

# $3\nu$ MASSES AND MIXING: THE GLOBAL PICTURE

Concha Gonzalez-Garcia

(ICREA U. Barcelona & YITP Stony Brook )

**Neutrino Town-Meeting: Oct 22-24 CERN**



# The Status of Neutrino Parameters (3f)

See e.g. Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Schwetz

	Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 0.83$ )		Any Ordering
	bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range	$3\sigma$ range
$\sin^2 \theta_{12}$	$0.306^{+0.012}_{-0.012}$	0.271 → 0.345	$0.306^{+0.012}_{-0.012}$	0.271 → 0.345	0.271 → 0.345
$\theta_{12}/^\circ$	$33.56^{+0.77}_{-0.75}$	31.38 → 35.99	$33.56^{+0.77}_{-0.75}$	31.38 → 35.99	31.38 → 35.99
$\sin^2 \theta_{23}$	$0.441^{+0.027}_{-0.021}$	0.385 → 0.635	$0.587^{+0.020}_{-0.024}$	0.393 → 0.640	0.385 → 0.638
$\theta_{23}/^\circ$	$41.6^{+1.5}_{-1.2}$	38.4 → 52.8	$50.0^{+1.1}_{-1.4}$	38.8 → 53.1	38.4 → 53.0
$\sin^2 \theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	0.01934 → 0.02392	$0.02179^{+0.00076}_{-0.00076}$	0.01953 → 0.02408	0.01934 → 0.02397
$\theta_{13}/^\circ$	$8.46^{+0.15}_{-0.15}$	7.99 → 8.90	$8.49^{+0.15}_{-0.15}$	8.03 → 8.93	7.99 → 8.91
$\delta_{CP}/^\circ$	$261^{+51}_{-59}$	0 → 360	$277^{+40}_{-46}$	145 → 391	0 → 360
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.50^{+0.19}_{-0.17}$	7.03 → 8.09	$7.50^{+0.19}_{-0.17}$	7.03 → 8.09	7.03 → 8.09
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.524^{+0.039}_{-0.040}$	+2.407 → +2.643	$-2.514^{+0.038}_{-0.041}$	-2.635 → -2.399	$[+2.407 \rightarrow +2.643]$ $[-2.629 \rightarrow -2.405]$

UPDATE REQUIRED

Absolute mass limits from Mainz and Troitsk:  $m_1 < 2.2 \text{ eV}$

Limits from cosmology: 0.15-0.2 eV

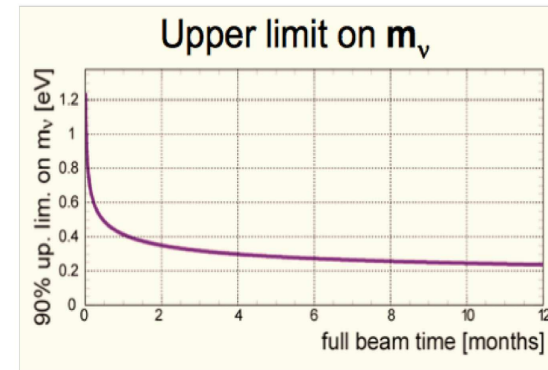
KATRIN: started operation → 0.2eV ; Project8, ...

**Important:**

- Active  $\nu$  unitarity already tested > 95%

- origin of  $\nu$  masses unknown

→ Talks by E. Martinez, T. Schwetz, Ch. Weinheimer and others



## Data to be Described

### Solar experiments

- Chlorine total rate, 1 data point.
- Gallex & GNO total rates, 2 points.
- SAGE total rate, 1 data point.
- SK1 E and zenith spect, 44 bins.
- SK2 E and D/N spect, 33 bins.
- SK3 E and D/N spect, 42 bins.
- SK4 2055-day E and D/N spect , 46 bins.
- SNO combined analysis, 7 points.
- Borexino Ph-I 740.7-day low-E spect 33 bins.
- Borexino Ph-I 246-day high-E spect ,6 bins.
- Borexino Ph-II 408-day low-E spect, 42 bins.

### Reactor experiments

- KamLAND DS1,DS2&DS3 spectra with Daya-Bay fluxes 69 bins
- D-Chooz FD-I/ND and FD-II spectral ratios with 455-day (FD-I), 363-day (FD-II) and 285 ND exposures , 56 bins.
- **SUMMER 18** Daya-Bay 1958-day EH2/EH1 & EH3/EH1 spectral ratios, 70 bins.
- **SUMMER 18** Reno 2200-day spec ratios 26 bins.

### Atmospheric experiments

- IceCube/DeepCore 3-year data, 64 bins.
- SK I-IV  $\chi^2$  table provided by the experiment.

### Accelerator experiments

- MINOS  $10.71 \times 10^{20}$  pot  $\nu_\mu$ -disapp data, 39 bins.
- MINOS  $3.36 \times 10^{20}$  pot  $\bar{\nu}_\mu$ -disapp data , 14 bins.
- MINOS  $10.6 \times 10^{20}$  pot  $\nu_e$ -app data , 5 bins.
- MINOS  $3.3 \times 10^{20}$  pot  $\bar{\nu}_e$ -app data , 5 bins.
- T2K  $14.93 \times 10^{20}$  pot  $\nu_\mu$ -disapp data, 55 bins.
- T2K  $14.93 \times 10^{20}$  pot  $\nu_e$ -app data, 39 bins.
- **SUMMER 18** T2K  $11.2 \times 10^{20}$  pot  $\bar{\nu}_\mu$ -disapp, 55 bins.
- **SUMMER 18** T2K  $11.2 \times 10^{20}$  pot  $\bar{\nu}_e$ -app, 23 bins.
- NO $\nu$ A  $8.85 \times 10^{20}$  pot  $\nu_\mu$ -disapp data , 76 bins.
- NO $\nu$ A  $8.85 \times 10^{20}$  pot  $\nu_e$ -app data , 13 bins.
- **SUMMER 18** NO $\nu$ A  $6.9 \times 10^{20}$  pot  $\bar{\nu}_\mu$ -disapp, 76 bins.
- **SUMMER 18** NO $\nu$ A  $6.9 \times 10^{20}$  pot  $\bar{\nu}_e$ -app, 12 bins.

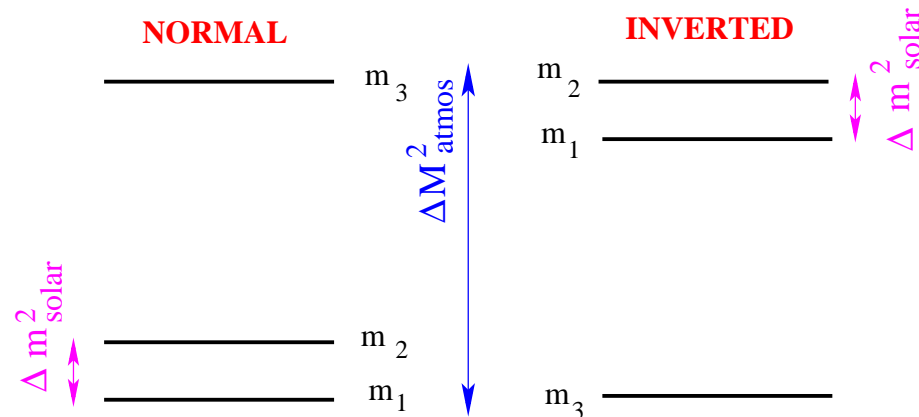
# 3ν Flavour Parameters

Concha Gonzalez-Garcia

- For for 3 ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\text{LEP}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\text{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Two Possible Orderings

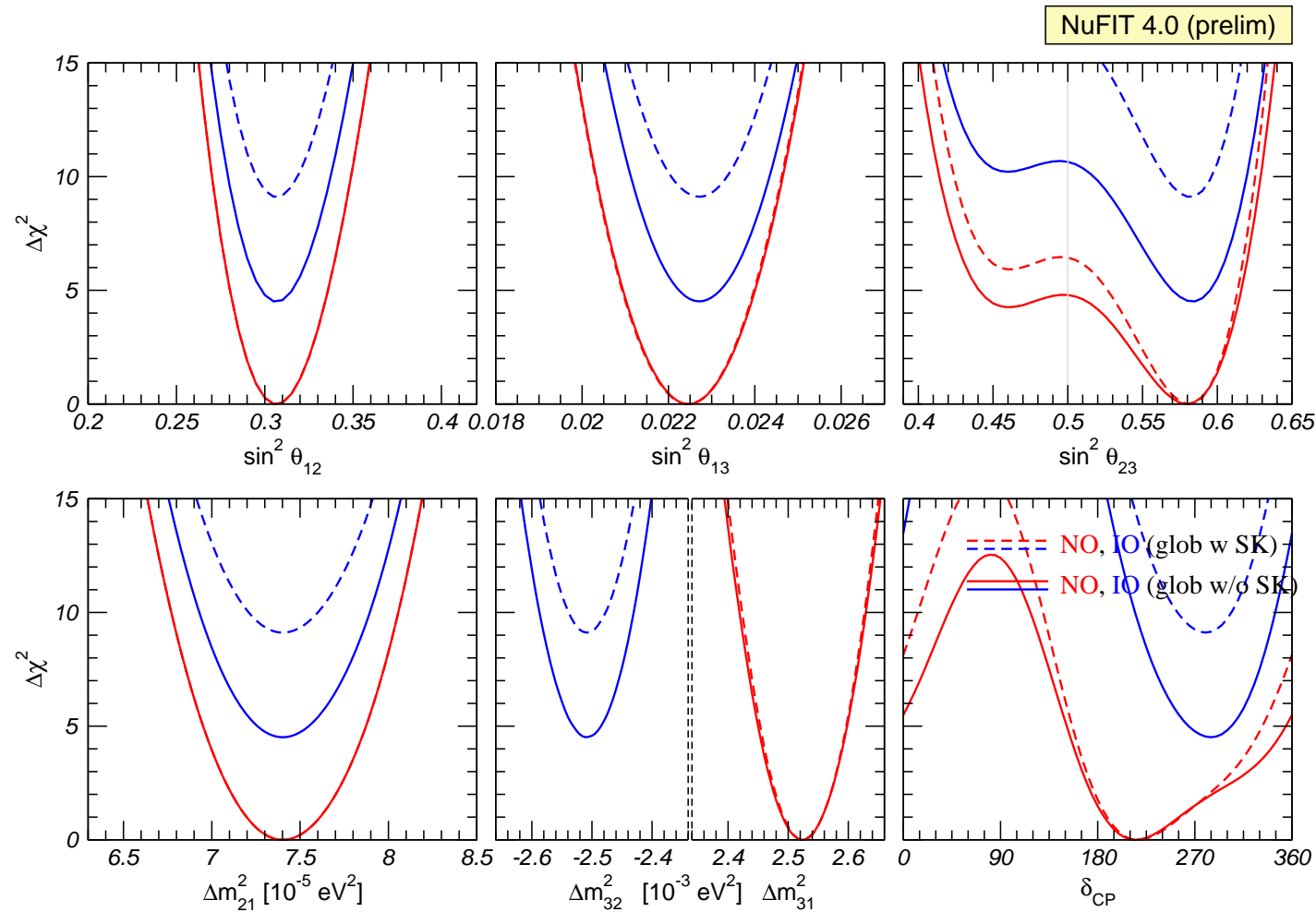


Experiment	Dominant Dependence	Important Dependence
Solar Experiments	$\rightarrow \theta_{12}$	$\Delta m^2_{21}, \theta_{13}$
Reactor LBL (KamLAND)	$\rightarrow \Delta m^2_{21}$	$\theta_{12}, \theta_{13}$
Reactor MBL (Daya Bay, Reno, D-Chooz)	$\rightarrow \theta_{13}$	$\Delta m^2_{\text{atm}}$
Atmospheric Experiments	$\rightarrow \theta_{23}$	$\Delta m^2_{\text{atm}}, \theta_{13}, \delta_{\text{CP}}$
Acc LBL $\nu_\mu$ Disapp (Minos, T2K, NOvA)	$\rightarrow \Delta m^2_{\text{atm}}$	$\theta_{23}$
Acc LBL $\nu_e$ App (Minos, T2K, NOvA)	$\rightarrow \theta_{13}$	$\delta_{\text{CP}}, \theta_{23}$

# 3 $\nu$ Flavour Parameters: Status OCT 2018

Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Hernandez-Cabezudo, Maltoni, Schwetz, MCG-G PRELIMINARY

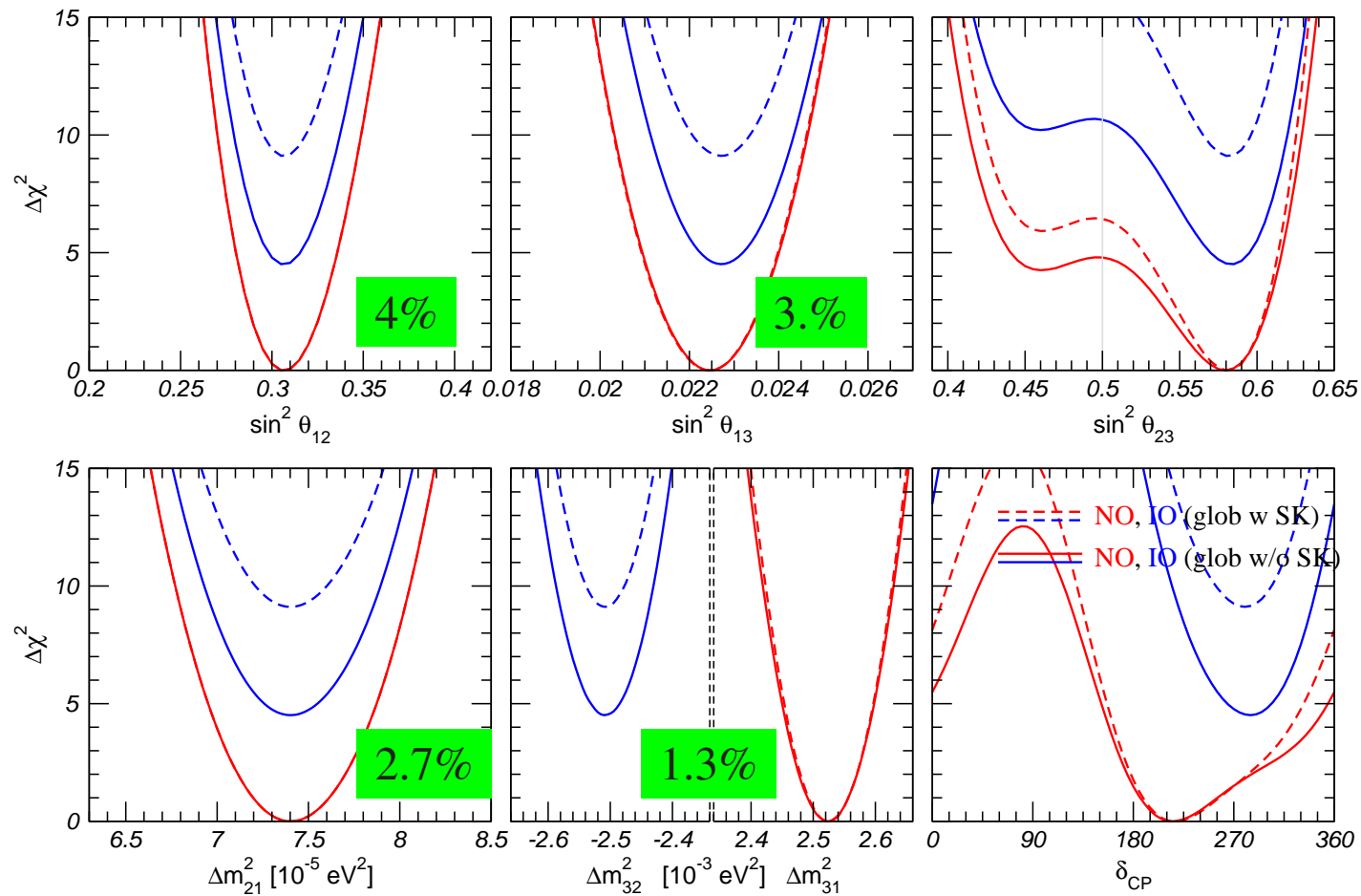


# 3 $\nu$ Flavour Parameters: Status OCT 2018

Global 6-parameter fit <http://www.nu-fit.org>  
 Esteban, Hernandez-Cabezudo, Maltoni, Schwetz, MCG-G PRELIMINARY

Precision  $3\sigma/3$

NuFIT 4.0 (prelim)



• Best determined:

$$\theta_{12}, \theta_{13}, \Delta m_{21}^2, |\Delta m_{3l}^2|$$

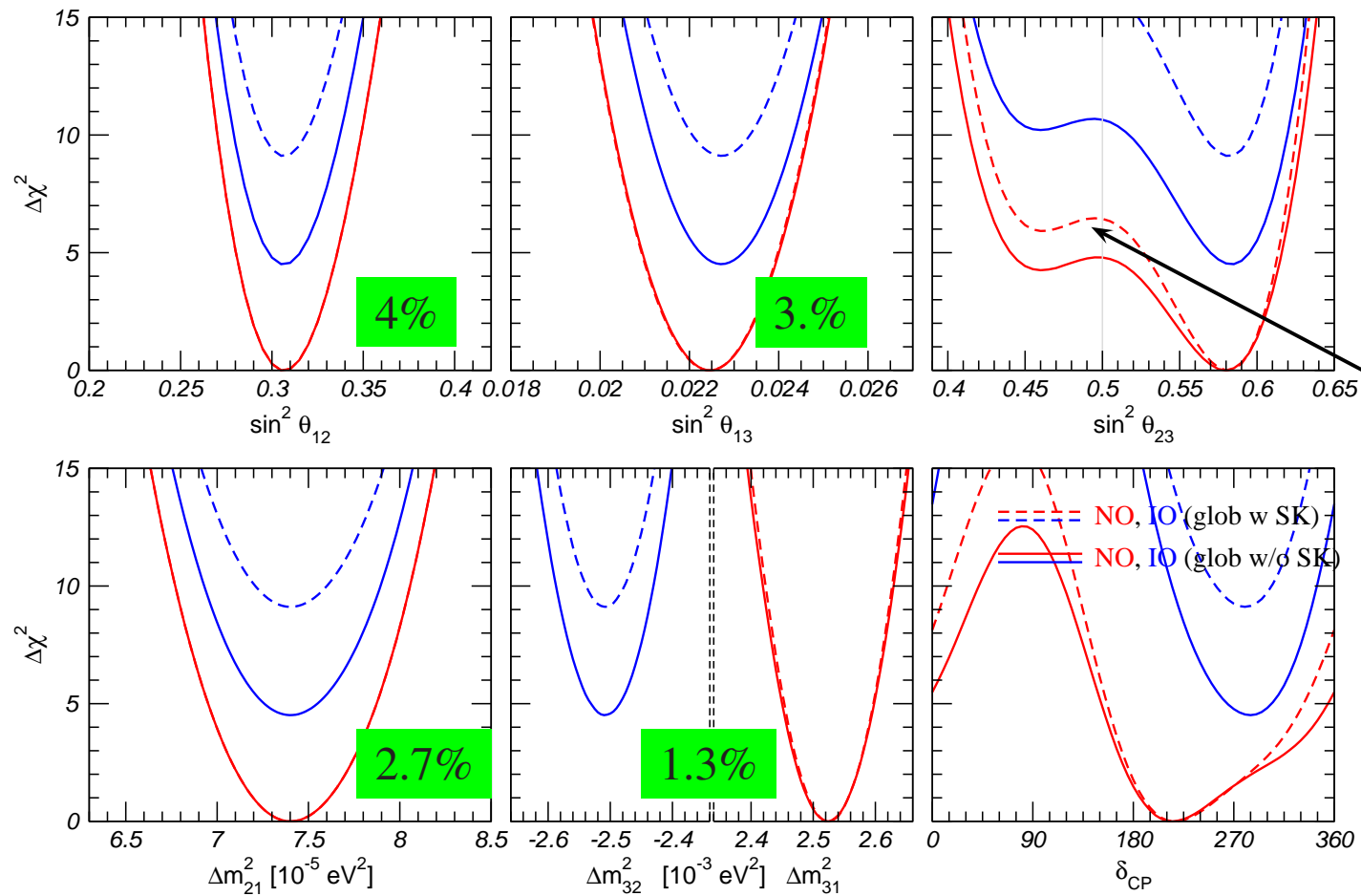
$[\Delta m_{3l}^2 = \Delta m_{31}^2 > 0 \text{ (NO)}$   
 $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0 \text{ (IO)}$

# 3 $\nu$ Flavour Parameters: Status OCT 2018

Global 6-parameter fit <http://www.nu-fit.org>  
 Esteban, Hernandez-Cabezudo, Maltoni, Schwetz, MCG-G PRELIMINARY

Precision  $3\sigma/3$

NuFIT 4.0 (prelim)



• Best determined:

$$\theta_{12}, \theta_{13}, \Delta m_{21}^2, |\Delta m_{3l}^2|$$

$[\Delta m_{3l}^2 = \Delta m_{31}^2 > 0 \text{ (NO)}$   
 $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0 \text{ (IO)}]$

• Pending issues:

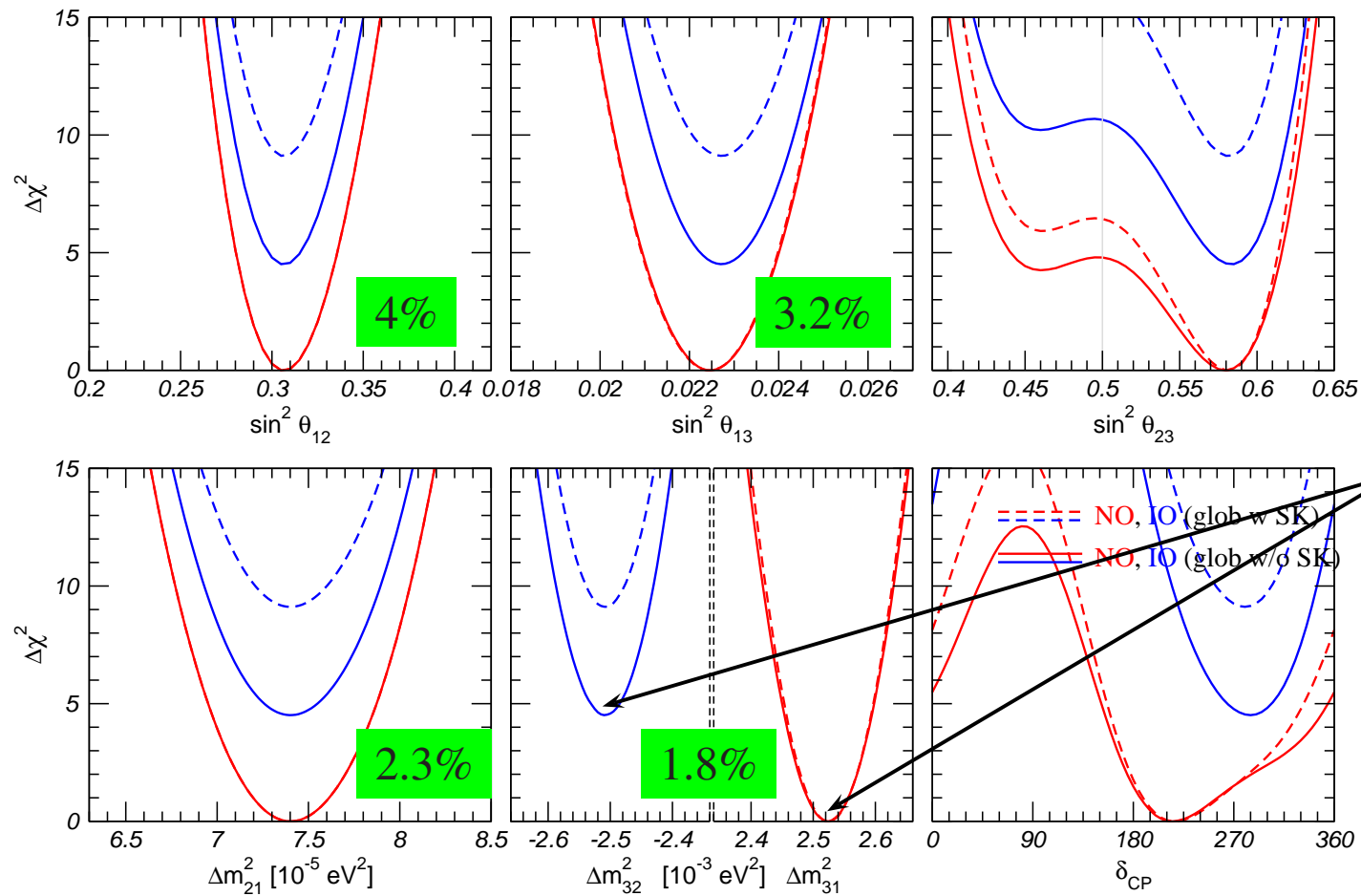
\*  $\theta_{23}$ : Maximality/Octant

# 3 $\nu$ Flavour Parameters: Status OCT 2018

Global 6-parameter fit <http://www.nu-fit.org>  
 Esteban, Hernandez-Cabezudo, Maltoni, Schwetz, MCG-G PRELIMINARY

Precision  $3\sigma/3$

NuFIT 4.0 (prelim)



- Best determined:
  - $\theta_{12}, \theta_{13}, \Delta m_{21}^2, |\Delta m_{3l}^2|$
  - $[\Delta m_{3l}^2 = \Delta m_{31}^2 > 0 \text{ (NO)}$
  - $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0 \text{ (IO)}$
- Pending issues:
  - \*  $\theta_{23}$ : Maximality/Octant
  - \* Mass Ordering

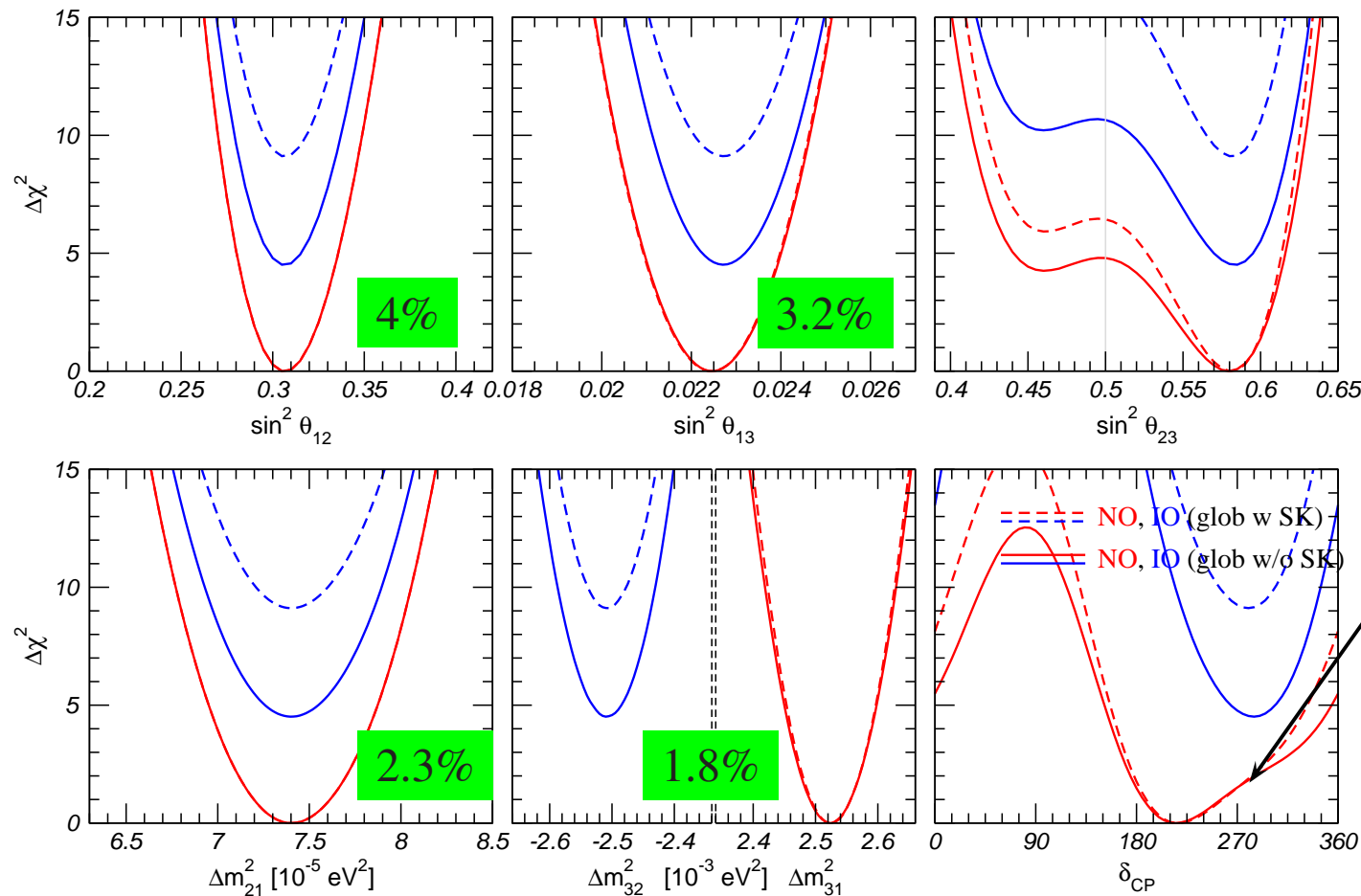


# 3 $\nu$ Flavour Parameters: Status OCT 2018

Global 6-parameter fit <http://www.nu-fit.org>  
 Esteban, Hernandez-Cabezudo, Maltoni, Schwetz, MCG-G PRELIMINARY

Precision  $3\sigma/3$

NuFIT 4.0 (prelim)



• Best determined:

$$\theta_{12}, \theta_{13}, \Delta m_{21}^2, |\Delta m_{3l}^2|$$

$[\Delta m_{3l}^2 = \Delta m_{31}^2 > 0 \text{ (NO)}$   
 $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0 \text{ (IO)}$

• Pending issues:

- \*  $\theta_{23}$ : Maximality/Octant
- \* Mass Ordering
- \* CP phase:  $> \pi$ ?

# 3 $\nu$ Analysis: “12” Sector

- $\Delta m_{13}^2 \gg E/L \Rightarrow P_{ee}^{2\nu}$  obtained by solving

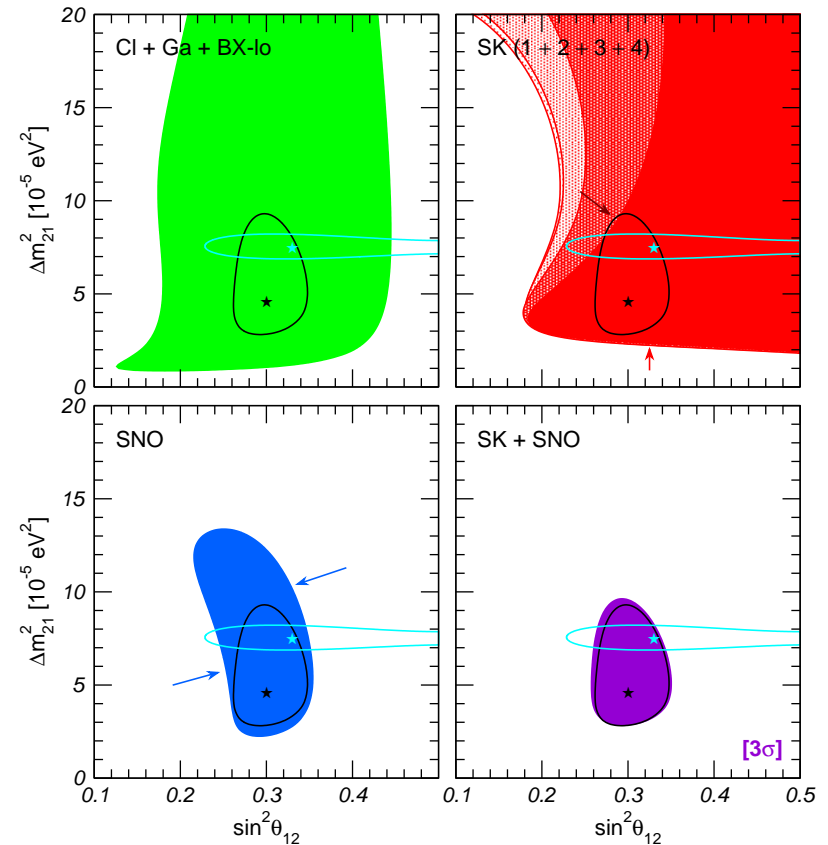
$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = \left[ \frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix} \pm \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}$$

For  $2\nu$ 's

$$P_{ee} \simeq \begin{cases} \text{Solar High E : } \sin^2 \theta_{12} \\ \text{Solar Low E : } (1 - \sin^2 2\theta_{12}/2) \\ \text{KLand : } \left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 L}{4E}\right) \end{cases}$$

- \* Solar region determined by High E data

- \* Param's  $\begin{cases} \theta_{12} \text{ SNO most sensitivity} \\ \Delta m_{21}^2 \text{ by KamLAND} \end{cases}$



## 2 $\nu$ Analysis: “12” Sector

- $\Delta m_{13}^2 \gg E/L \Rightarrow P_{ee}^{2\nu}$  obtained by solving

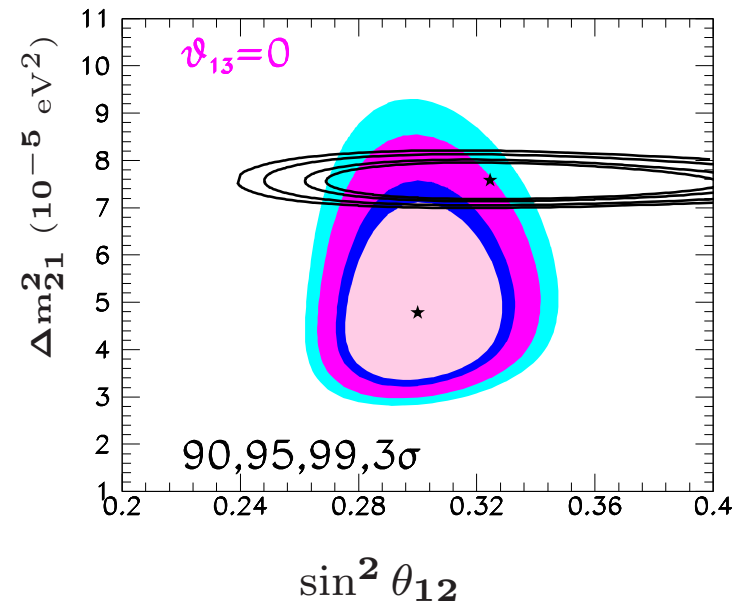
$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = \left[ \frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix} \pm \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}$$

$$P_{ee} \simeq \begin{cases} \text{Solar High E : } \sin^2 \theta_{12} \\ \text{Solar Low E : } (1 - \sin^2 2\theta_{12}/2) \\ \text{KLand : } (1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 L}{4E}) \end{cases}$$

- \* Solar region determined by High E data

- \* Param's  $\begin{cases} \theta_{12} \text{ SNO most sensitivity} \\ \Delta m_{21}^2 \text{ by KamLAND} \end{cases}$

$$\sin^2 \theta_{12} = \begin{cases} 0.3 \text{ From Solar} \\ 0.325 \text{ From KLAND} \end{cases}$$



## 3 $\nu$ Analysis: “12” Sector and $\theta_{13}$

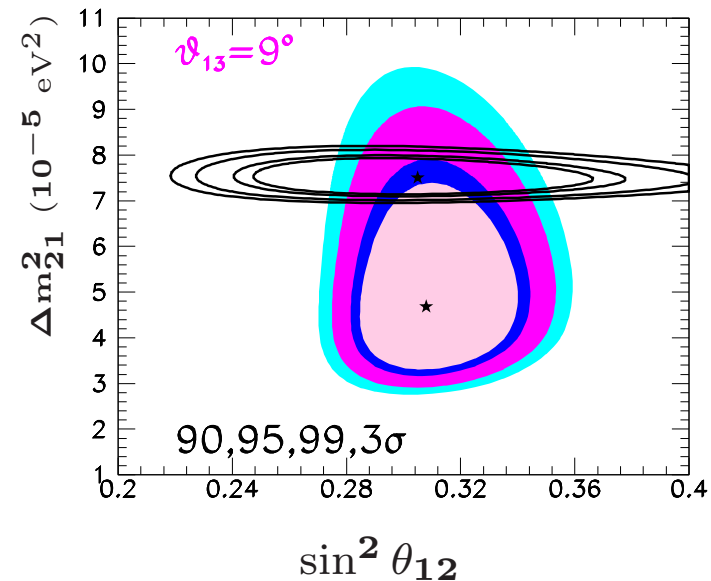
- $\Delta m_{13}^2 \gg E/L \Rightarrow P_{ee}^{3\nu} = c_{13}^4 P_{2\nu} + s_{13}^4$

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix} = \left[ \frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix} \pm \sqrt{2} G_F N_e \begin{pmatrix} c_{13}^2 & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_a \end{pmatrix}$$

$$P_{ee} \simeq \begin{cases} \text{Solar High E : } c_{13}^4 \sin^2 \theta_{12} \\ \text{Solar Low E : } c_{13}^4 \left( 1 - \frac{\sin^2 2\theta_{12}}{2} \right) \\ \text{Kam : } c_{13}^4 \left( 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{21}^2 L}{4E} \right) \end{cases}$$

$\Rightarrow$  KamLAND region shifts left

$\Rightarrow$  Solar slight shifts right (due to High E)

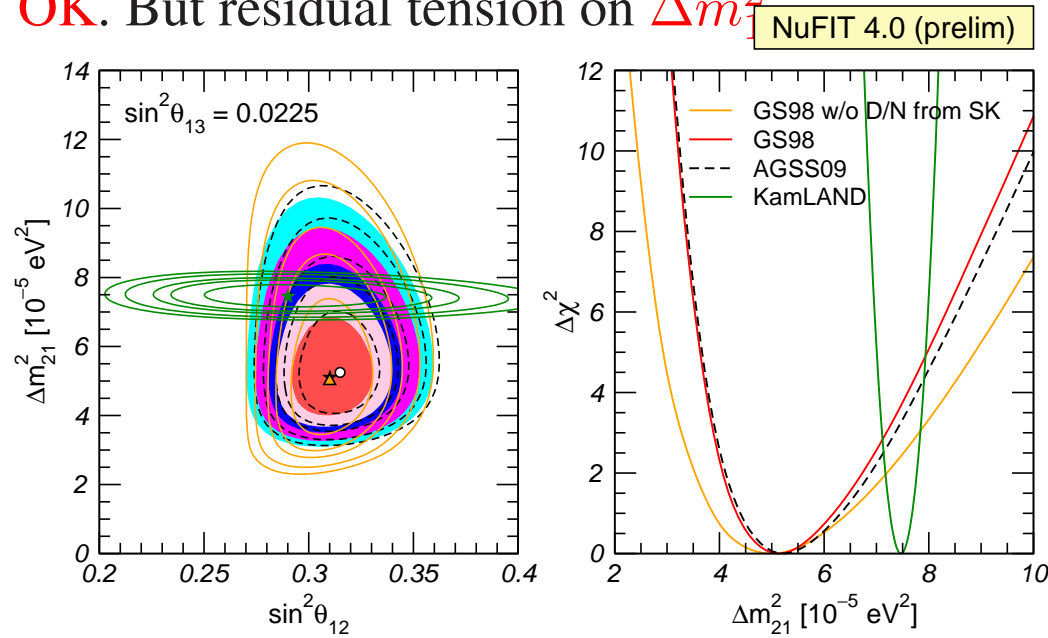


$\Rightarrow$  Good agreement of best fit  $\theta_{12}$

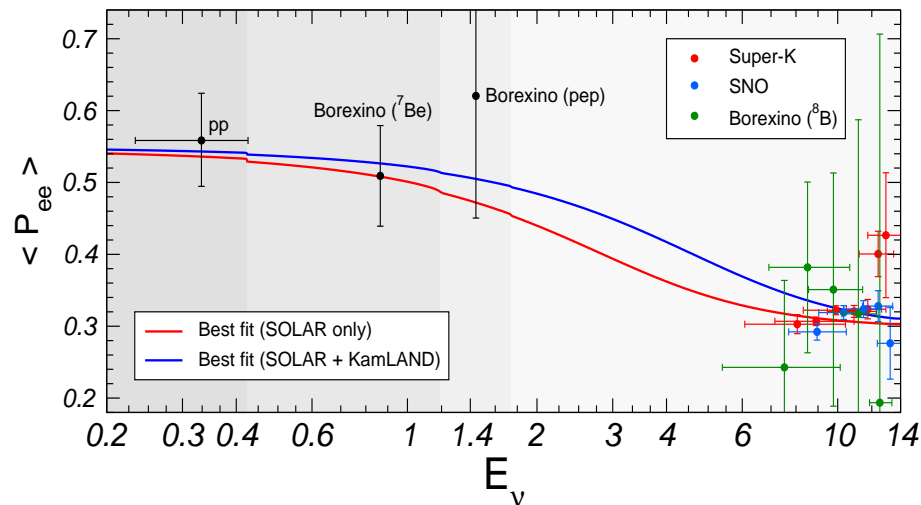
$\Rightarrow$  Residual tension on  $\Delta m_{21}^2$

# 3 $\nu$ Analysis: $\Delta m_{21}^2$ KamLAND vs SOLAR

For  $\theta_{13} \simeq 8.5^\circ$   $\theta_{12}$  OK. But residual tension on  $\Delta m_{21}^2$



Tension related to: a) “too large” of Day/Night at SK



b) smaller-than-expected low-E turn up from MSW at best global fit

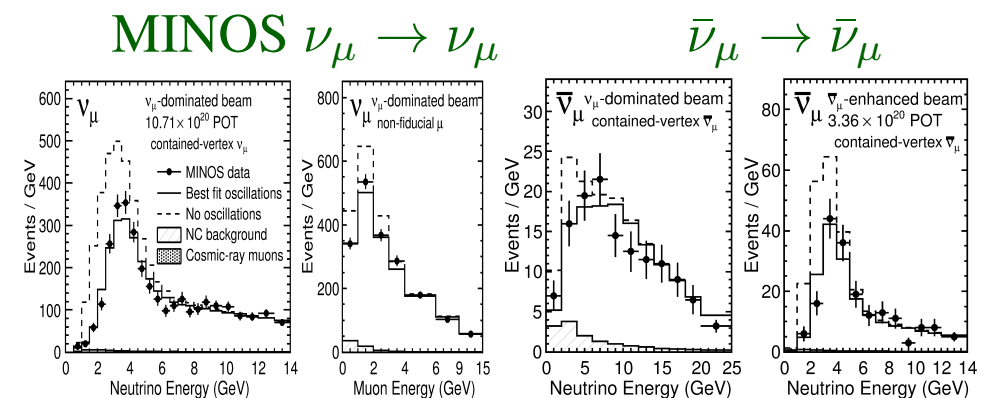
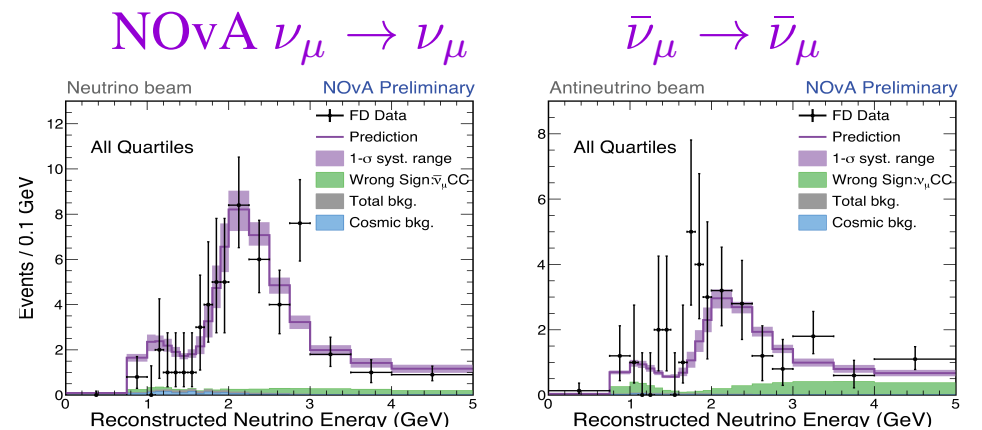
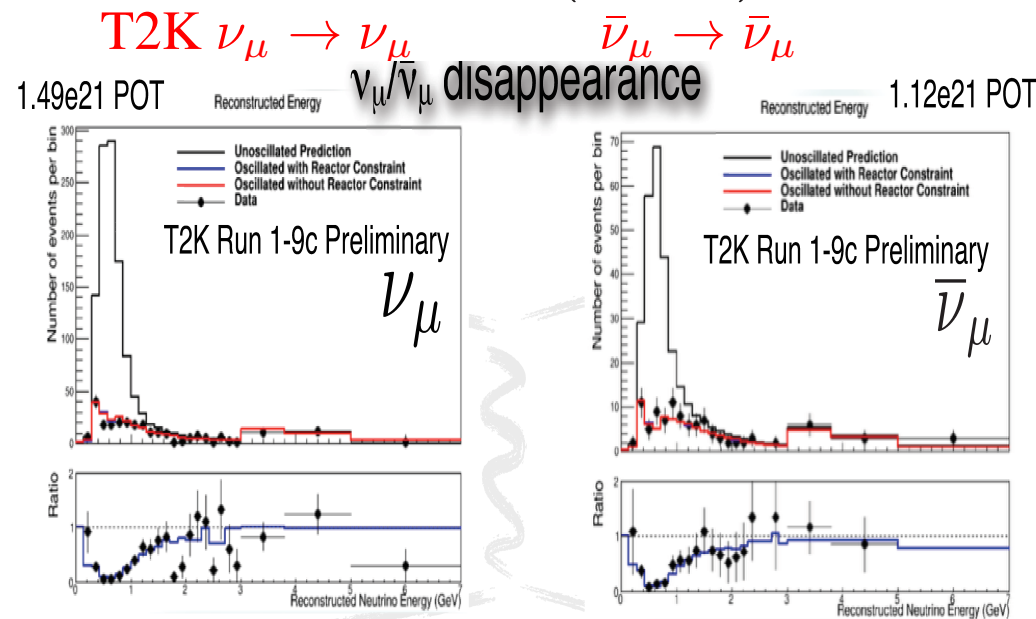
Modified matter potential?  
see Jordi Salvado's talk

# 3 $\nu$ Analysis: $\theta_{23}$

- Best determined in  $\nu_\mu$  and  $\bar{\nu}_\mu$  disappearance in LBL

$$P_{\mu\mu} \simeq 1 - (c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13}) \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + \mathcal{O}(\Delta m_{21}^2)$$

- At osc maximum  $\sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) = 1 \Rightarrow P_{\mu\mu} \simeq 0$  for  $\theta_{23} \simeq \frac{\pi}{4}$



T2K Fig M.Wascko  $\nu$ -2018 Talk

T2K data: T.Koga Phd Thesis

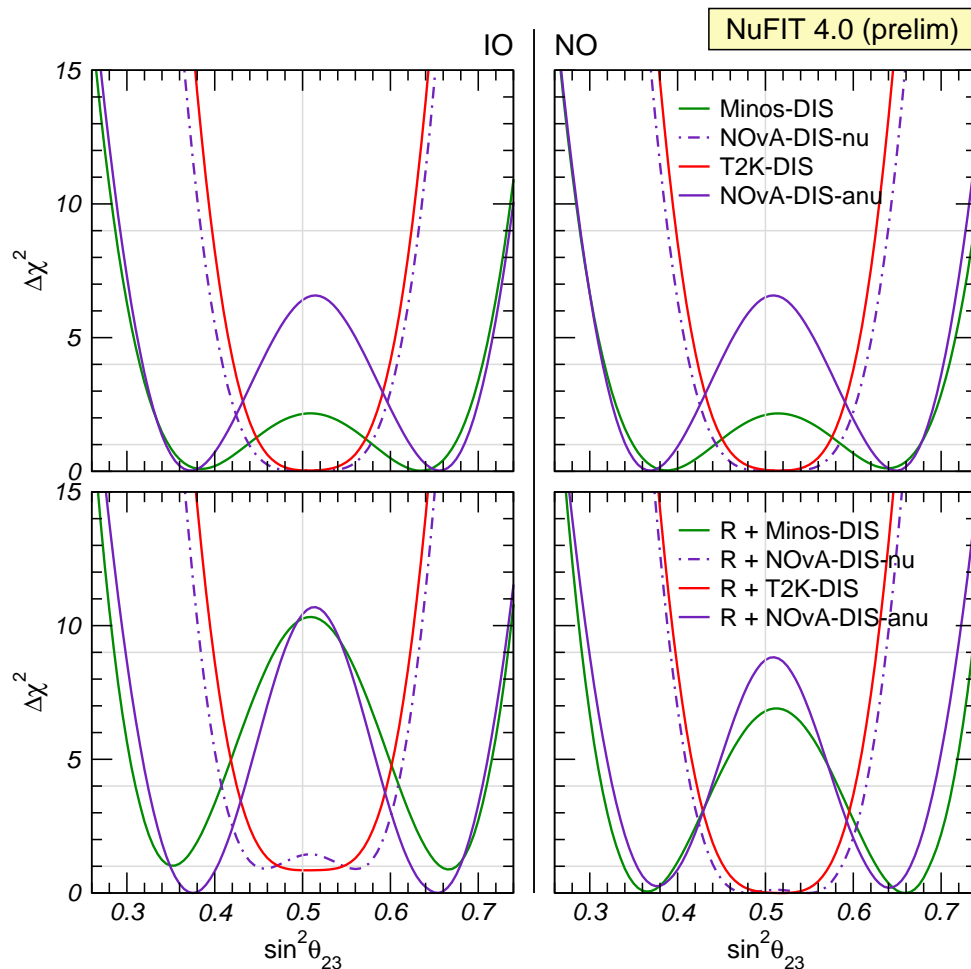
NOvA Fig/data: M. Sanchez M.Wascko  $\nu$ -2018 Talk

MINOS PRL 2013

# 3 $\nu$ Analysis: $\theta_{23}$

- Best determined in  $\nu_\mu$  and  $\bar{\nu}_\mu$  disappearance in LBL

$$P_{\mu\mu} \simeq 1 - (c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13}) \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + \mathcal{O}(\Delta m_{21}^2)$$



\* Upper panels use prior  $\theta_{13}$  from reactors  
(procedure in LBL experiment analysis)

\* Lower panels:

full combination with  $\chi_{\text{react}}^2(\theta_{13}, \Delta m_{31}^2)$

⇒ Non-maximality

driven by **NO $\nu$ A  $\bar{\nu}$**  and **MINOS**

⇒ More significant when full comb with React

⇒ Small contribution to NO/IO from disapp data

# 3 $\nu$ Analysis: $\Delta m_{3l}^2$ in LBL vs Reactors

- At LBL determined in  $\nu_\mu$  and  $\bar{\nu}_\mu$  disappearance spectrum

$$P_{\mu\mu} \simeq 1 - (c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13}) \sin^2 \left( \frac{\Delta m_{\mu\mu}^2 L}{4E} \right) + \mathcal{O}[(\Delta m_{21}^2)^2]$$

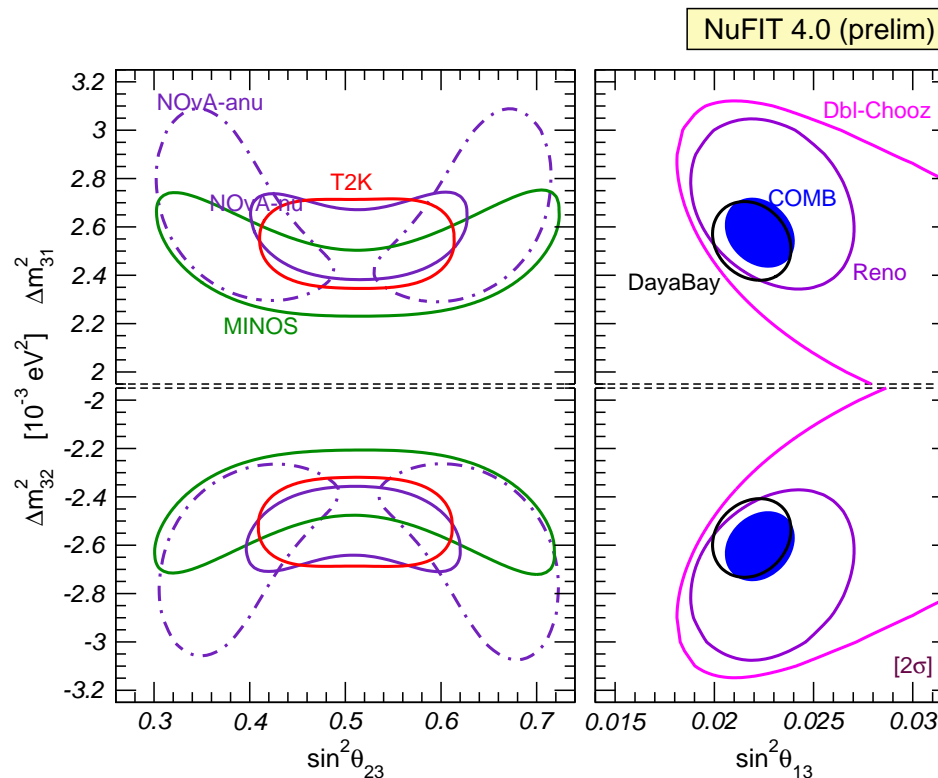
$$\Delta m_{\mu\mu}^2 \simeq \Delta m_{3l}^2 + \begin{matrix} c_{12}^2 \Delta m_{21}^2 & \text{NO} \\ s_{12}^2 \Delta m_{21}^2 & \text{IO} \end{matrix} + \dots$$

- At MBL Reactors (Daya-Bay, Reno, D-Chooz) determined in  $\bar{\nu}_e$  disapp spectrum

$$P_{ee} \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{ee}^2 L}{4E} \right) + \mathcal{O}[(\Delta m_{21}^2)^2]$$

$$\Delta m_{ee}^2 \simeq \Delta m_{3l}^2 + \begin{matrix} s_{12}^2 \Delta m_{21}^2 & \text{NO} \\ c_{12}^2 \Delta m_{21}^2 & \text{IO} \end{matrix}$$

Nunokawa, Parke, Zukanovich (2005)





# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

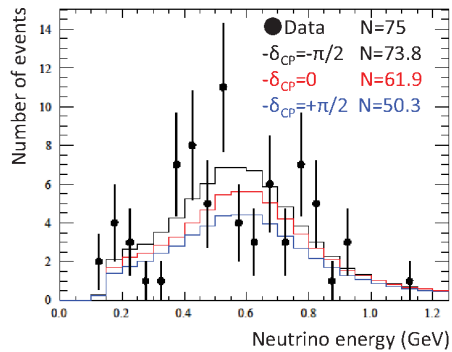
$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$

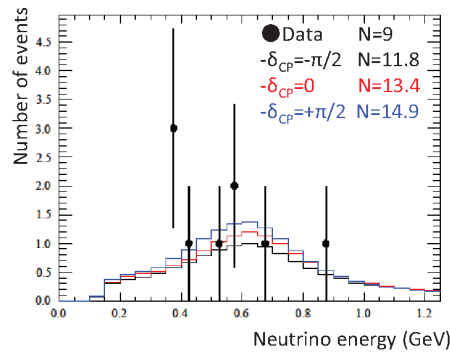
T2K  $\nu_{\mu} \rightarrow \nu_e$

$\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$

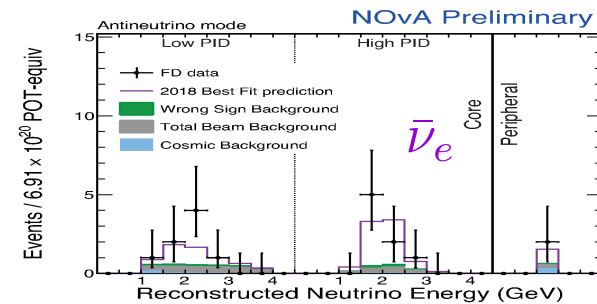
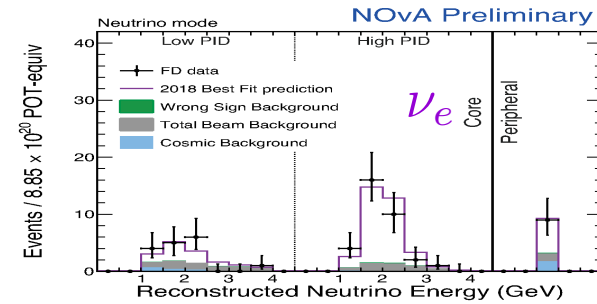
NOvA



FHC 1R $\nu_e$ ,  $\delta_{CP}$



RHC 1R $\nu_e$ ,  $\delta_{CP}$



SAMPLE	PREDICTED in NO				OBSERVED
	$\delta_{CP}=-\pi/2$	$\delta_{CP}=0$	$\delta_{CP}=\pi/2$	$\delta_{CP}=\pi$	
$\nu_{\mu}$ FHC 1R $\mu$	268.5	268.2	268.5	268.9	243
$\bar{\nu}_{\mu}$ RHC 1R $\mu$	95.5	95.3	95.5	95.8	102
$\nu_e$ FHC 1Re 0 decay-e	73.8	61.6	50.0	62.2	75
$\nu_e$ FHC 1Re 1 decay-e	6.9	6.0	4.9	5.8	15
$\bar{\nu}_e$ RHC 1Re 0 decay-e	11.8	13.4	14.9	13.2	9

NOVA OBSERVES: 58 EVENTS IN NEUTRINO, 18 EVENTS IN ANTINEUTRINO MODE.

EXPECT 30-75 EVENTS FOR NEUTRINO MODE AND 10-22 FOR ANTINEUTRINO MODE

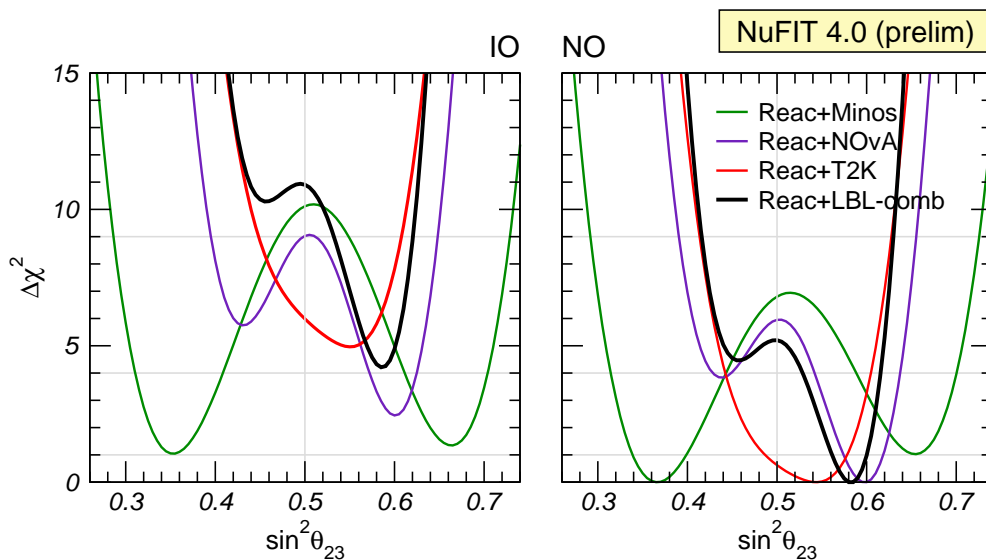
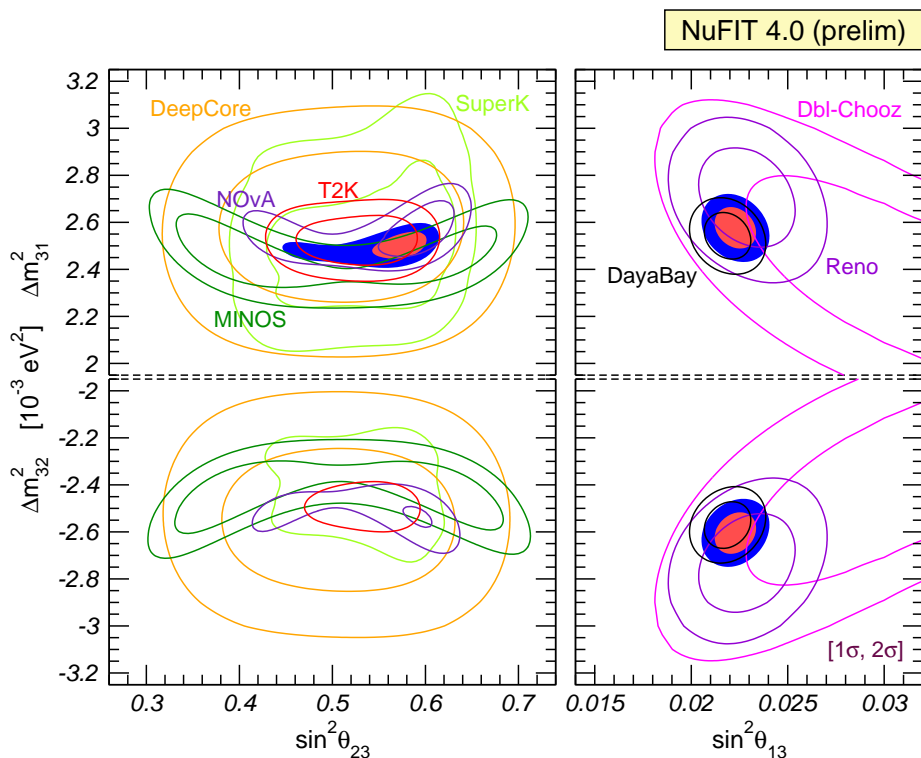
# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



REAC+LBL-COMB:

- In both orderings best fit  $\sin^2 \theta_{23} \sim 0.58$
- $\theta_{23} = \frac{\pi}{4}$  at  $\Delta\chi^2 \sim 5(6)$  of b.f. in NO(IO)

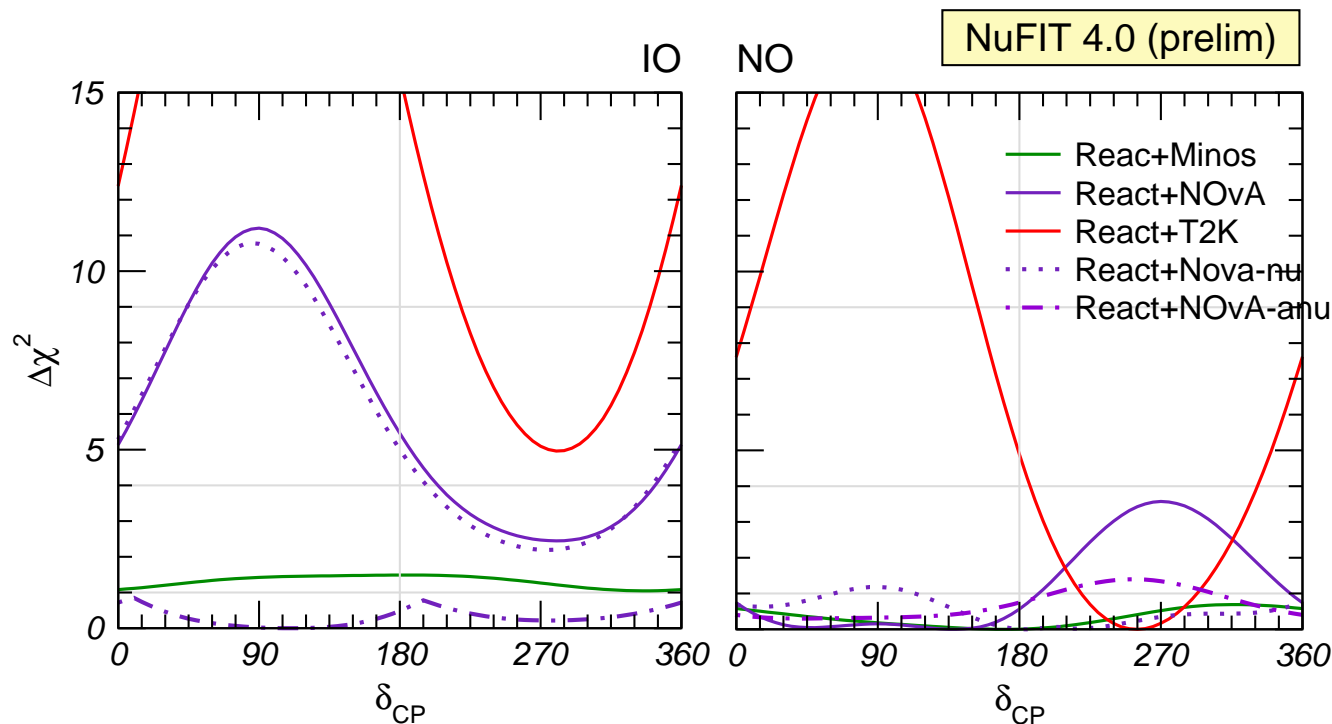
# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



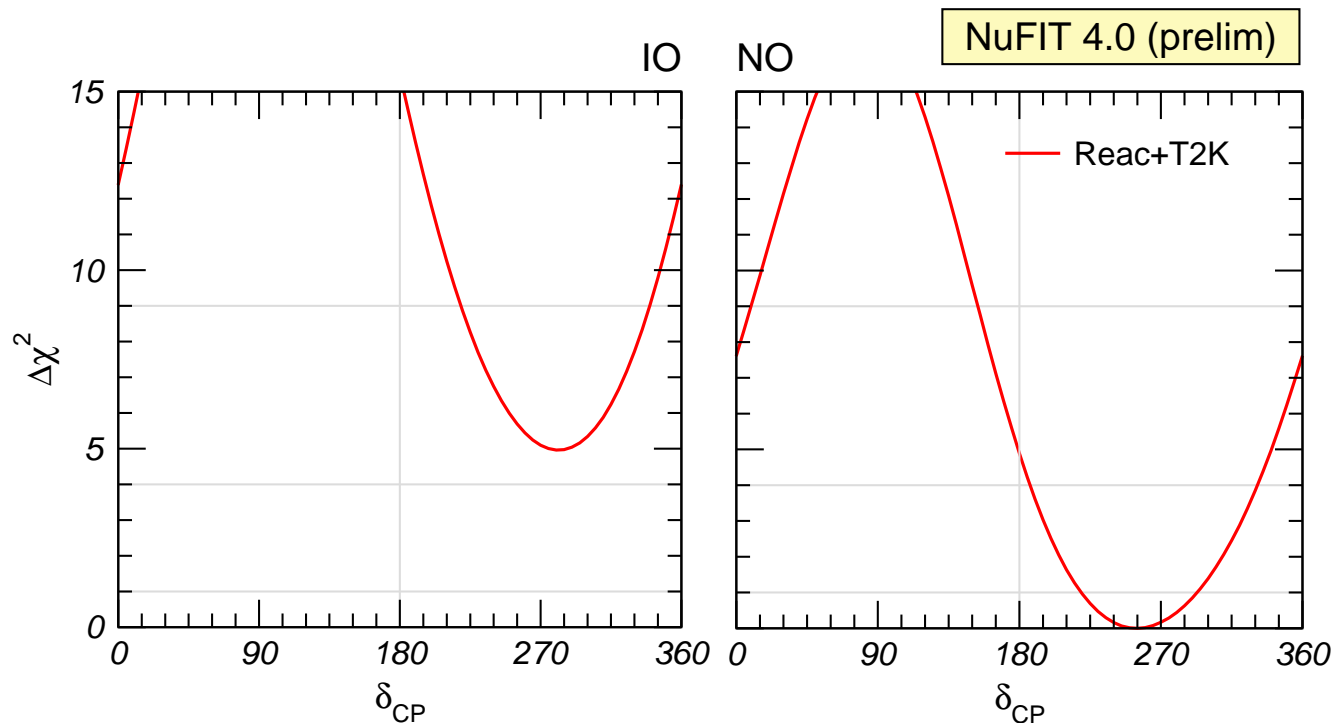
# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



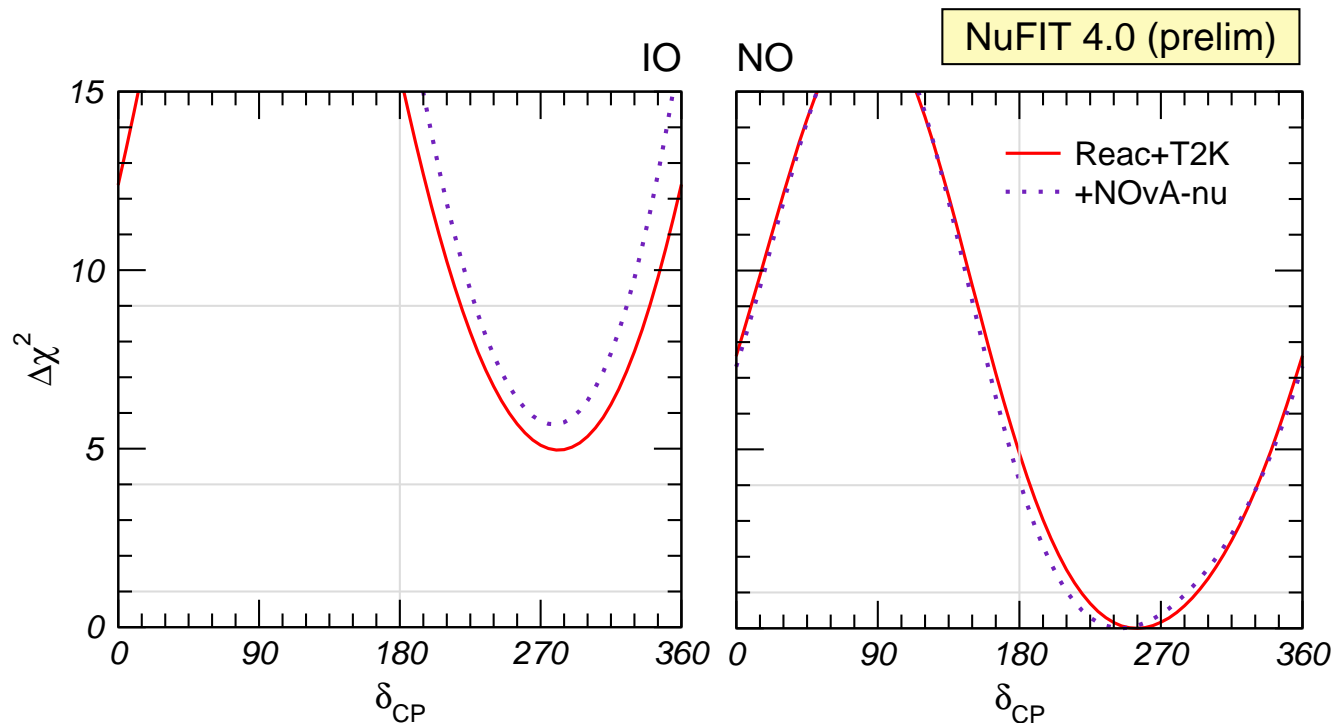
# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



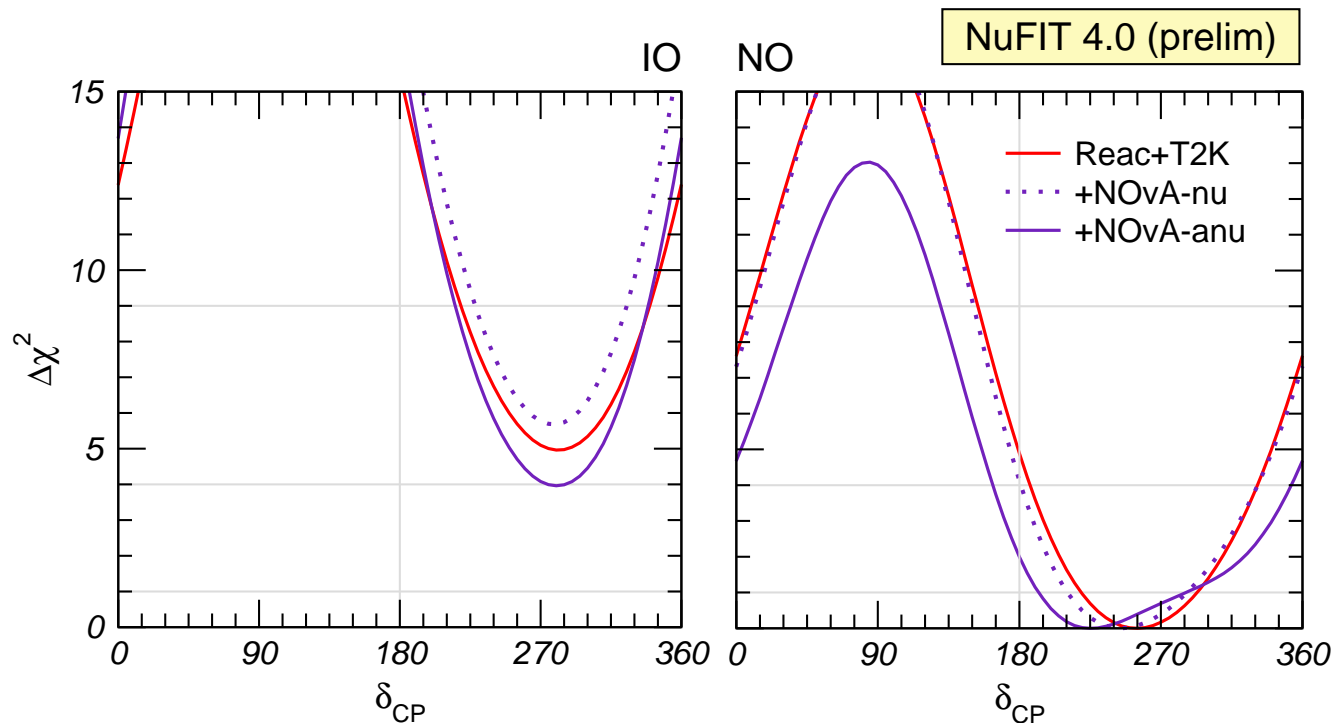
# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



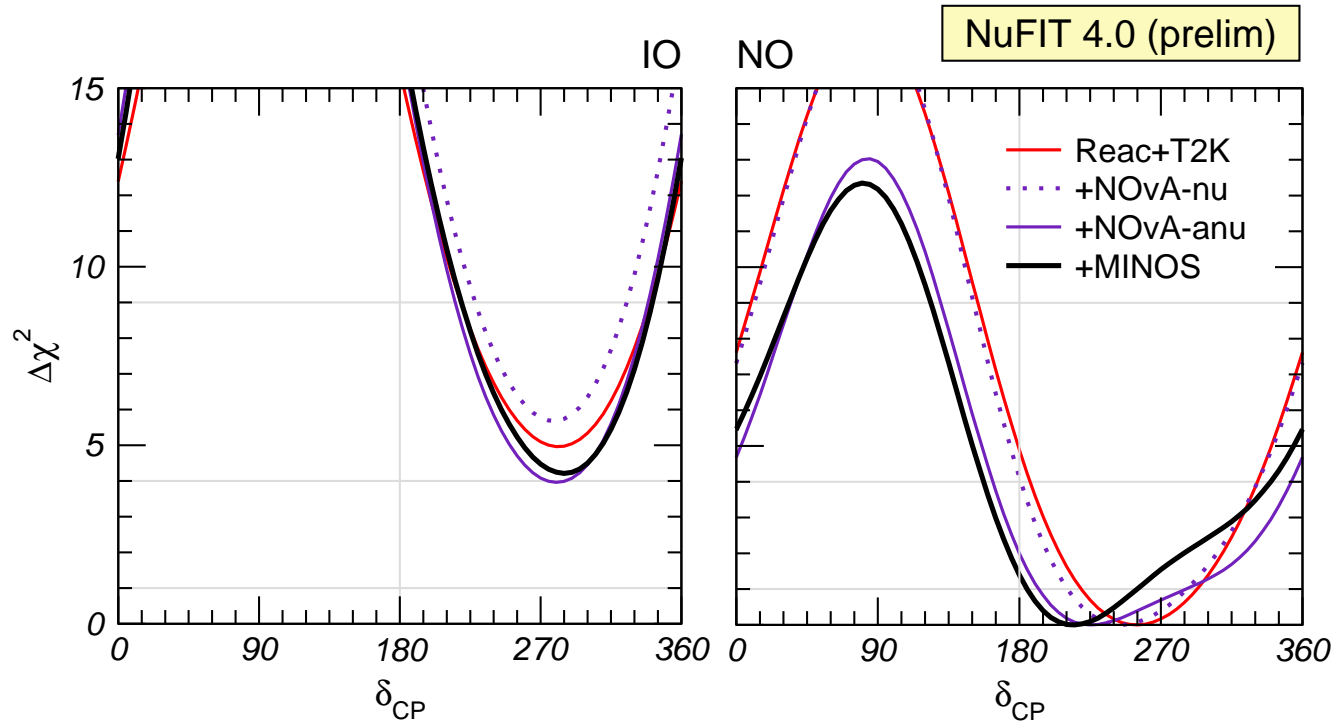
# 3 $\nu$ Analysis: $\theta_{23}$ Octant, Ordering, $\delta_{CP}$ in LBL

rcia

- Dominant information from  $\nu_e$  appearance in LBL

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left( \frac{\Delta_{31}}{B_{\mp}} \right)^2 \sin^2 \left( \frac{B_{\mp} L}{2} \right) + \tilde{J} \frac{\Delta_{21}}{V_E} \frac{\Delta_{31}}{B_{\mp}} \sin \left( \frac{V_E L}{2} \right) \sin \left( \frac{B_{\mp} L}{2} \right) \cos \left( \frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} \quad B_{\pm} = \Delta_{31} \pm V_E \quad \tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$



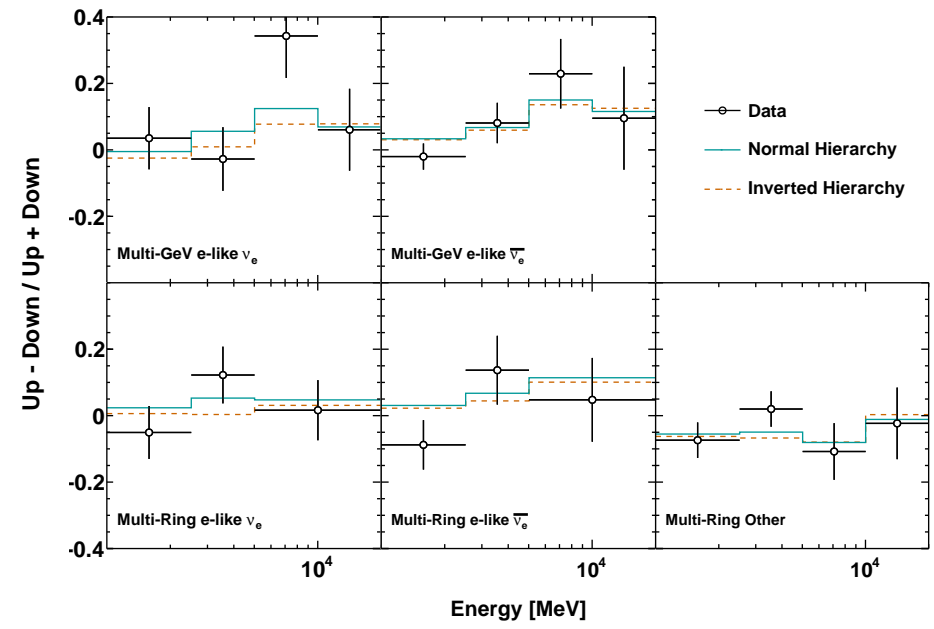
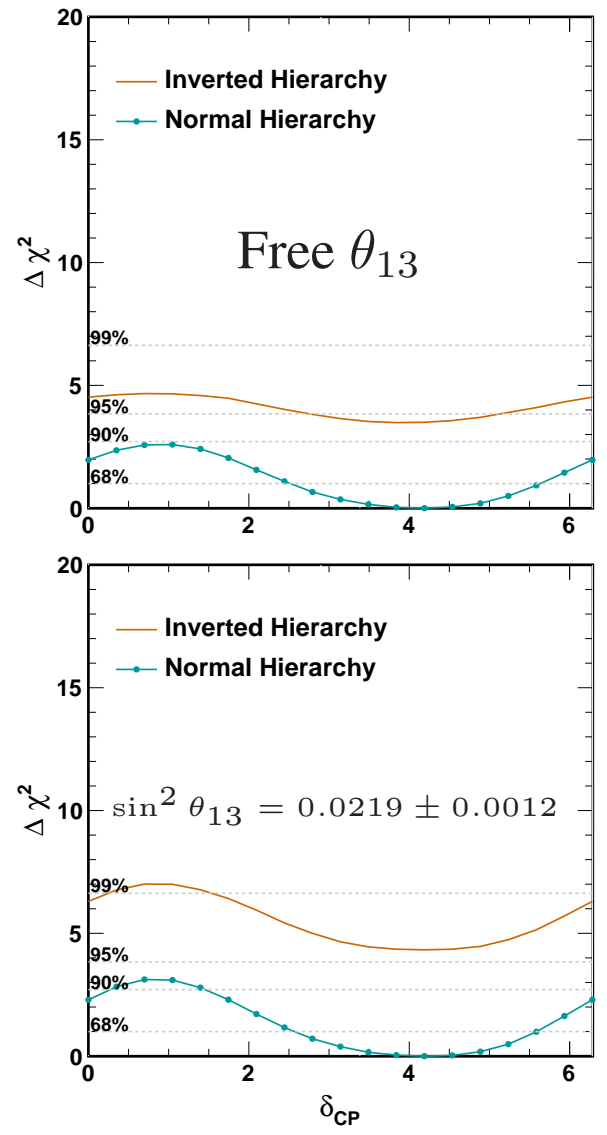
From LBL-COMB:

- \* **IO** disfavoured at  $2\sigma$
- \* **NO**: CP conservation allowed at  $\sim 1.2\sigma$
- \* **NO**:  $-20 \lesssim \delta_{CP} \lesssim 160$  ( $35 \lesssim \delta_{CP} \lesssim 125$ ) disfavoured at  $2\sigma$  ( $3\sigma$ )

# $\theta_{23}$ , CP & Ordering: Effect in SK ATM

Latest SK analysis (PRD 2018) yields:

From SK paper: *“Small excesses seen between a few and ten GeV in the Multi-GeV e-like  $\nu_e$  and the Multi-Ring e-like  $\nu_e$  and  $\bar{\nu}_e$  samples drive these preferences”.*



- No pheno group reproduces SK analysis
- Only possibility is combine with SK  $\chi^2$  tables



# Summary

• From Global Analysis w ATM IC/DC w/o SK :

	NO	
	bf $\pm 1\sigma$	$3\sigma$
$\sin^2 \theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \rightarrow 0.346$
$\sin^2 \theta_{13}$	$0.02246^{+0.00069}_{-0.00067}$	$0.02043 \rightarrow 0.02453$
$\frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \rightarrow 8.02$
$\frac{\Delta m_{3l}^2}{10^{-3} \text{eV}^2}$	$2.523 \pm 0.033$	$2.424 \rightarrow 2.623$
$\sin^2 \theta_{23}$	$0.580^{+0.018}_{-0.021}$	$0.417 \rightarrow 0.627$
$\delta_{\text{CP}}$	$215^{+41}_{-29}$	$125 \rightarrow 393$

	IO $\Delta\chi^2 = 4.5$	
	bf $\pm 1\sigma$	$3\sigma$
$\sin^2 \theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \rightarrow 0.346$
$\sin^2 \theta_{13}$	$0.02271^{+0.00070}_{-0.00068}$	$0.02068 \rightarrow 0.02480$
$\frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \rightarrow 8.02$
$\frac{\Delta m_{3l}^2}{10^{-3} \text{eV}^2}$	$-2.510^{+0.035}_{-0.032}$	$-2.610 \rightarrow -2.409$
$\sin^2 \theta_{23}$	$0.583^{+0.017}_{-0.020}$	$0.423 \rightarrow 0.629$
$\delta_{\text{CP}}$	$285^{+27}_{-30}$	$193 \rightarrow 360$

⇒ Including SK:

- **NO** vs **IO**:  $\Delta\chi^2 = 4.5 \Rightarrow 9.1$
- **NO**:  $\theta_{23} = \frac{\pi}{4}$ :  $\Delta\chi^2 = 4.4 \Rightarrow 6.2$
- **NO**: CP conserv:  $\Delta\chi^2 = 1.7 \Rightarrow 1.8$

