Latest results from electron beam simulations for the HEL

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Outline

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Target-setting for HEL parameters

The ideal hollow electron beam doesn’t induce any electric or magnetic fields inside, but generates strong nonlinear fields outside. The transverse kick applied to hollow particles can be expressed as:

$$\theta = \frac{2I_{er}L(1 \pm \beta_e \beta_p)}{r \beta_e \beta_p c^2 (B \rho)_p} \left( \frac{1}{4\pi \varepsilon_0} \right)$$

The whole problem, therefore, is to choose all parameters to maximize the efficiency of collimation and at the same time try to avoid different undesirable things which leading to any fields appearing inside hollow.
Stability analysis
HEL Current restrictions

First restriction - gun design and cathode type/size  (not a big problem)

Second restriction - Maximum current that can be transported through the vacuum chamber with radius R. In case current exceeds critical value, part of the beam is reflected, i.e. beam potential sags to zero.

\[ I_{max} = \frac{4 \cdot \pi \cdot \varepsilon_0 \cdot \sqrt{8}}{\sqrt{27}} \cdot \sqrt{\frac{e}{m}} \cdot \frac{U^{3/2}}{1 + 2\ln\left(\frac{R}{r}\right)} \]

Pipe radius \( R = 30 \text{ mm} \)
Outer beam radius \( r = 1.8 \text{ mm} \)

\( U = 12 \text{ kV} \)
\( U = 15 \text{ kV} \)
Diocotron instability consideration

On Fermilab test stand the destruction of hollow beam into specific microstructure was observed (very similar to diocotron instability):

**Origin of the diocotron instability:** Different angular velocities for different radii provide relative motion of layers (See (*) ). Small initial asymmetry may lead to the significant density equilibrium violation and cluster origin.

\[
\omega_e(r) = \frac{\omega_p}{\omega_c} \left(1 - \frac{r_{out}^2}{r^2}\right)
\]

\[
\frac{\partial \omega_e(r)}{\partial r} \neq 0 \quad (*)
\]

\[
r_{in} < r < r_{out}
\]
Davidson stability criteria*

\[\left\{-l \left[1-\left(\frac{r_1}{r_2}\right)^2\right] + 2 - \left[\left(\frac{r_1}{a}\right)^{2l} + \left(\frac{r_2}{a}\right)^{2l}\right]\right\}^2 \geq 4 \left(\frac{r_1}{r_2}\right)^{2l} \left[1-\left(\frac{r_2}{a}\right)^{2l}\right]\] \quad l = 1, 2, 3...

\(r_1\) – beam inner radius, \(r_2\) – beam outer radius, \(a\) – radius of the vacuum chamber

In case the beam is unstable, beam current and external magnetic field influence rate of the instability growth \(T\):

\[T \sim \frac{I_b}{B_z} M\]

\(I_b\) is the beam current, \(B_z\) is external longitudinal magnetic factor \(M\) is the geometry factor (depends on mode number \(l, r_1, r_2, a\))

* R. C. Davidson, "Physics of Non-neutral Plasmas"
“1” corresponds to the beam with radii $r_1 = 3\, \text{mm}$, $r_2 = 8\, \text{mm}$, “2” corresponds to the beam with radii $r_1 = 5.5\, \text{mm}$, $r_2 = 8\, \text{mm}$.
Gray region corresponds to the beam stable state (up to the stability criterion)
Checking of the stability criterion

Unstable up to the criterion
\[ r_1 = 5.5 \text{ mm}, \quad r_2 = 8 \text{ mm} \]

Stable up to the criterion
\[ r_1 = 3 \text{ mm}, \quad r_2 = 8 \text{ mm} \]

\[ I = 10 \text{ A} \]
\[ U = 15 \text{ keV} \]
\[ B = 0.2 \text{ T} \]

... after 3 m
Stability diagram (HEL parameters)

“1” corresponds to the beam with radii $r_1 = 0.9$ mm, $r_2 = 1.8$ mm
Gray region corresponds to the beam stable state (up to the stability criterion)
(Only the region above the 45° line has meaning because $r_2 > r_1$)
Particle tracking
At the beginning of the interaction space

At the middle of the interaction space

At the end of the interaction space
Main sol. with TRK and PIC solvers

Particles with this maximum potential has to be rotated on 190 deg, what one can see on the figures.

The difference of the potentials results in the beam shape distortion. But this is not unstable regime!
Potential changing due to the asymmetry of the vacuum chamber

Beam is not centered between walls of the vacuum tube

Walls of vacuum chamber influence the beam = beam is influenced by external electric field

Beam tries to compensate influence of external electric field, but it is “frozen” in high magnetic field of 5 T so that particle density cannot change

Beam potential becomes asymmetric to compensate external electric field
Potential asymmetry at the outer radius

\[ \Delta U_b \approx \frac{I}{2\pi \nu_b \varepsilon_0} \ln \frac{r_b}{a} - \frac{I}{2\pi \nu_b \varepsilon_0} \ln \frac{r_b}{2a + \sqrt{3}/2a} = \]

\[ \frac{I}{2\pi \nu_b \varepsilon_0} \ln \frac{2a + \sqrt{3}/2a}{a} \]

If the beam was solid with parameters \( U_0 = 15 \text{ keV}, \nu_b = 0.2 \text{ c} \)
\( I = 5 \text{ A}, \ a = 30 \text{ mm}, \ r_b = 1.8 \text{ mm}, \) potential difference is

\[ \Delta U_b \approx 1.3 \text{ kV} \]
Potential asymmetry and beam shape perturbation

Particles starting from different azimuthal angles have different rotation velocity

Perturbation of the azimuthally uniform electron density

Non-zero electric field in the hollow increasing with the beam motion through the vacuum chamber
According to the simulation the maximum field is about 0.8 MV/m and according to the theory is 0.7 MV/m. The difference can be provided by slightly difference of the beam size and beam current (in the simulation the current is about 5.3 A).

The field irregularity in the middle of the beam is not more than 5% (~4% or about 30 kV/m) at the end of the HEL.
Conclusion

Stability:
• The used beam parameters (current is not more than 5 A with voltage of 15 kV) are looked quite reliable. They should provide the stable regime of the HEL.
• With exact design parameters $r_1 = 0.9 \text{ mm}$, $r_2 = 1.8 \text{ mm}$ beam is not influenced by the diocotron instability.

Particle tracking:
• All simulations were performed for the irregular particles distribution with “peak” current density near the inner radius. The influence of this peak on the beam motion is not observed and this peak is saved during the motion.
• The beam simulation was carried out with different PIC and TRK solvers. With the same grid they give the same results.
• The amplitude of residual field in the center not more than 5% from the maximum field near the outer beam radius at the end of the HEL.
Potential asymmetry at the inner radius

Beam is off-centered = beam is influenced by the equivalent electric field

External electric field does not penetrate into the hollow because beam compensates it therein, changing the potential

$E_{ext}$

$U_2 > U_1$

$E_b$

$10.4 \text{ kV}$

$11.6 \text{ kV}$
Dependence of time of instability growth on beam outer radius

Fix inner radius \( r_1 = 0.9 \) mm, consider dependence on beam outer radius (which is more changeable because of beam field)

Design beam radii \( r_1 = 0.9 \) mm, \( r_2 = 1.8 \) mm correspond to the stable state

Example of deviation of designed parameter:
\( r_1 = 0.9 \) mm, \( r_2 = 1.6 \) mm
\( \tau \approx 40 \) ns
Time of flight \( \approx 50 \) ns
Dependence of time of instability growth vs beam outer radius

Modes with $l \geq 6$ are less dangerous

Fixed inner radius $r_1 = 0.9$ mm

$l = 4$

$l = 5$