Tools for NNLO and higher order computations

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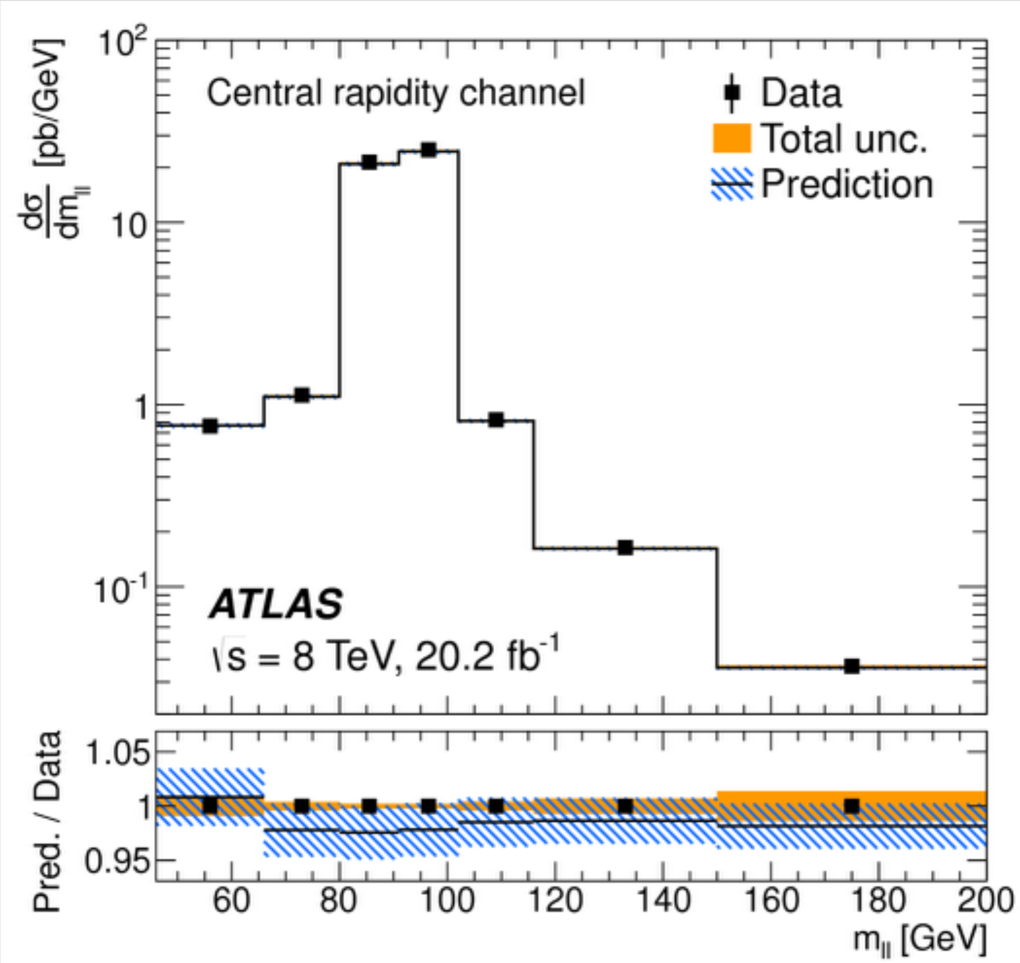
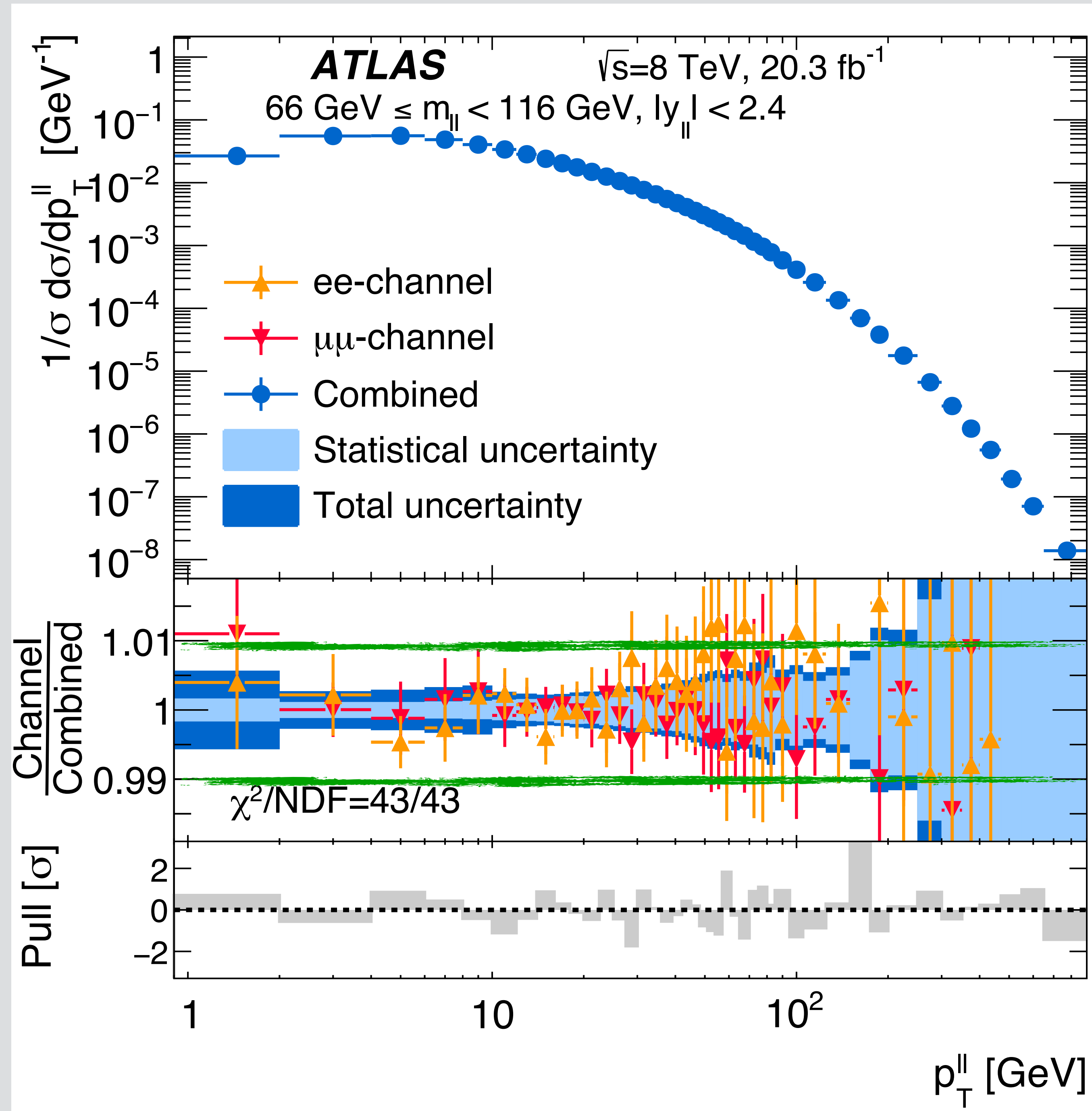
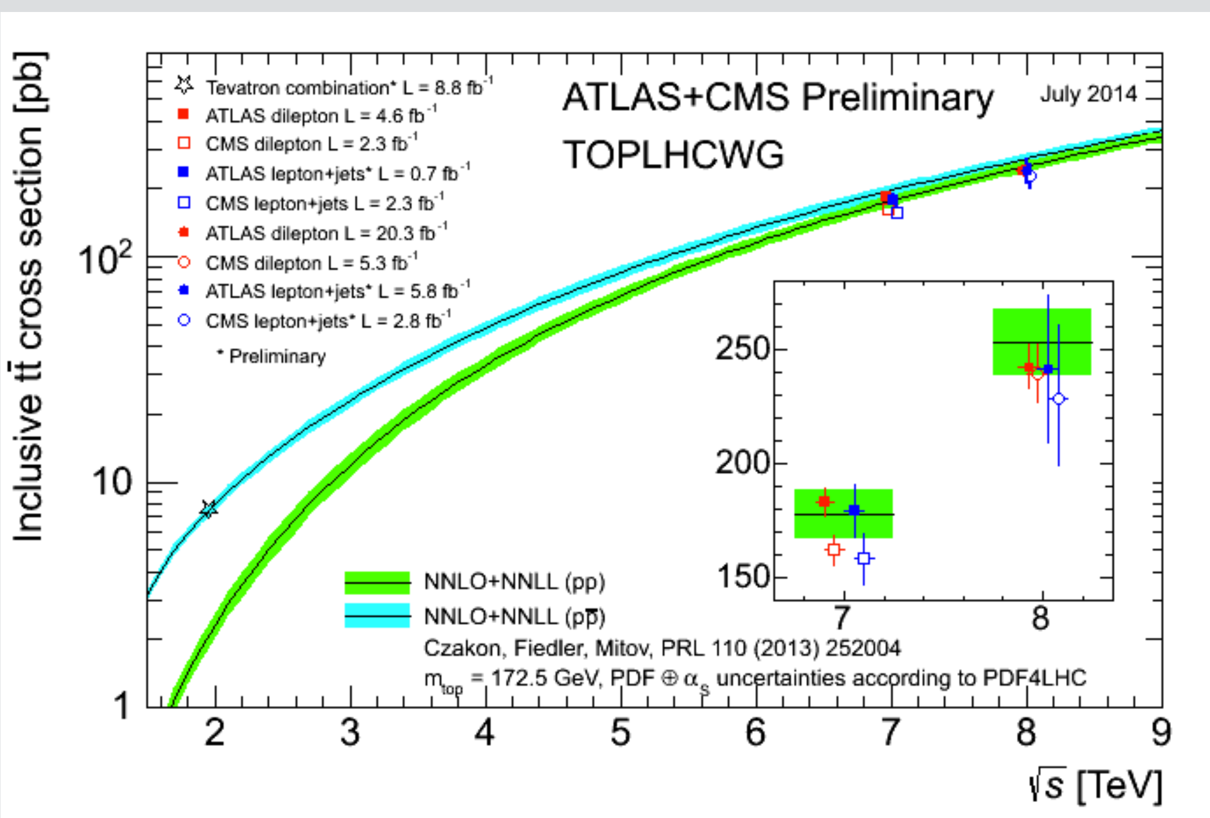
NNPDF & N³PDF kick-off meeting, Gargnano, 17 September 2018

LHC: a precision machine



Current experimental precision for standard candles: ~percent or less

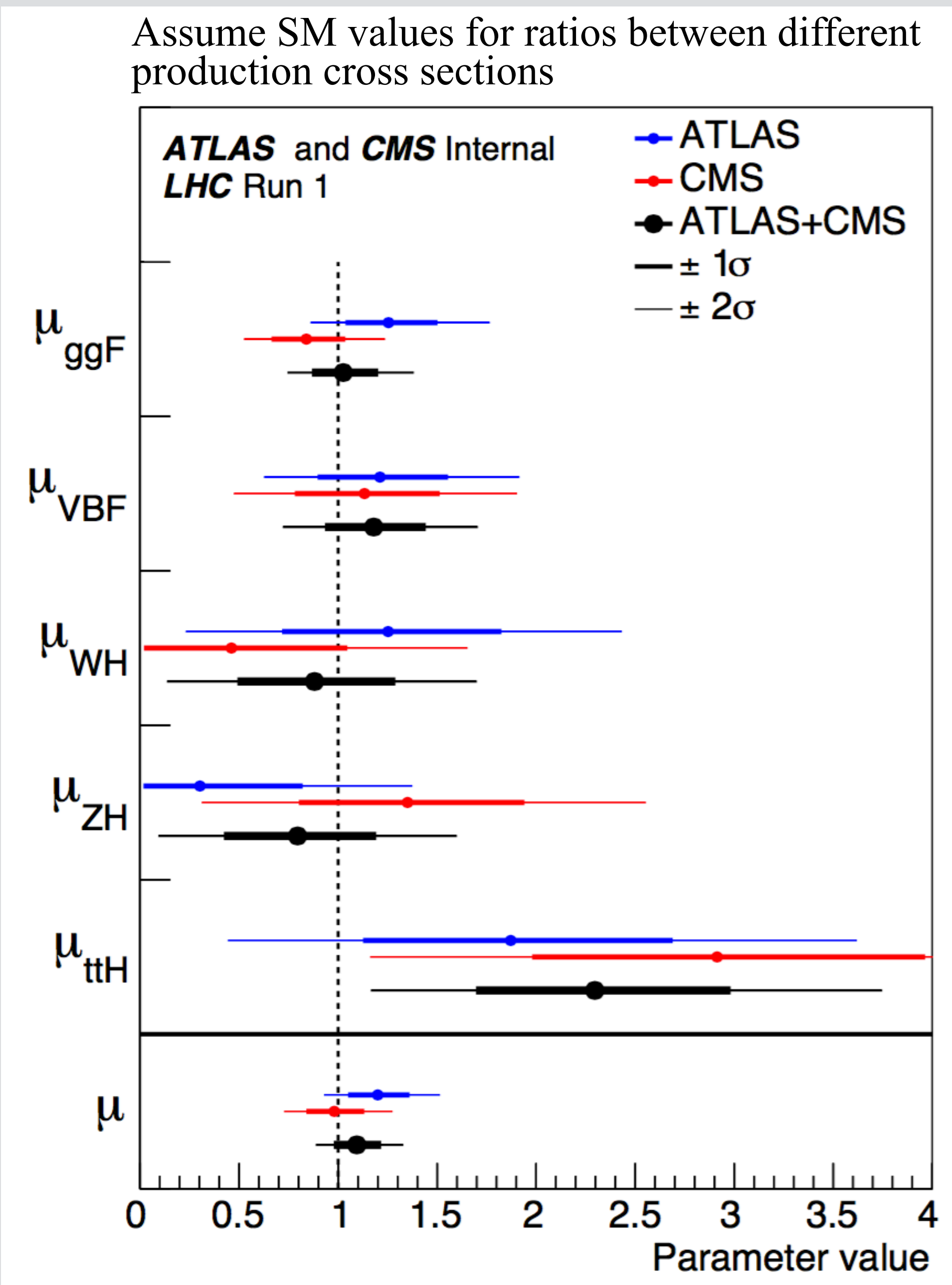


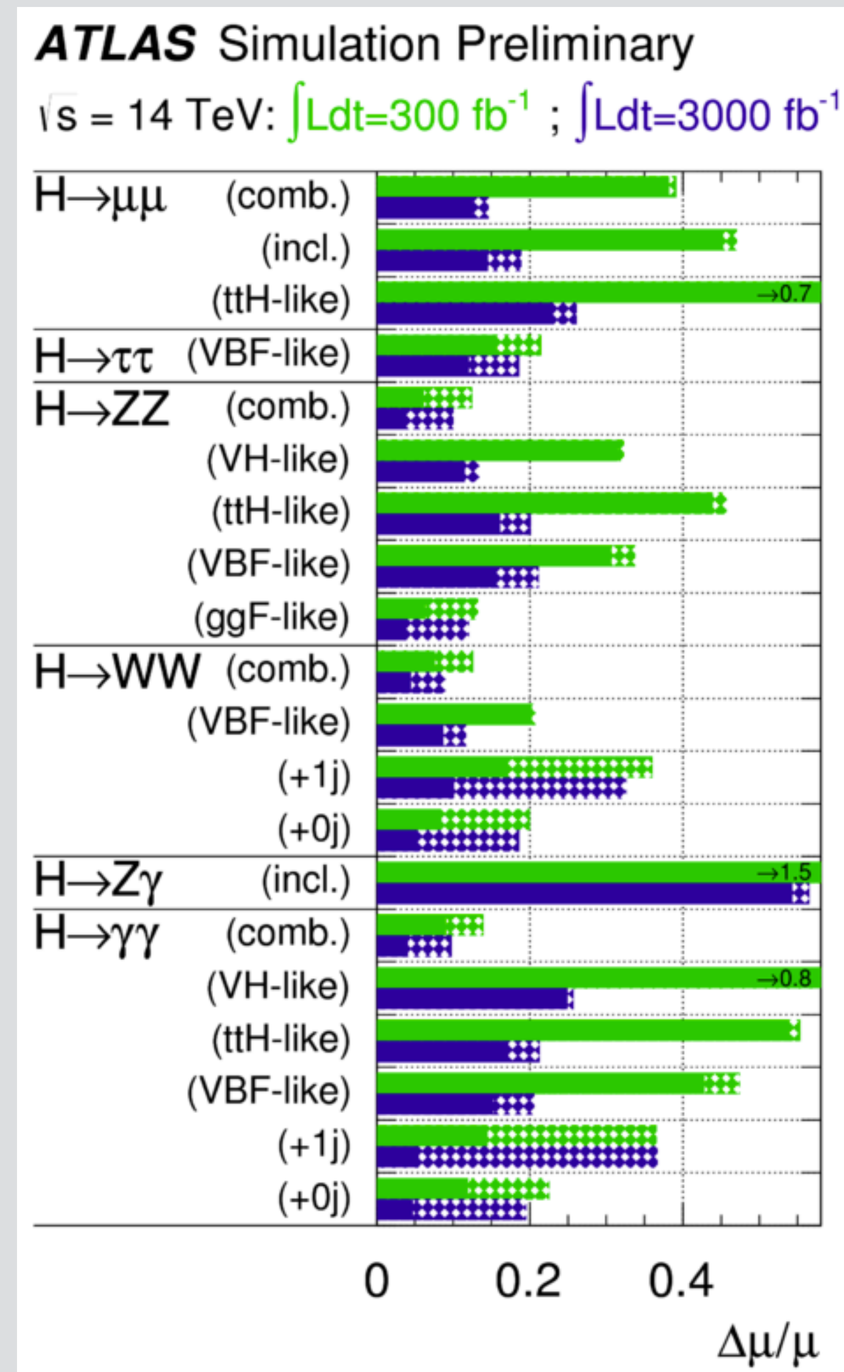


LHC: a precision machine

In the future: ~ few percent may be within experimental reach for a larger class of processes/observables

Example: *Higgs couplings*





What to do with precision

- STUDY SM PARTICLES AND THEIR DYNAMICS, at unprecedented level of scrutiny. Stress-test SM (and our understanding of QFT)

- PRECISION IS NOW ALSO TOOL FOR DISCOVERIES

  Imagine to have new physics at a scale Λ

  - if Λ small $\rightarrow$ should see it directly, bump hunting. So far: only Higgs, $\Lambda \gtrsim$ TeV
  - if Λ large, typical modification to observable w.r.t. standard model prediction: $\delta O \sim Q^2/\Lambda^2$
  - standard observables at the EW scale: $\Lambda \gtrsim$ TeV $\Rightarrow$ $\delta O \sim$ percent

Experimentally within reach, must match on the theory side

QCD at colliders: the factorization formula

$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2 f(x_1) f(x_2) \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q))$$

INPUT PARAMETERS: PDFS, α_s

- Accurate predictions for standard candles / evolution

HARD SCATTERING MATRIX ELEMENT

- large Q → theoretically clean
- $\alpha_s \sim 0.1$ → For TYPICAL PROCESSES, we need NLO for ~ 10% and NNLO for ~ 1% accuracy. Processes with large color charges (Higgs): $\alpha_s C_A \sim 0.3$ → N^3LO

NON PERTURBATIVE EFFECTS:

- typical observable: O(Λ/Q) ~ few percent
- No good control/understanding of them at this level. LIMITING FACTOR FOR FUTURE DEVELOPMENT [m_t, m_W…]

Where can we achieve high accuracy?

Focus on simple [*clean exp/th comparison, good control*] processes, high scale [*little non pert. contamination*] observables. Typical examples:

- V / V+j(j) → PDFs, backgrounds
- tt, single top → gluon and b PDF, V_{tb}, backgrounds…
- jj(j) → PDFs, jet dynamics, α_S…
- H / H+j(j) / VH → Higgs couplings / characterization
- VV → anomalous couplings, (Higgs) backgrounds…

Fixed order predictions:

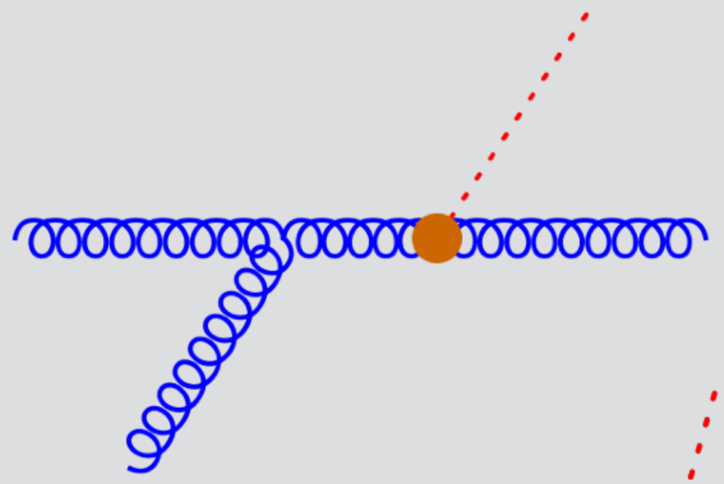
- Able to provide HIGH PRECISION while PROPERLY ACCOUNT FOR EXPERIMENTAL SETUP (cuts, fiducial region…)
- At high Q, typically processes are a multi-scale problem. However, no huge scale hierarchies → fixed (high enough) order predictions correctly capture all the relevant logs

NNLO: the big picture

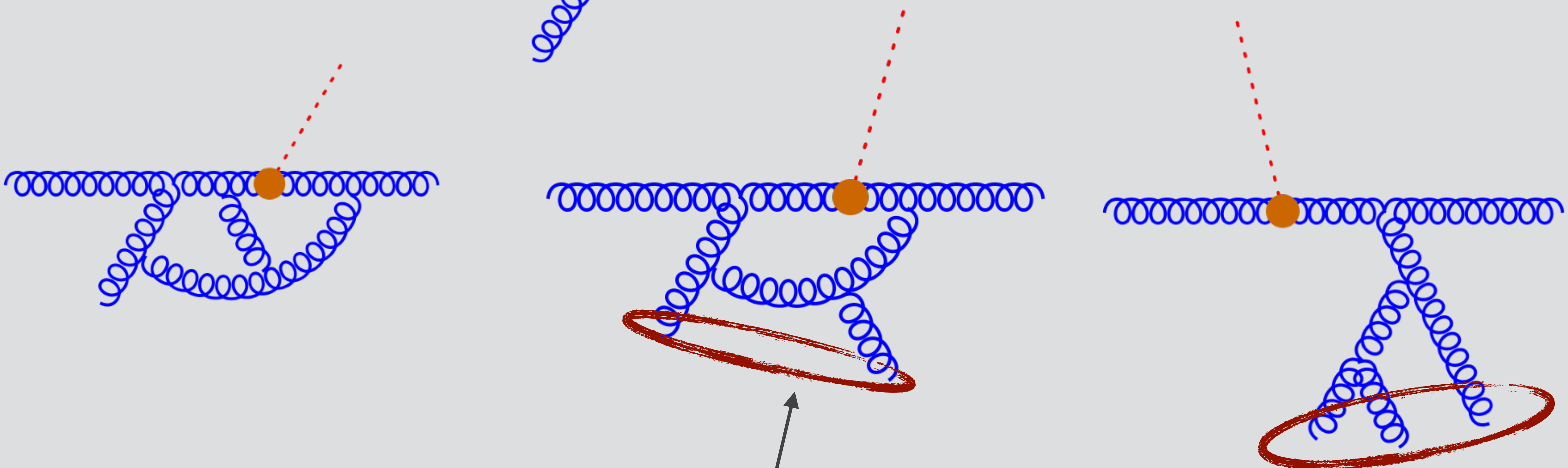
NNLO computations in a nutshell

$O(\alpha_s^2)$ corrections: two-loop (VV), one-loop+j (RV), tree+jj (RR)

E.g. Higgs p_t: LO



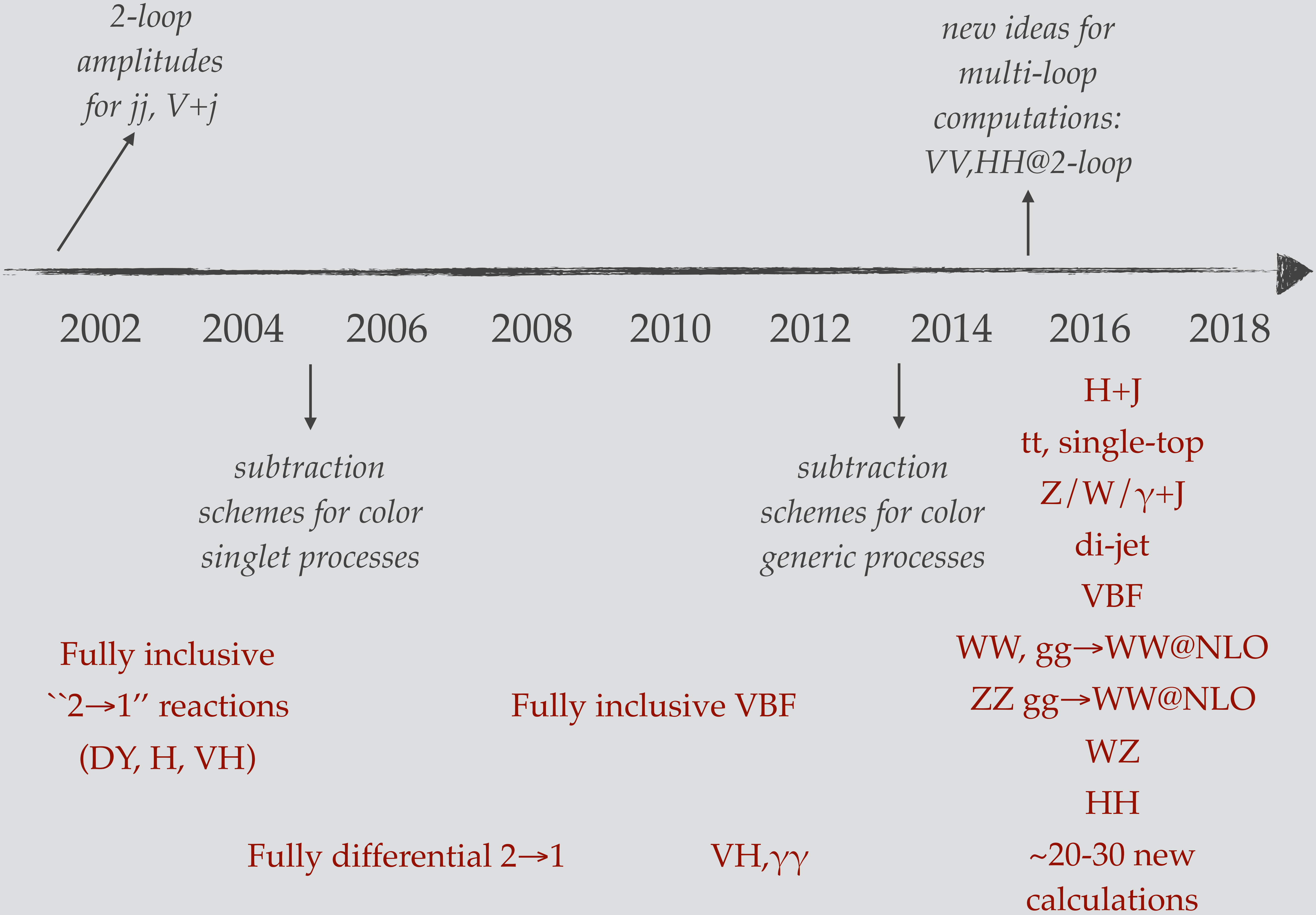
NNLO →



TWO BIG PROBLEMS:

- loop amplitudes
- non trivial soft/collinear radiation patterns
  - ✶must be properly dealt with (``subtracted'')
  - ✶especially challenging in presence of **realistic cuts on final states**

A NNLO timeline



NNLO at hadron colliders: the big picture

Higgs

- $gg \rightarrow H$ ● ● ● ● ● *public*
- VBF_{DIS} ● ● *public*
- VH, $H \rightarrow bb$ ● ●
- HH_{HEFT} ●
- $H+j/p_{t,H}$ ● ● ●

DY

- $pp \rightarrow V$ ● ● ● ● *public*
- $W+j/p_{t,W}$ ● ●
- $Z+j/p_{t,Z}$ ● ●
- $\gamma+j$ ●

APPLgrid

Top

- tt ● *partially public (grids, fastNLO)*
- $t_{t\text{-channel, DIS}}$ ● ● [+decay]

Jets

- single inclusive ●
- di-jet ●

VV

- $\gamma\gamma$ ● ● *public*
- WW,WZ,ZZ,HH ● *public*

DIS

- *ep→jet* ● [also massive] ● [+N3LO]
- *ep→2jet* ●

● *antenna* ● *FKS+sector decomposition (STRIPPER, nested subtraction…)* ● *q_T (+COLORFULL)*
● *$N_{jettiness}$/SCET-based slicing*
● *Projection to Born*

NNLO at hadron colliders: the big picture

Hig
- g
- V
- V
- H
- H

Top
- tt
- $t_{t\text{-}}$

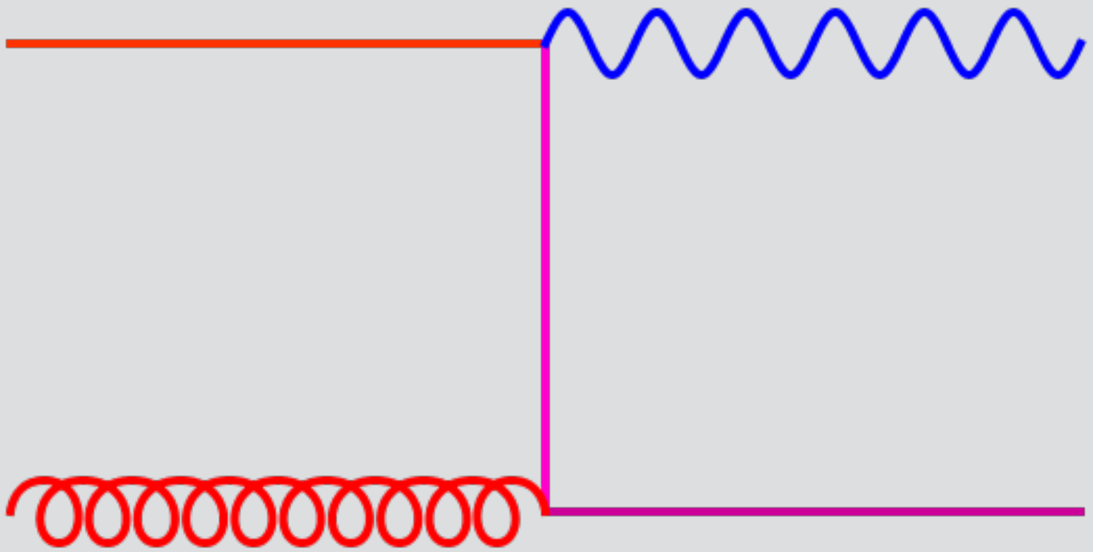
VV
- γ
- W

DIS
- *ep*–
- *ep→2jet* ●

The upshot:

- $2 \rightarrow 2$ processes basically done
- In most cases, different calculations/techniques → proper validation
- Very complicated calculations → no ``generic" public codes yet
- Investigations on different ways of disseminating results (fast tables, NTuples…)
- Color singlet processes: new general purposes codes available (MCFM, Matrix…)
- MCFM for non color-singlet (V/H+j) could be available in the near future
- NNLOJet → see Juan's talk

A striking omission: Wc



- In principle, subset of W+j → simple to compute
- However, it actually depends a lot on the proper definition of the process
  - charm jet? Flavor Algorithm? Massive charm [complicated amplitude, large logarithms]
  - D mesons? [fragmentation…]
  - Wc/Wcc separation? [Wcc beyond current reach…]

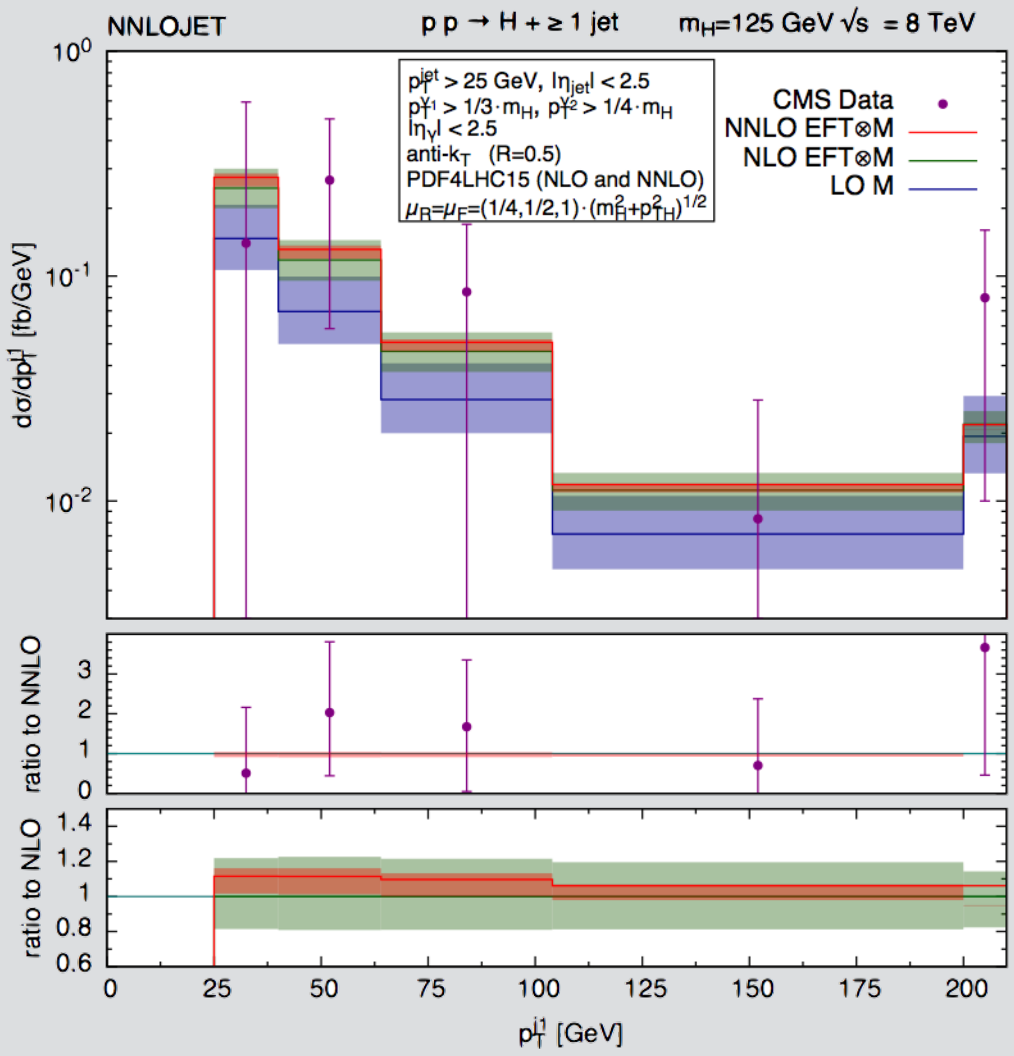
$2 \rightarrow 2$ NNLO phenomenology: a quick overview

$2 \to 2$ pheno @ NNLO: the global picture

Greatly reduced theoretical uncertainties, perturbative convergence established

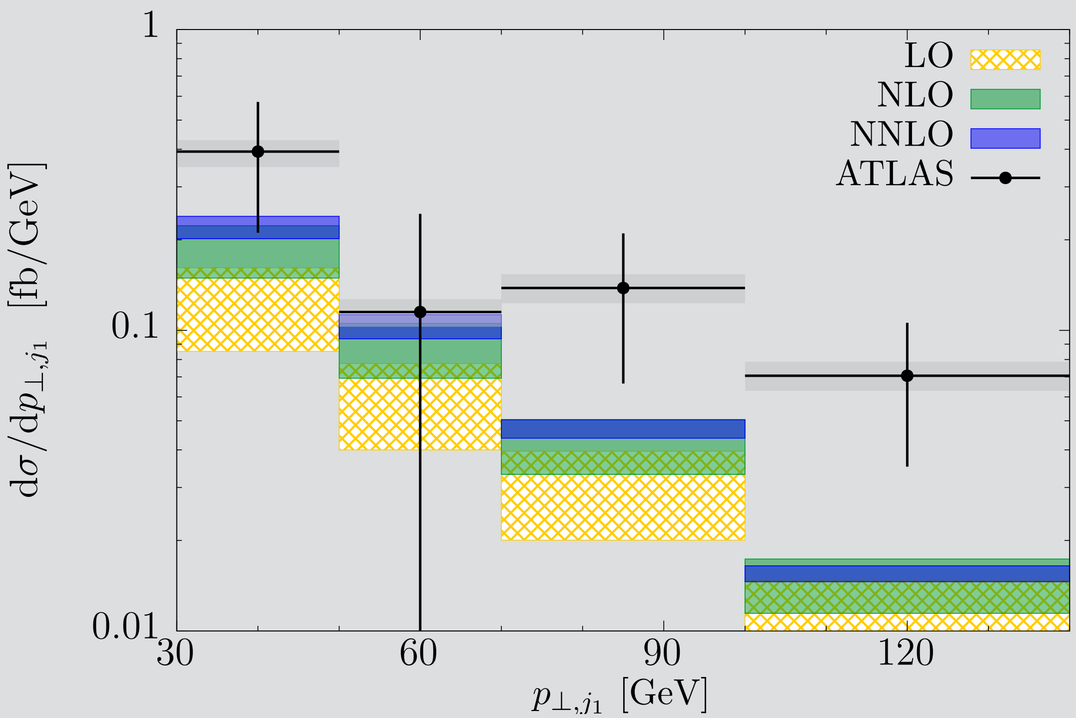
$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

Inclusive H@N³LO
[Anastasiou, Duhr, Dulat, Herzog, Mistlberger]



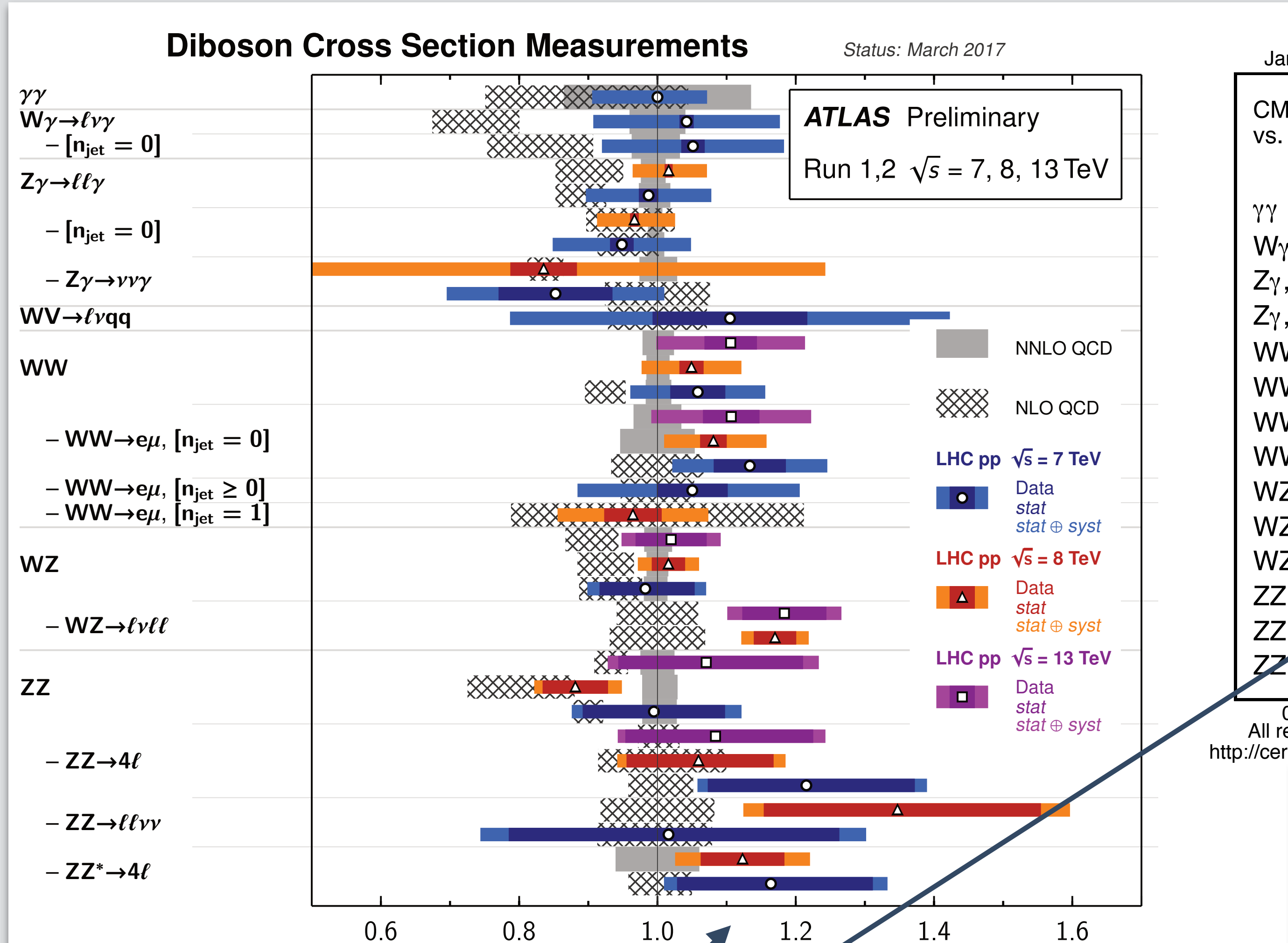
Exclusive Higgs + jet

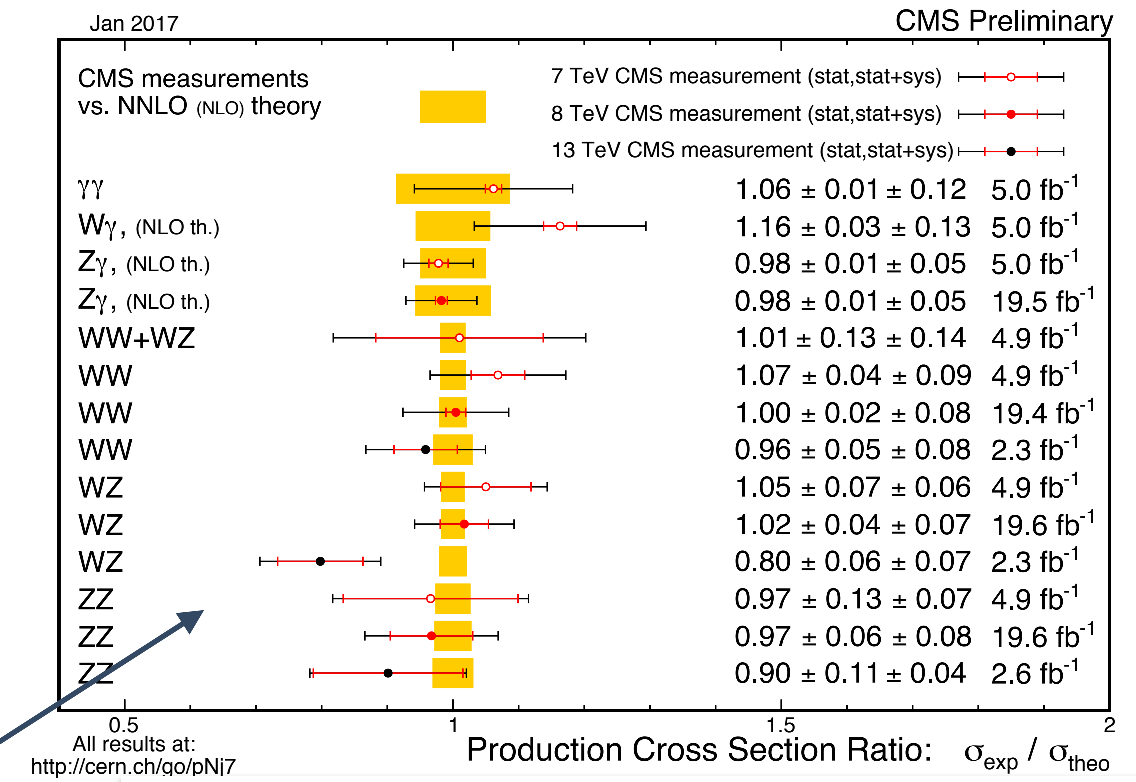
[Boughezal, et al; Chen et al; FC, Melnikov, Schulze]

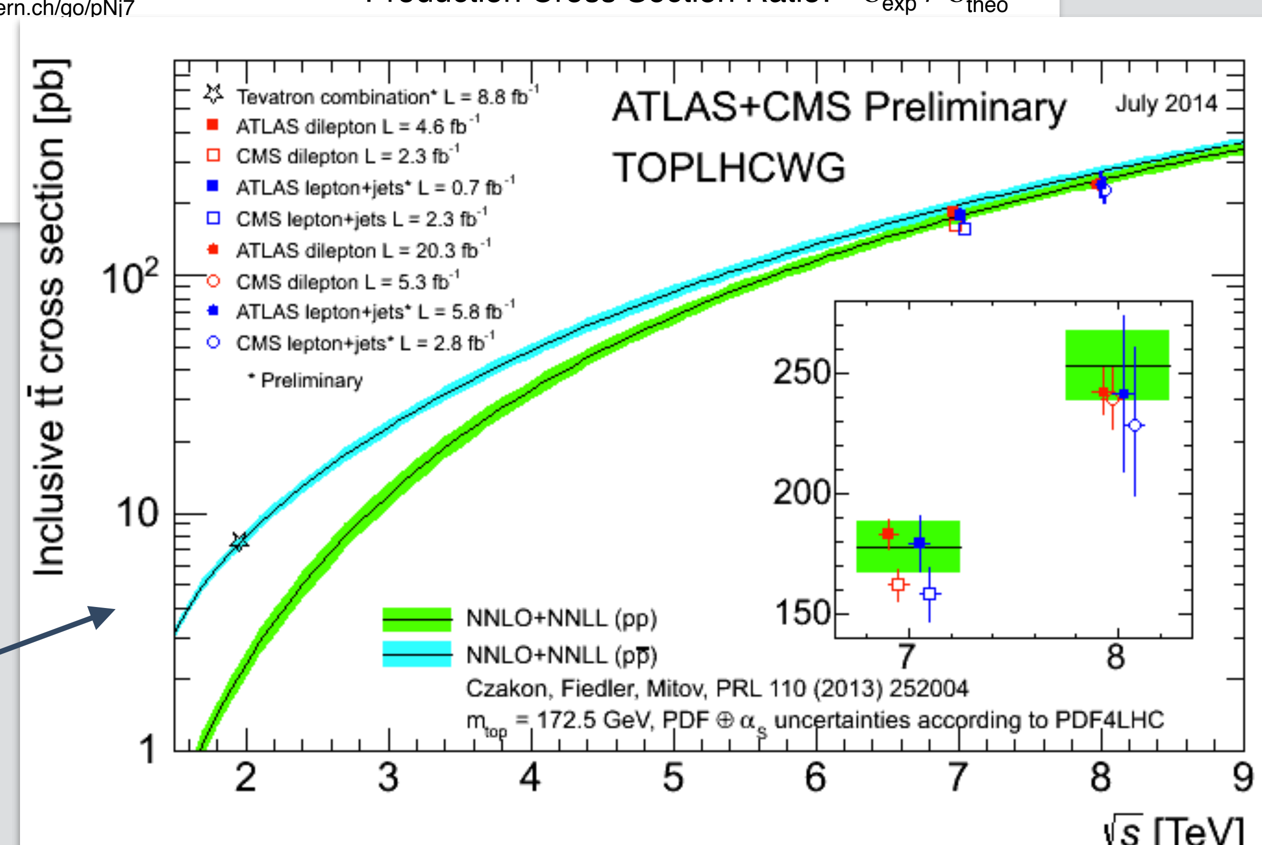


2→2 pheno @ NNLO: the global picture

Very good / improved data-theory comparison





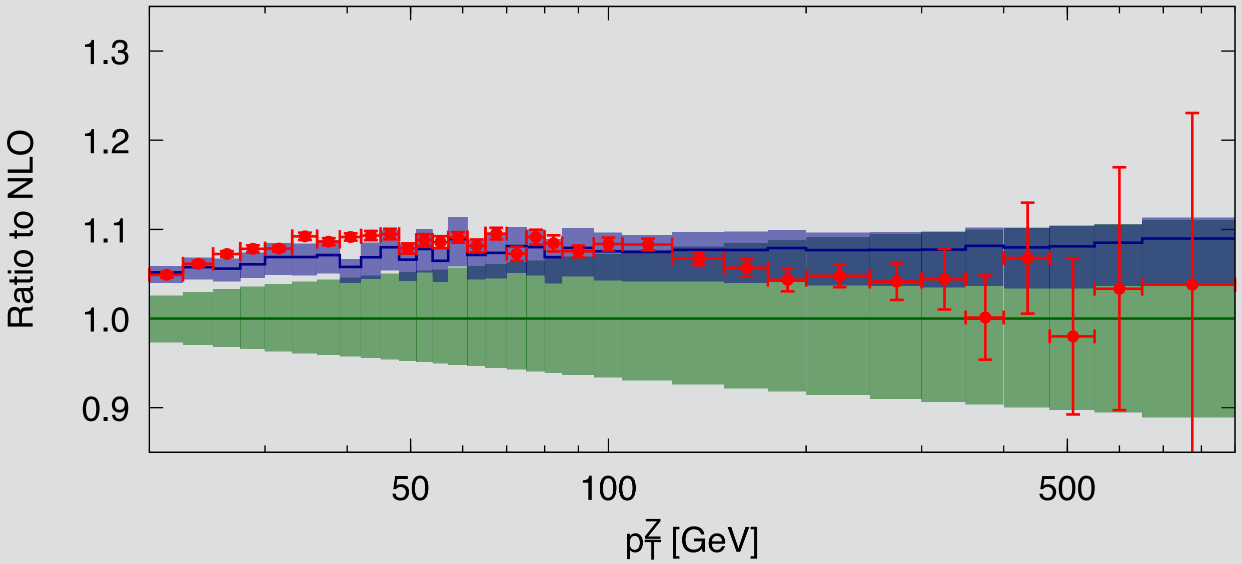


Di-bosons
[Catani, Grazzini; Campbell, Ellis, Williams; Grazzini et al (2015-2017)]

Top pairs [Czakon, Fiedler, Mitov]

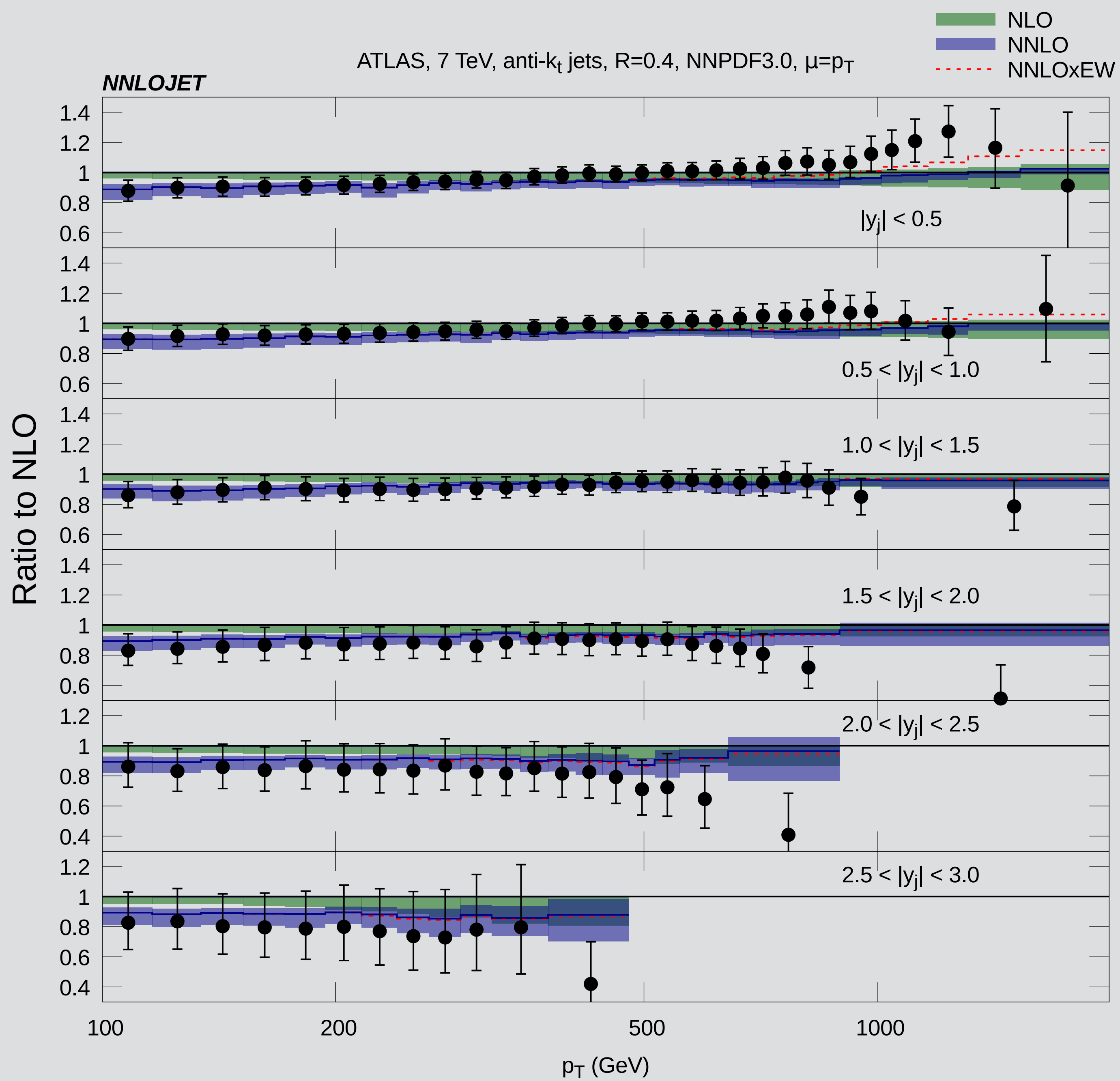
2→2 pheno @ NNLO: the global picture

Very good / improved data-theory comparison



Z+J/Z p_T *shape*

[Gehrmann-de Ridder et al; Boughezal et al]



Inclusive jet production

For a particular scale choice

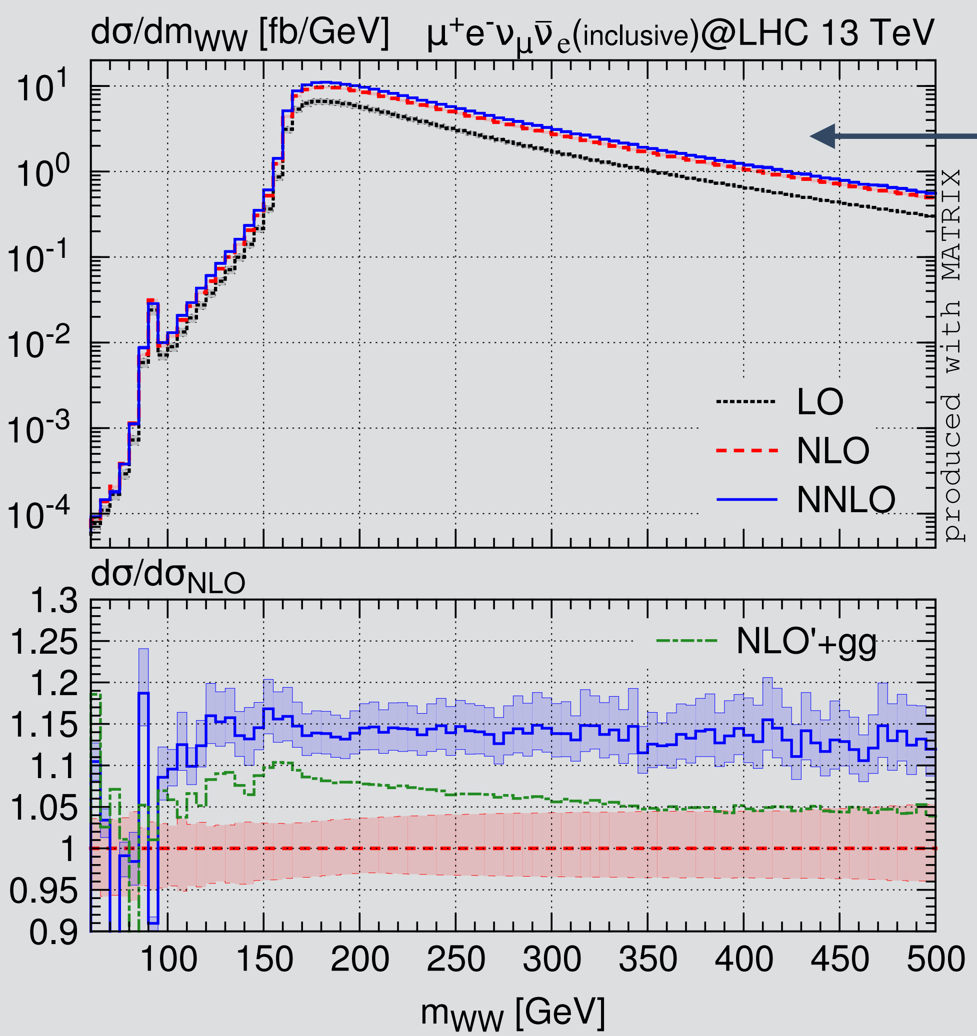
[Currie, Glover, Gehrmann, Gehrmann-de Ridder, Huss, Pires (2017)]

NNLO: what have we learned so far?

- Properly modeling the actual experimental setup is crucial
  (especially for cuts constraining QCD radiation)

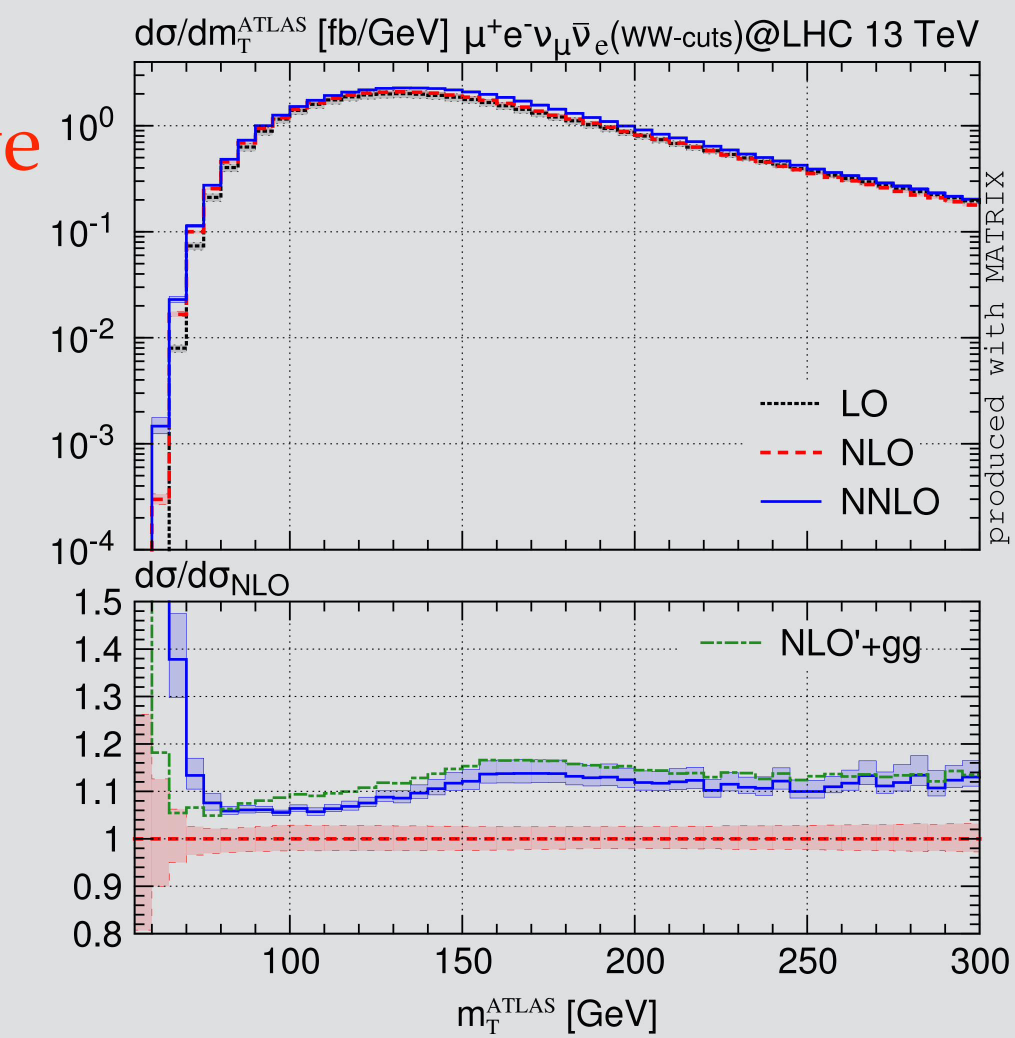
Example: WW, 13 TeV: qq- vs gg-initiated sub-processes

- full inclusive [*unobservable*]: qq@NNLO +7%, gg + 4%
- WW fiducial region: qq@NNLO -2%, gg +9% (similar result for Higgs-cuts)



Inclusive

Fiducial



[Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann (2016)]

[higher order corrections to gg component: FC, Dowling, Melnikov, Röntsch, Tancredi (2016)]

NNLO: what have we learned so far?

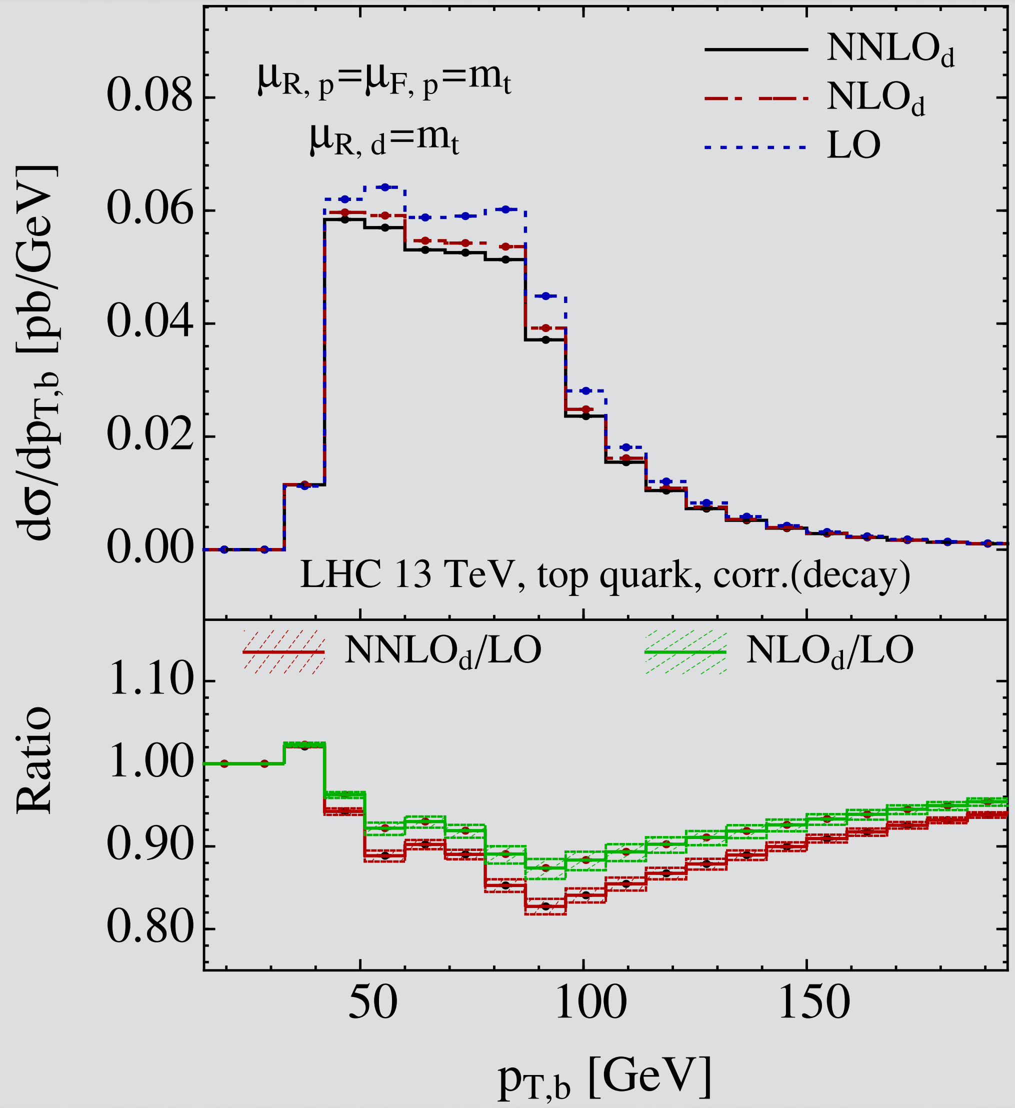
Example: modeling TOP decay

tt, approx $\text{NNLO}_{\text{prod}}$ *(carefully benchmarked against [Czakon et al])* x $\text{NNLO}_{\text{decay}}$

CMS setup, $e^{\pm}\mu^{\mp}, e^+e^-, \mu^+\mu^-$ channel [25], 2 b-jets required (anti-k_t algorithm [66], $R = 0.5$)

energy	fiducial volume	LO [pb]	NLO [pb]	$\hat{\text{N}}$NLO [pb]	$\delta_{\text{dec.}}$	CMS [pb]
8 TeV	$p_T(l^{\pm}) > 20$ GeV, $\lvert\eta(l^{\pm})\rvert < 2.4$, $p_T(J_b) > 30$ GeV, $\lvert\eta(J_b)\rvert < 2.4$	$3.780^{+37.4\%}_{-25.3\%}$	$4.483^{+9.0\%}_{-11.5\%}$	$4.874^{+2.5\%}_{-6.8\%}$	-8.0%	$4.73^{+4.7\%}_{-4.7\%}$

[Papanastasiou, Gao (2017)]

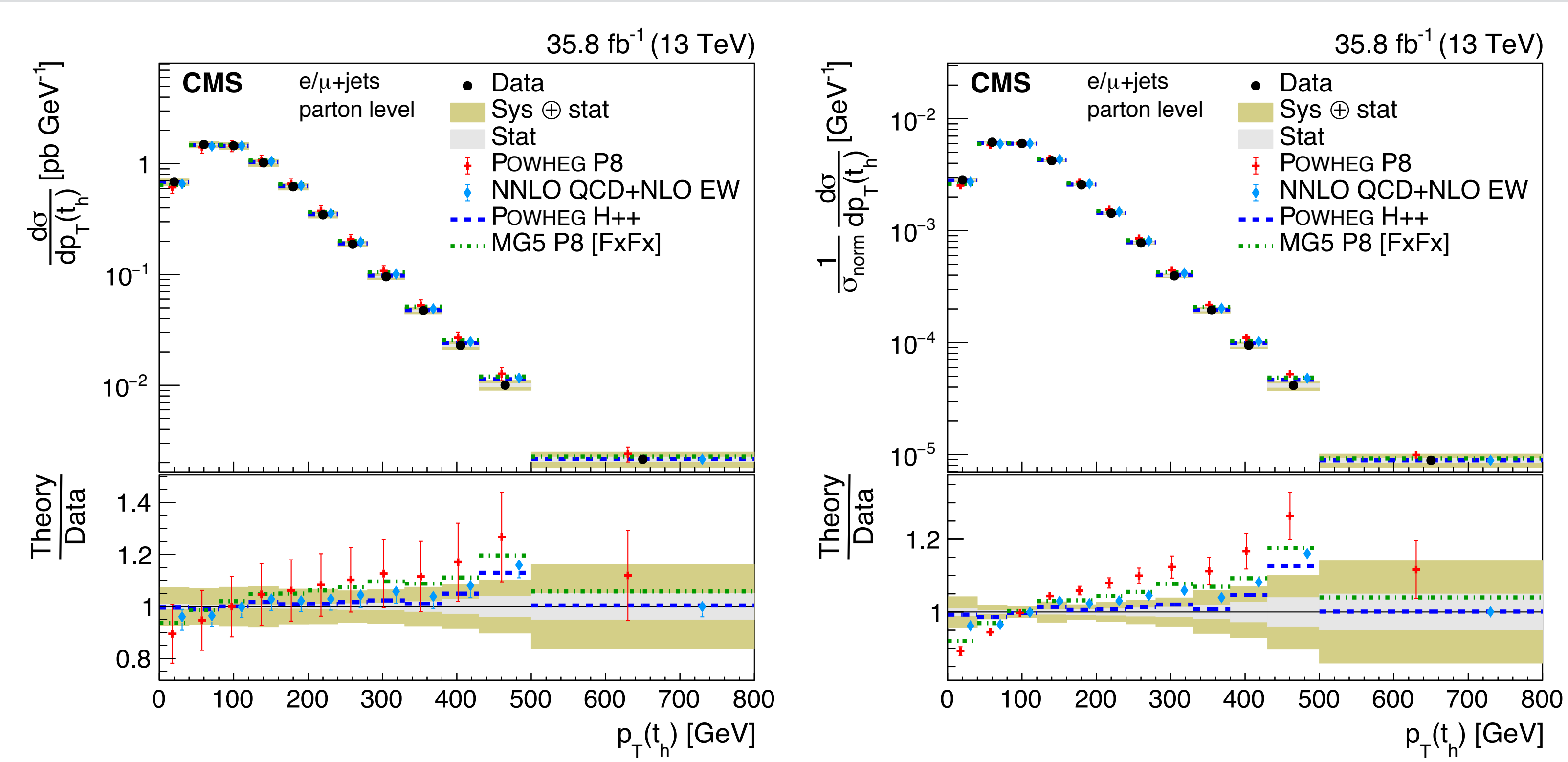


t-channel single-top plus top-decay (NWA)

[Berger, Gao, Yuan, Zhu (2016)]

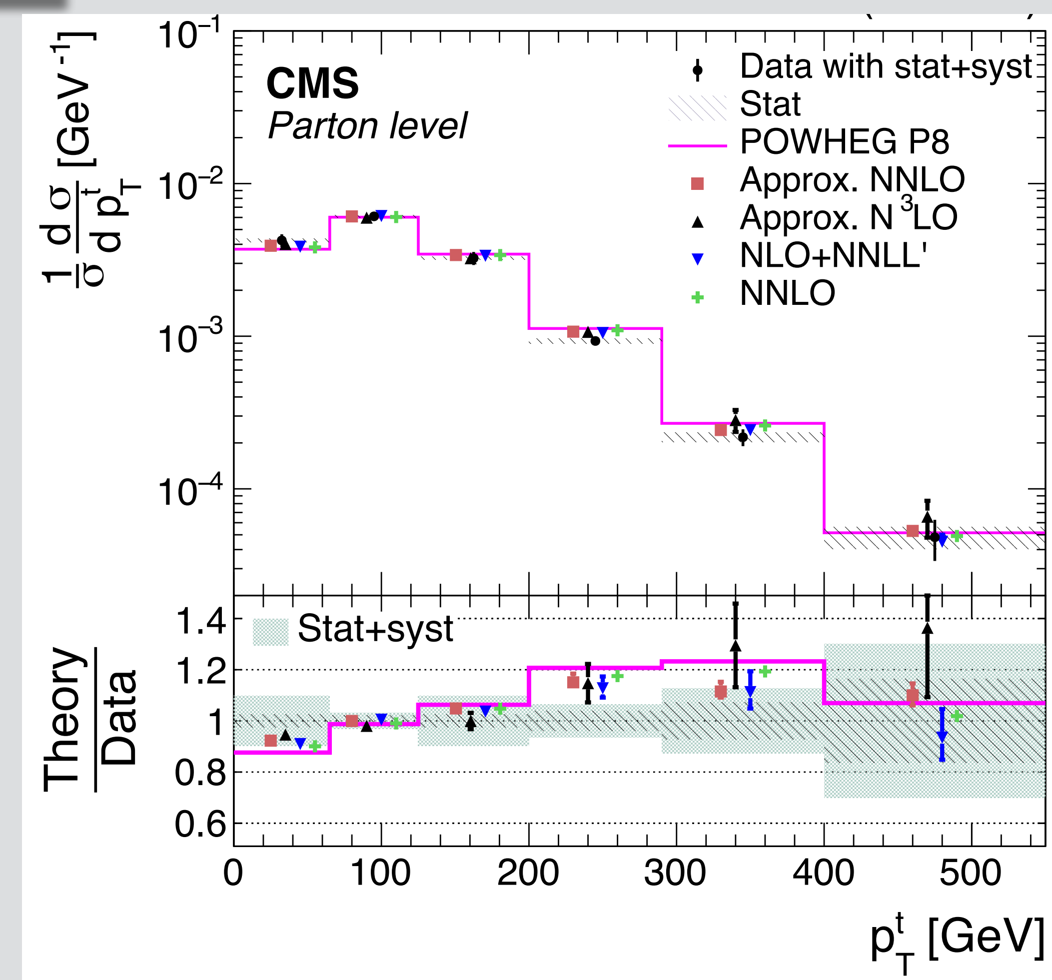
- Small inclusive corrections
- LARGE CORRECTIONS in exclusive region

NNLO: what have we learned so far?

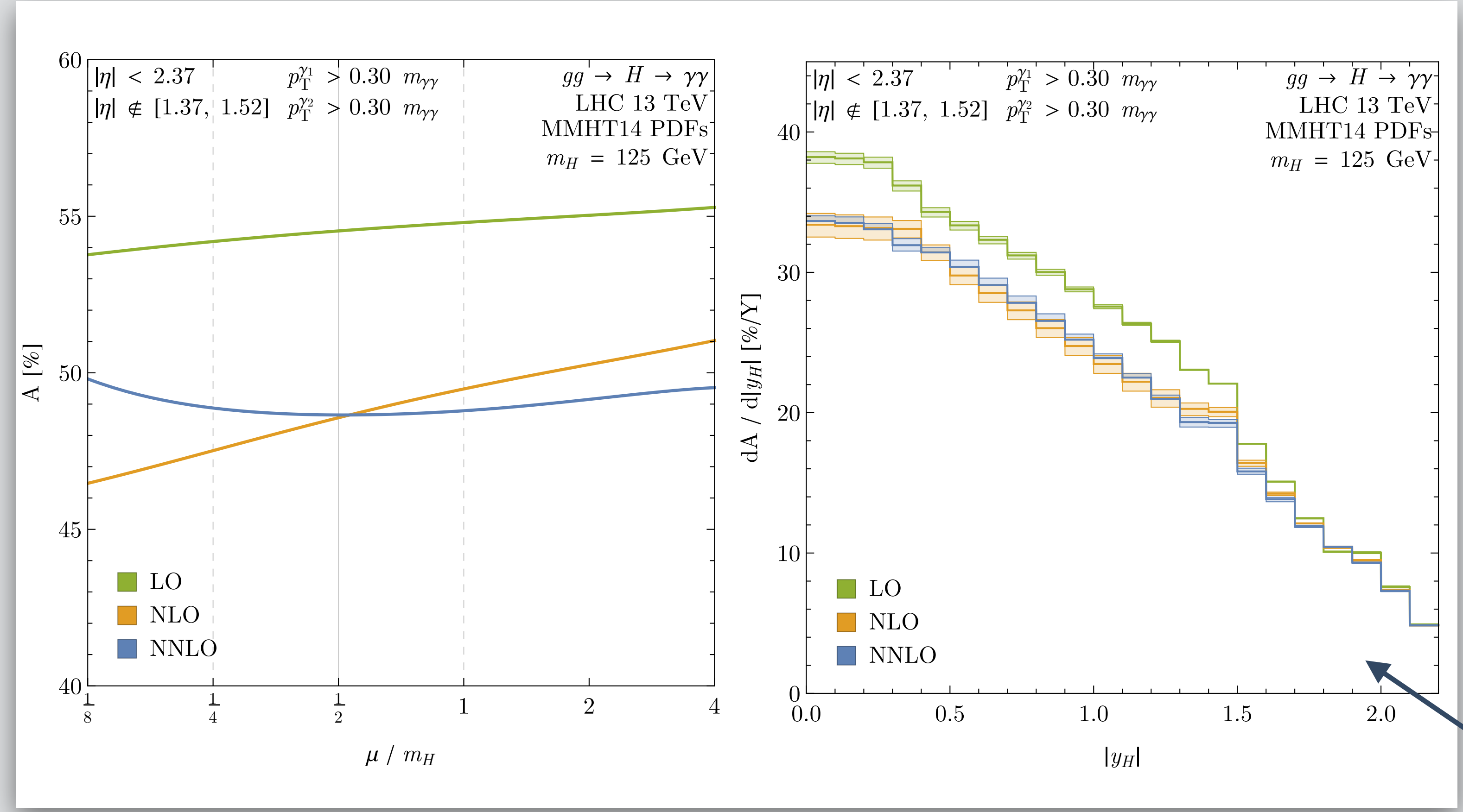


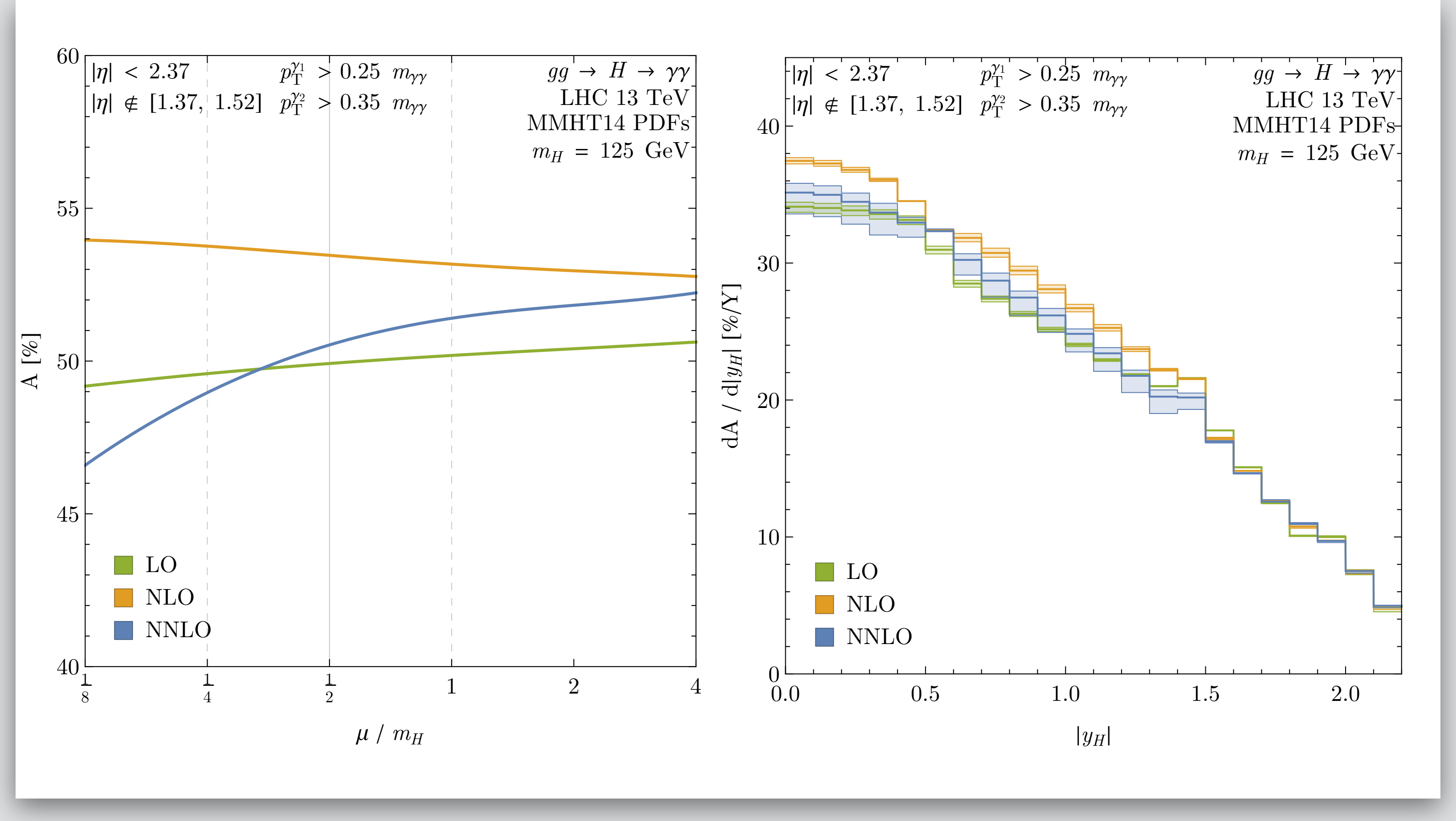
Is tension in $p_{t,\ top}$ partially due to reconstruction?

- shape difference, non very significant but persistent at different energies
- effect seems smaller for correlated lepton-observables…
- crucial for R3 to properly understand top p_t spectrum…



NNLO: what have we learned so far?





[Lionetti et al. (2018)]

H→γγ in the fiducial region: very different picture for different cuts

ATLAS-like cuts (asymmetric)

Symmetric cuts

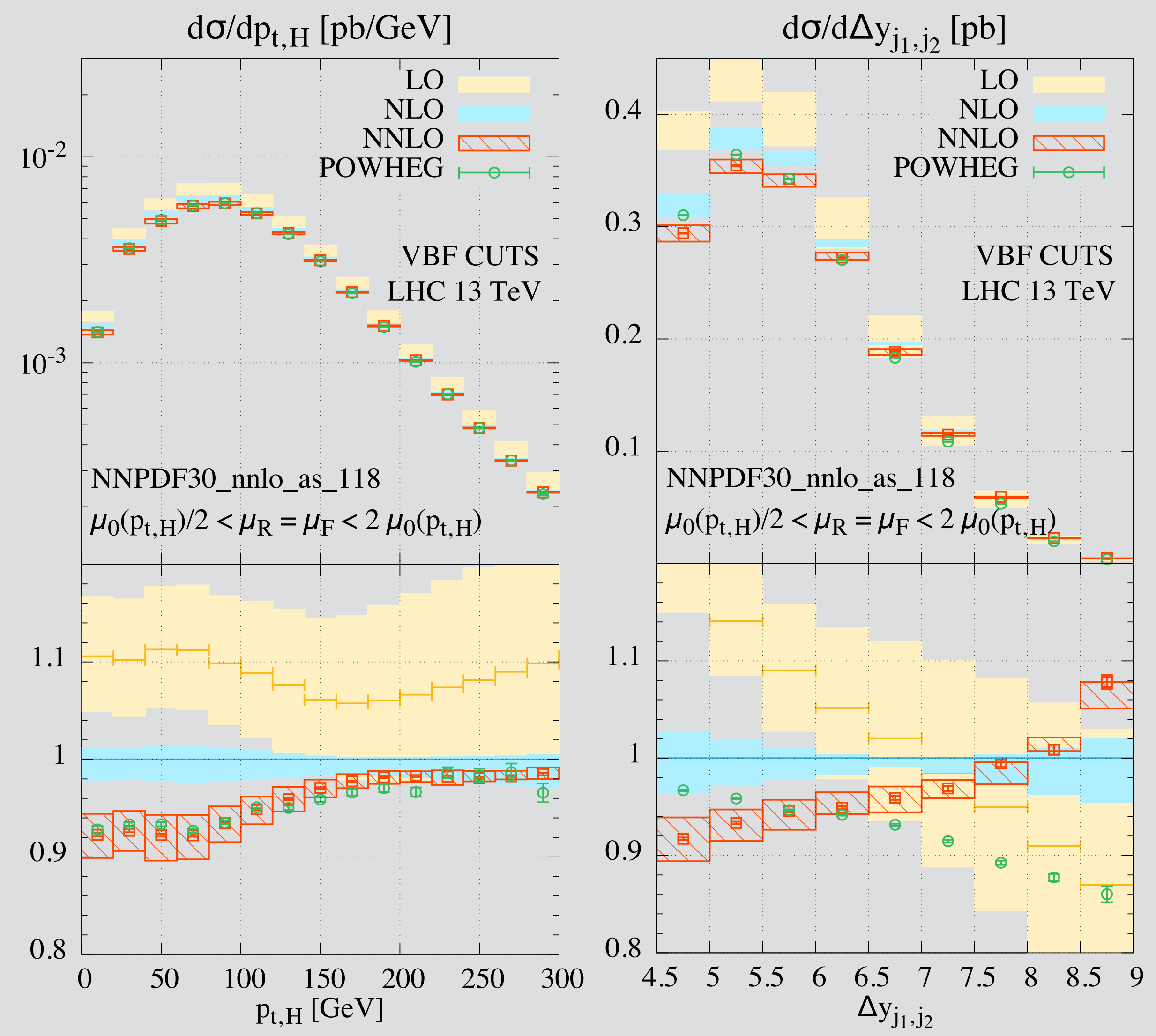
[similar studies for di-photon final states: Catani et al (2018)]

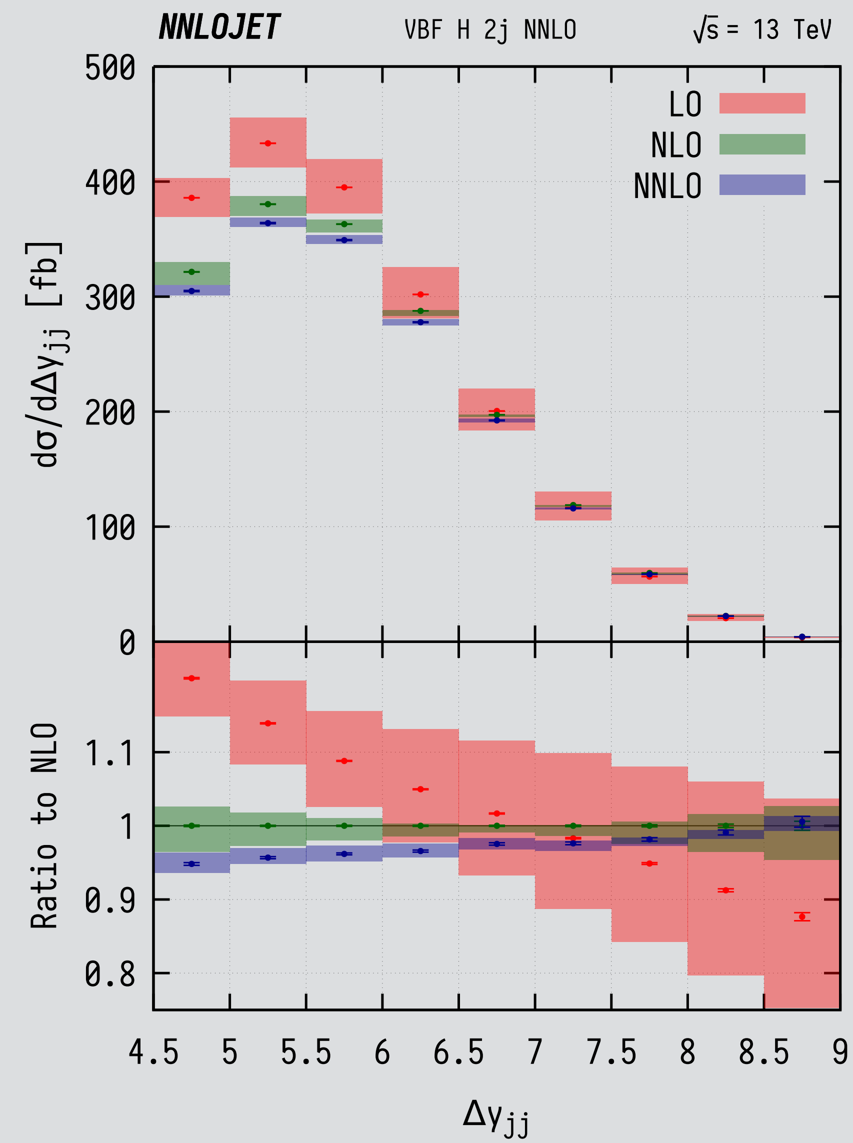
NNLO: what have we learned so far?

Extra parton dynamics play a significant role

Not always captured by parton showers. E.g.: VBF

[Cacciari et al. (2016)]





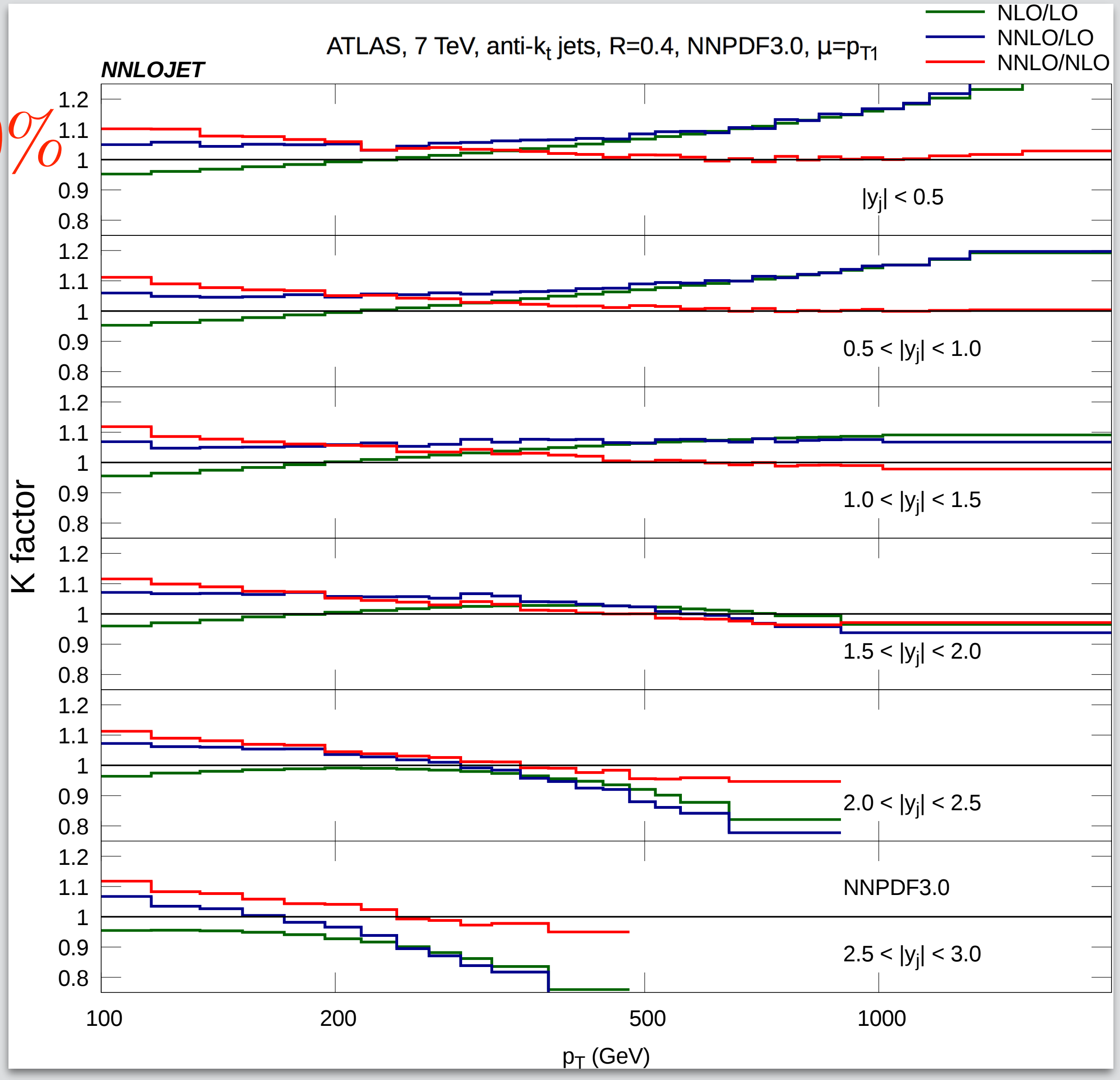
[Cruz-Martinez et al (2018)]

- corrections in the fiducial region much larger than inclusive
- for some observables, PS goes in the opposite direction

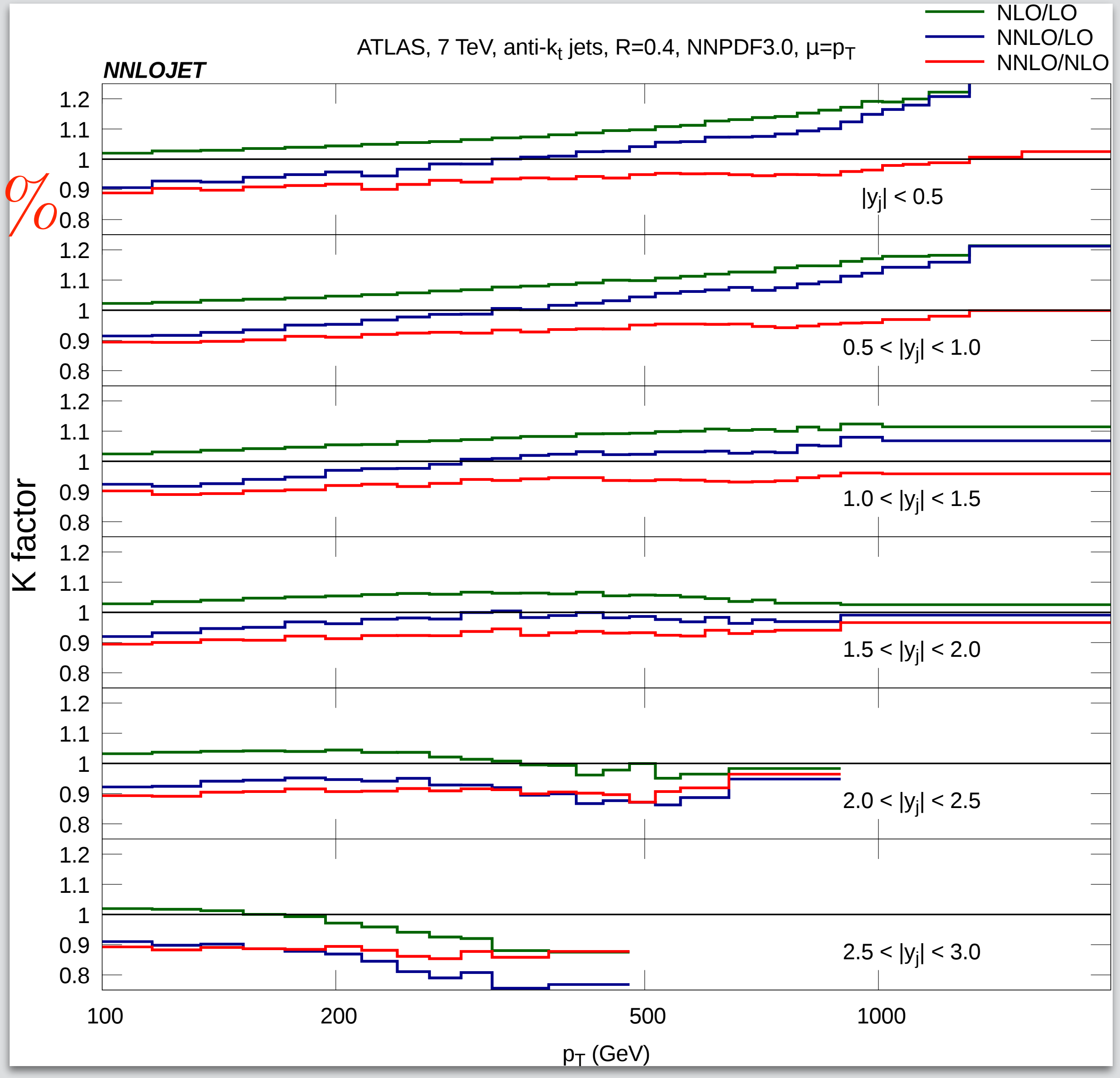
Again on jet dynamics: single inclusive jet

- Inclusive jet spectrum: $\mu = p_{t,L}$ vs p_t

+10%



-10%

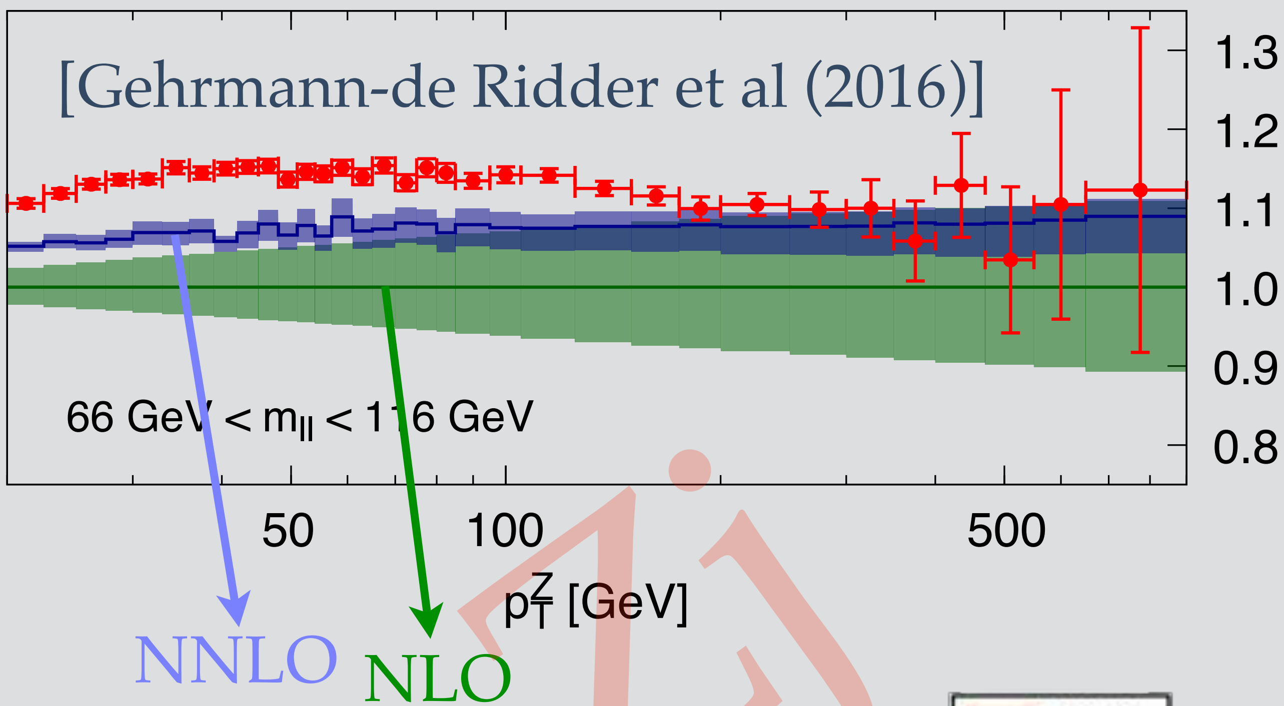


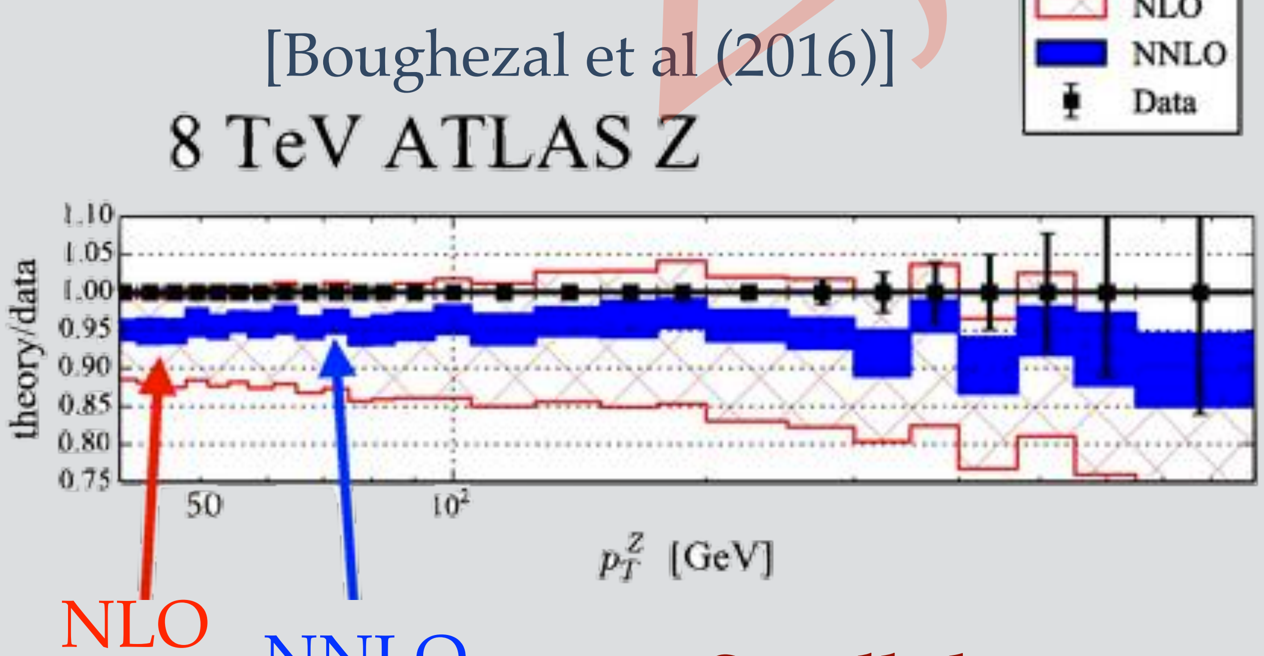
[Currie, Glover, Gehrmann, Gehrmann-de Ridder, Huss, Pires (2017)]

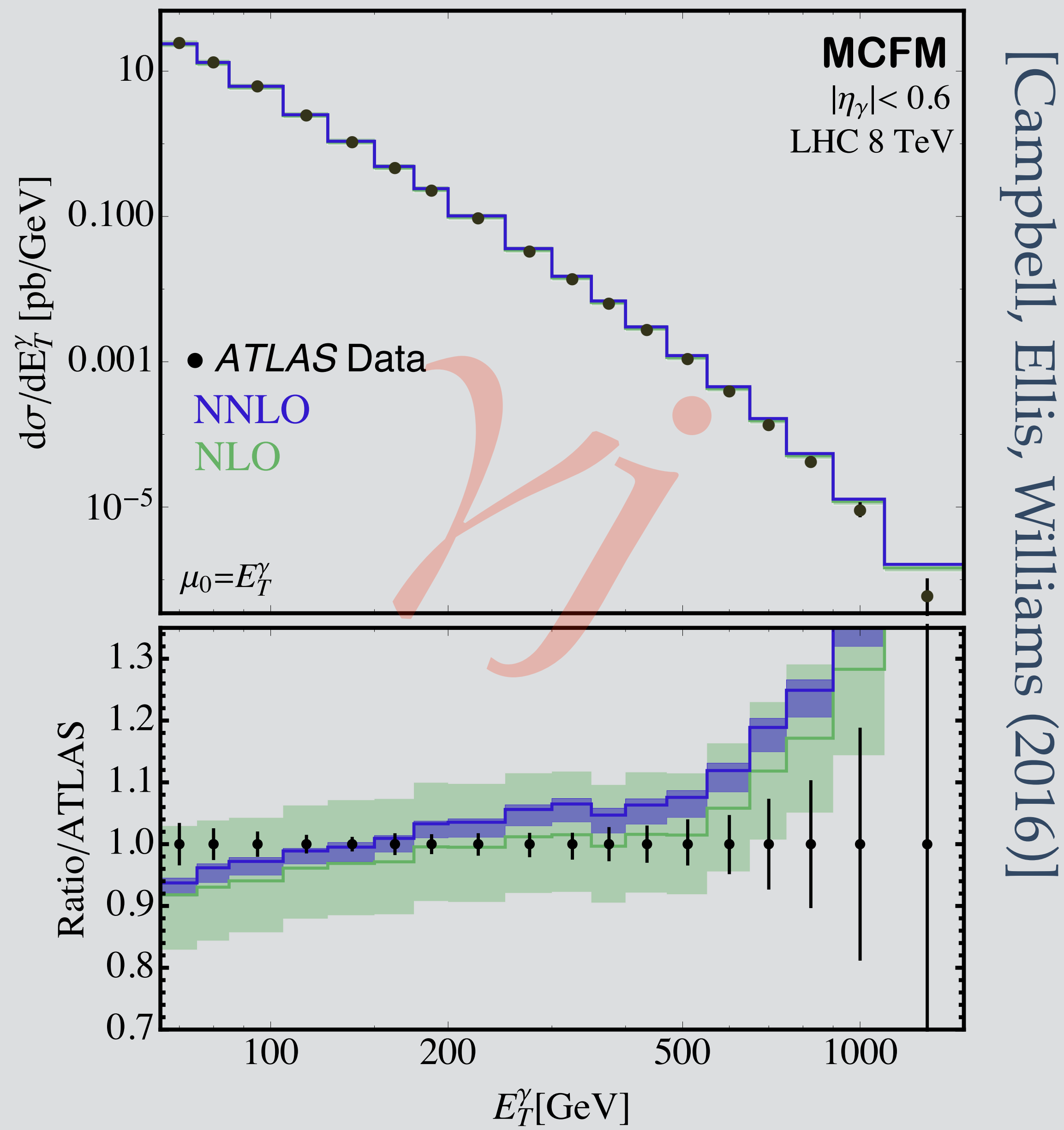
- Despite small scale variation, very large dependence on scale choice (hardest jet in the event vs individual jet). Non trivial jet dynamics

NNLO: open puzzles

- V+j: unexpected disagreement even with high precision / clean data





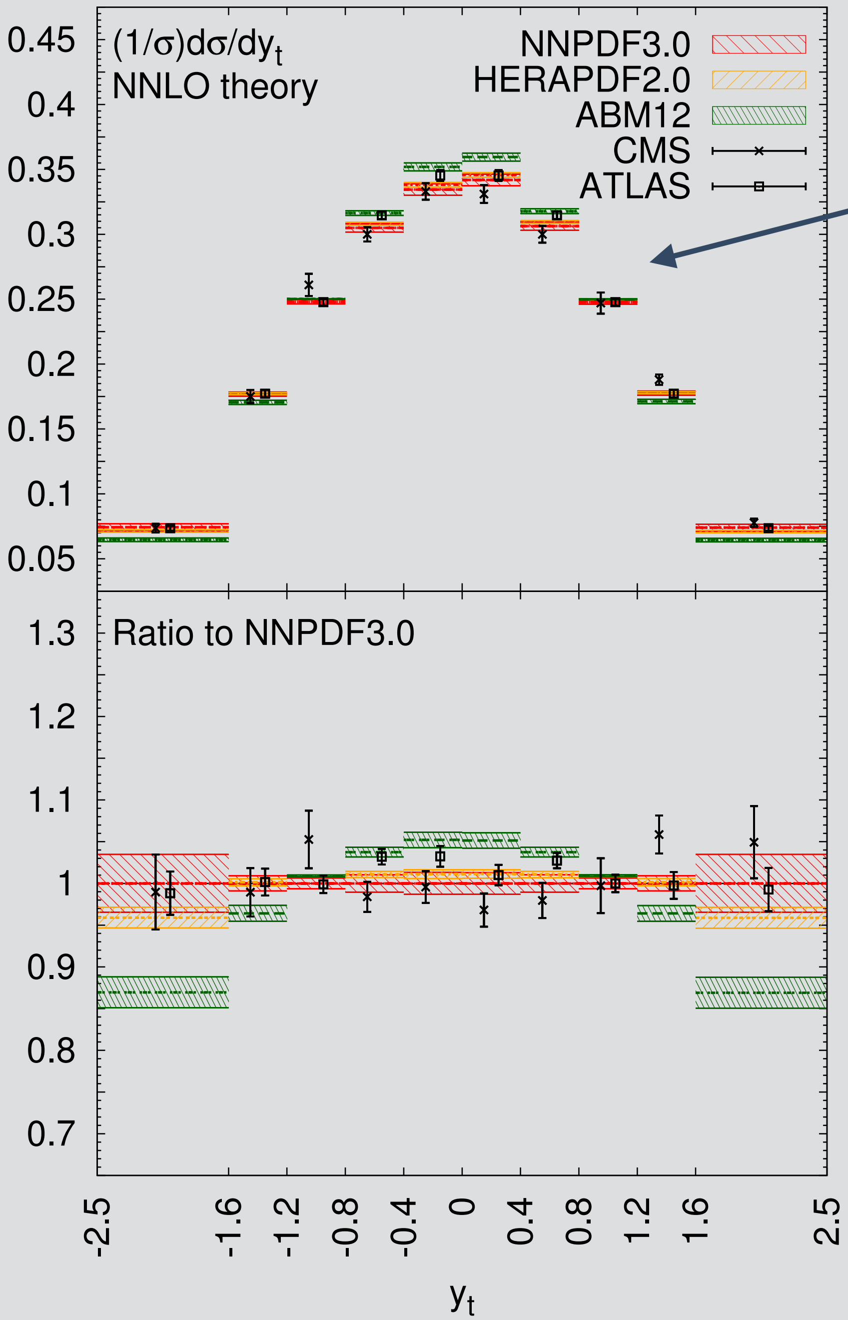


- Small deviations evident in the overall normalization ($Z\ p_t$) and shape ($\gamma\ E_T$). Calibration? *Non pert*?

NNLO: a few applications

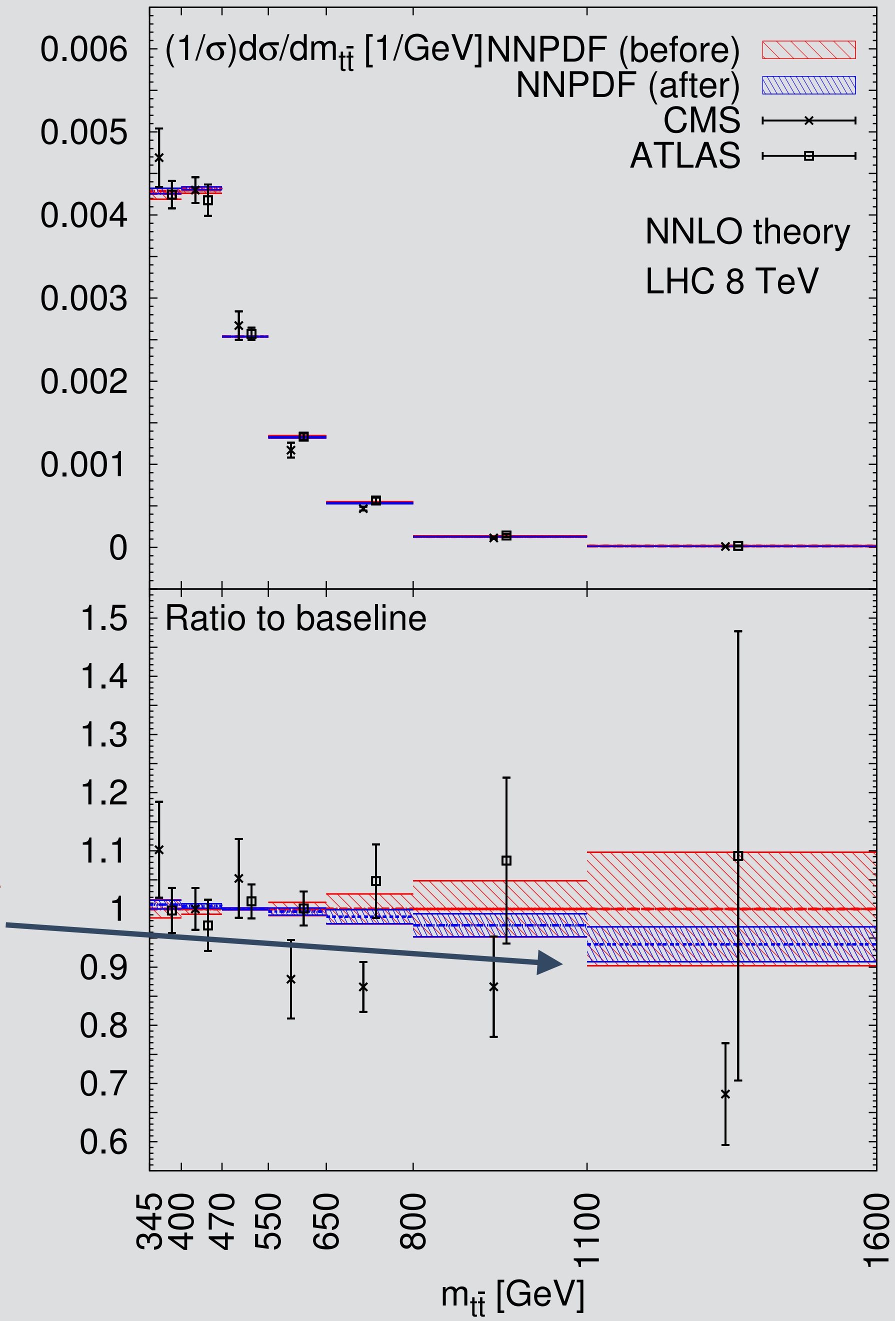
NNLO: PDFs

Top Differential Distributions and PDFs



Discriminating power between different PDF sets

Better control on high mass tail (BSM…)

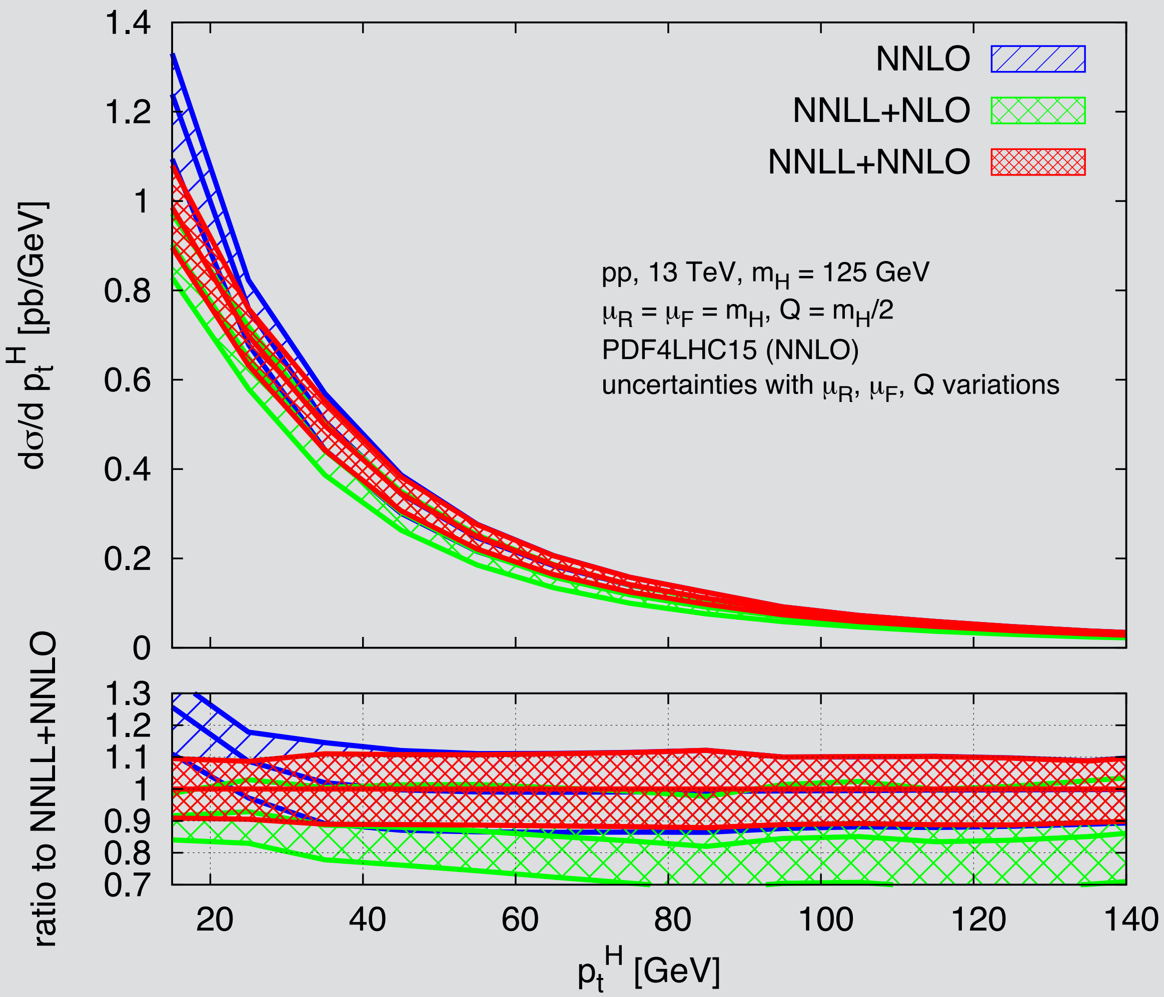


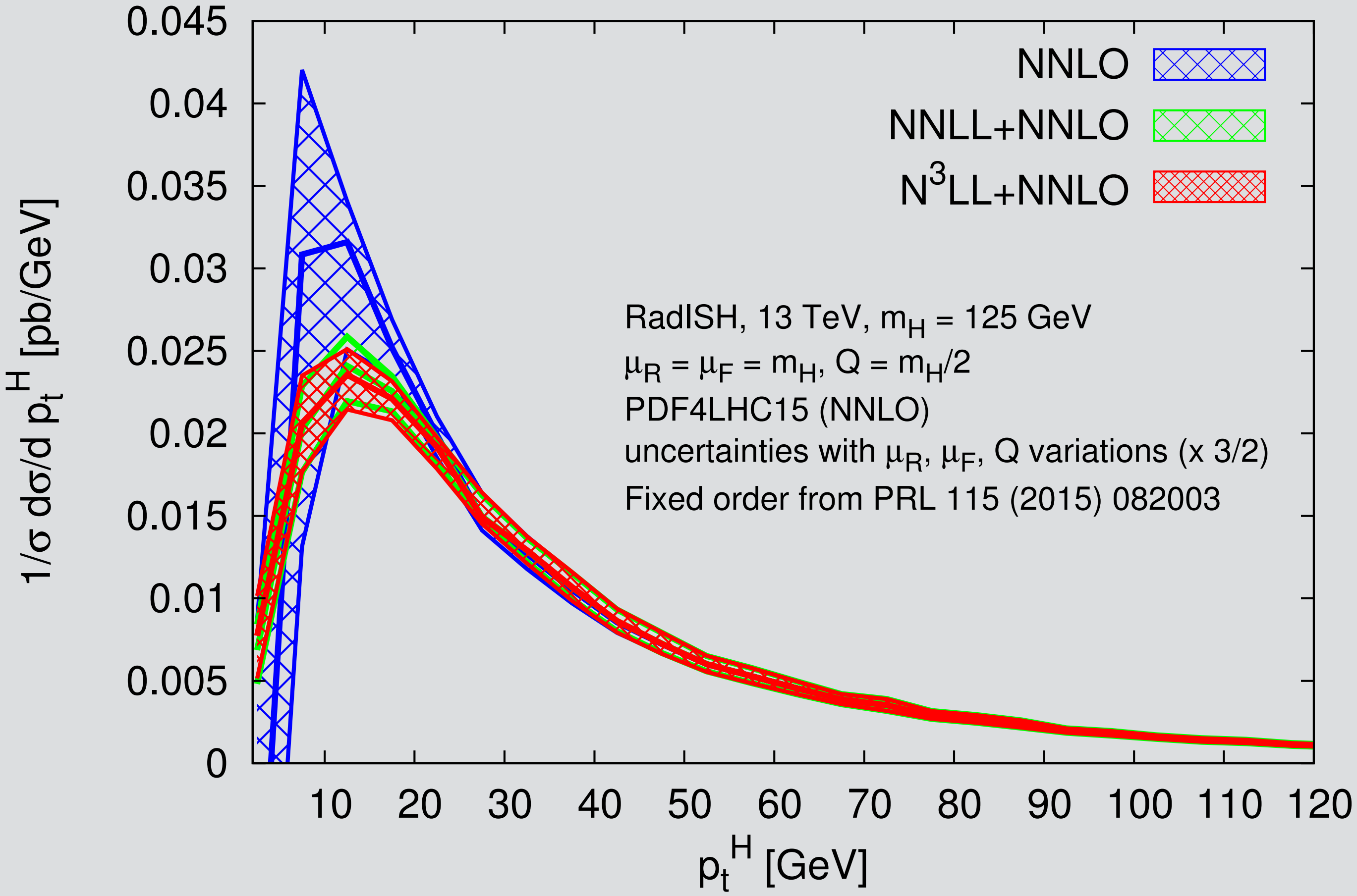
[Czakon, Hartland, Mitov, Nocera, Rojo (2017)]

- Similar results from Z p_t, di-jet
- Di-jet → large-x gluon. Can disentangle different aspects (high p_t: potential new physics! Forward region cleaned, but is f.o. good enough?)

Application of NNLO results: H p_T

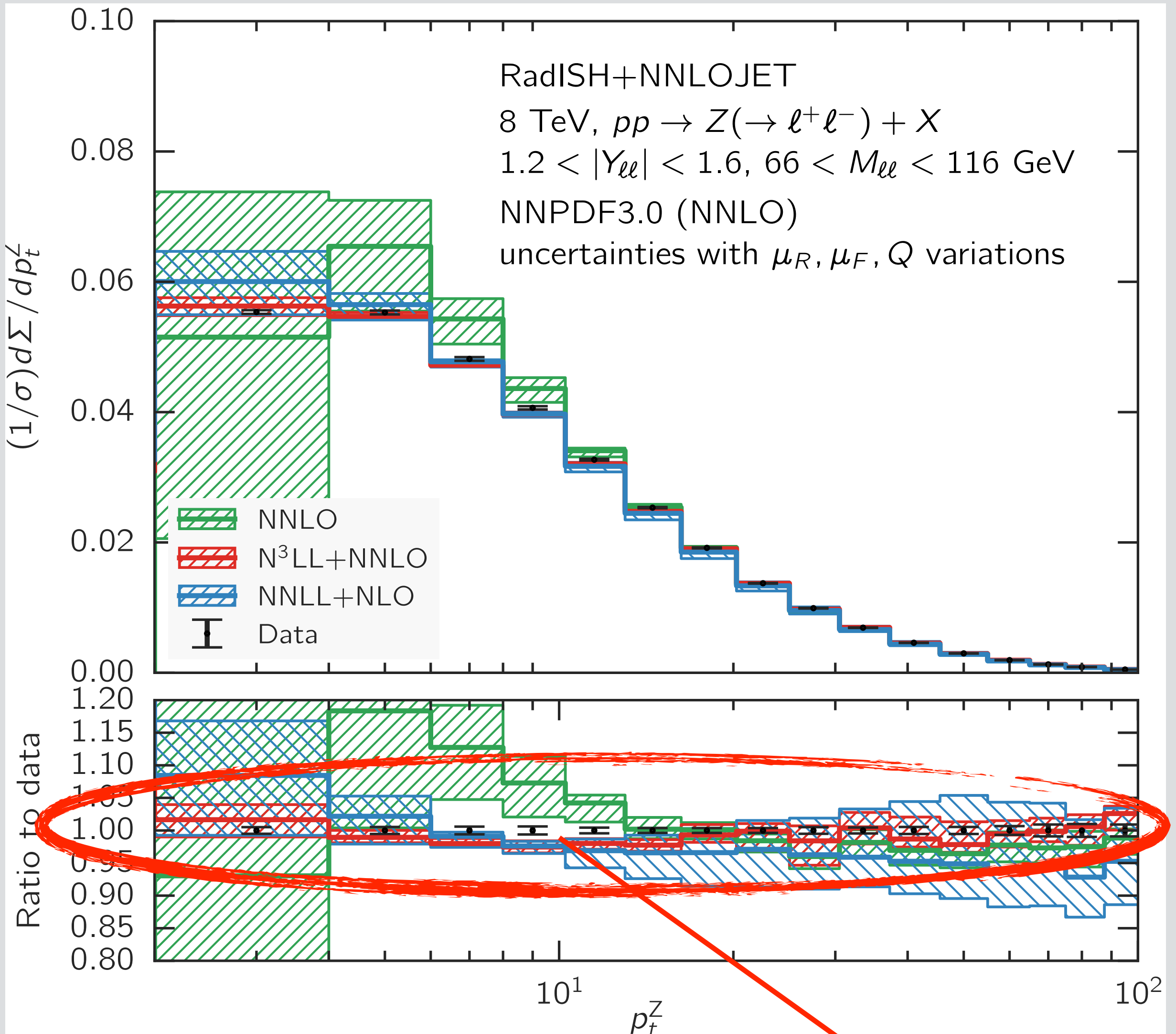
[Bizon, Monni, Re, Rottoli, Torrielli (2017), similar results from Chen et al (2018)]



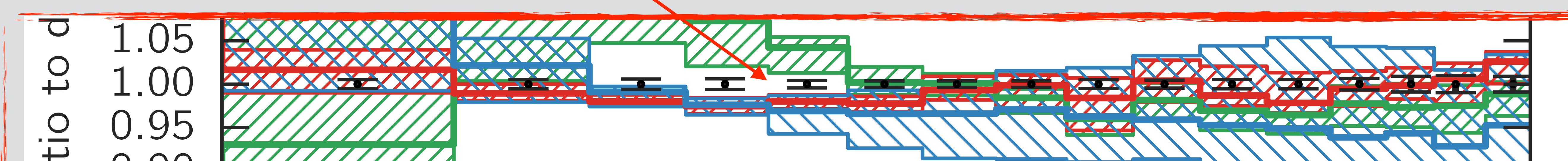


- Matching of NNLO H+J with N^3LL Higgs p_T resummation
- Significant reduction of perturbative uncertainties from NLO+NNLL to NNLO+NNLL, no large N^3LL effect
- No breakdown of perturbation theory until very low scales (resummation effects: 25% at p_T = 15 GeV, ~0% at p_T = 40 GeV)

Application of NNLO results: Z p_t



[Bizon et al, 2018]



- Tiny uncertainties
- At face value, slight data / theory tension. In this plot non significant, but systematically there over different rapidity / invariant mass bins
- Underestimate uncertainty, PDFs, non perturbative, …?

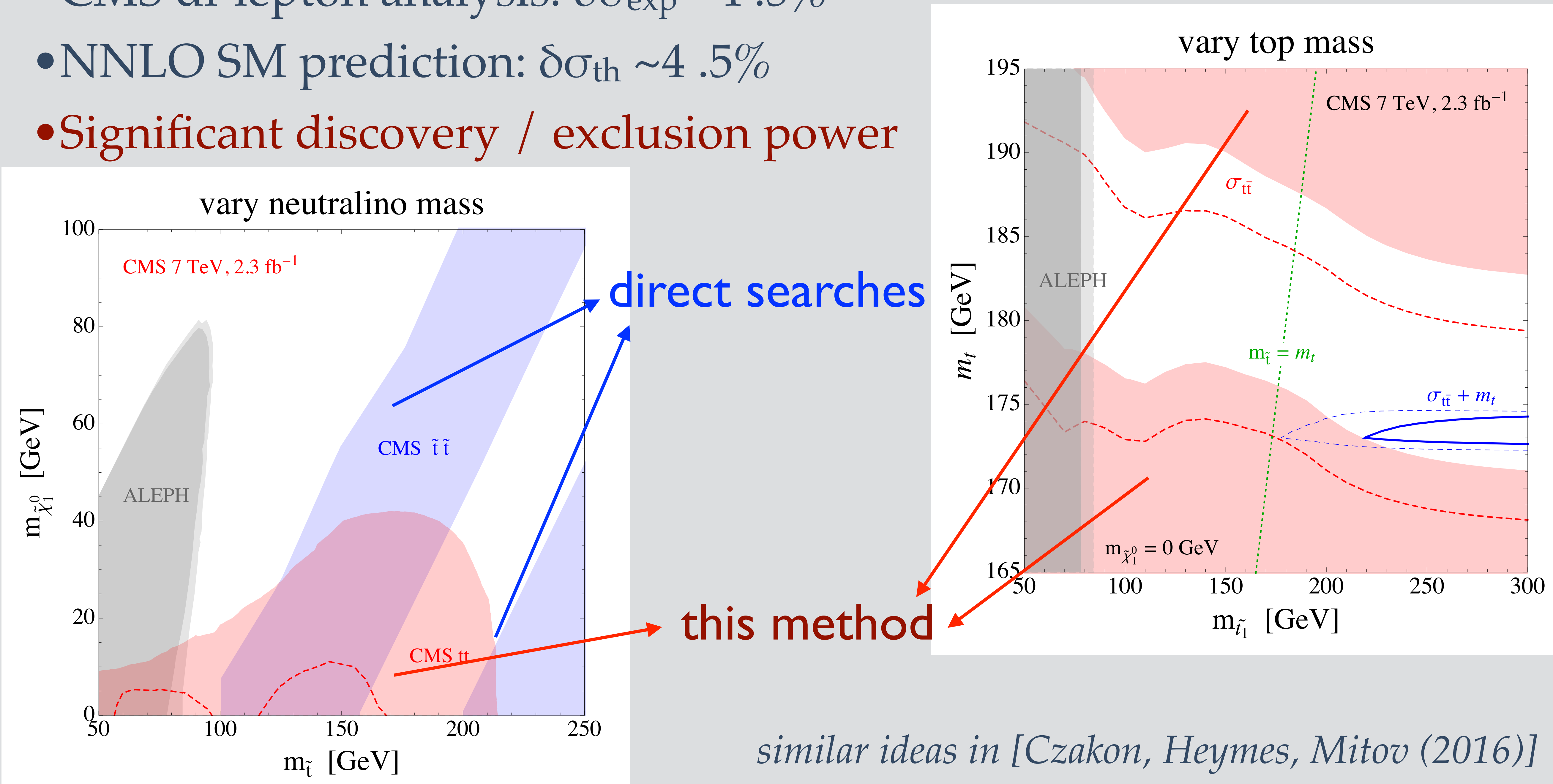
Nice ``test case" for precision targets for HL/future colliders

Application: hunting elusive BSM signals

[Czakon, Mitov, Papucci, Ruderman, Weiler (2014)]

$$\tilde{t} \to t + \chi_1^0/\tilde{G}, \quad m_{\tilde{t}} \sim m_t \gg m_{\chi_1^0,\tilde{G}}, \quad \sigma_{\tilde{t}} \approx 0.15 \cdot \sigma_t$$

- Hunting for stealthy stop
- CMS di-lepton analysis: $\delta\sigma_{exp}$ ~4 .5%
- NNLO SM prediction: $\delta\sigma_{th}$ ~4 .5%
- Significant discovery / exclusion power

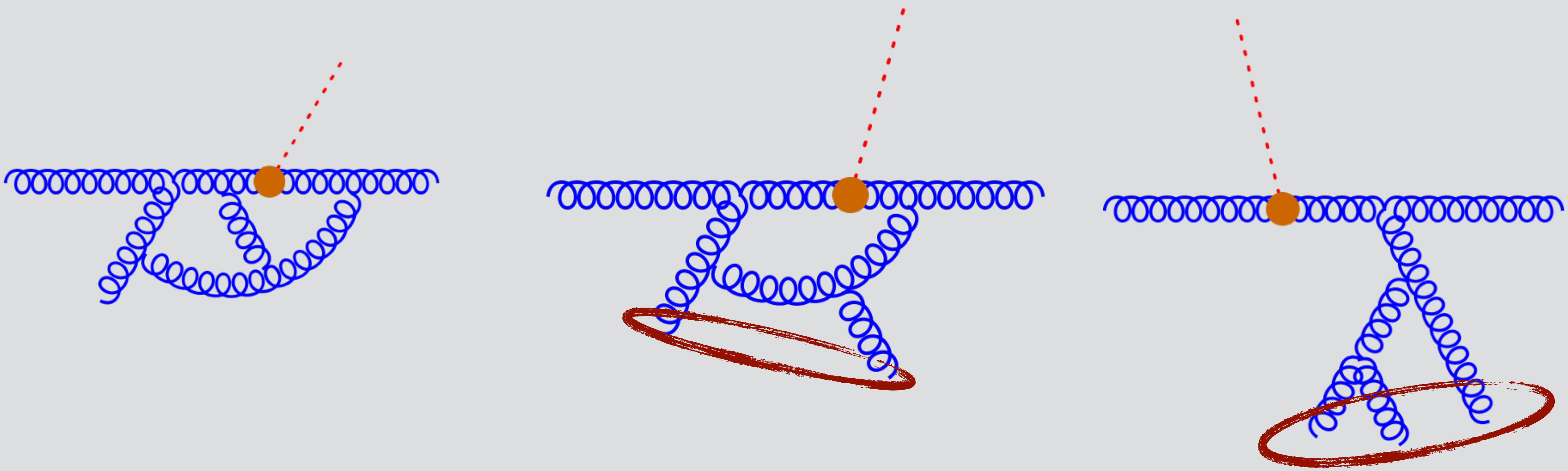


similar ideas in [Czakon, Heymes, Mitov (2016)]

NNLO: going forward

Back to the start…

O(α_s^2) corrections: two-loop (VV), one-loop+j (RV), tree+jj (RR)



GOING BEYOND WHAT I HAVE SHOWN SO FAR

- loop amplitudes
- better subtraction schemes

Loop amplitudes: status

- Amplitude COMPLEXITY GROWS VERY FAST with the number of scales: invariants (~# legs) and particle masses

$$
\begin{aligned}
F^{\mathrm{L}}_{--++} = & -(x^2+y^2)\Big[4\mathrm{Li}_4(-x) + \frac{1}{48}Z_+^4 \\
& +(\tilde{Y}-3\tilde{X})\mathrm{Li}_3(-x) + \Xi\mathrm{Li}_2(-x) \\
& +i\frac{\pi}{12}Z_+^3 + i\frac{\pi^3}{2}X - \frac{\pi^2}{12}X^2 - \frac{109}{720}\pi^4\Big] \\
& +\frac{1}{2}x(1-3y)\Big[\mathrm{Li}_3(-x/y) - Z_-\mathrm{Li}_2(-x/y) \\
& -\zeta_3 + \frac{1}{2}Y\tilde{Z}\Big] + \frac{1}{8}\left(14(x-y) - \frac{8}{y} + \frac{9}{y^2}\right)\Xi \\
& +\frac{1}{16}(38xy-13)\tilde{Z} - \frac{\pi^2}{12} - \frac{9}{4}\left(\frac{1}{y}+2x\right)\tilde{X} \\
& +\frac{1}{4}x^2\Big[Z_-^3 + 3\tilde{Y}\tilde{Z}\Big] + \frac{1}{4} + \Big\{t \leftrightarrow u\Big\},
\end{aligned}
$$

$gg \to \gamma\gamma$

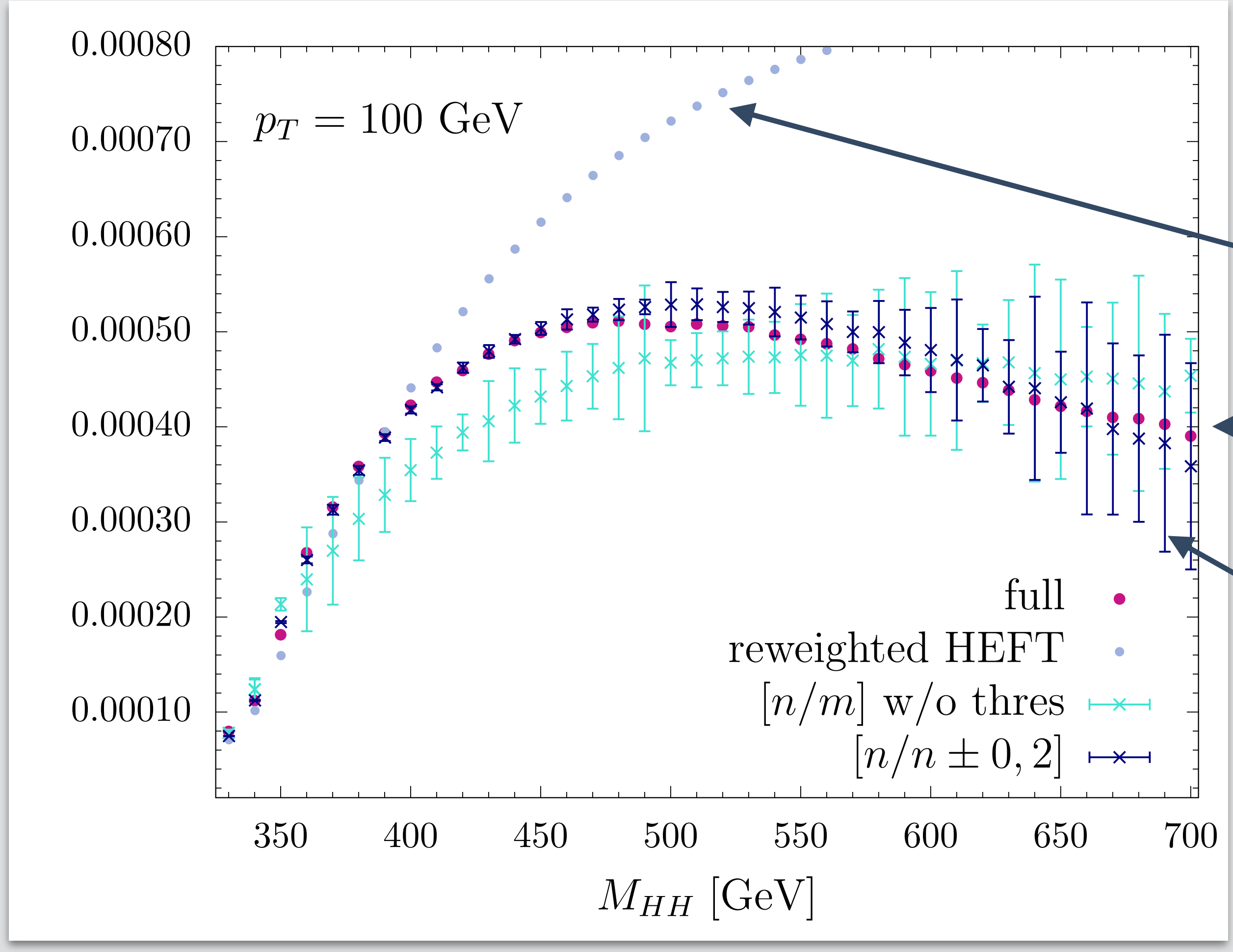
[Bern, De Freitas, Dixon [2002]

$gg \to VV$: ~ 10 MB expression

- Despite a lot of recent progress still pretty limited knowledge. State of the art:
  - Analytically: 2 -> 2, external masses (pp->VV*) [FC, Henn, Melnikov, Smirnov, Smirnov (2014-15); Gehrmann, Manteuffel, Tancredi (2014-15)]
  - Numerically: 2->2, internal/external masses (pp-> tt, pp->HH) [Czakon; Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
  - Lot of recent progress: towards 2->3 [Badger et al (2016-18)]; numerical unitarity [Abreu et al (2017-18)], many-scales integrals [Gehrmann, Henn, Lo Presti (2015); Papadopoulos, Tommasini, Wever (2016); Tancredi, Remiddi (2016); Weinzierl et al (2017); Bonciani et al (2016)]

Loop amplitudes: prospects

- For complicate amplitudes, at least a semi-numerical approach seems unavoidable to get the full result
- 2→2 processes: functions of 2 variables (*s, scattering angle*) + parameters (*masses)* → natural to tabulate
- How can we deal with the multi-dimensional case?
- Can we systematically construct and merge approximations over the whole kinematic region?



Example: double-Higgs
[Gröber, Maier, Rauh (2017)]

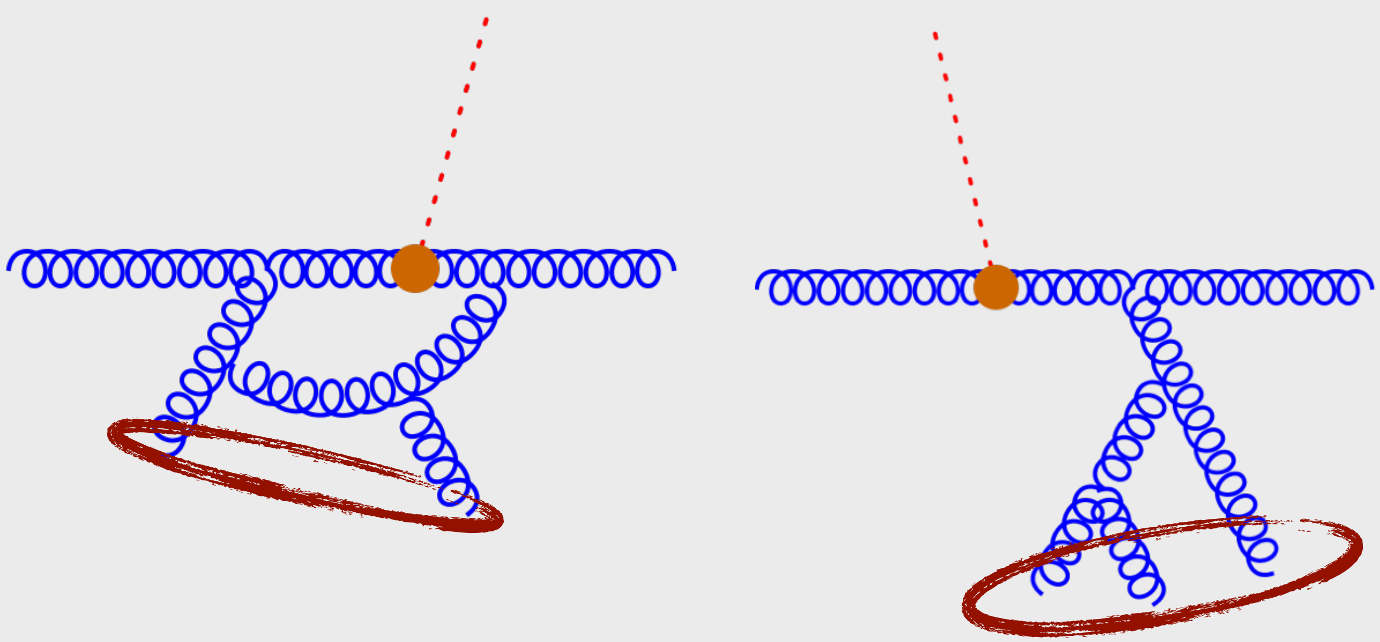
$1/m_t$

Exact [Heinrich et al]

$1/m_t$ + threshold

Subtraction: status+challenges

Higher order: non trivial soft/collinear radiation patterns



Explosion in complexity: CPU hours

	2→1	2→2	2→3
NNLO	**100**	**10^5-10^6**	**out of reach**
N^3LO	**~10^7**	**-**	**-**

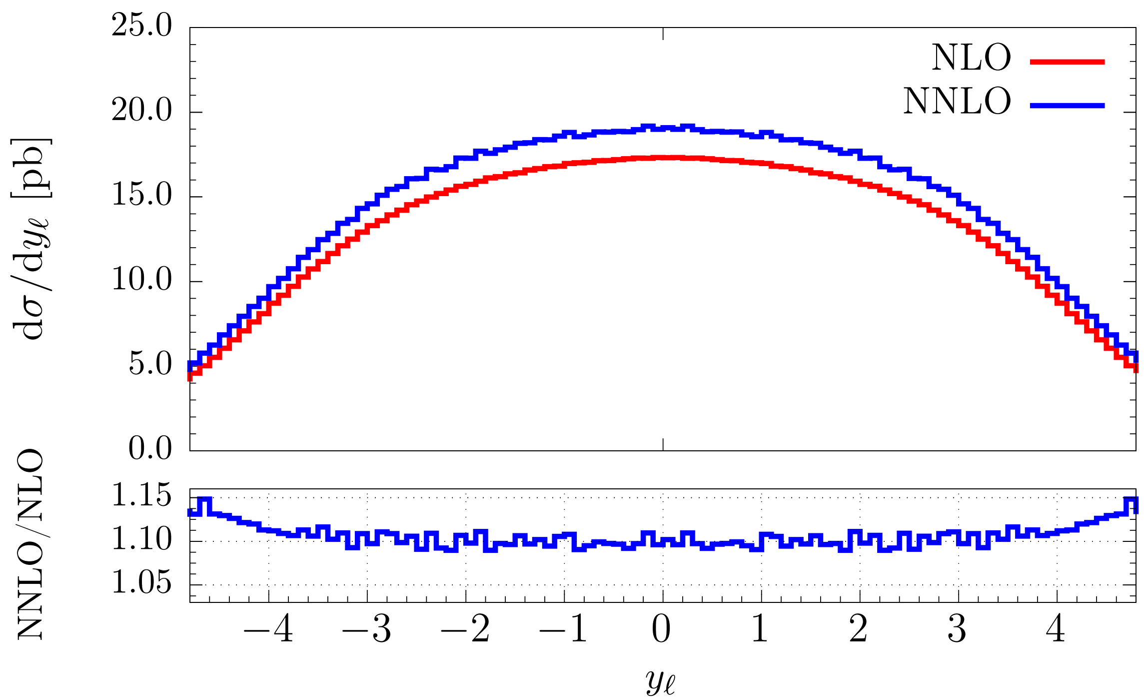
- Problem of finding NNLO subtraction solved
- Problem of finding *good subtraction* far from over
- A lot of work, new ideas ([FC, Melnikov, Röntsh (2018); Magnea et al (2018); Herzog (2018),…]

New ideas at work: nested soft-collinear subtraction

[FC, Melnikov, Röntsh, 2017-2018]

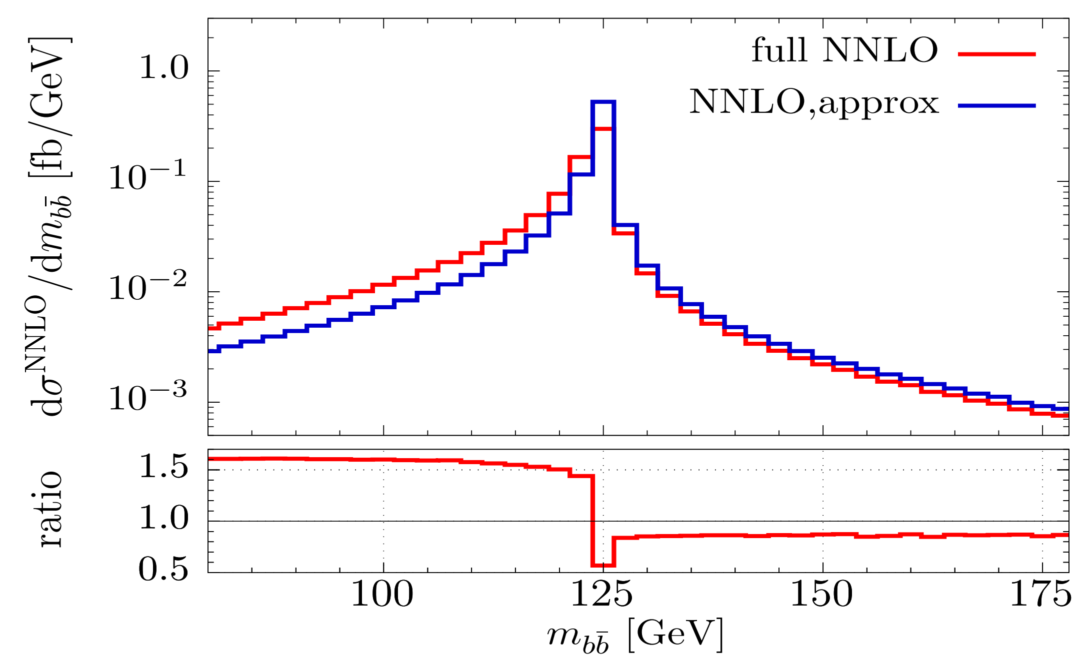
Insight from resummation → simplify radiation patterns (color coherence)

$$pp \to V \to l^+ l^-$$



O(10) CPU hours

$$pp \to W(\to l\nu)\ H(\to b\bar{b})$$



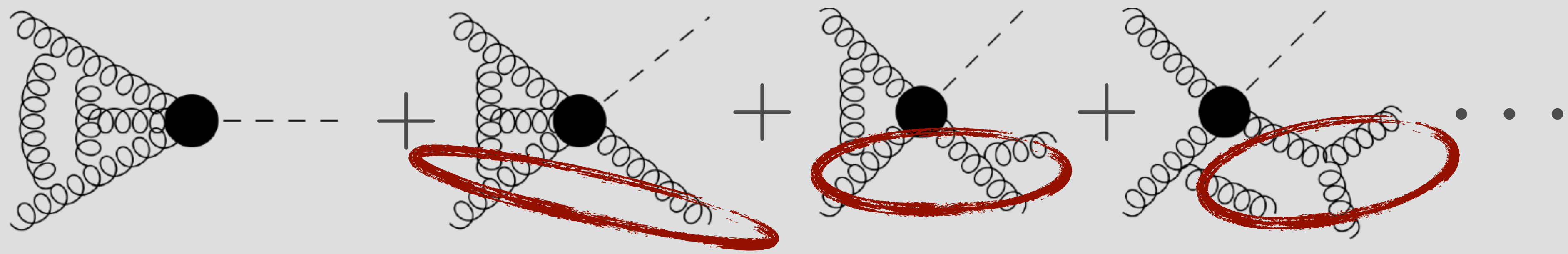
O(100) CPU hours

- very good convergence, reduced run time
- would be interesting to stress-test
- *in principle* there for arbitrary processes [FC, Delto, Frellesvig, Melnikov (2018)], in practice no implementation yet

Beyond NNLO?

N^{3}LO for simple processes

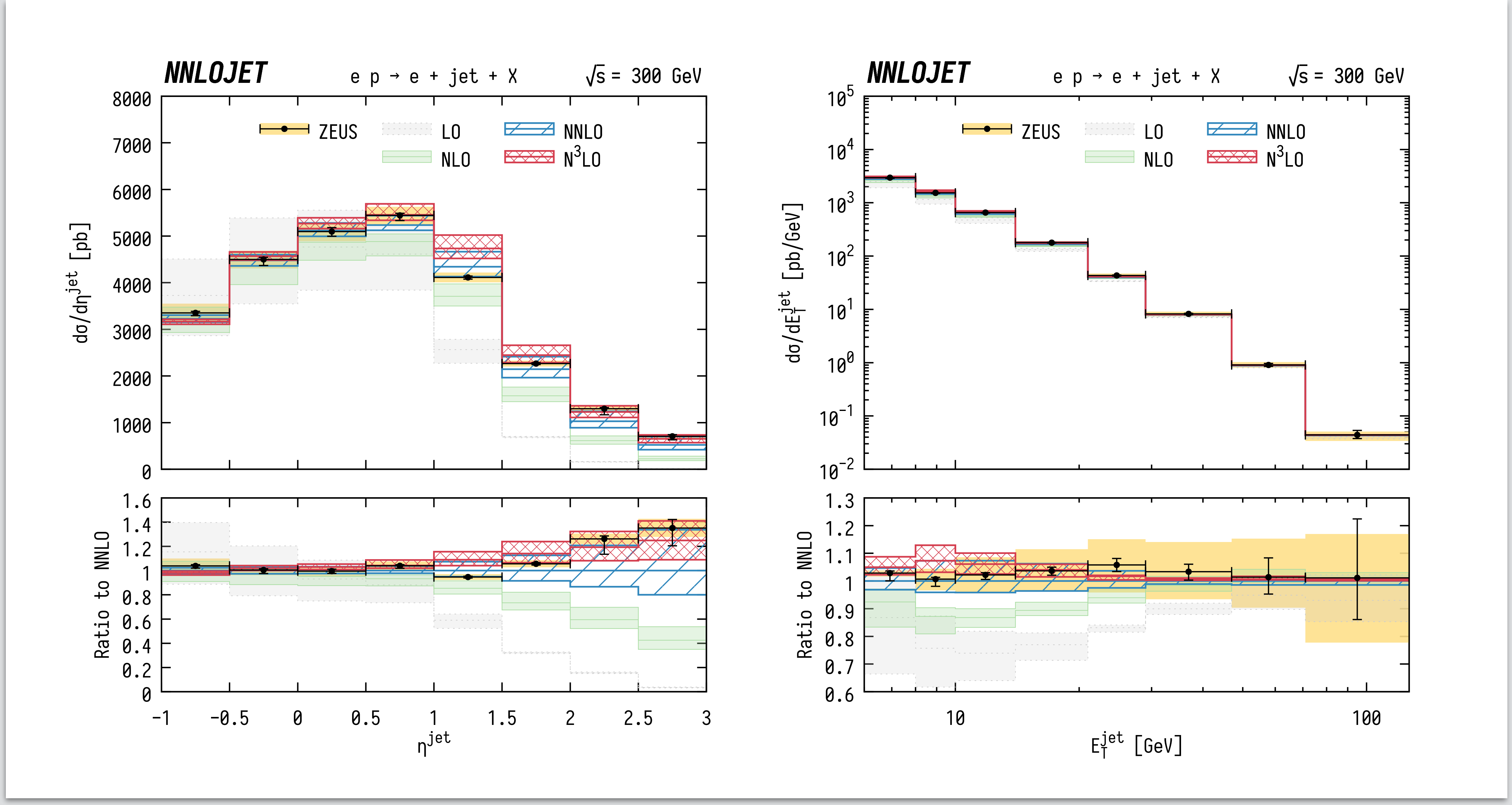
- We still do not master fully differential NNLO → generic N^{3}LO out of the question
- Still, we can imagine having fully differential N^{3}LO predictions in the near future for selected processes: DIS (✓), Higgs, DY

$\Delta\sigma =$ 

- X+J@NNLO contains most of the X-N^{3}LO information. Missing parts:
  - 3-loop purely virtual → ~ trivial
  - ``missing jet'': non-trivial zero-p_t rapidity dependence. If this is known, can combine with X+jet to obtain full result

N³LO for DIS

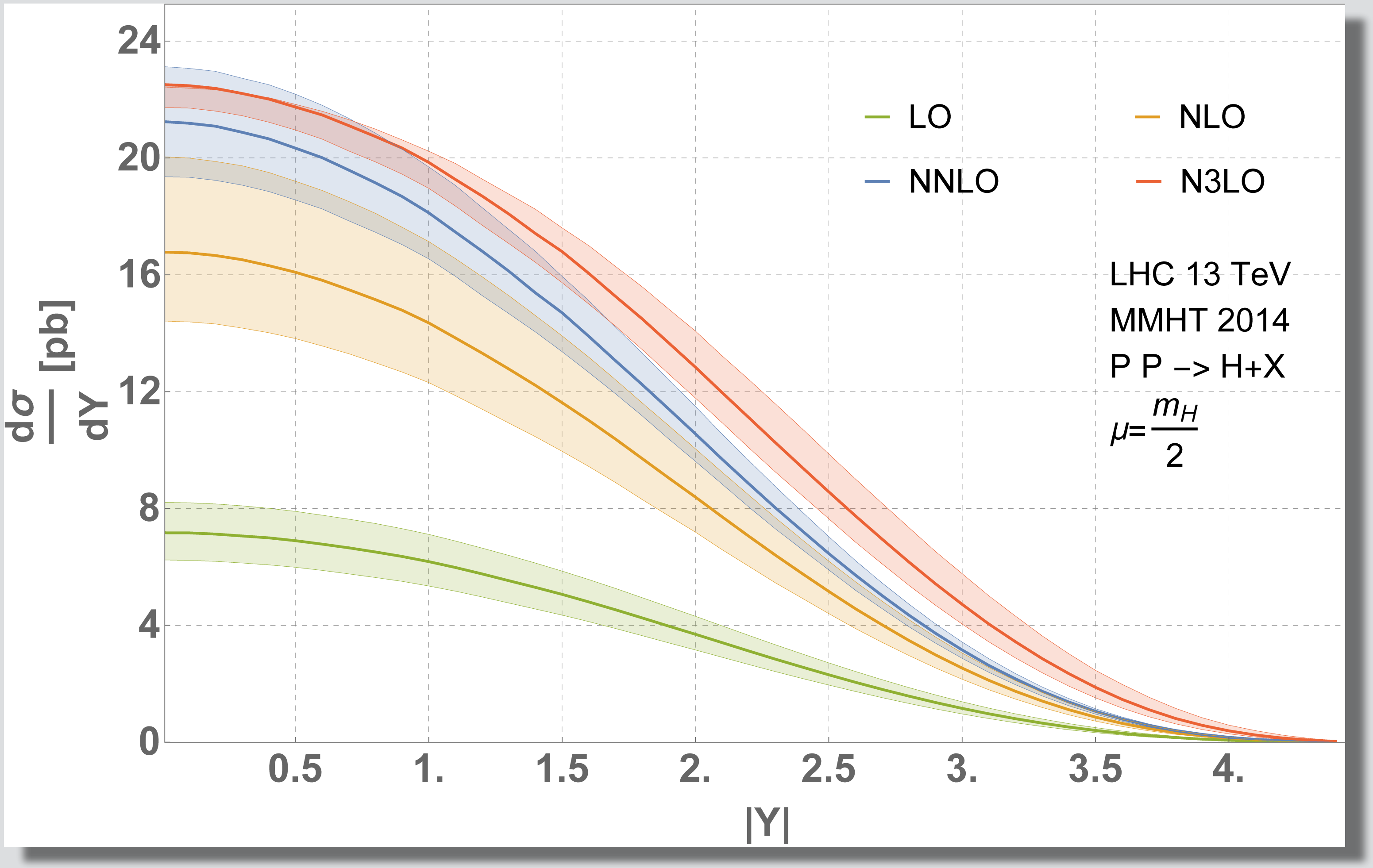
[Currie, Gehrmann, Glover, Huss, Niehues, Vogt (2018)]



- F_i@N³LO known → combine with ep→2jet@NNLO
- Computation of N⁴LO evolution also under the way

N^3LO: Higgs

- Non-trivial missing information: Higgs rapidity
- Calculation under way, similar strategy of inclusive (soft expansion)



[Mistlberger et al (2018)]

- Much more complicated than inclusive, but getting there…

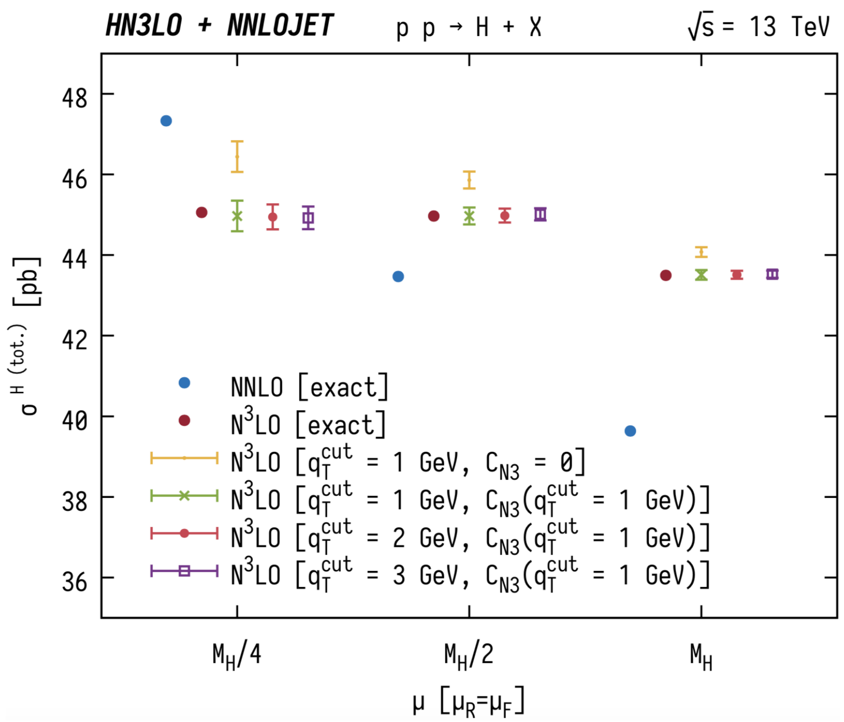
N^3LO: Higgs

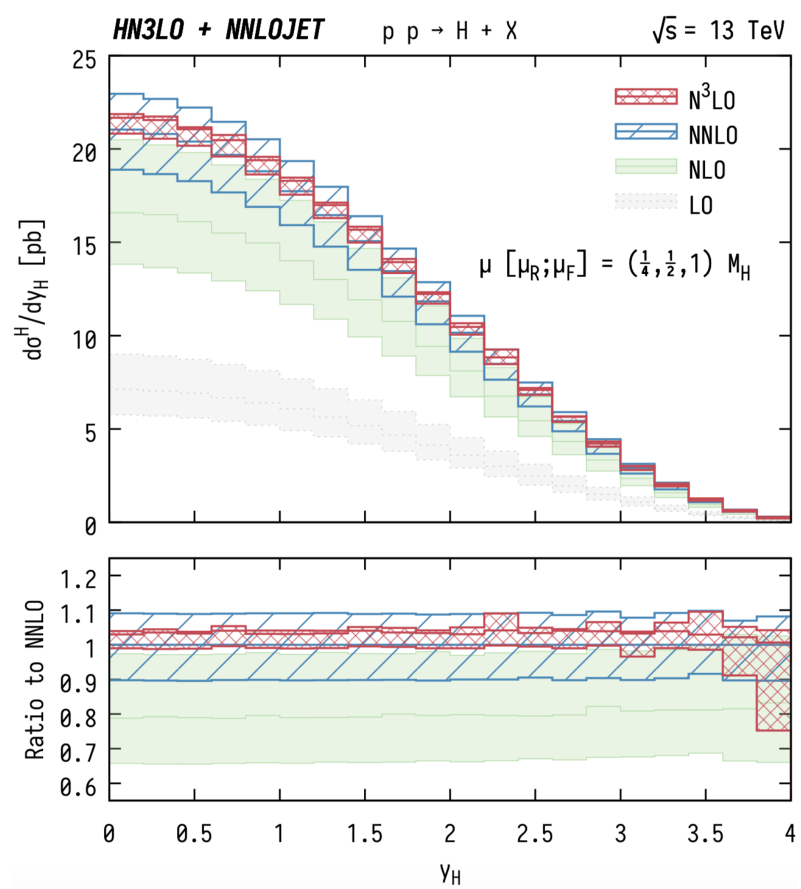
- Everything else is ready…

N^3LO Higgs total cross section and rapidity distribution

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- With C_{N3} approximation, the $\sigma^H_{N^3LO}$ and $d\sigma^H_{N^3LO}/dy^H$ distributions are:





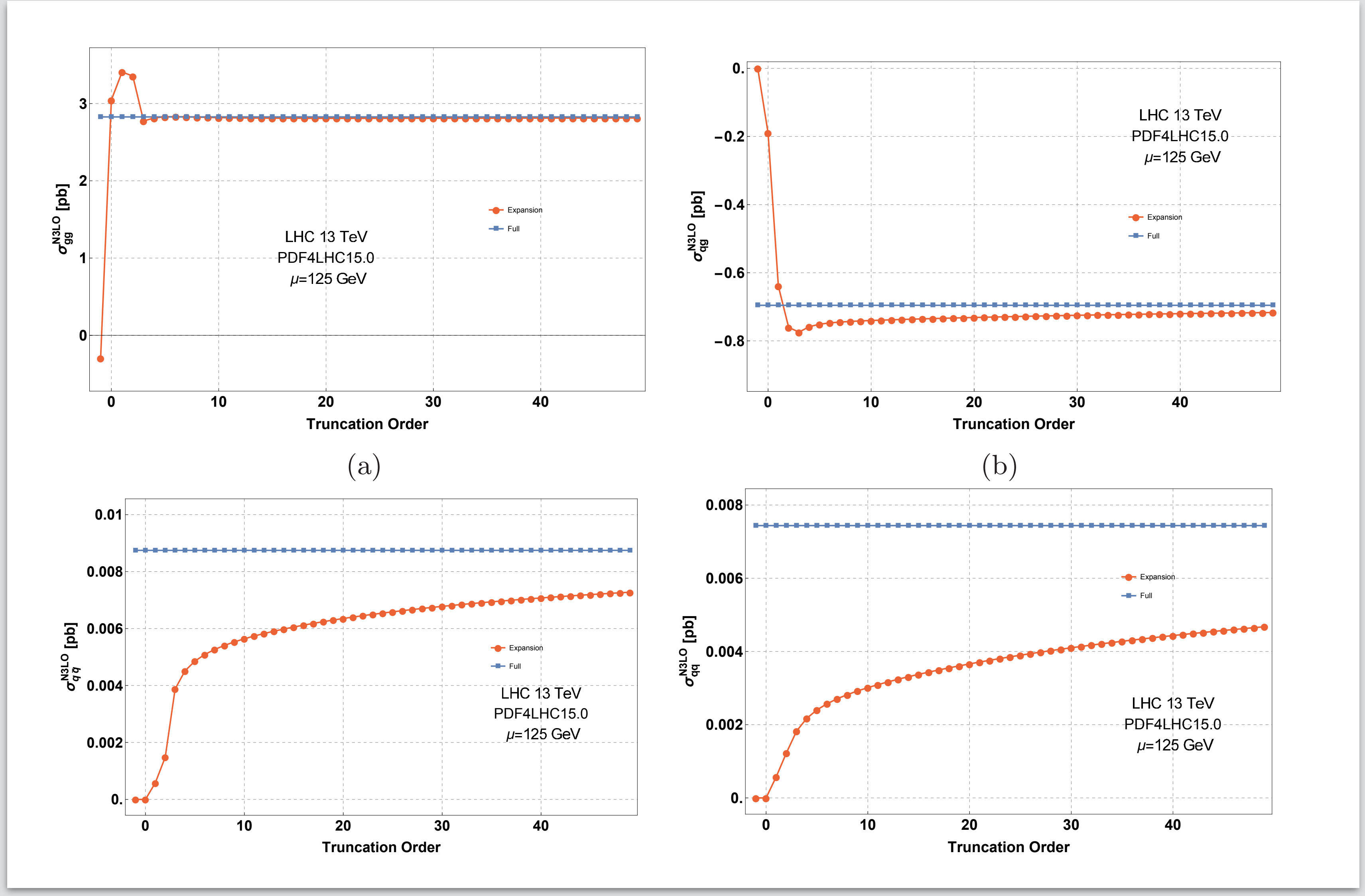
- Total XS agree with exact results at level of 0.2%
- y^H distribution take uncertainties from p_T^{cut}, 7-scales and C_{N3} uncertainty
- Uncertainty reduction $> 50\%$, flat k factor (~ 1.04 central) same as total XS

[slide by X. Chen, MITP Workshop 2018]

- Approximate N^3LO Higgs cross-section *assuming* ~trivial rapidity distribution

N^{3}LO beyond Higgs

For DY, it may be more tricky: soft expansion for quarks does not work very well. Also: non trivial spin correlations



[Mistlberger (2018)]

- More generic techniques may needed for DY
- Beyond DY: *hic sunt leones* [factorization violation, bound states and not integrable PDF integrals…]

NNLO: status and future

- A lot of theoretical progress in the recent past
- This lead to realistic *2→2 Phenomenology at NNLO*
- Many interesting features
  - Greatly reduced th. uncertainties (expected)
  - Stability w.r.t. logarithmic corrections (not so obvious) → fiducial region
- And a few surprises
  - Non trivial jet dynamics (larger than naively expected corrections)
  - Curious data/theory discrepancies
- A lot more to explore
  - More pheno: e.g. jet dynamics @ NNLO vs mergedPS…
  - 2→2 in ``extreme'' kinematics (boosted/off-shell H+j and pp→VV)
  - better understanding of jet dynamics: pp → 3j. Also: α_s, maybe some extra handle to understand NP effects?
  - Important backgrounds / precision tests: Hjj (VBF contamination, jet-bin correlations…), Vjj, ttj

NNLO: status and future

- This will require significant improvement on stat-of-the art
- Breaking the 2 → 2 barrier highly non trivial
  - 2-loop amplitudes
  - more efficient IR subtraction
  - even if the goal is ≠ from NLO, at least some degree of automation
- Beyond NNLO?
  - Exclusive Higgs at N^3LO
  - N^3LO beyond the Higgs?

The LHC provides constant Motivation and Inspiration

Exciting times ahead!

Thank you
very much for
your attention!