
CERN

Electron Cloud Effects in the Circulant Matrix Model

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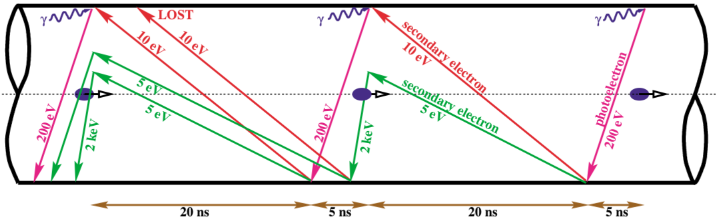
Electron cloud instabilities



- Cloud buildup mechanism leads to **high electron densities** in the pipe.
- Several **negative effects** : losses, vacuum degradation, heating
- 2 types of tranverse instabilities : multi or **single** bunch.
- Magnetic field affects the density pattern.

Project goals and scope(1)

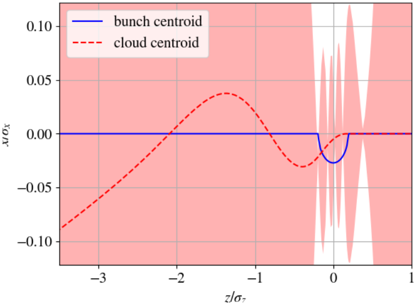
2 approaches for electron cloud effects simulation

- **Macroparticles** model (Computationally intensive)
- **Broadband resonator** model (Phenomenological model)

Purpose of this project :

- Simplified **linearized** model
- Taking into account the movement of the electrons along their interaction with the cloud

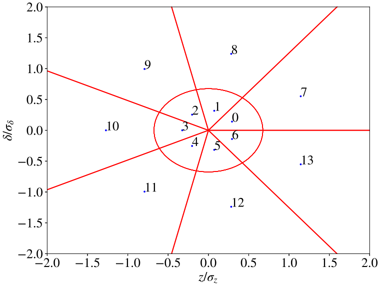
Project goals and scope(2)



Project goals and scope(2)

- Analytical derivation of the **electron cloud transverse kick**
- **Numerical implementation** in Circulant Matrix Model Python algorithm. (BimBim)
- Algorithm **benchmarking**.
- Preliminary **results** in High-Luminosity LHC at injection parameters.

Circulant Matrix Model (CMM)



$$\begin{pmatrix} x_{1,k+1} \\ x'_{1,k+1} \\ x_{2,k+1} \\ x'_{2,k+1} \\ \vdots \\ x_{N_r N_s,k+1} \\ x'_{N_r N_s,k+1} \end{pmatrix} = M \cdot \begin{pmatrix} x_{1,k} \\ x'_{1,k} \\ x_{2,k} \\ x'_{2,k} \\ \vdots \\ x_{N_r N_s,k} \\ x'_{N_r N_s,k} \end{pmatrix}$$

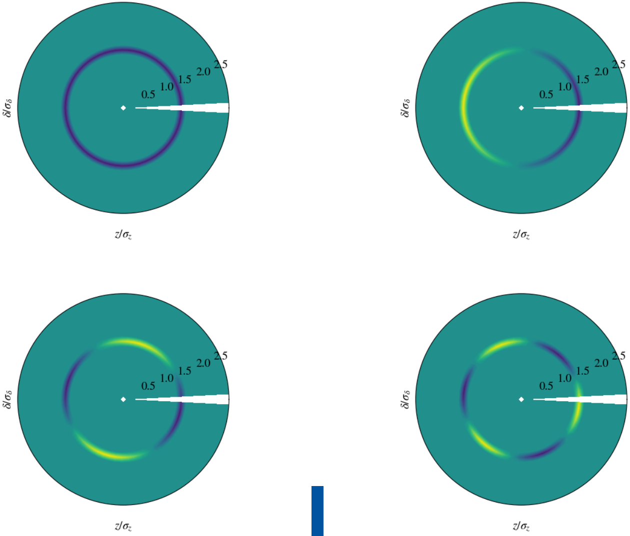
- Decomposition in **polar cells**.
- Cells : **Transverse Gaussian** distribution around x_i
- Purpose : diagonalizing the one turn matrix.
- **Tunes and stability** can be assessed from the real and imaginary part of the eigenvalues

Normal Mode Analysis

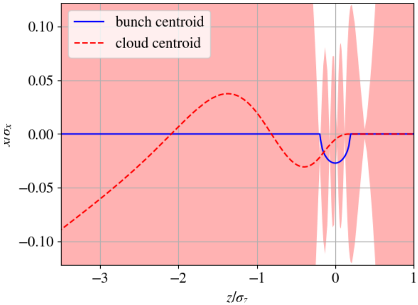
Decomposition in N_s slices, N_r rings, with synchro-betatron transport only:

- Unperturbed spectrum : $N_s \cdot N_r$ **stable** modes.
- Defined by **azimuthal** and **radial** numbers
- Radial modes are degenerated.

Normal Mode Analysis (2)



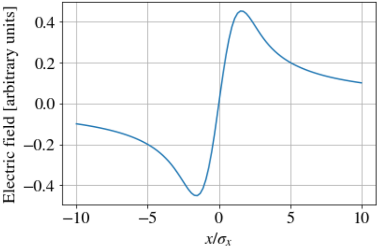
Electron Cloud effects modelisation



General idea:

- Resolve the equation for the **variable cloud size**
- Obtain the **cloud centroid** position from **numerical integrator**
- Deduce the kick received by each cell

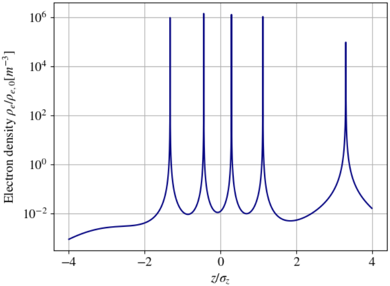
Variable cloud size



- Electron equation of motion around a **non oscillating** bunch :

$$\ddot{x}(t) = \frac{1}{m_e}\frac{\lambda_b(t)e^2}{2\pi\epsilon_0}\frac{1}{x}[1-exp(-\frac{x^2}{2\sigma_x^2})] \Longrightarrow -\frac{\lambda_b(t)e^2}{4\pi m_e\epsilon_0\sigma_x^2}x = -\omega_e^2(t)x(t)\,,$$

Variable cloud size(2)



- Result: **Gaussian** electron density with **variable size** $\sqrt{D(t)}$

$$n_e(x,t) = \frac{\lambda_e}{2\pi D(t)} e^{-\frac{x^2}{2D(t)}} \,,$$

- Electrons undergo **pinching**

Oscillating bunch

- **Oscillating bunch** centered around $\langle x \rangle_b(t)$,
  $\longrightarrow$ oscillating electron cloud centroid $\langle x \rangle_e(t)$
- $n_e(x,t) = \frac{\lambda_e}{2\pi D(t)} e^{-\frac{x^2}{2D(t)}} \longrightarrow \frac{\lambda_e}{2\pi D(t)} e^{-\frac{(x-\langle x \rangle_e(t))^2}{2D(t)}}$
- Cloud centroid linearised equation of motion:

$$\ddot{\langle x \rangle}_e(t) = -\omega_{e,coh}^2(t)(\langle x \rangle_e(t) - \langle x \rangle_b(t)) \,,$$

Transverse kick

Gaussian Electron density → force felt by a **single proton**:

$$F(x,z) \propto \frac{1}{(x - \langle x \rangle_e(z))}[1 - exp(-\frac{(x - \langle x \rangle_e(z))^2}{2D(z)})]$$

We can deduce the **kick** received over a complete revolution :

$$\Delta x'(x,z) = \frac{1}{m_p c \beta \gamma} \frac{L_c}{c} F(x,z)$$

Electron Cloud Matrix(1)

We want to compute:

$$\begin{pmatrix} x_{1,k+1} \\ x'_{1,k+1} \\ x_{2,k+1} \\ x'_{2,k+1} \\ \vdots \\ x_{N_rN_s,k+1} \\ x'_{N_rN_s,k+1} \end{pmatrix} = M_{EC} \cdot \begin{pmatrix} x_{1,k} \\ x'_{1,k} \\ x_{2,k} \\ x'_{2,k} \\ \vdots \\ x_{N,k} \\ x'_{N,k} \end{pmatrix} .$$

Electron Cloud Matrix(2)

Next step: implementing the kick in the CMM.

- **Coherent kick**: integration over an entire cell

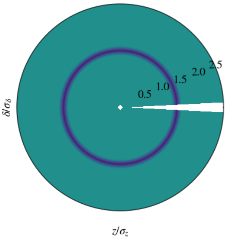
$$\Delta x'_{coh}(x_j; z) \propto \frac{1}{(x_j - \langle x\rangle_e(z))}[1 - exp(-\frac{(x_j - \langle x\rangle_e(z))^2}{2(D(z)+\sigma_x^2)})]$$

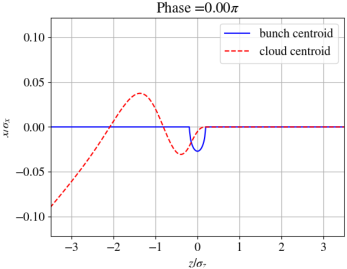
- **Linearisation**:

$$\begin{aligned}\Delta x'_{coh}(x_j; z) &\propto \frac{1}{[D(z)+\sigma_x^2]}(x_j - \langle x\rangle_e(z)) \\ &= K_{EC}(z)[x_j - \langle x\rangle_e(z)]\end{aligned}$$

- Equivalent to **focusing** lens centered around cloud centroid
- Last missing piece : CMM expression of the centroids.

Electron Cloud Matrix(3)





Solution : **CMM operators**

- **Bunch centroid** :
$$\langle x\rangle_b(z) = \int_{-\infty}^{\infty} d\delta\ x(z,\delta)\Psi(z,\delta) \approx \sum_{i=1}^{n} x_i \int d\delta \Psi_i(\delta)$$
- **Cloud centroid** : apply **finite differences** on the equation of motion (non uniform grid spacing) ⟶ recurrent relation :

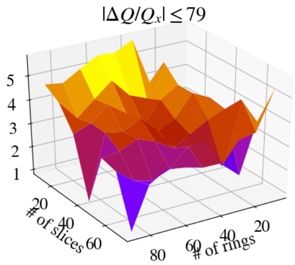
$$\langle x\rangle_{e,n+1} \approx c^2[\langle x\rangle_{e,n} h_n(-\frac{1}{\,}\omega^2_{e,coh,n}(h_n + h_{n-1}) + \frac{1}{\,} + \frac{1}{\,}) - \langle x\rangle_{e,n-1}\frac{h_n}{\,}$$

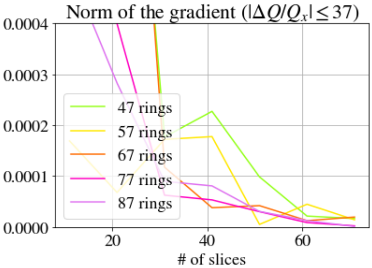
Convergence analysis(1)

Before analyzing results: we need to ensure that the polar grid is **precise enough**.

- CMM **artifacts** : upper limit on the number of slices. (unexplained)
- Sufficient longitudinal resolution for **stability of numerical integrator**
- Sufficient longitudinal resolution for **resolving the pinching peaks**

Convergence analysis(2)

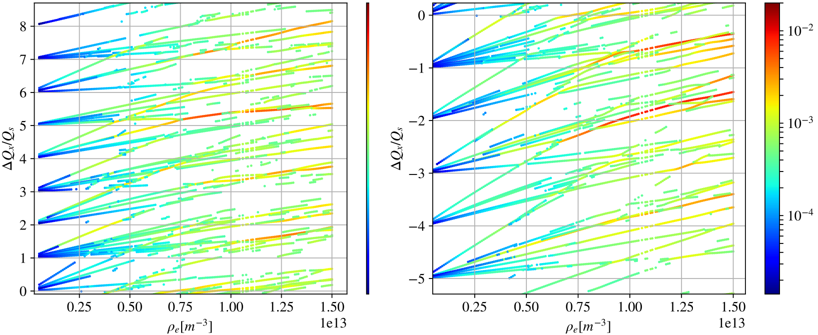




Convergence obtained for $N_a \leq 37$, for 61 rings, 77 slices.

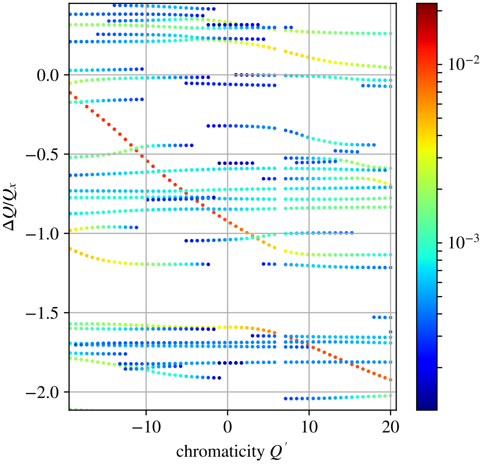
Note: high order modes converge slower than low order ones.

Simulated TMCI Threshold



Predicted TMCI for HL-LHC at injection around $7.5\ 10^{12}$ electrons $/m^3$

Influence of chromaticity



- Chromaticity and transverse feedback are **used in practice** to mitigate e-cloud instabilities.
- Some modes are stabilized by chromaticity while others get unstable.
- Investigation should be pushed further on.

Conclusion and future work

- We developped a model for the prediction of **single bunch electron cloud instabilities in drift sections.**
- Based on **linearised** equation of motions.
- The **convergence** of the model was checked.
- **Preliminary results** were obtained in the case of HL-LHC at injection.

Future work:

- Extension to **dipolar** and **quadrupolar** sections
  $\longrightarrow$ **Comparison** with measurement data.
- **Comparison** with macroparticles and broadband resonator simulations.
- Investigation of the **CMM artifacts**.
- Investigation of the impact of the initial cloud size.
- Further investigation of the impact of **chromaticity**

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