Istituto SPIN-CNR

INFN Istituto Nazionale di Fisica Nucleare

Universita' di Napoli Federico II
Dipartimento di Fisica

Ruggero Vaglio

A simple model for the RF field dependence of the trapped flux sensitivity based on a non-linear pinning force

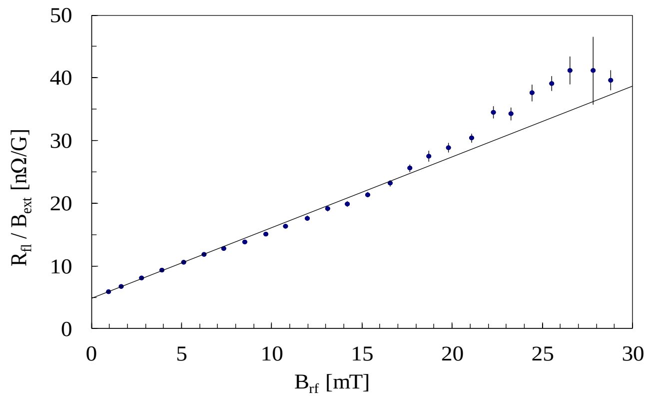
In collaboration with:

Sergio Calatroni

TTC Workshop, CERN 9-11-2018

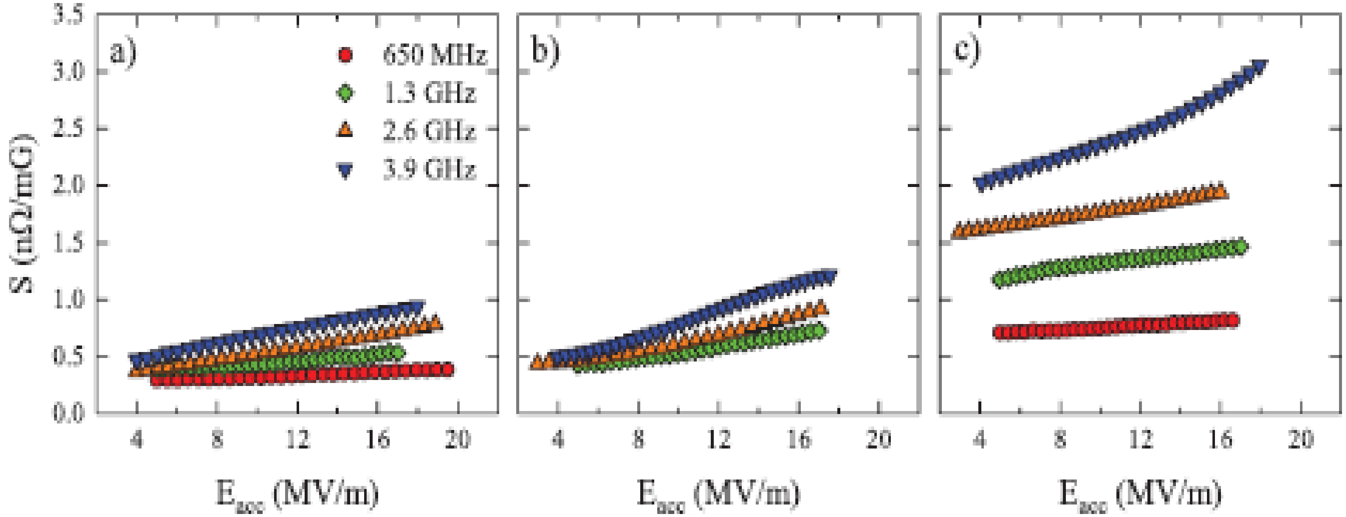
The improvement of SRF cavities has recently motivated a renewed research effort on the effect of trapped magnetic flux on the surface resistance

A linear dependence of the flux sensitivity is generally experimentally observed, for many different cavities shape and operating frequency



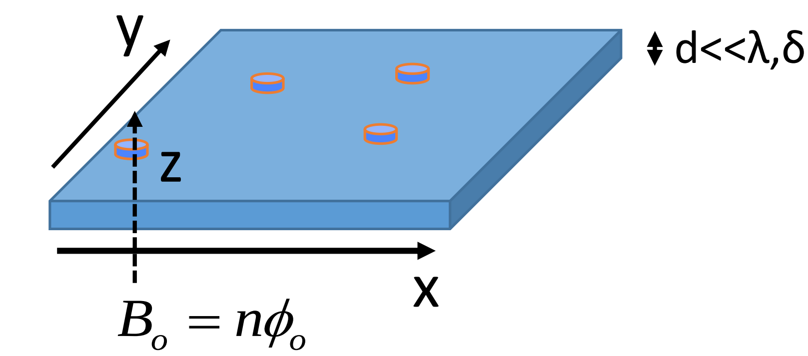
$$S = \frac{R_{fl}}{B_o} = (R_{fl}^o + R_{fl}^1 B_{rfo})$$

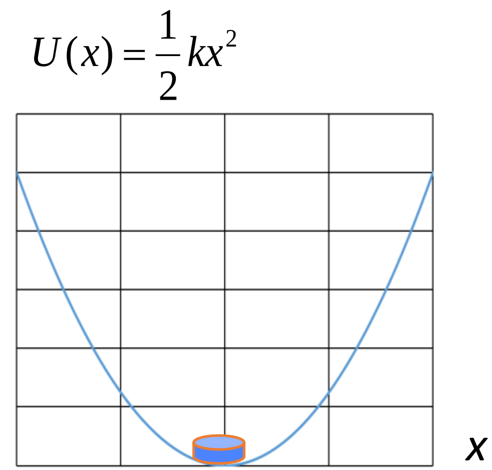
Benvenuti et al 1999 Nb film



Checchin et al 2018 Nb bulk

The first model describing the surface impedance due to flux flow is due to Gittleman and Rosenblum (GL, Phys Rev. Lett. 16, 734, 1966)





$$\cancel{ma} = f_L + f_\eta + f_p$$

$$f_L = J_{rf}\phi_o \; ; \; f_\eta = -\eta v \; ; \; f_p = -kx$$

(all quoted forces are per unit lenght)

$$\boxed{\eta v(t) + kx(t) = J_{rfo}\varphi_o \sin \omega t}$$

$$P_s = n\frac{1}{T}\int_0^T \left[d\phi_o J_{rf}(t) \right] v(t)dt$$

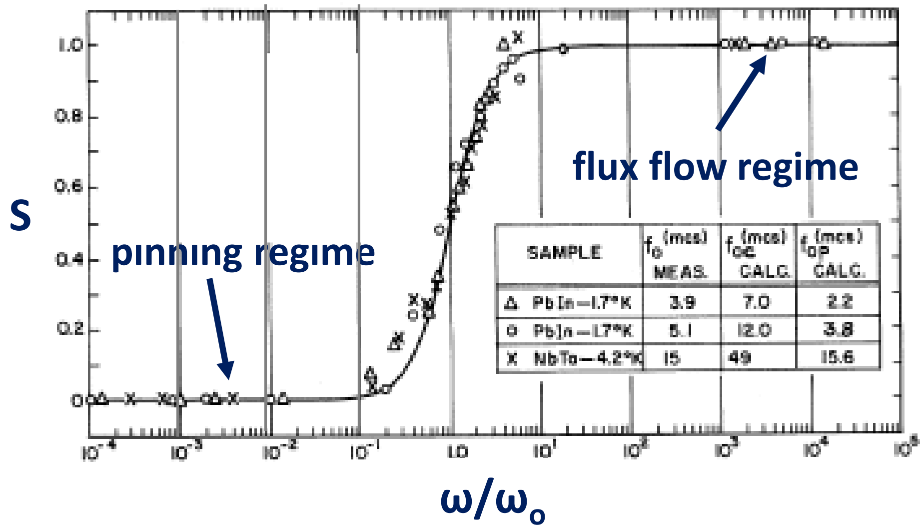
$$P_s = \frac{1}{2} R_{fl} H_{rfo}^2 \Rightarrow R_{fl} = R_n \frac{B_o}{B_{c2}} \frac{\omega^2}{\omega^2 + \omega_o^2}$$

$$S = R_{fl}^o = \frac{R_n}{B_{c2}} \frac{\omega^2}{\omega^2 + \omega_o^2}$$

$$(R_{fl}^1 = 0)$$

$$\omega_o = \frac{k}{\eta}$$

depinning frequency



SAMPLE	f_o (mcs) MEAS.	f_{oc} (mcs) CALC.	f_{op} (mcs) CALC.
△ PbIn—1.7°K	3.9	7.0	2.2
o PbIn—1.7°K	5.1	12.0	3.8
X NbTa—4.2°K	15	49	15.6

$$\omega \gg \omega_o \Rightarrow R_{fl} = R_n \frac{B_o}{B_{c2}}$$ this result represents the so called «static model»

that well describes the experimental results for bulk SRF in that regime

($R_n = \sqrt{\frac{\mu_o \omega}{2} \rho_n}$)

Some model evolutions

- Keeping the GR approach, if the condition d<<λ,δ is removed, one can calculate the average resistivity due to flux oscillations :

$$\rho_{fl} = \frac{E_{rfL}}{J_{rf}} = \frac{vB_o}{J_{rf}} = \rho_n \frac{B_o}{B_{c2}} \left(\frac{\omega^2}{\omega^2 + \omega_o^2} + i \frac{\omega\omega_o}{\omega^2 + \omega_o^2} \right)$$

$$R_{sf} = \mathrm{Re}\left[Z_{sf} \right] = R_n \, \mathrm{Re}\left[(1+i)\sqrt{\rho_{fl}} \right]$$

many authors (es. Calatroni, Vaglio IEEE TAS 27 2017)

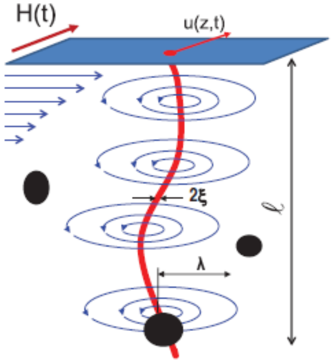
- A further improvement is to consider a non uniform Lorentz force along the z direction, and an angle θ between field and current :

$$\eta v + kx = \phi_o J_{rf} e^{-z/\lambda} \sin\theta$$

(Checchin et al SUST 30 2017)

- It is also possible to consider non-rigid fluxons:

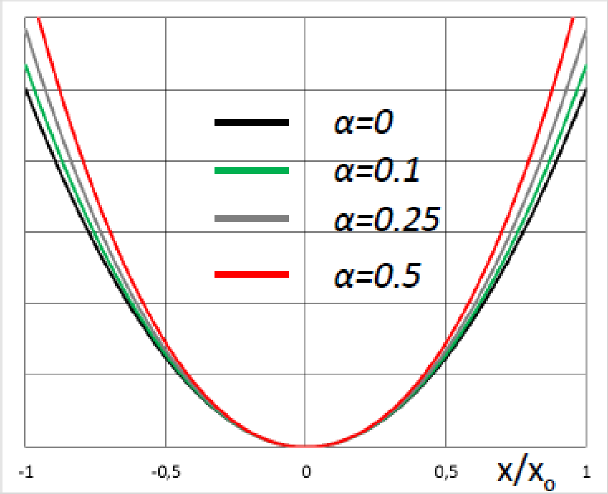
(Gurevitch, Ciovati PRB 87 2013)

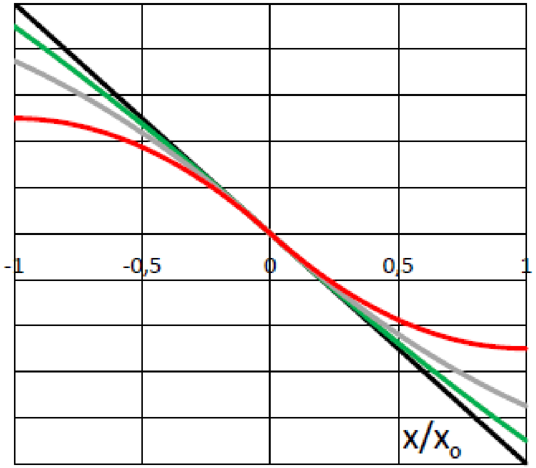


In all cases R_{fl} is independent of B_{rfo} , as expected for linear harmonic oscillations

Proposed non-linear model (simple extension of the GL approach)

S. Calatroni and R.Vaglio, submitted PRE, Arxiv 1810.00540





$$U(x) = \frac{1}{2}kx^2 - \frac{1}{3}\gamma\left|x^3\right|$$

$$f_p = -kx + \gamma\left|x\right|x$$

$$\alpha = \frac{\gamma x_o}{k}$$

$$\eta\dot{x} + kx - \gamma\left|x\right|x = J_{rf}\varphi_o$$

Duffing approximation

$$\alpha = \frac{\gamma x_o}{k} << 1$$

$$v(t) = \dot{x} = \frac{J_{rfo}\phi_o}{\eta}\sin\omega t - \frac{k}{\eta}x(t) + \frac{\gamma}{\eta}\left|x(t)\right|\cdot x(t)$$

with

$$\begin{cases} x(t) = x_o \sin(\omega t - \varphi) \\ tg\varphi = \omega / \omega_o \\ x_o = \dfrac{J_{rfo}\varphi_o}{\eta\sqrt{\omega^2 + \omega_o^2}} \end{cases}$$

(as in the linear case)

$$P_s = n\frac{1}{T}\int_0^T \left[d\phi_o J_{rf}(t) \right] v(t)dt =$$

$$= \frac{nd\phi_o^2 J_{rfo}^2}{\eta}\frac{1}{T}\int_0^T \sin^2 \omega t dt - d\omega_o\phi_o J_{rfo} x_o \frac{1}{T}\int_0^T \sin\omega t \cdot \sin(\omega t - \phi)dt +$$

$$\frac{\gamma d\omega_o\phi_o J_{rfo} x_o^2}{k}\frac{1}{T}\int_0^T \sin\omega t \cdot \sin(\omega t - \phi)\left|\sin(\omega t - \phi)\right| dt$$

$$\begin{cases} \frac{1}{T}\int_0^T \sin^2 \omega t \cdot dt = \frac{1}{2} \quad ; \quad \frac{1}{T}\int_0^T \sin\omega t \cdot \sin(\omega t - \phi)dt = \frac{1}{2}\cos\varphi \\ \frac{1}{T}\int_0^T \sin\omega t \cdot \sin(\omega t - \phi)\left|\sin(\omega t - \phi)\right| dt = \frac{4}{3\pi}\cos\varphi \end{cases}$$

$$P_s = \frac{1}{2}\frac{nd\phi_o^2 J_{rfo}^2}{\eta}\frac{\omega^2}{\omega^2 + \omega_o^2} + \frac{4}{3\pi}\frac{\gamma x_o}{k}\frac{nd\phi_o^2 J_{rfo}^2}{\eta}\frac{\omega_o^2}{\left(\omega^2 + \omega_o^2\right)}$$

$$\eta = \frac{\phi_o B_{c2}}{\rho_n} \; ; \; H_{rfo} = dJ_{rfo}$$

$$P_s = \frac{1}{2} R_n \frac{B_o}{B_{c2}} \frac{\omega^2}{\omega^2 + \omega_o^2} \left(1 + \frac{8\alpha}{3\pi} \frac{\omega_o^2}{\omega^2} \right) H_{rfo}^2 = \frac{1}{2} R_{fl} H_{rfo}^2$$

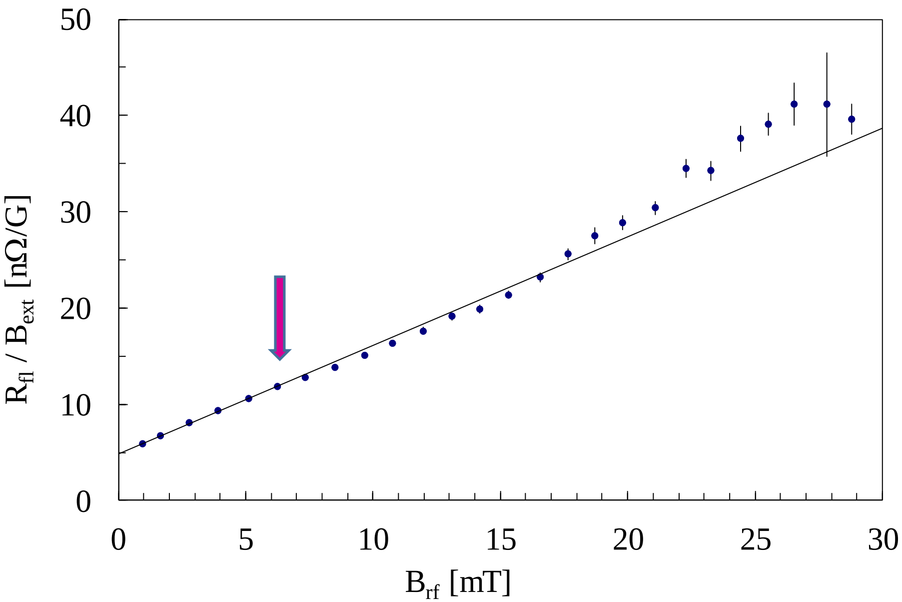
$$R_{fl}(B_{rfo}) = R_{fl}(0) \left(1 + \frac{8\alpha}{3\pi} \frac{\omega_o^2}{\omega^2} \right) \qquad (\alpha = \frac{\gamma x_o}{k} = \frac{\gamma}{k} \frac{R_n}{\mu_o} \frac{B_{rfo}}{B_{c2}} \frac{1}{\sqrt{\omega^2 + \omega_o^2}})$$

$$S = \frac{R_{fl}}{B_o} = (R_{fl}^o + R_{fl}^1 B_{rfo})$$

$$R_{fl}^0 = \frac{R_n}{B_{c2}} \frac{\omega^2}{\omega^2 + \omega_o^2}$$

$$R_{fl}^1 = R_{fl}^o \frac{8}{3\pi} \frac{\gamma}{k} \frac{R_n}{\mu_o B_{c2}} \frac{1}{\sqrt{\omega^2 + \omega_o^2}} \frac{\omega_o^2}{\omega^2}$$

Validity limits



C. Benvenuti, S. Calatroni, I.E. Campisi, P. Darriulat, M.A. Peck, R. Russo and A-M Valente, "Study of the surface resistance of superconducting niobium films at 1.5 GHz", Physica C 316 (1999) 153-188

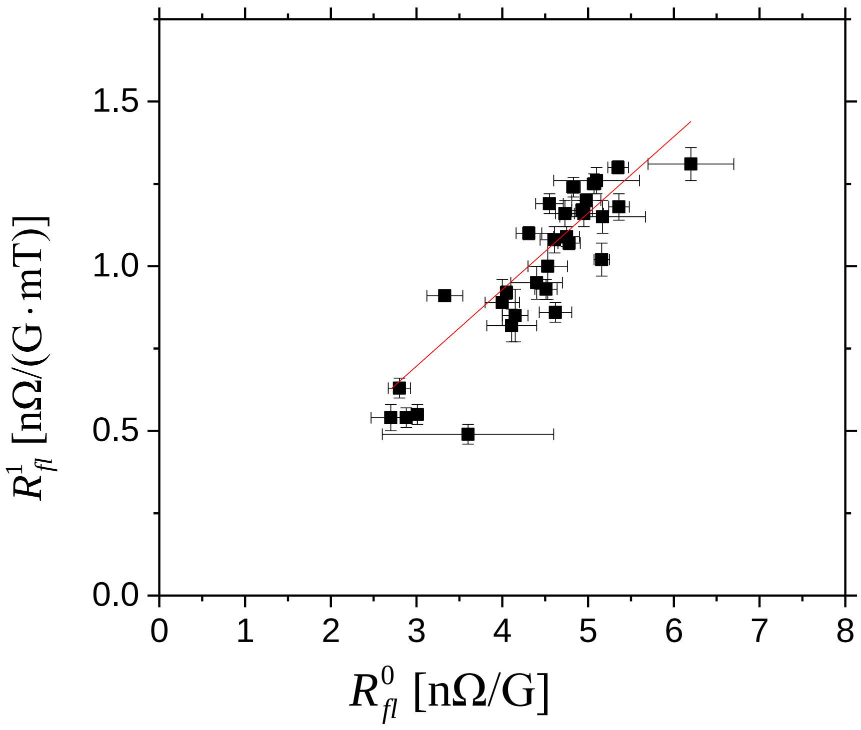
$$\omega < \omega_0$$

$$R_{fl}(B_{rfo}) = R_{fl}(0)\left(1 + \frac{8\alpha}{3\pi}\frac{\omega_o^2}{\omega^2}\right)$$

$$\alpha << 1 \Rightarrow R_{fl}(B_{rf0}^{max}) \approx 2R_{fl}(0)$$

$$B_{rf0}^{max} \approx 6mT$$

Model vs experimental results : R_{fl}^1 vs R_{fl}^0 correlation



Benvenuti et al 1999 Nb film

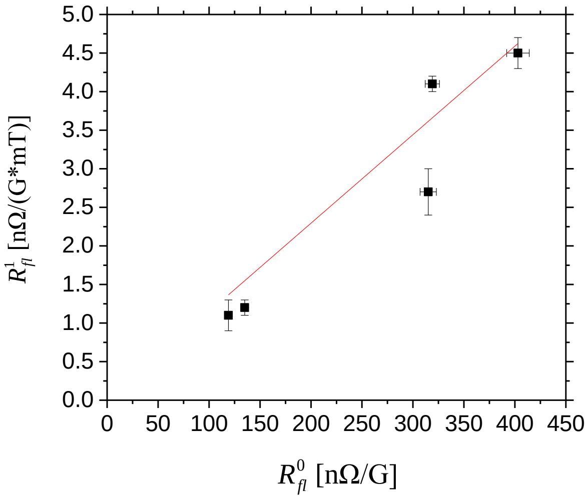
$$R_{fl}^0 \cong \frac{R_n}{B_{c2}} \frac{\omega^2}{\omega_o^2} \qquad \omega < \omega_0$$

$$R_{fl}^1 \cong R_{fl}^o \frac{\gamma}{k} \frac{8}{3\pi} \frac{R_n}{\mu_o \omega_o B_{c2}} \frac{\omega_o^2}{\omega^2}$$

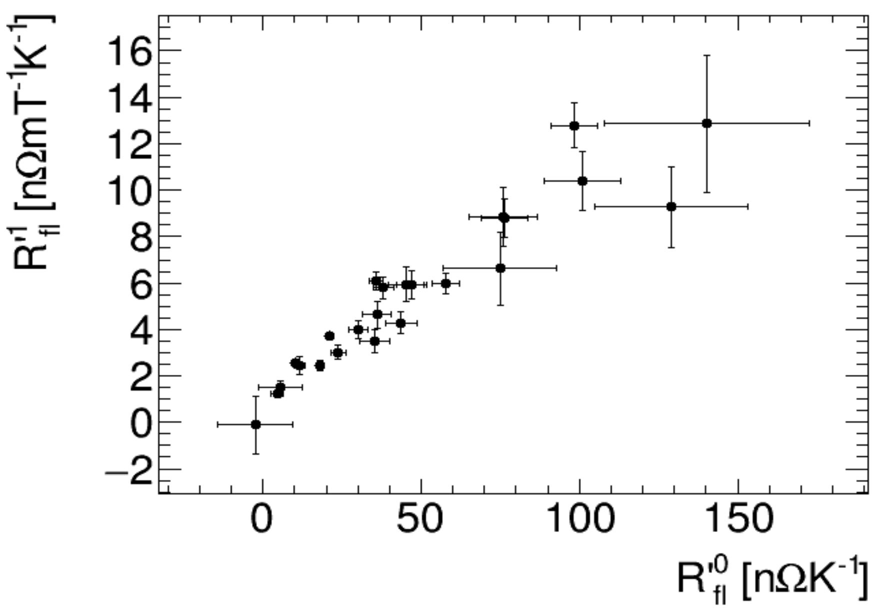
$$R_{fl}^0 = R_{fl}^0(k, \rho_n)$$

In the model the relation is strictly linear only if the R_{fl}^0 change is due to a change in the pinning force *k*

Other examples of linear correlation

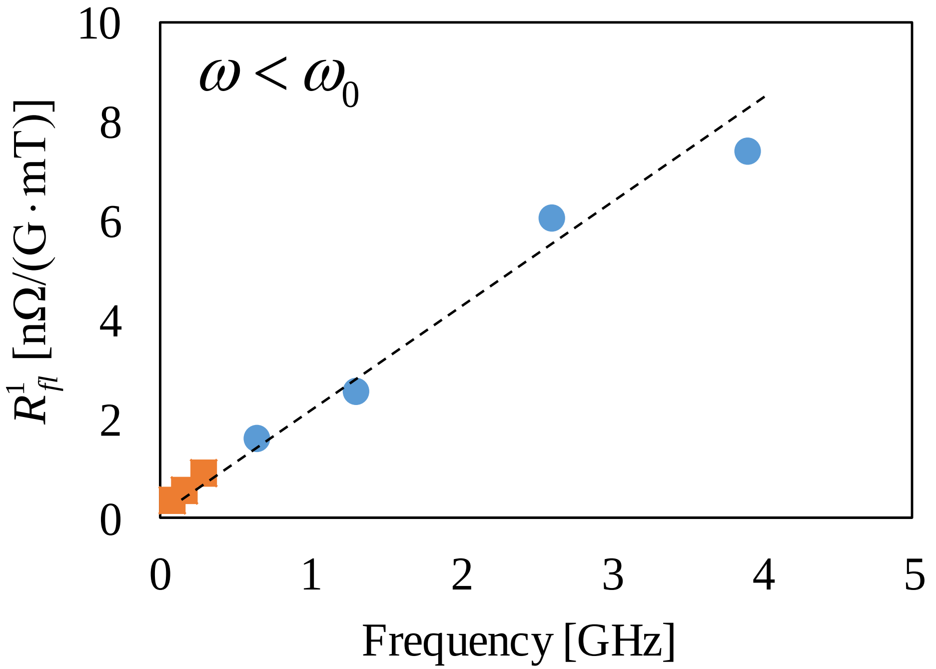


Benvenuti et al 1999 Nb bulk



Miyazaki, Venturini et al HIE-ISOLDE cavities

Model vs experimental results : frequency dependence



■ B. Piosczyk, P. Kneisel, O. Stoltz, J. Halbritter, IEEE Trans. *Nucl. Sci.* 20 (1973) 108-112

● M. Checchin, M. Martinello, A. Grassellino, S. Aderhold, S. K. Chandrasekaran, O. S. Melnychuk, S. Posen, A. Romanenko, and D. A. Sergatskov, Appl. Phys. Lett. 112, 072601 (2018)

model result for $\omega < \omega_0$

$$R^1_{fl} \cong \frac{\gamma}{k} \frac{8}{6\pi} \frac{1}{\omega_o} \frac{R_n^2}{\mu_o B_{c2}^2} \propto \omega$$

(a different behavior is predicted for $\omega \geq \omega_o$)

Conclusions :

- **Linear GL-type approach to the flux flow dissipation give a rf field amplitude (B_{rfo}) independent surface resistance R_{fl}**

- **Experiments clearly show a linear dependence of R_{fl} on B_{rfo}, at least at low fields**

- **A simple non-linear extension of the GL-type approach reproduces the correct R_{fl} linear dependence on B_{rfo}**

- **The observed strong correlation between R^o_{fl} and R^1_{fl} naturally emerges from the introduced non-linear model**

- **The model predicts the linear frequency dependence of R^1_{fl} , observed in the experiments, but only in the limit $\omega<\omega_o$**

- **The model is far too semplified to fully account for the flux flow dissipation complex phenomena, but can give useful hints towards the developement of a more complete treatment.**