m_D^\dagger m_D)_{ij}^2 \right] \times \left[ f_V \left( \frac{M_j^2}{M_i^2} \right) + f_S \left( \frac{M_j^2}{M_i^2} \right) \right] \]

It does not depend on U!
Flavoured decay parameters:

\[ K_{1\alpha} = P_{1\alpha}^0 K_1 = \sum_k \sqrt{\frac{m_k}{m_{\alpha k}}} U_{\alpha k} \Omega_{k1} \leq K_1 \]

\[ \sum_{\alpha} K_{1\alpha} = K_1 \]

\[ \Rightarrow \epsilon_{1\alpha} \equiv -\frac{P_{1\alpha} \Gamma_1 - \bar{P}_{1\alpha} \bar{\Gamma}_1}{\Gamma_1 + \bar{\Gamma}_1} = P_{1\alpha}^0 \epsilon_1 + \Delta P_{1\alpha} (\Omega, U) / 2 \]

\[ \Rightarrow N_{B-L}^{\text{fin}} = \sum_{\alpha} \epsilon_{1\alpha} \kappa_{1\alpha}^{\text{fin}} \approx 2 \epsilon_1 \kappa_{1}^{\text{fin}} + \frac{\Delta P_{1\alpha}}{2} [\kappa_f (K_{1\alpha}) - \kappa_f^{\text{fin}} (K_{1\beta})] \]

Flavour effects introduce an explicit dependence on \( U \) and can in some case greatly enhance the asymmetry compared to the unflavoured case.

3 MAIN APPLICATIONS AND CONSEQUENCES OF FLAVOUR EFFECTS:

- Lower bound on \( M_1 \) (and therefore on \( T_{\text{RH}} \)) is \textbf{not} relaxed.
  - Upper bound on \( m_1 \) is slightly relaxed to \( \sim 0.2 \text{eV} \) but if wash-out is strong then
    Low energy phases can strongly affect the final asymmetry (second term).

- In the case of real \( \Omega \) \( \Rightarrow \) all CP violation stems from low energy phases;
  - if also Majorana phases are CP conserving only \( \delta \) would be responsible for the asymmetry:
    \[ \Rightarrow \text{DIRAC PHASE LEPTOGENESIS}: \eta_{B0} \propto |\sin \delta| \sin \Theta_{13} \]

- Asymmetry produced from heavier RH neutrinos also contributes to the asymmetry and has to be taken into account:
  \[ \text{IT OPENS NEW INTERESTING OPPORTUNITIES} \]
Remarks on the role of $\delta$ in leptogenesis

**Dirac phase leptogenesis:**

- It could work but only for $M_1 \geq 5 \times 10^{11}$ GeV (plus other conditions on $\Omega$)
  \[ \Rightarrow \text{density matrix calculation needed!} \]

- No reasons for $\Omega$ to be real except when it is a permutation of identity (from discrete flavour models) but then all CP asymmetries vanish! So one needs quite a special $\Omega$

- In general the contribution from $\delta$ is *overwhelmed* by the high energy phases in $\Omega$

**General considerations:**

- **CP violating value of $\delta$** is strictly speaking neither necessary nor sufficient condition for successful leptogenesis and no specific value is favoured model independently but....

- ....it is important to exclude CP conserving values since from $m_D = U \sqrt{D_m} \Omega \sqrt{D_M}$ one expects for generic $m_D$ that if there are phases in $U$ then there are also phases in $\Omega$, vice-versa if there are no phases in $U$ one might suspect that also $\Omega$ is real (disaster!):
  \[ \text{discovering CP violating value of $\delta$ would support a complex $m_D$} \]
A parameter reduction would help and can occur in various ways:

- $\eta_B = \eta_B^{\text{CMB}}$ is satisfied around “peaks”;
- some parameters cancel out in the asymmetry calculation;
- imposing independence of the initial conditions (strong thermal leptog.);
- imposing model dependent conditions on $m_D$ (e.g. SO(10)-inspired);
- additional phenomenological constraints (e.g. Dark Matter).