Dissipation caused by oscillating vortices in the SRF cavities

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Outline

- Trapped vortices can produce significant RF power in hotspots. Vortices contribute to the residual surface resistance which can exceed the BCS surface resistance.

- How much vortex dissipation can be tolerated? Theory of RF power produced by oscillating flexible vortex lines. Effect of pinning. Residual surface resistance.

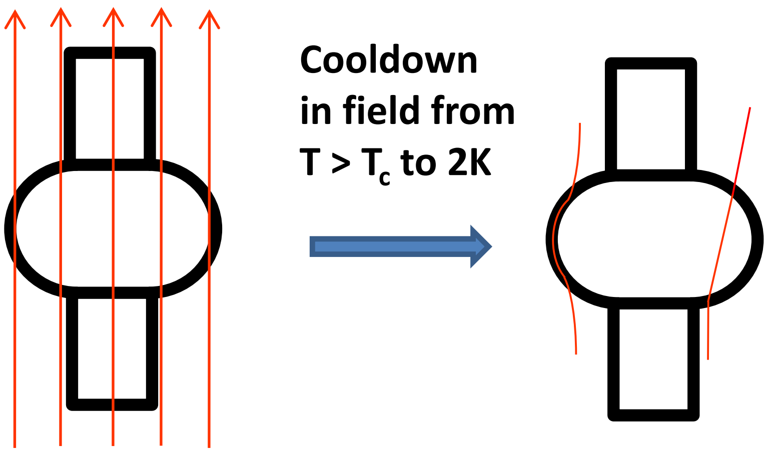
- Intricate dependencies of R_i on frequency, pin spacing and the mean free path. Optimization of R_i

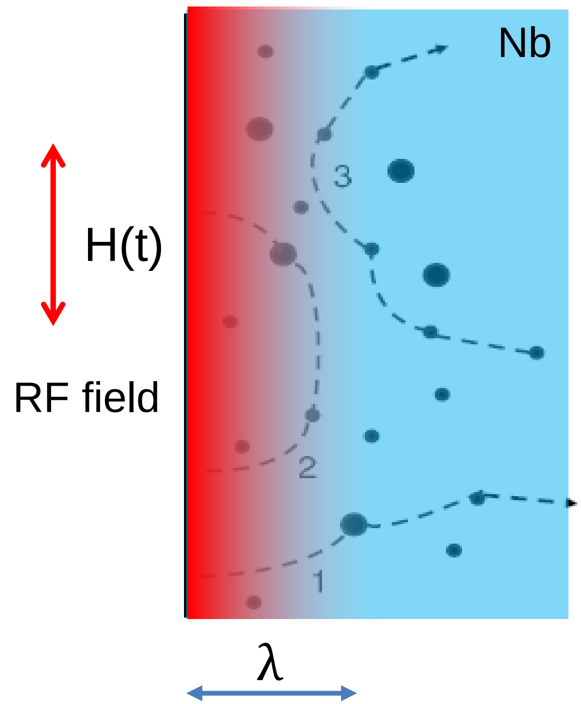
- To what extent can vortex RF dissipation be reduced by strong pinning?
  - Is the optimum pinning good for SRF?

- How fast can vortices move in the SRF cavities?
  - Viscous drag coefficient in dirty and clean limits
  - Nonlinear vortex viscosities and terminal velocities
  - What happens when a hypersonic vortex enters a superconductor?

- Conclusions

How do trapped vortices appear?





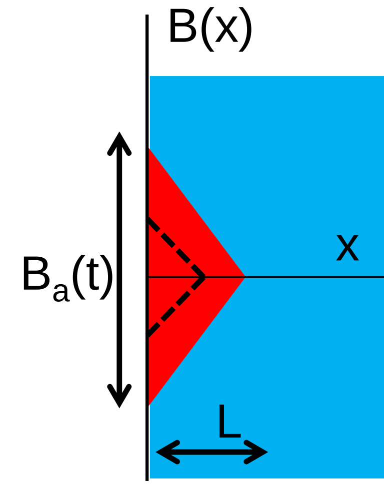
London penetration depth of superconducting rf currents ≈ 40 nm $<<$ d $= 3$mm

- Trapped appear during cooling through T_c due to any stray magnetic fields
- Trapped vortices due to the Earth magnetic field can give 2-3 orders of magnitude higher R_i than R_{BCS} at 2K.
- Even good screening (1% of H_E) cannot fully eliminate trapped vortices
- Flushing vortices out by strong thermal gradients: G. Ciovati and A Gurevich, PRAB 11, 122001 (2008); A. Romanenko et al, JAP 115, 184903 (2014); S. Posen et al. JAP 119, 213903 (2016); M. Checchin et al, PR Appl. 5, 044019 (2016)

Why are vortices so deadly for SRF?

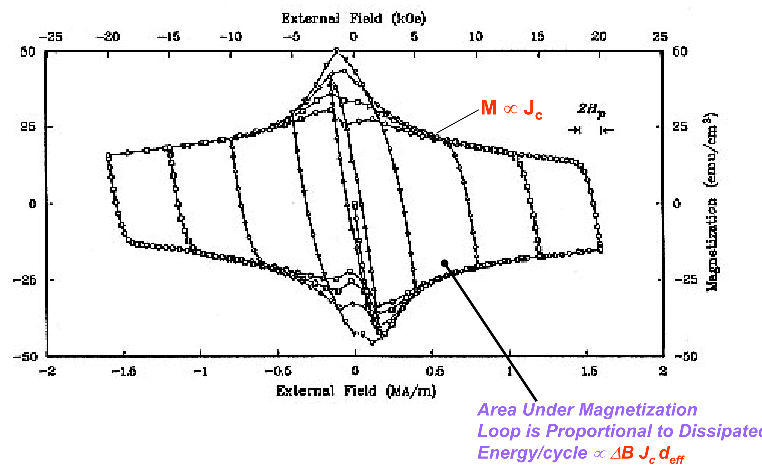
Bean's critical state of pinned vortices parallel to the surface with the flux gradient equal to the critical current density J_c:

$$B(x) = B_a(t) - \mu_0 J_c x$$



Flux penetration depth $L = B_a(t)/\mu_0 J_c$ is orders of magnitude greater than the London λ = 30-40 nm

Low-f remagnetization hysteretic losses can be orders of magnitude higher than Ohmic losses in Cu



$$P = f \oint M dB_a = \frac{2fB_a^3}{3\mu_0^2 J_c} = \frac{R_i H_a^2}{2}$$

R_i is linear in B_a, independent of resistivity and can be decreased by stronger pinning, BUT:

For Nb at f = 2GHz, and $J_c = 10^8$ A/m^2, we get R_i = 0.27 Ohm at B_a = 20 mT !

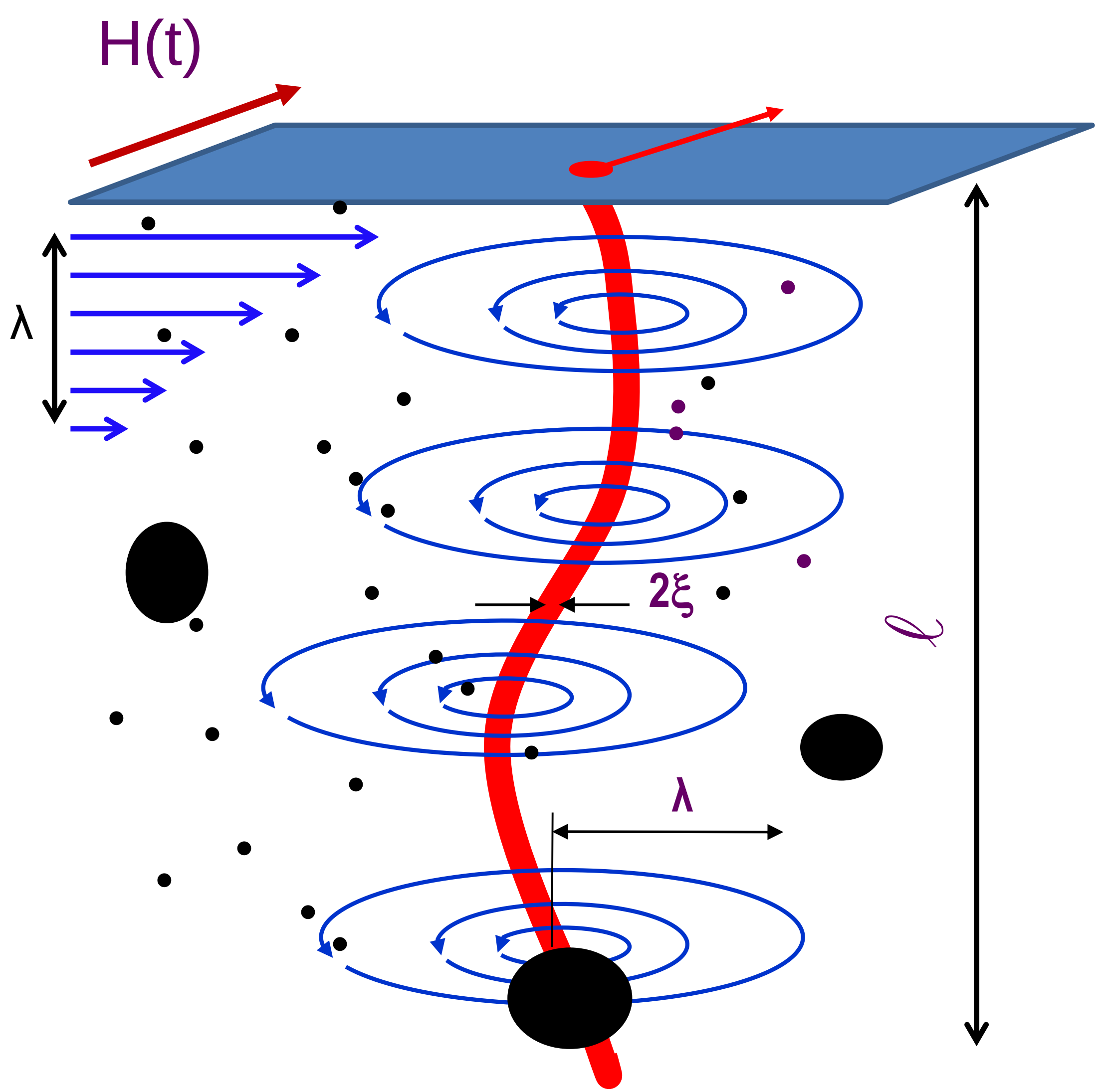
$$\boxed{R_i = \frac{4\alpha f B_a}{3J_c}}$$

Surface fraction of vortices $\alpha < 10^{-7}$

RF response from collective and strong pinning theories:
R. Willa et al, PRB 92, 134501 (2015); D. Liarte et al, arXiv:1808.01293

Pinning of vortices in Nb

Typical J_c = 1-10 kA/cm^2 in clean Nb are some 4-6 orders of magnitude lower than the depairing current density $J_c = H_c/\lambda$, = 500 MA/cm^2, indicating weak pinning:



Nanoprecipitates spaced by L >> the vortex core diameter 2ξ = 30-80 nm

Clusters of Impurities with the m.f.p. either smaller or greater than 2ξ.

Dislocation networks caused by plastic deformation

Compositional inhomogeneities

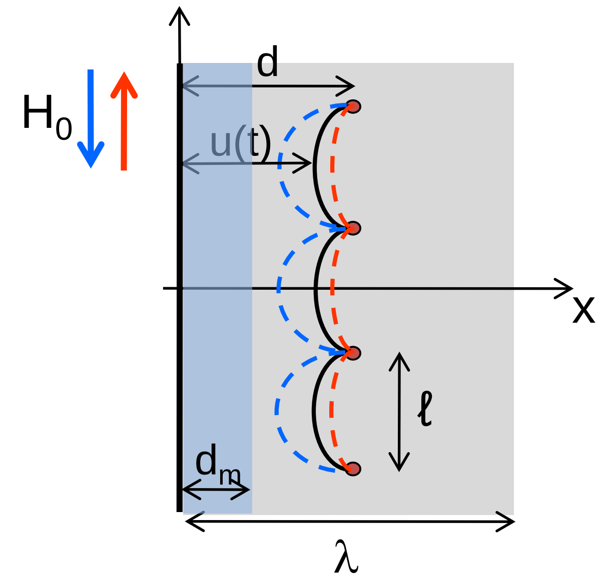
No direct correlation between the m.f.p. and the pin spacing

Coalescence or appearance of pins during heat treatments changes both the pin spacing and the m.f.p.

Parallel vortices near the oscillating surface barrier

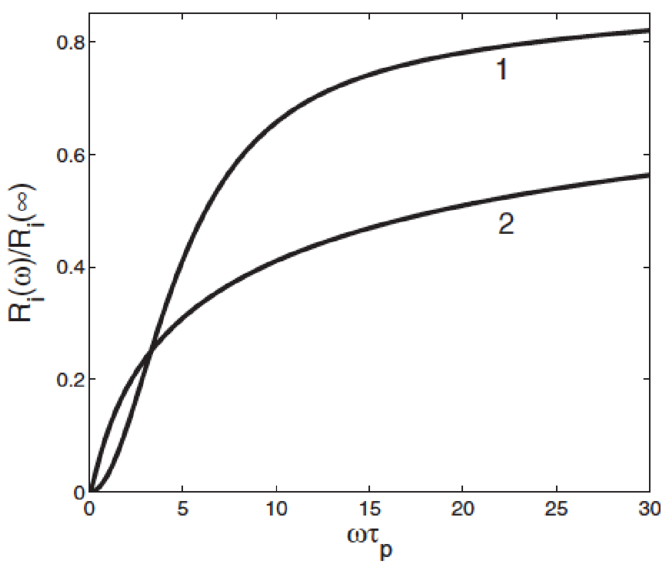
A. Gurevich and G.Ciovati, PRB 77, 104501 (2008)

$$\eta\partial_t u = \epsilon\partial_{zz}u + \frac{\phi_0 H_a}{\lambda}e^{-u/\lambda}\sin\omega t - \\ -\frac{\phi_0^2}{2\pi\mu_0\lambda^3}K_1\left(\frac{2u}{\lambda}\right) + \sum_m f_p(u, z - m\ell)$$

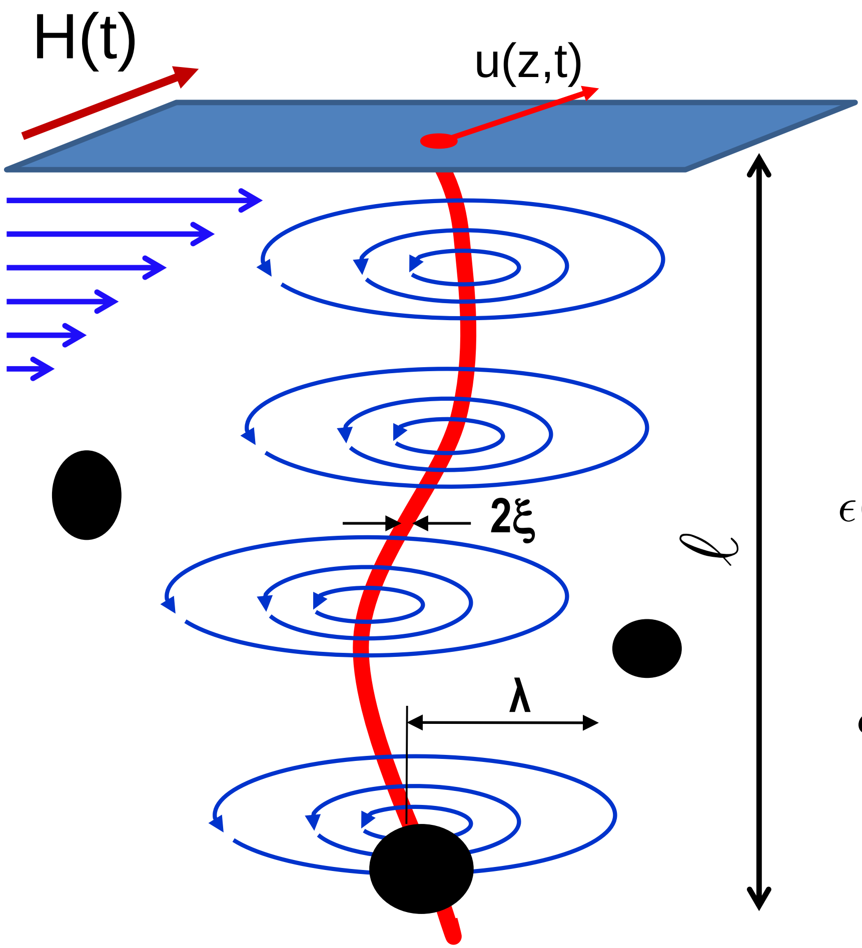


- Vortex pinned by a periodic chain of pins
- Vortex viscous drag: $\eta = \phi_0 B_{c2}/\rho_n$
- Trapping layer where a vortex is pushed out due to attraction to the surface
- Low-field residual resistance at $\omega\tau_p < 1, \quad \tau_p = \eta\ell^2/2\epsilon$

$$R_i \simeq \frac{\pi\omega^2\mu_0^2\ell^2\kappa^2}{15\rho_n \ln^2\kappa}\left(\frac{B_0}{\phi_0}\right)^{1/2} e^{-2d/\lambda}$$



Wagging tails of perpendicular vortices



$$\eta\frac{\partial u}{\partial t} = \hat{\epsilon}\frac{\partial^2 u}{\partial z^2} - \frac{\phi_0 H_a}{\lambda} e^{-z/\lambda}\cos\omega t$$

Nonlocal vortex line tension

$$\epsilon(k_z) = \frac{\epsilon_0}{\Gamma^2}\ln\frac{\lambda_c^2}{\xi^2(1+\lambda^2 k_z^2)} + \frac{\epsilon_0}{2\lambda^2 k_z^2}\ln(1+\lambda^2 k_z^2),$$

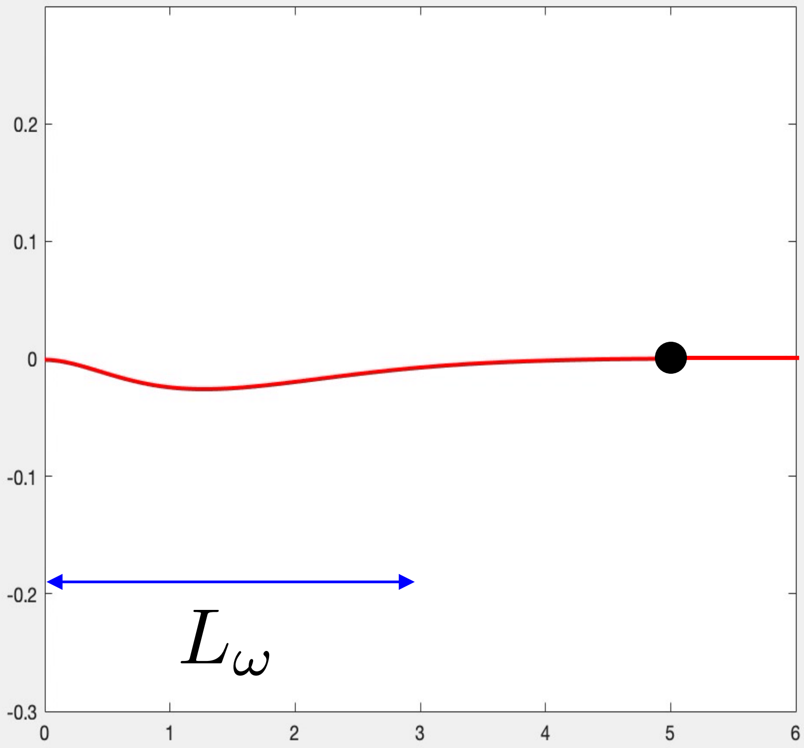
$$\epsilon_0 = \frac{\phi_0^2}{4\pi\mu_0\lambda^2}, \qquad g = \frac{\ln(\kappa\Gamma)}{\Gamma^2} + \frac{1}{2}, \qquad \kappa = \frac{\lambda}{\xi}$$

All vortex bending modes:

$$u(z,t) = \sum_{n=0}^{\infty} A_n \cos(k_n z) e^{i\omega t}, \qquad k_n = \frac{\pi}{l}\left(n+\frac{1}{2}\right)$$

A. Gurevich and G. Ciovati, PRB 87, 054502 (2013)

RF elastic ripple length of a vortex line



- L_ω is practically independent of T and decreases as the m.f.p. decreases

- Depending on ω and the m.f.p. L_ω can be either larger or smaller than the pin distance from the surface. If $L_\omega < \ell$ the effect of pinning is weak

Clean Nb

$$\lambda \approx \xi, \quad \rho_n = 1\ n\Omega m, \quad f = 2\ GHz$$

$$L_\omega \approx 180\ nm$$

Nb_3Sn

$$\lambda/\xi \approx 20, \quad \rho_n = 0.2\ \mu\Omega m, \quad f = 2\ GHz$$

$$L_\omega \approx 126\ nm$$

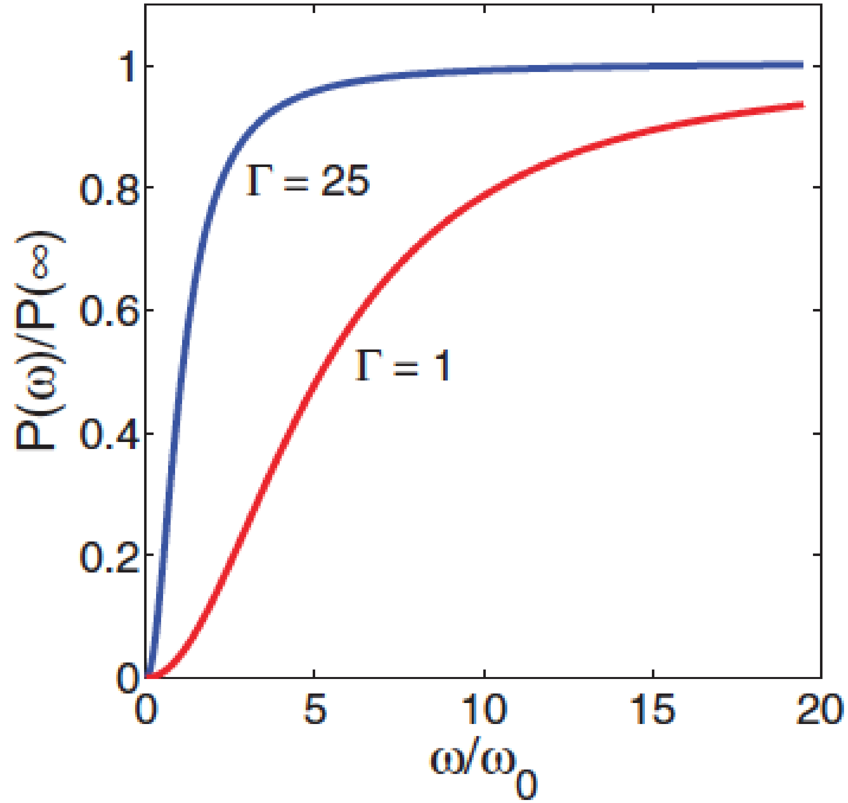
$$L_\omega = \sqrt{\frac{\epsilon}{\eta\omega}} = \frac{\xi}{2\lambda}\sqrt{\frac{\rho_n}{\pi\mu_0 f}}$$

General expression for the low-field RF power

$$P = \frac{H^2\omega^2}{4\lambda^2}\sum_{n=0}^{\infty}\frac{\eta\phi_0^2\ell I_n^2}{\eta^2\omega^2 + k_n^4\varepsilon^2(k_n)},$$

$$I_n = \frac{\pi(-1)^n(2n+1)e^{-a} + 2a}{a^2 + \pi^2(n+1/2)^2}, \qquad a = \frac{\ell}{\lambda}$$

Anisotropy increases P at low and intermediate ω but does not affect P(∞)



For a long vortex segment l > λ, P(ω) becomes:

$$P = \frac{H^2\phi_0^2\chi^2}{2\eta\lambda(1+\chi^2)^2}\left[\frac{\chi^2}{2} + \frac{5}{2} + \frac{(1-2\chi-\chi^2)\sinh\sqrt{2\nu} - (1+2\chi-\chi^2)\sin\sqrt{2\nu}}{\sqrt{2}\,\chi^{3/2}(\cosh\sqrt{2\nu}+\cos\sqrt{2\nu})}\right]$$

$$\chi = \omega\eta\lambda^2/\varepsilon, \qquad \nu = \omega\eta\ell^2/\varepsilon$$

Characteristic frequencies

Low frequencies
$\omega \ll \omega_l$

Entire vortex segment swings

$$f_l = \frac{\rho_n \xi^2}{4\pi\mu_0\lambda^2\ell^2}$$

For clean Nb, f_l becomes smaller than 2 GHz for the pin spacing > 150 nm

RF power depends on the pinned segment length

Intermediate frequencies
$\omega_\lambda \ll \omega \ll \omega_l$

Only the vortex tail of length L(f) >> λ swings

$$f_l = \frac{\rho_n \xi^2}{4\pi\mu_0\lambda^4}$$

$$L(f) = \lambda\sqrt{\frac{f_\lambda}{f}}$$

For clean Nb, $f_\lambda \approx$ 40 GHz, but it decreases rapidly with the m.f.p. in the dirty limit:

$$f_l \simeq \frac{\rho_n l_i^3}{4\pi\mu_0\lambda_0^4\xi_0} \propto l_i^2$$

High frequencies
$\omega \gg \omega_\lambda$

Only the vortex tip of length λ swings

RF power is independent of the pin spacing

RF power

- Low frequencies. Entire vortex segment swings

$$P \simeq \frac{4\pi B_p^2 \ell^3 \omega^2}{3\rho_n \xi^2}$$

Decreases strongly as the pin spacing l decreases

- Intermediate ω.

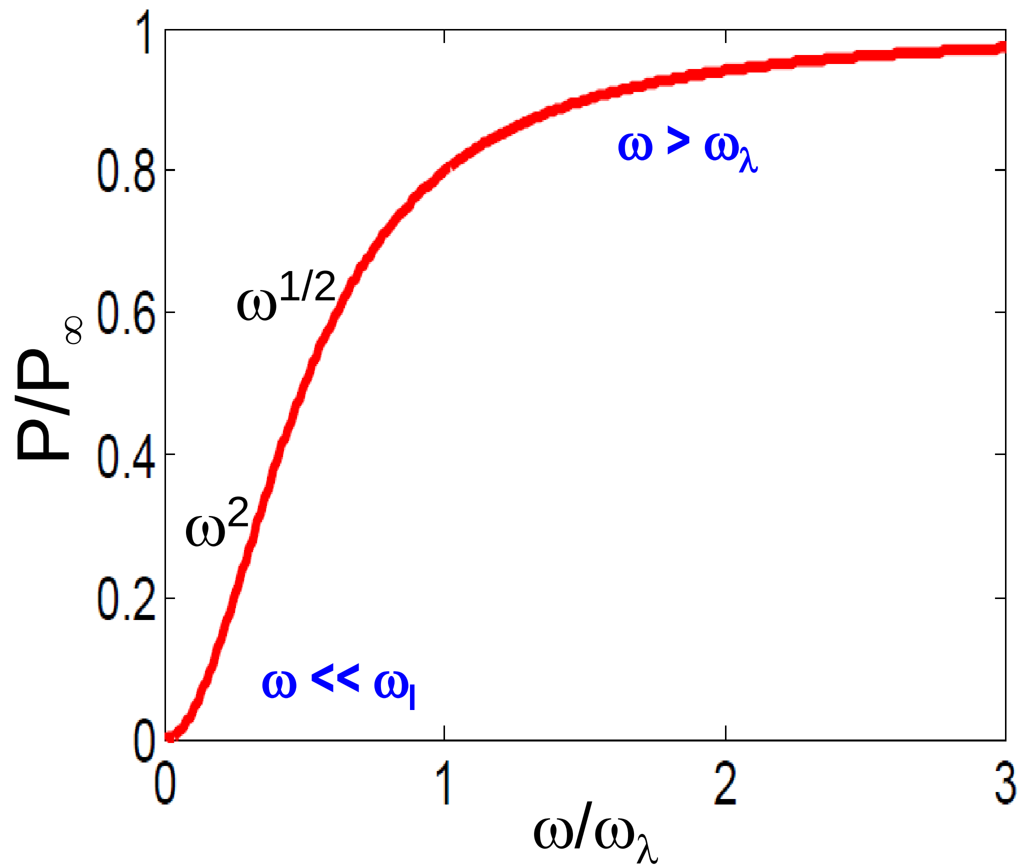
$$P \simeq \pi \mu_0^{-3/2} B_p^2 \lambda \xi \sqrt{\omega \rho_n}$$

No dependence on the pin spacing

- High ω.

$$P_{¥} = pH^2 r_n x^2 / 2l$$

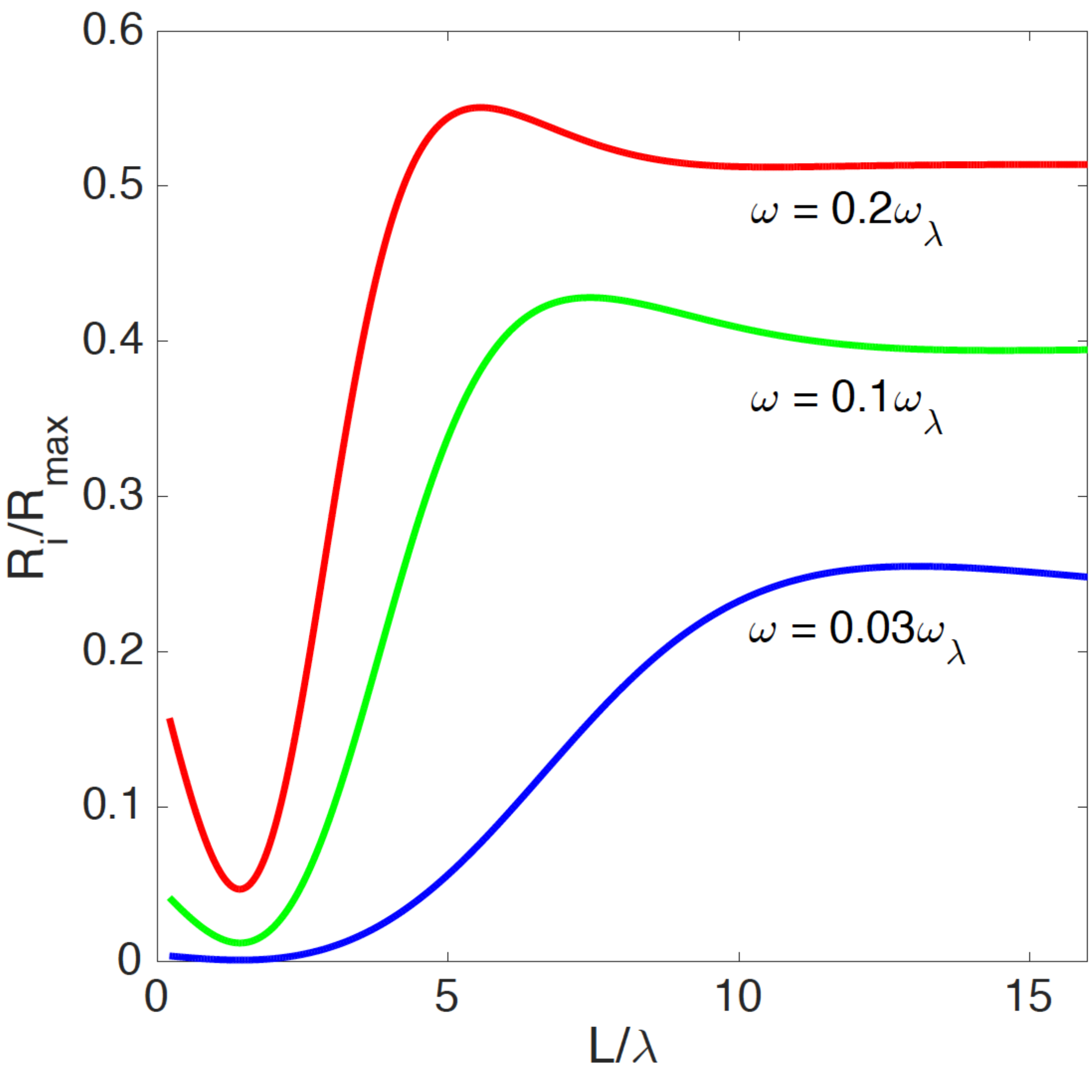
No dependence on the pin spacing



P ~ 0.13 μW at B = 100 mT and 2 GHz.

Hotspots revealed by thermal maps require regions ~ few mm with ~ 10^6 vortices

Dependencies of P on the pinned length



For random distribution of pins, P should be averaged over pin spacings

$$\langle P \rangle = \int_0^\infty P(\ell, \omega) F(\ell) d\ell$$

Here F(l) is the distribution function of pin spacings:

$$\int_0^\infty F(\ell) d\ell = 1$$

Residual resistance due to trapped vortices

$$R_i = \frac{B_0}{B_c}\left(\frac{\mu_0 \rho_n \omega}{2\gamma}\right)^{1/2}, \qquad \omega_l < \omega < \omega_l$$

For Nb with $\rho_n = 10^{-9}$ Ωm, B_c = 200 mT, the observed R_i = 5 nΩ at 2GHz can be produced by the residual field **$B_0 \sim 0.3$ μT** much smaller than the Earth field **B_E = 20-60 μT.**

- Vortex hotspots contribute to the field dependence of Q(H)
- Vortex hotspots can ignite thermal instability and lateral quench propagation

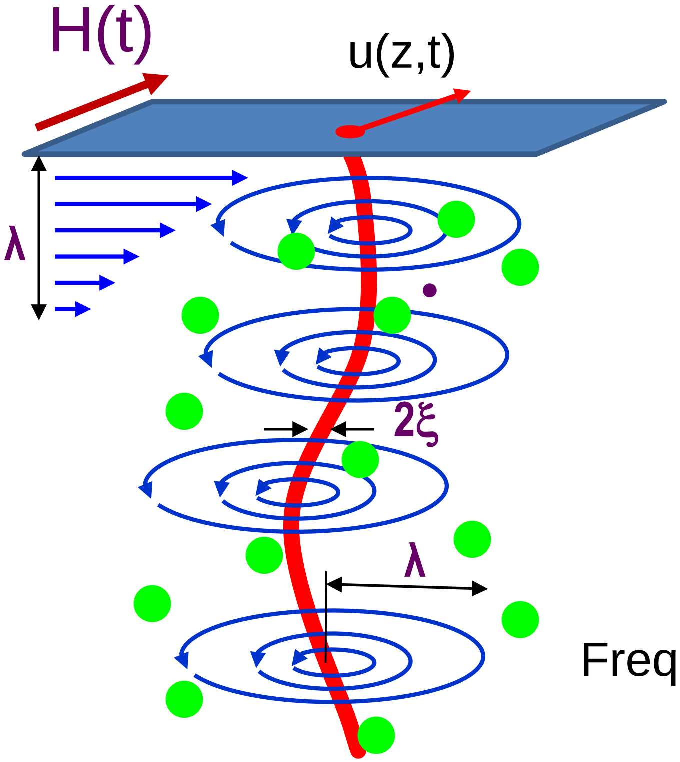
Field dependent R_i due to thermal feedback controlled by thermal conductivity κ and the Kapitza thermal conductance α_K between the film and the substrate/coolant

$$R_i(H) = \frac{R_i}{1 - H^2(1/2\alpha_K + d/2\kappa)[\partial R_s/\partial T]_{T_m}}$$

Thermal feedback makes R_i(H) interconnected with R_{BCS} and causes thermal instability at the rf field at which the denominator goes to zero

Weak collective pinning

A. Gurevich, SUST 30, 034004 (2017)



Small oscillations of elastic vortex line interacting with randomly distributed weak pins and weak rf field

$$\eta\partial_t u = \epsilon\partial_{xx}u - \alpha u - (\phi_0 H_a/\lambda)e^{-z/\lambda + i\omega t}$$

Labusch pinning spring constant $\alpha \sim n_p r_p f_p$

$$u(z,t) = \frac{H_a\phi_0 e^{i\omega t}}{\alpha\lambda^2 - \epsilon + i\omega\eta\lambda^2}\left(\lambda e^{-z/\lambda} - \lambda_c e^{-z/\lambda_c}\right)$$

Frequency and pinning-dependent complex Campbell length:

$$\lambda_c = \left(\frac{\epsilon}{k + i\omega\eta}\right)^{1/2}$$

The RF disturbance propagates along the vortex line over the length $\lambda_c > \lambda$ which increases as pinning gets weaker or frequency decreases

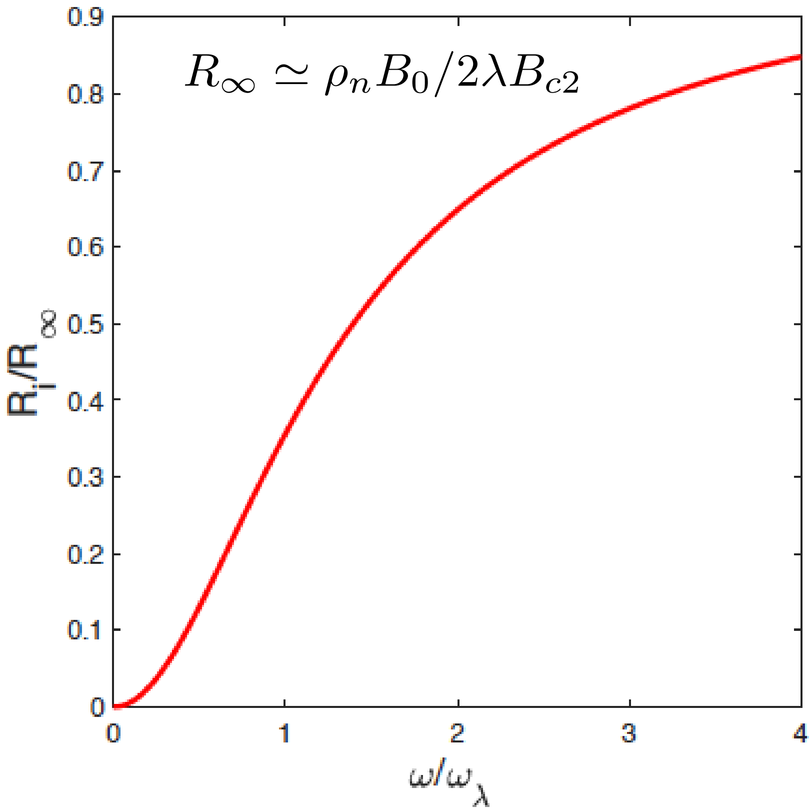
At k = 0 (no pinning) the Campbell length equals the RF elastic ripple length

RF power and residual resistance

RF power per vertex line: $$p = 0.5\mathrm{Re}\left(i\omega F\int_0^\infty e^{-z/\lambda}u_\omega(z)dz\right)$$

Surface resistance due to trapped vortices of areal density: B_0/ϕ_0

$$R_i = \frac{2pB_0}{\phi_0 H_a^2} = -\frac{2\pi\mu_0 B_0\lambda^2}{\phi_0 g}\mathrm{Im}\left\{\frac{\lambda_c^2(\lambda+2\lambda_c)}{(\lambda+\lambda_c)^2}\right\}$$



Three frequency domains:

- Long-range swings affected by pinning:

$$R_i \propto \omega^2, \qquad \omega < \omega_p = \alpha/\eta$$

- Free vortex ripples at intermediate length

$$R_i \propto \sqrt{\omega}, \qquad \omega_p < \omega < \omega_\lambda$$

- Ripples localized in the RF current layer

$$R_i \simeq R_\infty, \qquad \omega > \omega_\lambda$$

Why can't we just use the Gittleman-Rosenblum model for the SRF cavities?

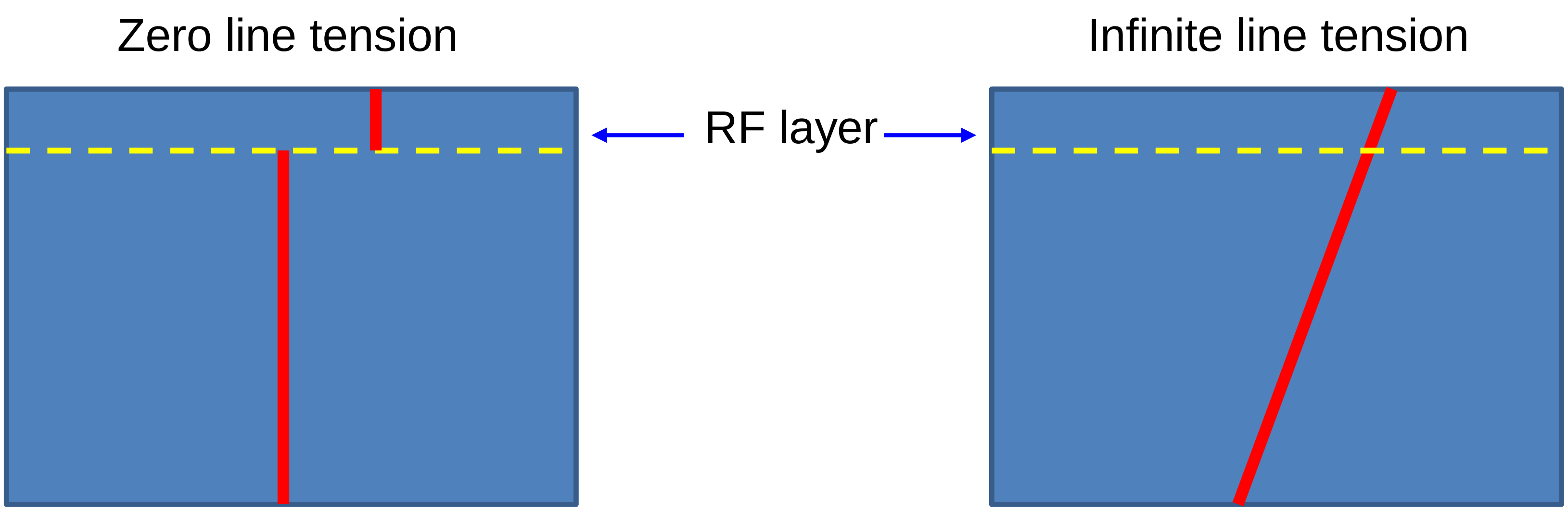
A short vortex driven by a uniform rf current in a thin film

M. Checchin et al, SUST 30, 034003 (2017)

$$\eta\dot{u} - \alpha u = F\cos\omega t$$

- Assumes no bending of a vortex
- For a long vortex in the SRF cavities, this implies either zero or infinite line tension

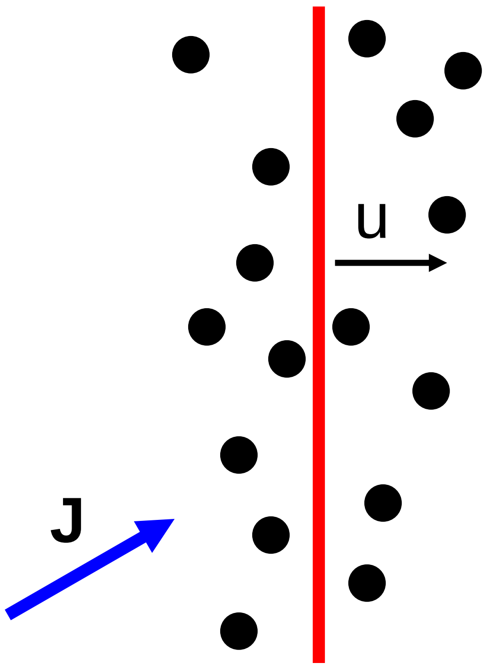
Problems in the SRF cavities

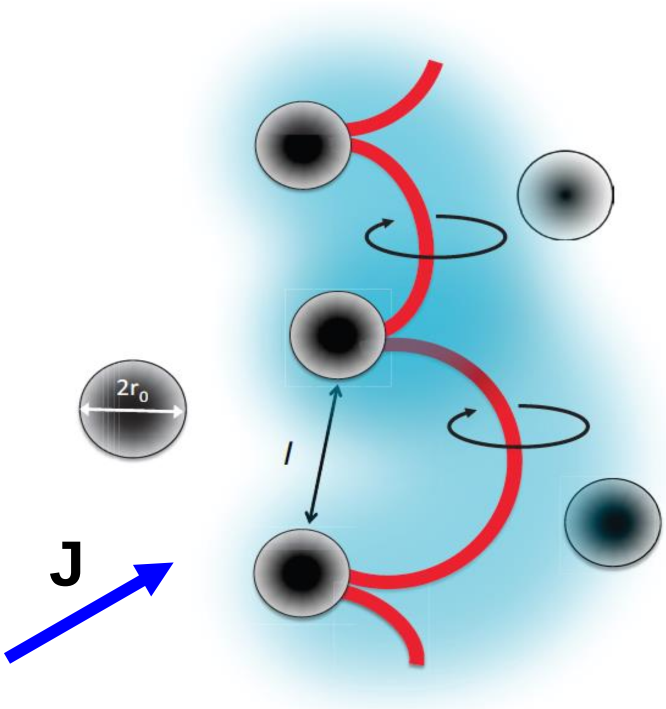


Vortex tear-down

Vortex stick swings through the entire cavity wall

A long vortex with either zero or infinite line tension cannot be pinned





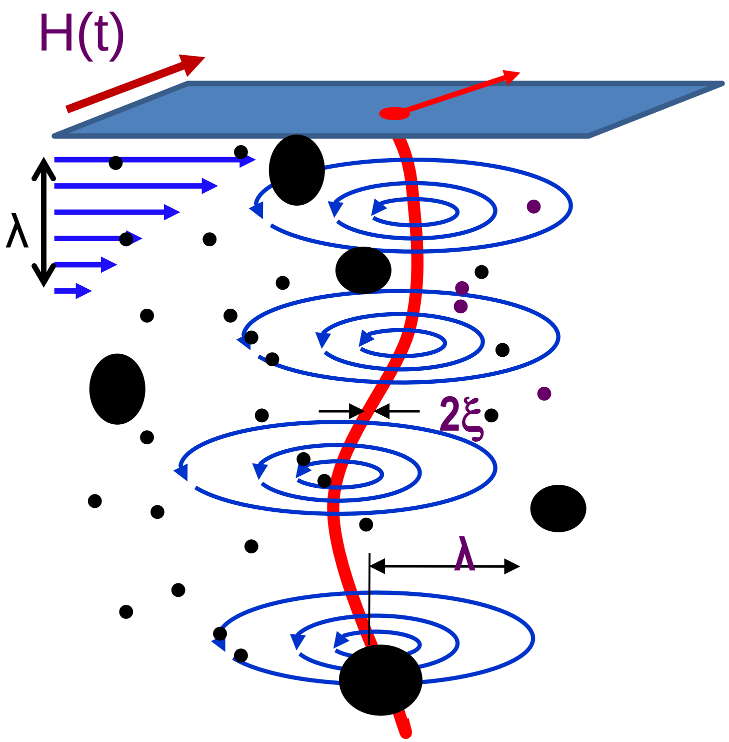
Energy of a long straight vortex does not change upon any shift through randomly-distributed pins

Net pinning force = 0

Soft vortex filaments between pins bow out and reconnect with the neighbors

Pinning requires finite bending rigidity

How effective pinning can be in the SRF cavities?



RF current density flowing at the surface

$$J(z) = (H_a/\lambda)e^{-z/\lambda}\cos\omega t$$

At the superheating field H_s = 240 mT
J(0) at the surface reaches the depairing current density

$$J_d = H_s/\lambda \simeq 500\ MA/cm^2 \qquad for\ Nb$$

At H = 24 mT we get J(0) = 50 MA/cm^2

In Nb typical depinning J_c are about 10 kA/cm^2

What kind of pinning defect structure could produce J_c = 10%J_d in Nb?

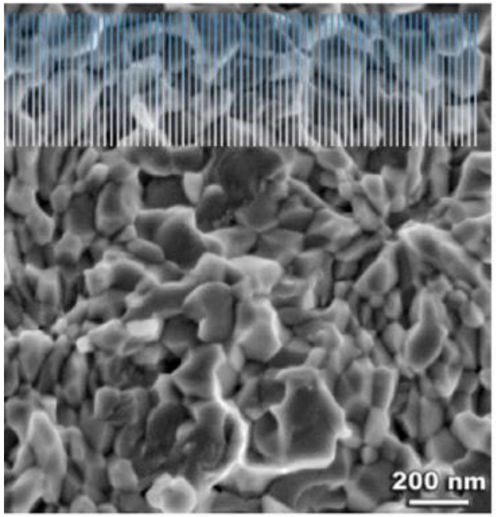
Would such materials be useful for SRF?

What it takes to get $J_c = 10\%J_d$ in NbTi

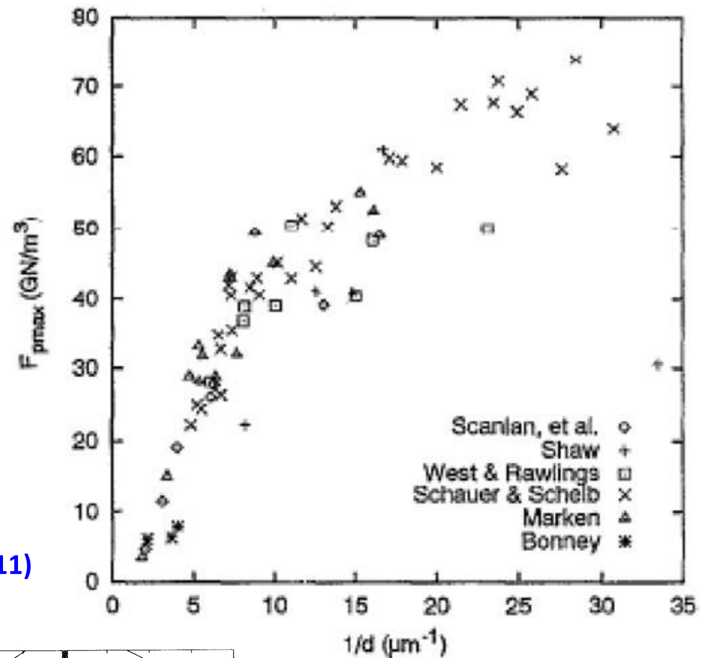
α-Ti ribbons in a Nb-Ti alloy (D. Larbalestier & P. Lee)

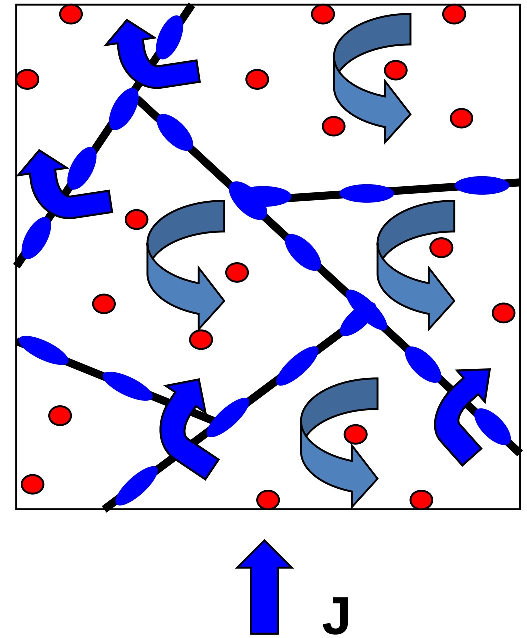
- Vortices are chopped into short strongly pinned segments
- From weakly pinned vortex spaghetti to strongly pinned vortex pasta.
- Does work in LTS, producing very high $J_c \sim 0.1J_d$. Here $J_d \approx H_c/\lambda$ is the depairing current density.

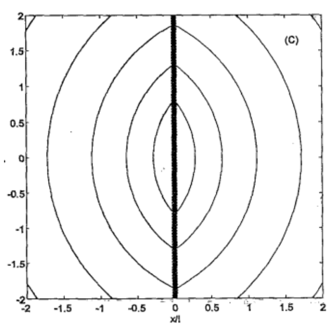
Strong pinning of vortices by grain boundaries in Nb_3Sn

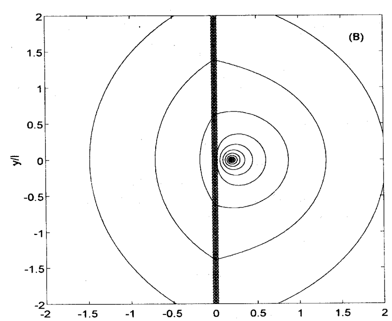


J. Durrell et al, Rep. Prog. Phys. 74, 124511 (2011)









GB cuts currents circulating around a vortex causing its attraction to the GB at r < L:

$$J(r) \simeq J_d \frac{\xi}{r}, \quad L \simeq \xi \frac{J_d}{J_c}$$

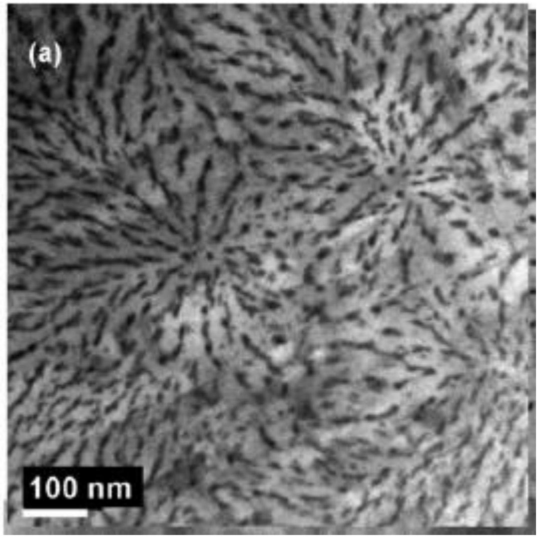
A. Gurevich and L. Cooley, PRB 50, 13563 (1994)

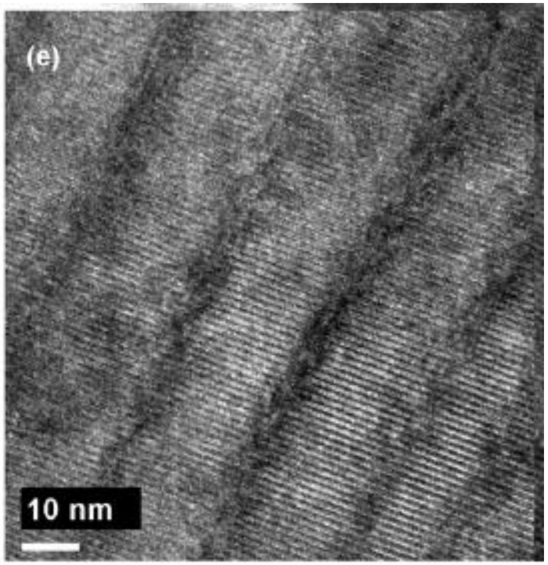
- Vortices caged in the grains and pinned by GBs
- GBs both pin vortices and impede the net transport current
- Bulk pinning force depends on the grain size:

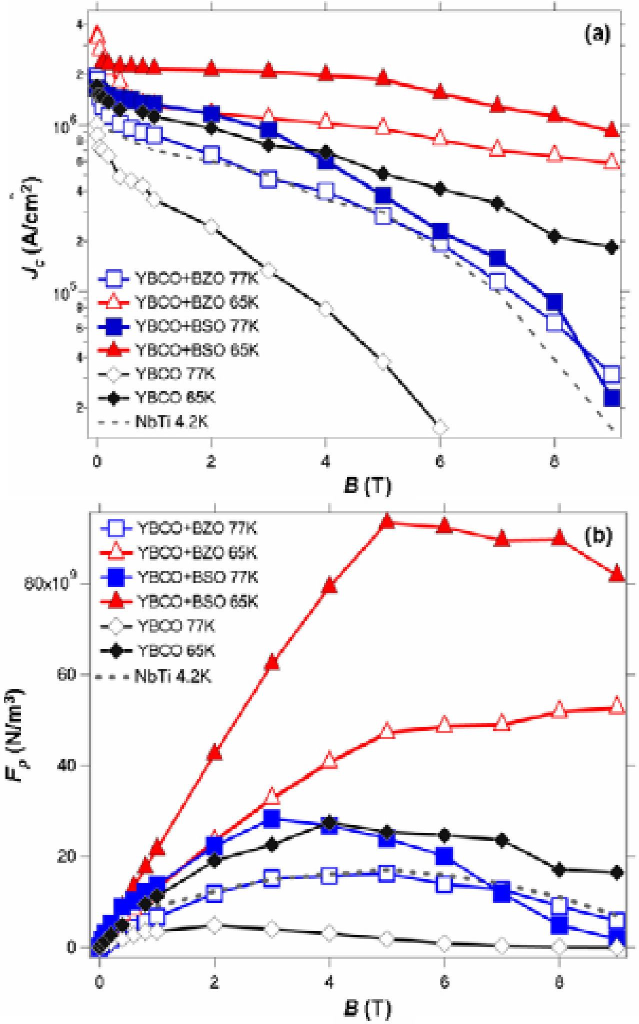
$$F_p = BJ_p \propto 1/d^2, \qquad d > \lambda,$$
$$F_p = BJ_p \propto 1/d, \qquad d < \lambda$$

Designer nanoparticle structures in cuprates

Self-assembled chains of BZO nanoparticles

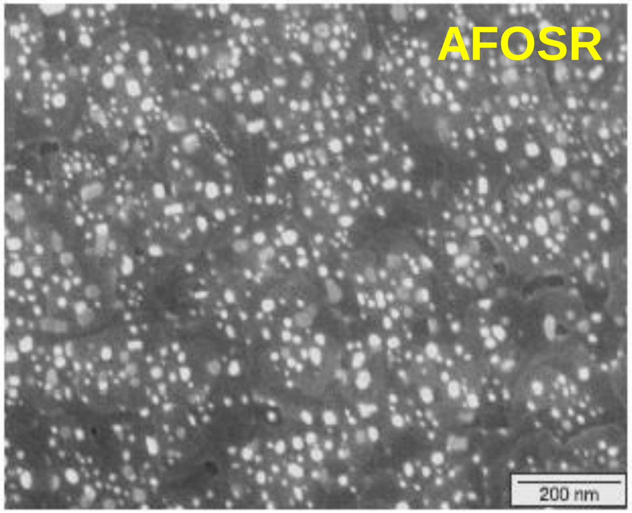


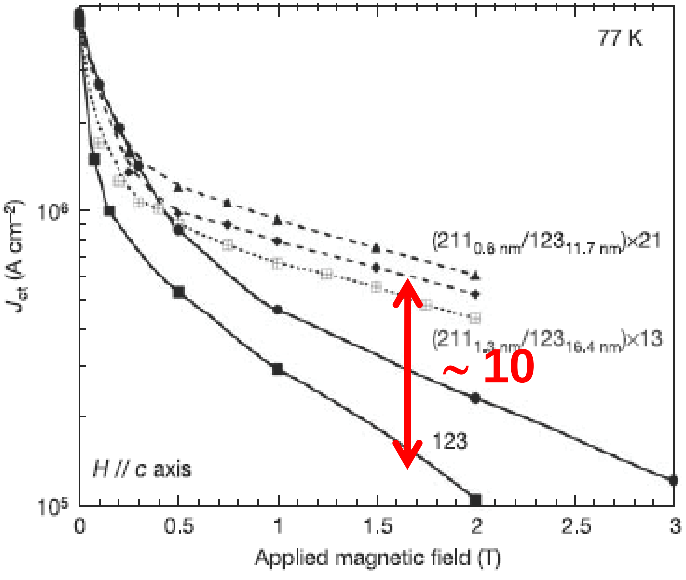




P. Mele, K. Matsumoto, T. Horide, A. Ichinose, M. Mukaida, Y. Yoshida, S. Horii, R. Kita SUST 21, 032002 (2008)

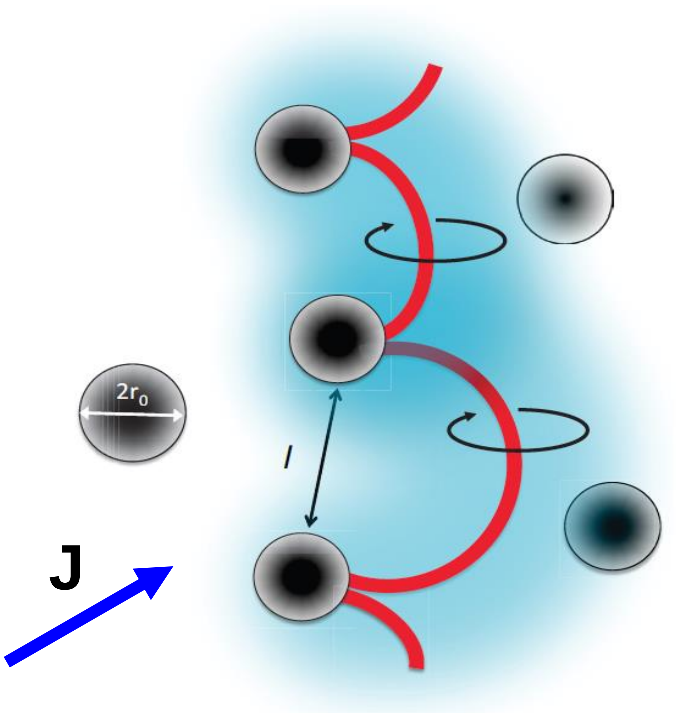
8 nm YBa_2CuO_5 nanoparticles





T. Haugan, et al. Nature 430, 867 (2004)

Strong pinning by nano-particles/pores



- Vortex segments bowing out by the Lorenz force:

$$\frac{\epsilon}{R} = \phi_0 J$$

- Depinning due to reconnection of neighboring vortex segments

$$J_c \simeq \frac{\phi_0}{2\pi\mu_0\lambda^2\ell} \ln\frac{\ell}{\xi}$$

- The larger the vortex line tension the higher J_c
- The smaller the pin spacing the higher J_c
- Too many pins cause T_c suppression and current blocking. Optimum pin spacing
- Too many pins cause strong RF dissipation similar to that of trapped vortices

How can all these things be good for the SRF cavities?

Optimum pin density: pinning vs current blocking

A. Gurevich, SUST 20, S128 (2007); Annu. Rev. Cond. Mat. Phys. 5, 35 (2014)

- Randomly-distributed insulating particles of radius r_0 spaced by ℓ

$$J_c \simeq \underbrace{\frac{\phi_0}{2\pi\mu_0\lambda^2\ell} \ln\frac{\ell}{\xi}}_{\text{Pinning}} \underbrace{\left(1 - \frac{4\pi r_0^3}{3x_c\ell^3}\right)}_{\text{Current-carrying cross-section}}$$

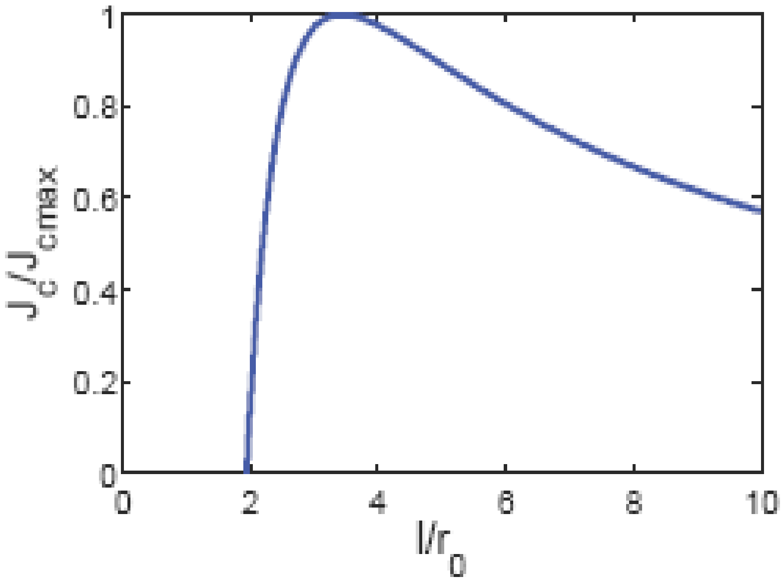
- $x_c \approx 1/3$ is the 3D percolation threshold
- Optimum pin spacing and volume fraction:

$$\ell_m \approx 3-4r_0, \qquad x_m = \frac{4\pi r_0^3}{3\ell_m^3} \approx 8-12\%$$

- Optimum J_c in Nb:

$$\ell_m = 3r_0 = 3\xi = 120\ nm:$$

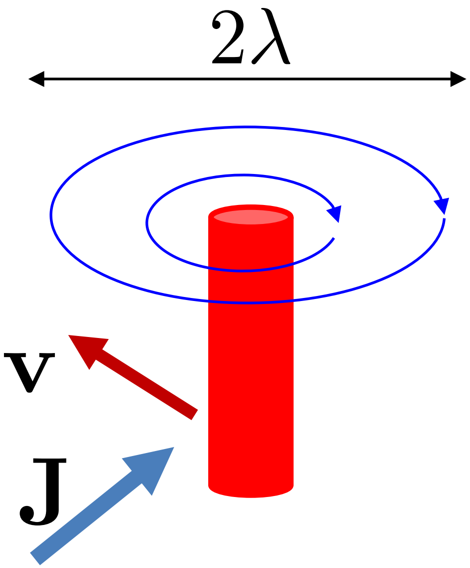
$$J_c \simeq 80\ MA/cm^2 \approx J_d/6,$$
$$B_b \approx B_s/6 \approx 40\ mT$$

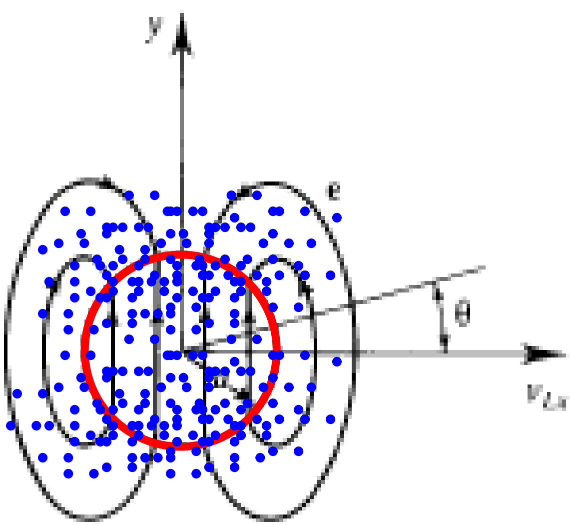


Upper limit for small pins, no proximity effect T_c suppression

ANL large-scale TDGL simulations of interacting vortices pinned by arrays of metallic nano-particles
W-K. Kwok et al, Rep. Prog. Phys. 79, 116501 (2016)

Bardeen-Stephen vortex drag (dirty limit)





Viscous drag force balanced by the Lorentz force:

$$\eta \mathbf{v} = \phi_0 \mathbf{J} \times \mathbf{z}$$

Viscous drag coefficient η results from Ohmic losses of eddy currents in a moving normal core of radius ξ

$$\eta = \frac{\phi_0^2}{2\pi\xi^2\rho_n}$$

Dirty limit: the core diameter is larger than the m.f.p. and the Drude formula is applicable:

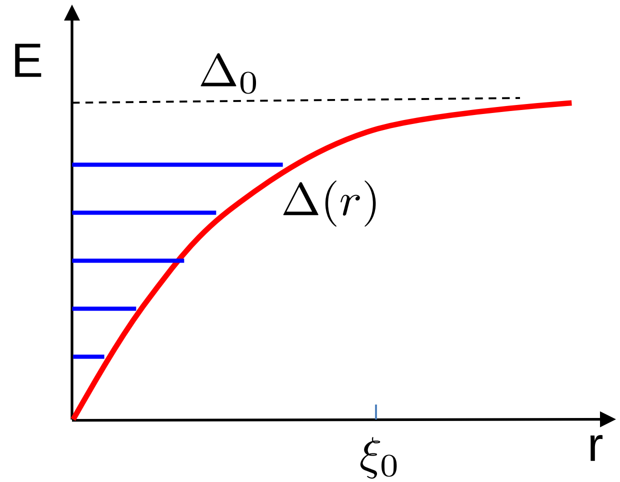
$$\xi = \sqrt{\xi_0 \ell} > \ell \quad \rightarrow \quad \ell < \xi_0 = \hbar v_F / \pi\Delta$$

$$\rho_n = p_F / n e^2 \ell$$

Drag coefficient is independent of the m.f.p.

$$\eta = \frac{\pi^2 \hbar n \Delta}{4E_F}$$

Vortex drag (clean limit)



DeGennes-Matricon bound states in the vortex core

$$E_\mu = \frac{\mu\Delta^2}{E_F}, \quad \mu = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

Levels are weakly broadened due to scattering on impurities

Kramer-Pesch core shrinkage at T << T_c in the clean limit

$$\xi \sim \xi_0 T/T_c$$

- Moderately clean limit at T << T_c:

$$\eta \simeq 0.23 \frac{\phi_0^2 \ln(\Delta/T)}{2\pi\xi_0^2\rho_n} \propto l$$

$$l \ll l_c \sim \frac{\xi_0 E_F}{\Delta \ln(\Delta/T)}$$

Discrepancies with experiment

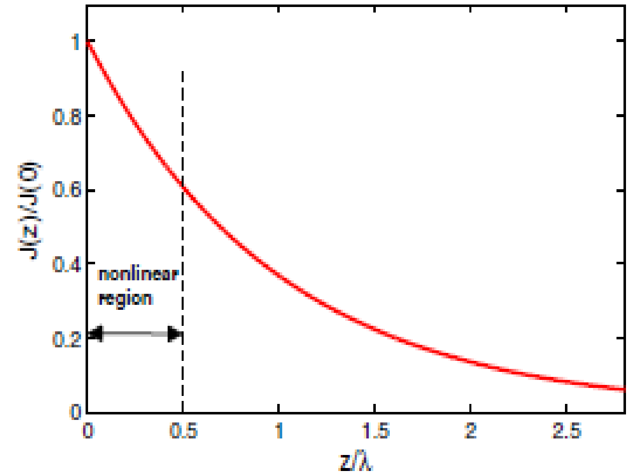
- Super-clean limit, l > l_c

- Vortex viscosity becomes very small
- Vortex moves nearly parallel to the driving current
- The Hall angle approaches 90º
- Ultra-low temperatures $T < T_c^2/T_F$

N.B. Kopnin, Theory of Nonequilibrium Superconductivity, Oxford U Press, 2001

Vortex dynamics nonlinearities

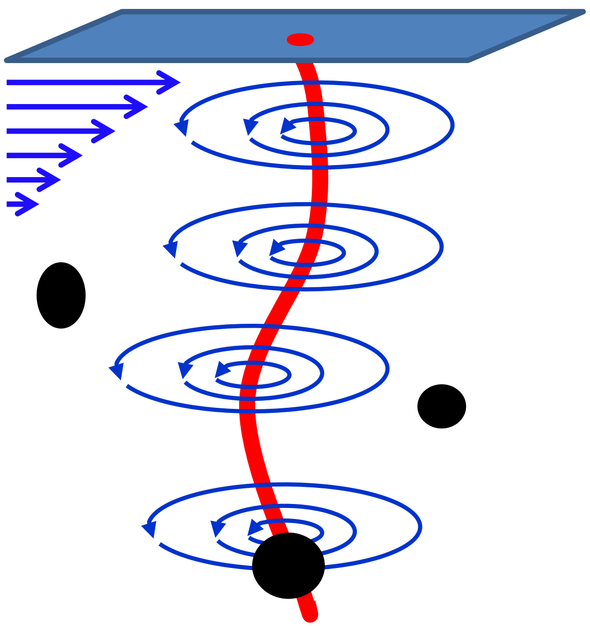
At H = H_c, the velocity of Cooper pairs reaches the critical pairbreaking value $v_c = \Delta/p_F$.



How fast could the vortex tip move?

$$v \simeq \frac{J_d \phi_0}{\eta} \simeq \frac{\rho_n \xi}{2\mu_0 \lambda^2}$$

This yields v = 10 km/s, which exceeds both the speed of sound and $v_c = \Delta/p_F$ = 1 km/s

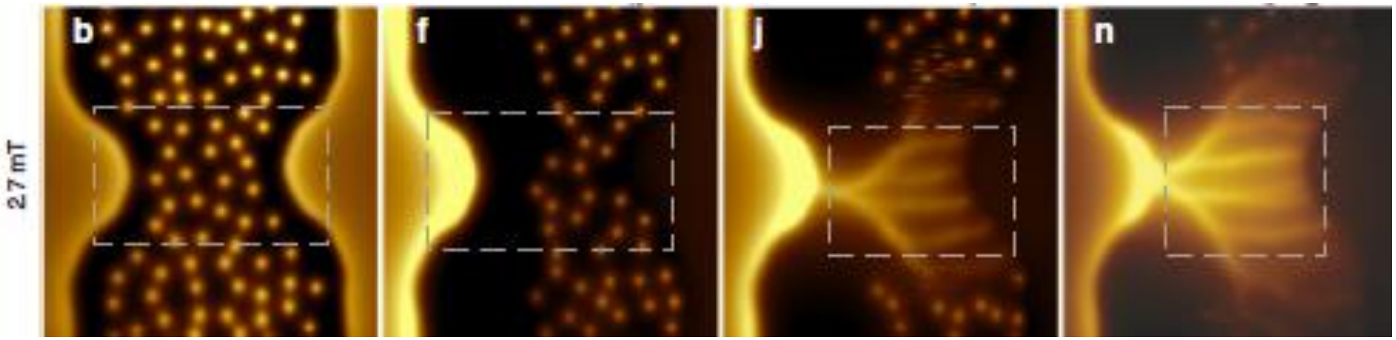


Linear Bardeen-Stephen vortex dynamics fails.
Larkin-Ovchinnikov and/or thermal instabilities

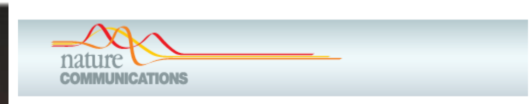
$$\eta = \frac{\eta_0}{1 + (v/v_0)^2}$$

Vortex jumps at
$v > v_0 \sim 0.1$ km/s
as was measured on Nb films

How fast can vortices penetrate and can they be stopped?



Nature Comm. 8, 85 (2017)



ARTICLE

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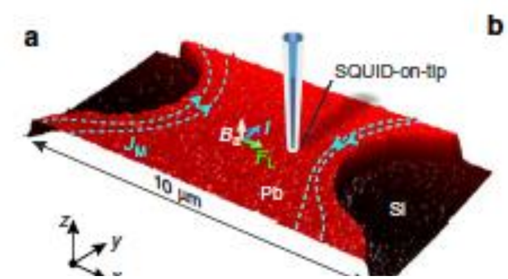
Imaging of super-fast dynamics and flow instabilities of superconducting vortices

L. Embon[1], Y. Anahory[1,2], Ž.L. Jelić[3,4], E.O. Lachman[1], Y. Myasoedov[1], M.E. Huber[5], G.P. Mikitik[6], A.V. Silhanek[4], M.V. Milošević[3], A. Gurevich[7] & E. Zeldov[1]

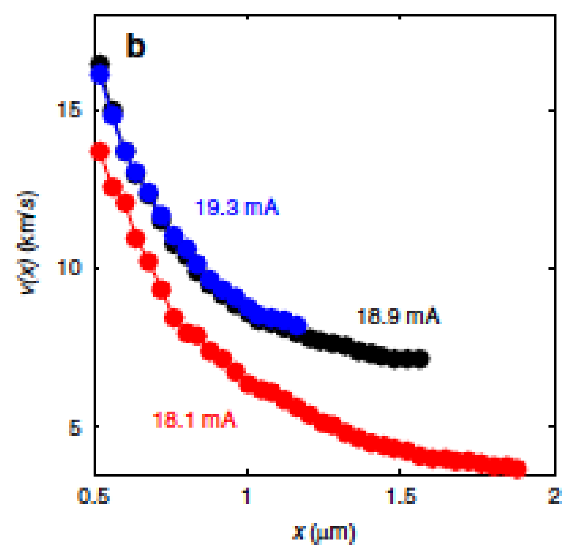
75 nm thick Pb film: imaging of penetrating vortices with a nanoscale SQUID on tip



Velocities can reach **10—20 km/s** as J(x,y) at the edge reaches J_d ($H = H_s$ for the SRF cavities)

If v = 10 km/s, a vortex penetrates by the distance

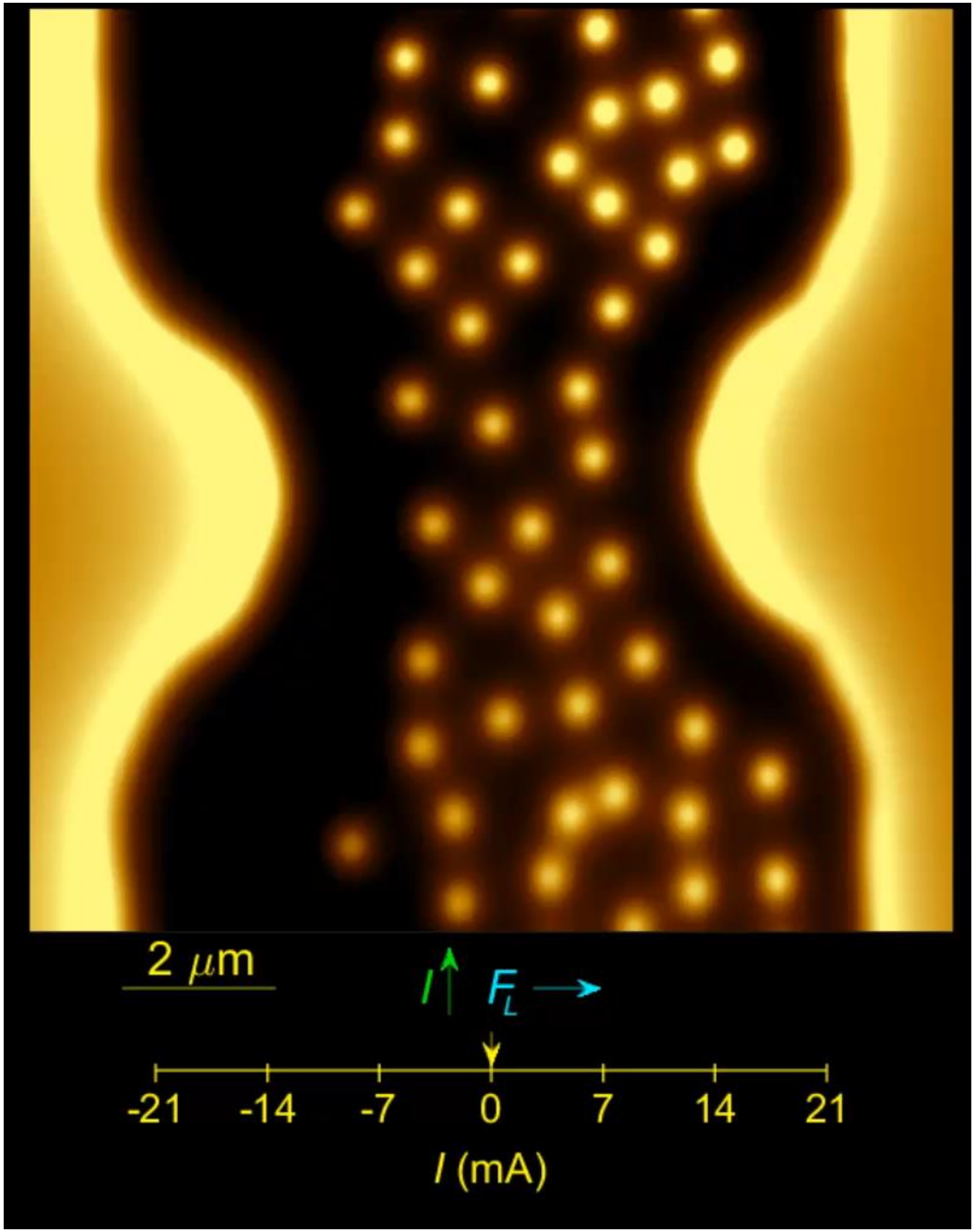
$$L/f \simeq 10\ \mu m \gg \lambda \quad @1\ GHz$$



Vortices penetrate almost instantaneously through the Meissner RF layer

Hot vortex branching trees. No “natural” materials defects can pin such superfast vortices.

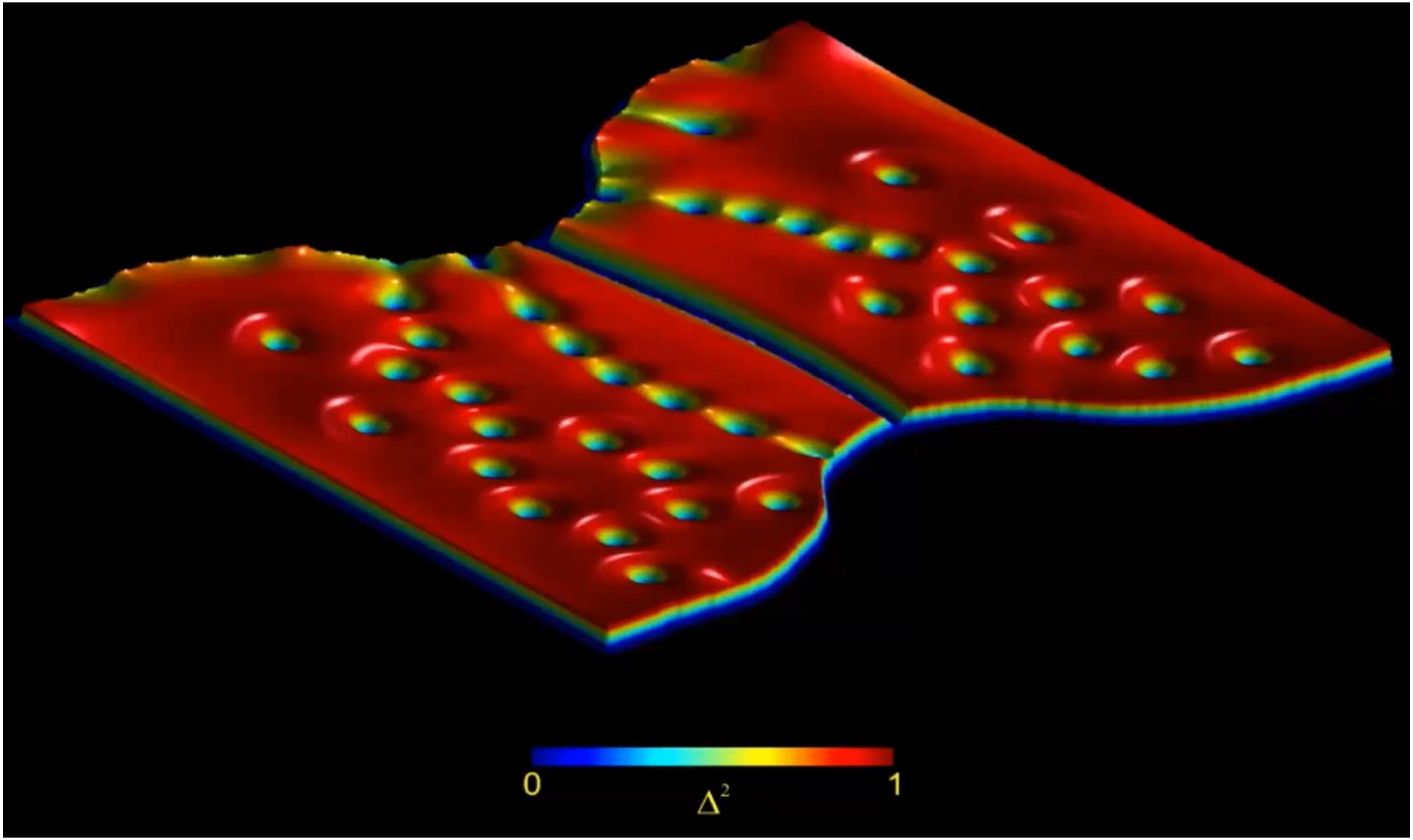
Dynamics of vortex branching observed by SOT micsoscope



Pb bridge at B_a = 27 G
SOT diameter: 225 nm
Scan area: 12 × 12 μm^2
Pixel size: 40nm
Scan time: 4 min/frame
T = 4.2 K

L. Embon et al, Nature Comm. 8, 85 (2017)

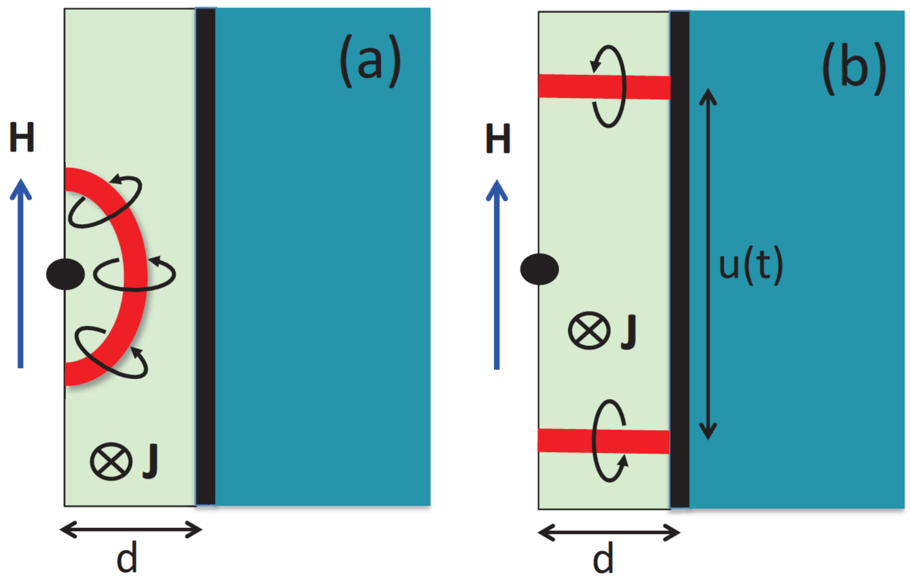
TDGL simulations of ultra-fast vortex dynamics



Elongated vortex cores along the direction of motion. **L. Embon et al, Nature Comm. 8, 85 (2017)**
Transition of Abrikosov to mixed Abrikosov-Josephson vortices in self-induced weak links

Arresting penetration of vortices in a multilayer

I layer intercepts propagating vortex loops.
No propagation in the bulk if h(d) < H_{c1}



Great reduction of the RF vortex power q localized in a thin S layer:

$$q \le \frac{\beta^2 d_s \phi_0 H_c \rho_n}{\kappa \mu_0^2 \lambda^2}$$

For Nb_3Sn, $u_m \approx 4$ μm, and $q \approx 2$ μW

Nb_3Sn: $\rho_n = 0.2$ μΩm, $d/\lambda = 0.2$, $\kappa = 20$, $\lambda = 100$ nm, $\beta = H_p/H_s = 1/2$, f = 2GHz

Unlike thick Nb_3Sn films, a thin ML only slightly (by ≈ 5%) increases the thermal impedance of the cavity wall. No deterioration of thermal quench stability.

A. Gurevich, APL 88, 012511 (2006); AIP Adv. 5, 017112 (2015); SUST 30, 034004 (2017)

Conclusions

- Theory of dissipation of oscillating flexible vortex lines driven by low-amplitude RF currents gives a complicated dependencies of R_i on ω, pin spacing and m.f.p.
- Strong pinning and weak collective pinning: similar dependencies of R_i on frequency and the m.f.p. details of pin distribution at the surface are crucial
- The observed residual resistance R_i = 2-5 nOhm can be produced by low vortex density corresponding to $B_0 \sim 0.1$-$0.3\ \mu$T much smaller than the Earth field 20-60 μT.
- Pinning can only reduce vortex dissipation at low RF fields << H_s.
- Strong pinning defect nanostructure impedes RF currents and causes significant extra dissipation even with no vortices
- Problem with vortex drag coefficient: dependence on the m.f.p. and the behavior at high velocities relevant to SRF
- Vortex core elongation and A –AJ transition at high velocities
- Observation of hypersonic vortex velocities exceeding 10 km/s.