NNLO QCD predictions in NNLOJET

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Before August 31st:

IP3
Durham University
higgstools

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After October 1st:

Machine Learning • PDFs • QCD

Outline

What is NNLOJET exactly?

NNLOjet is a fixed order parton-level event generator for inclusive or jet processes at higher orders in QCD.
It can compute differential quantities at NNLO for any partonic channel for several different processes.

The codebase is formed by about 3 millions lines of Fortran code with access to external libraries such as LHAPDF or FastJet. The ingredients of NNLOJET are:

- Matrix elements
- Antenna subtraction method
- Phase space integrator
- Analysis routines

Who is NNLOJET?

NNLOJET is an ever-growing collaboration which spans several universities across Europe. The code has received contributions from:

Xuan Chen, **J CM**, James Currie, Rhorry Gauld, Aude Gehrmann-De Ridder, Thomas Gehrmann, Marius Höfer, Alexander Huss, Nigel Glover, Imre Majer, Jonathan Mo, Tom Morgan, Jan Niehues, Joao Pires, Duncan Walker and James Whitehead.

NNLO

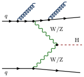
As you saw in this morning talk NNLO calculations can be quite involved:

- Complicated infrared structure.
- Not all amplitudes are known.
  - Specially multiloops!
- Hard to make subtraction numerically stable

See Fabrizio's slides for a summary of the state of the art for higher order calculations.

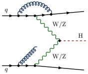
NNLO

- Double Radiation matrix elements (M^0_{n+2})



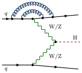
  - Implicit double unresolved singularities arise during phase space integration
  - Very challenging computationally

- Single Radiation one loop matrix elements (M^1_{n+1})



  - Explicit IR poles arising from loop integration
  - Single unresolved singularities arise during phase space integration

- Two loops matrix elements (M^2_n)



  - Only explicit IR poles arise, coming from loop integration
  - Theoretical bottleneck of most NNLO calculations

Antenna subtraction

NNLOJET uses Antenna Subtraction in order to cancel infrared singularities.

The antenna subtraction method is based in the factorisation properties of QCD in the infrared limits, schematically we find in soft limits:

$$\lim_{p_j \to 0} M_{n+1}(\{p_{n+1}\}) = S_{ijk} M_n(\{\bar{p}_n\}),$$

and, similarly, in collinear limits:

$$\lim_{p_i // p_j} M_{n+1}(\{p_{n+1}\}) = P_{f_i,f_j}(p_i, p_j) M_n(\{\bar{p}_n\}).$$

Antenna subtraction

A set of antenna functions can be constructed for each type of singularity from ratios of matrix elements:

$$X_3^{(0)} = \frac{M_3(\{p_3\})}{M_2(\{\bar{p}_2\})}$$

so that we can build a subtraction term for a matrix element M_{n+1} as a combination of antenna functions ($X_3^{(0)}$):

$$\mathrm{d}\sigma^S = -\sum_{\{p_{n+1}\}} X_3^{(0)}(p_i, p_j, p_k) M_n(\{\bar{p}_n\})$$

Note: the (0) stands for the number of loops. This discussion is true at NLO. Beyond NLO new types of singularities arise and more complicated antenna functions have to be used. See `1301.4693` for a more detailed discussion.

Divergence cancellation

In order to ensure the cancellation of divergences in the infrared limit the subtraction term must behave as the physical contribution does.
If the subtraction term is behaving correctly, as we reduce the technical cut (x) we should find $d\sigma^S \to d\sigma^R$.
Ie,

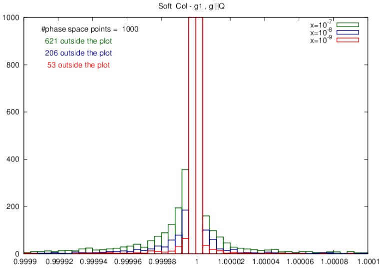
$$\lim_{x \to 0} \left(\frac{\mathrm{d}\sigma^S}{\mathrm{d}\sigma^R} \right) (x) \longrightarrow 1$$

Once the divergences have been cancelled, the subtraction term must be integrated back at the virtual level,

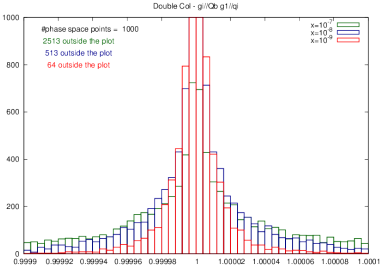
$$\sigma^{NLO} = \int_{n+1} (\mathrm{d}\sigma^R - \mathrm{d}\sigma^S) + \int_n (\mathrm{d}\sigma^V - \mathrm{d}\sigma^T),$$

where $\mathrm{d}\sigma^T = -\int_1 \mathrm{d}\sigma^S$.

Divergence cancellation



Limit $g_1||q_f, g_f \to 0$



Limit $g_1||Q_f, q_i||g_f$

Limits for a process $gq \to gqQ\bar{Q}$H in VBF.

Logistical challenges of NNLO calculations

The numerical implementation of a NNLO calculation can be very challenging even beyond the construction of Matrix Elements and Subtraction Terms

- ✗ Slow convergence of the integrand.
- ✗ Numerical stability in the singular regions.
- ✗ Very complex calculations $\rightarrow$ bugs.

These issues can't never be completely avoided, but we can do our best to minimise their effect.

- ✓ Grid computing and scalability of the code.
- ✓ Validation tests all calculations need to go through.
- ✓ Daily regression test ensure bugs are not accidentally introduced in already-tested calculations.

Vegas

Instablities can be tackled from the numerical side:

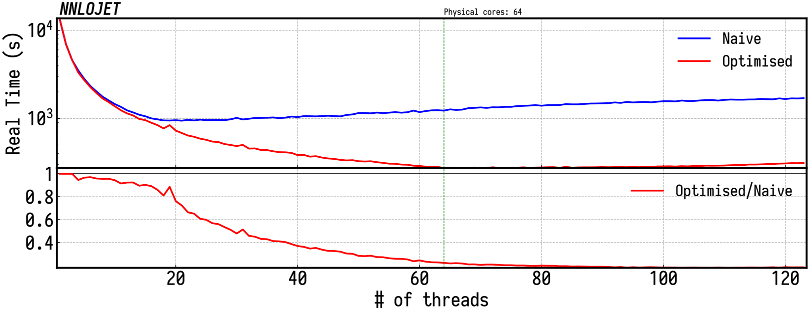
- The phase space integrator is usually optimised for each particular process or application.
- The numerical integration is performed using the Monte Carlo algorithm Vegas.

It has been modified to run over different CPU cores as well as over different nodes in remote clusters*.

*This does not improve on the number of CPU hours it takes to get a result (and even makes it worse!) but it considerably reduces the number of hours we human have to wait to obtain a prediction.

Vegas benchmark

Being careful with the optimisation can have a tremendous effect on the efficiency of the calculation:



Comparison between two different implementations of parallelisation for the (same) Vegas algorithm.

CPU cost (VBF Higgs calculation)

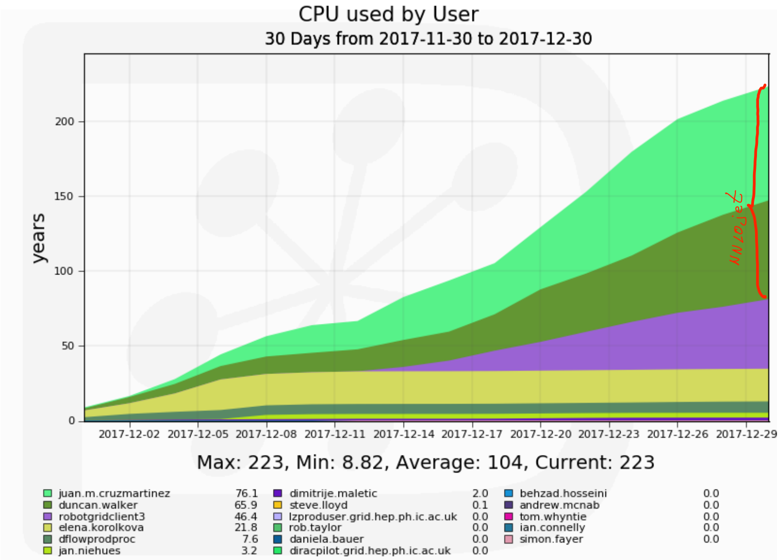
LO and NLO results are below 1000 CPU hours and thus are negligible in comparison with NNLO.

	Warmup (cpu h)	Production (cpu h)	% error in σ
RV	200	20000	0.1
RR region 1	1000	50000	3
RR region 2	3000	100000	5

Each thread ran from in processors with a range of speed from 1.4 to 4 Ghz. Warmups ran across several nodes. The production runs for particular observables would be even more expensive!

RR is separated into two regions for better optimisation

CPU cost, preparation of 1712.07543 (W+J) and 1802.02445 (VBF)



NNLOJET validation suite

Red ticks refer to tests against external tools, green ticks are internal NNLOJET tests. Non-applicable tests are marked with an hyphen.

	Level	ME	Subt	Tech cut	$\frac{1}{\epsilon}$	Layer	Scale	P. Space	Incl.
LO	B	✓ (red)	-	✓ (green)	-	-	-	✓ (green)	✓ (red)
NLO	R	✓ (red)	✓ (green)	✓ (green)	-	✓ (green)	✓ (green)	✓ (green)	✓ (red)
	V	✓ (red)	-	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (red)
NNLO	RR	✓ (red)	✓ (green)	✓ (green)	-	✓ (green)	✓ (green)	✓ (green)	✓ (red)
	RV	✓ (red)	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (red)
	VV	-	-	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (green)	✓ (red)

Example of tests table for VBF Higgs calculation

Processes currently published

Processes included in NNLOJET with already published NNLO results include:

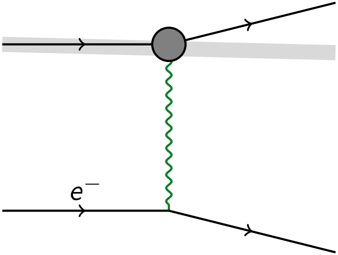
- Single/di-jet production in hadron-hadron collisions (dijet)
- Single/di-jet production in lepton-handron collisions (for both neutral and charged-current DIS)
- Three-jet production in e^+e^- annihilation
- Vector boson plus jet production
- Higgs plus jet production
- Higgs production in VBF

Note: see final slides for references!

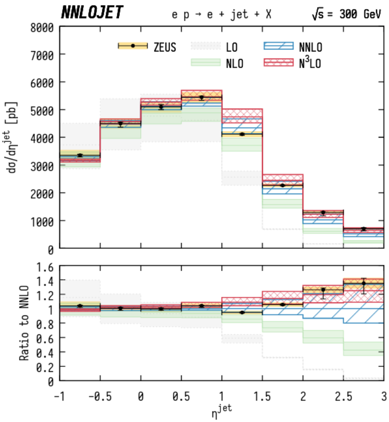
DIS: 1606.03991; 1703.05977; 1803.09973; 1807.02529

The Deep Inelastic Process (commonly known as DIS) is of uttermost importance for PDF fitting.
It offer access to the internal structure of the proton.

This processes is implemented up to N^{3}LO for one jet production and to NNLO for di-jet production.



DIS 1 jet at N3LO: 1803.09973

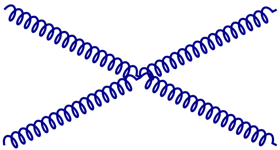


Recently using Projection To Born alongside antenna subtraction: N3LO differential distributions were produced.
N3LO predictions improve agreement with data with respect to NNLO.

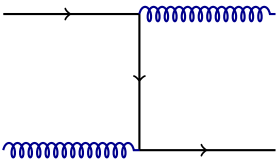
Plot from hep-ph/1803.09973
J. Currie, T. Gehrmann, N. GLover, A. Huss, J. Niehues, A. Vogt.

dijet and single jet inclusive: 1611.01460; 1705.10271; 1807.03692

Jet inclusive cross section: one of the most basic and well measured observables, obtained by summing the contribution of all jets in an event with at least one jet.

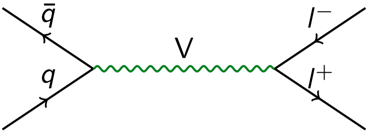


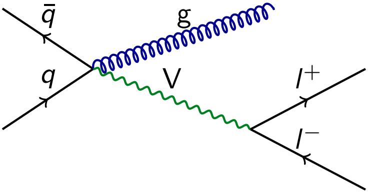
QCD corrections are only known for the leading colour/leading N_F contributions. This is a $2 \to 2$ with an α_s dependence already at the lowest order



Different scale choices can lead to different NNLO behavior.

V+J: 1507.02850; 1605.04295; 1610.01843; 1708.00008; 1712.07543

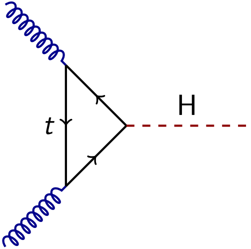




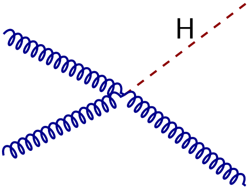
The Drell-Yan process is theoretically very well known. It corresponds to gauge vector boson production (which sub-sequentially goes to a lepton pair).
At the lowest order, the vector boson is always produced along the line of the beam: 0 transverse momentum (p_T^V).

Corrections to the transverse momentum of V only start when an extra particle is radiated: V + jet production.

H+J: 1408.5325; 1607.08817; 1805.00736; 1805.05916; 1807.11501

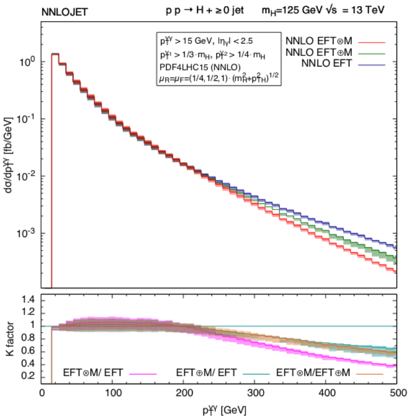


Gluon fusion is similar to the Drell-Yan processes in that a jet is necessary in order for the Higgs boson to gain a momentum fraction.



This is a one-loop process at the lowest order for which QCD NNLO corrections are only known in the $m_t \to \infty$ limit, with a ggH effective vertex. In the full theory NLO was only computed recently (1802.00349).

H+J, 1607.08817



An estimation of our lack of knowledge on the full result (notated M) can be obtained by reweighting the observables bin-by-bin.

NNLO EFT $\otimes$ M:

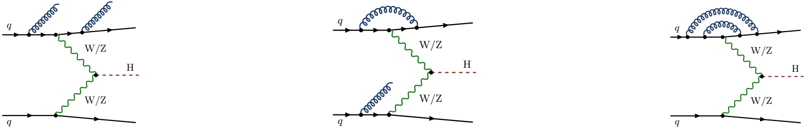
$$\frac{d\sigma^{est}_{NNLO}}{dp^H_t} = \frac{d\sigma^{EFT}_{NNLO}}{dp^H_t}\left(\frac{d\sigma^{M}_{LO}}{dp^H_t} \Big/ \frac{d\sigma^{EFT}_{LO}}{dp^H_t}\right)$$

NNLO EFT $\oplus$ M:

$$\frac{d\sigma^{est}_{NNLO}}{dp^H_t} = \frac{d\sigma^{EFT}_{NNLO}}{dp^H_t} + \left(\frac{d\sigma^{M}_{LO}}{dp^H_t} - \frac{d\sigma^{EFT}_{LO}}{dp^H_t}\right)$$

Vector Boson Fusion Higgs production: 1802.02445

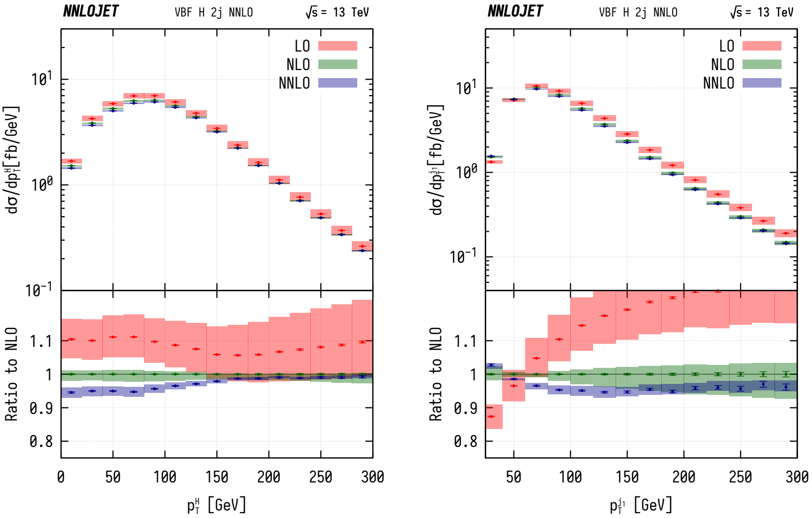
Differential results for VBF are obtained in what the DIS or "structure function" approach, which means defining Vector Boson Fusion as diagrams in which the vector boson is exchanged in the t channel and without QCD interference between the two quark currents:



Effectively, this mean forbidding contributions which are estimated to be negligible when VBF cuts are applied:

- Gluon exchange between the upper and lower quark currents (either real or virtual).
- Interference from same flavour quarks.
- Interference between different production channels.

VBF


NNLOJET
VBF H 2j NNLO
√s = 13 TeV
LO
NLO
NNLO
dσ/dp$_T^H$[fb/GeV]
Ratio to NLO
p$_T^H$ [GeV]
NNLOJET
VBF H 2j NNLO
√s = 13 TeV
LO
NLO
NNLO
dσ/dp$_T^{j_1}$[fb/GeV]
Ratio to NLO
p$_T^{j_1}$ [GeV]

Summary

- Some of the difficulties and numerical challenges of Higher Order calculations relevant for the processes included in NNLOJET has been highlighted.
- NNLOJET can provide differential predictions at NNLO QCD for many $2 \to 2$ processes in leptonic and hadronic collisions.

Thanks

NNLOJET publications I

[1] Wojciech Bizoń et al. "Fiducial distributions in Higgs and Drell-Yan production at $N^3LL+NNLO$". In: (2018). arXiv: 1805.05916 [hep-ph].

[2] X. Chen et al. "NNLO QCD corrections to Higgs boson production at large transverse momentum". In: *JHEP* 10 (2016), p. 066. DOI: 10.1007/JHEP10(2016)066. arXiv: 1607.08817 [hep-ph].

[3] X. Chen et al. "Precise QCD predictions for the production of Higgs + jet final states". In: *Phys. Lett.* B740 (2015), pp. 147–150. DOI: 10.1016/j.physletb.2014.11.021. arXiv: 1408.5325 [hep-ph].

NNLOJET publications II

[4] Xuan Chen et al. "Precise QCD Description of the Higgs Boson Transverse Momentum Spectrum". In: (2018). arXiv: 1805.00736 [hep-ph].

[5] Leandro Cieri et al. "Higgs boson production at the LHC using the q_T subtraction formalism at N^{3}LO QCD". In: (2018). arXiv: 1807.11501 [hep-ph].

[6] J. Cruz-Martinez et al. "Second-order QCD effects in Higgs boson production through vector boson fusion". In: (2018). arXiv: 1802.02445 [hep-ph].

[7] J. Currie et al. "N^{3}LO corrections to jet production in deep inelastic scattering using the Projection-to-Born method". In: *JHEP* 05 (2018), p. 209. DOI: 10.1007/JHEP05(2018)209. arXiv: 1803.09973 [hep-ph].

NNLOJET publications III

[8] James Currie et al. "Infrared sensitivity of single jet inclusive production at hadron colliders". In: *Submitted to: JHEP* (2018). arXiv: 1807.03692 [hep-ph].

[9] James Currie et al. "Infrared Structure at NNLO Using Antenna Subtraction". In: *JHEP* 04 (2013), p. 066. DOI: 10.1007/JHEP04(2013)066. arXiv: 1301.4693 [hep-ph].

[10] James Currie et al. "NNLO QCD corrections to jet production in deep inelastic scattering". In: *JHEP* 07 (2017), p. 018. DOI: 10.1007/JHEP07(2017)018. arXiv: 1703.05977 [hep-ph].

NNLOJET publications IV

[11] James Currie et al. "Precise predictions for dijet production at the LHC". In: *Phys. Rev. Lett.* 119.15 (2017), p. 152001. DOI: 10.1103/PhysRevLett.119.152001. arXiv: 1705.10271 [hep-ph].

[12] James Currie et al. "Precise QCD predictions for the production of dijet final states in deep inelastic scattering". In: *Phys. Rev. Lett.* 117.4 (2016), p. 042001. DOI: 10.1103/PhysRevLett.117.042001. arXiv: 1606.03991 [hep-ph].

[13] J Currie et al. "Next-to-Next-to Leading Order QCD Predictions for Single Jet Inclusive Production at the LHC". In: *Phys. Rev. Lett.* 118.7 (2017), p. 072002. DOI: 10.1103/PhysRevLett.118.072002. arXiv: 1611.01460 [hep-ph].

NNLOJET publications V

[14] R. Gauld et al. "Precise predictions for the angular coefficients in Z-boson production at the LHC". In: *JHEP* 11 (2017), p. 003. DOI: 10.1007/JHEP11(2017)003. arXiv: 1708.00008 [hep-ph].

[15] A. Gehrmann-De Ridder et al. "NNLO QCD corrections for Drell-Yan p_T^Z and ϕ^* observables at the LHC". In: *JHEP* 11 (2016), p. 094. DOI: 10.1007/JHEP11(2016)094. arXiv: 1610.01843 [hep-ph].

[16] A. Gehrmann-De Ridder et al. "NNLO QCD corrections to the transverse momentum distribution of weak gauge bosons". In: (2017). arXiv: 1712.07543 [hep-ph].

NNLOJET publications VI

[17] A. Gehrmann-De Ridder et al. "Precise QCD predictions for the production of a Z boson in association with a hadronic jet". In: *Phys. Rev. Lett.* 117.2 (2016), p. 022001. DOI: 10.1103/PhysRevLett.117.022001. arXiv: 1507.02850 [hep-ph].

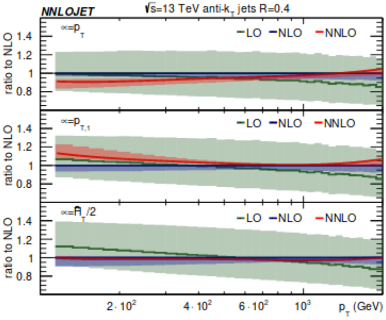
[18] Aude Gehrmann-De Ridder et al. "The NNLO QCD corrections to Z boson production at large transverse momentum". In: *JHEP* 07 (2016), p. 133. DOI: 10.1007/JHEP07(2016)133. arXiv: 1605.04295 [hep-ph].

[19] J. Niehues et al. "NNLO QCD Corrections to Jet Production in Charged Current Deep Inelastic Scattering". In: (2018). arXiv: 1807.02529 [hep-ph].

dijet and single jet inclusive: 1611.01460; 1705.10271; 1807.03692

For inclusive jet cross section two choices of scale, equivalent for configurations with 1 or 2 jets can be made: to evaluate each contribution at $\mu_R = p_T^j$ of the given jet, or $\mu_R = p_T^{j_1}$, the hardest jet.

These different choices can make an important difference, still not under control at NNLO. A criteria of stability and perturbative convergence suggest the usage of $\sum_i p_T^i$ where the sum is over partons and not reconstructed jets.



Plot and discussion from from 1807.03692