Rare B Decays and Penguins with Charm

G. Buchalla
LMU München

Outline

- Charm Penguins in $B \to X_s l^+ l^-$
- A Toy Model
- Quark-Hadron Duality
- Discussion
- Conclusions

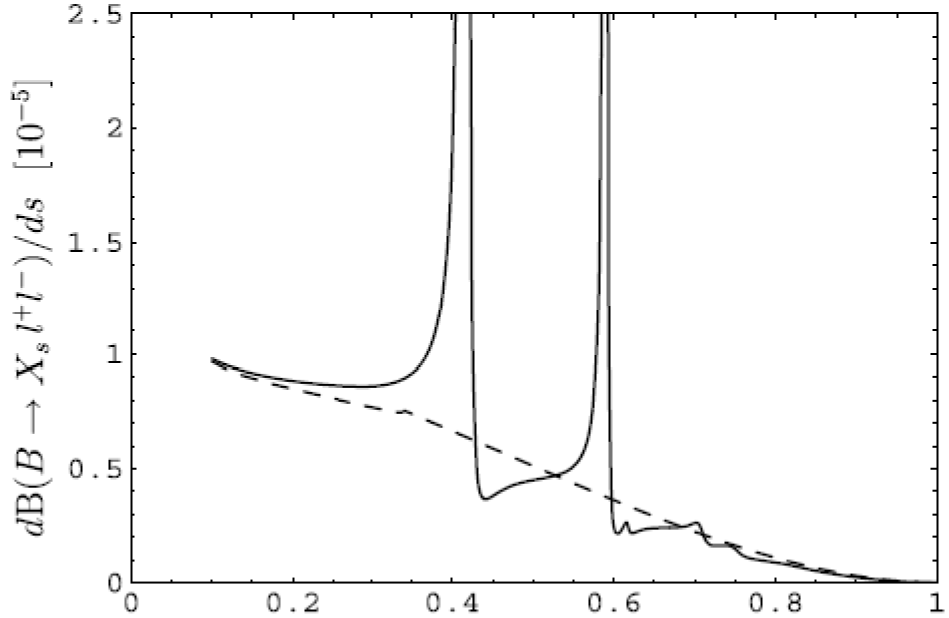
Beneke, G.B., Neubert, Sachrajda, Eur. Phys. J. C (2009) 61, 439

$B \to X_s l^+ l^-$

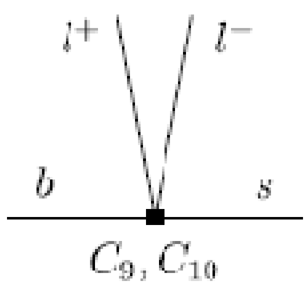
$7.8 \cdot 10^{-3}$ $6 \cdot 10^{-2}$

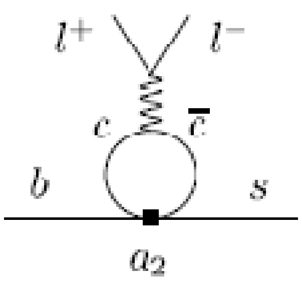
$$\frac{B(B \to X_s \psi \to X_s l^+ l^-)}{B(B \to X_s l^+ l^-)_{SD}} \approx 90$$

$5.3 \cdot 10^{-6}$



$s = q^2 / m_b^2$



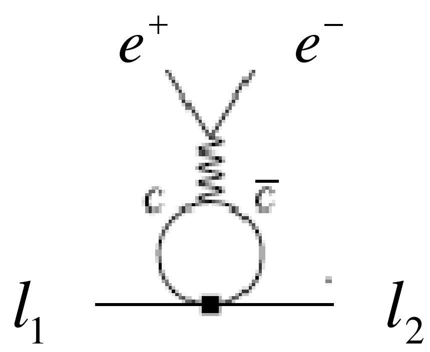


$$C_9 = C_9^{NDR} + 3 a_2 \, h(m_c / m_b, s)$$ SD coefficient

$$h_{KS} = -\frac{8}{9} \ln \frac{m_c}{\mu} - \frac{4}{9} + \frac{16\pi^2}{9} \left[\Pi(q^2) - \Pi(0) \right]$$

Krüger-Sehgal model

Toy Model



$$H_{eff} = \frac{G}{\sqrt{2}}\left[(\bar{l}_2 l_1)_{V-A}(\bar{c}c)_{V-A} - (\bar{l}_2 l_1)_{V-A}(\bar{t}t)_{V-A}\right]$$

$$A(l_1 \to l_2 e^+ e^-) = -\frac{G}{\sqrt{2}} e_c\, e^2 \, \boxed{\Pi(q^2)} \, \bar{l}_2 \gamma^\mu (1-\gamma_5) l_1 \, \bar{e} \gamma_\mu e$$

$$\boxed{\Pi_{\mu\nu} = i\int d^4x\, e^{iq\cdot x} \langle 0 | T\, j_\mu(x)\, j_\nu(0) | 0 \rangle \equiv (q_\mu q_\nu - q^2 g_{\mu\nu})\, \Pi(q^2)}$$

$$j_\mu = \bar{c}\gamma_\mu c$$

Properties of $\Pi(q^2)$

dispersion relation

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{\pi}\int_0^\infty \frac{dt}{t}\,\frac{\mathrm{Im}\,\Pi(t)}{t - q^2 - i\varepsilon}$$

quark loop

$$\Pi(q^2) - \Pi(0) = -\frac{N}{12\pi^2}\ln\frac{-q^2 - i\varepsilon}{m_c^2}$$

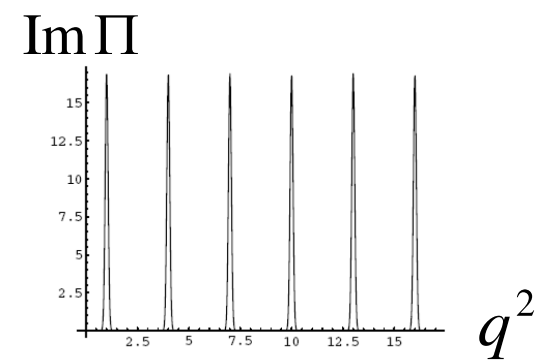
in toy model

$$\Pi(0) \equiv \Pi_c(0) - \Pi_t(0) = \frac{N}{12\pi^2}\ln\frac{m_t^2}{m_c^2}$$

Quark-Hadron Duality in a Nutshell

Shifman

exact: $$\mathrm{Im}\,\Pi = \frac{1}{12\pi}\sum_{n=1}^{\infty}\delta(q^2/\lambda^2 - n)$$



$$\Pi = \frac{1}{12\pi^2}\sum_{n=1}^{\infty}\frac{1}{z+n-i\varepsilon} + const. = -\frac{1}{12\pi^2}\Psi(z+1) + const.$$

$$z \equiv -q^2/\lambda^2$$

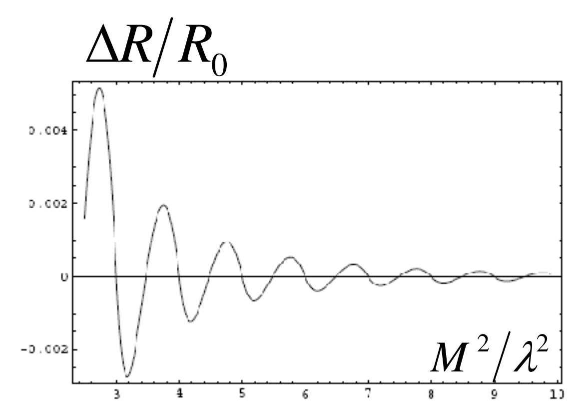
OPE: $$-\Psi(z+1) = -\ln z - \frac{1}{2z} + \sum_{k=1}^{\infty}\frac{B_{2k}}{2k}\frac{1}{z^{2k}} = -\ln z - \frac{1}{2z} + \frac{1}{12z^2} - \frac{1}{120z^4} + \ldots$$

$$12\pi\,\mathrm{Im}\,\Pi(z\le 0) = 1 - \delta(z)/2 - \delta'(z)/12 + \delta'''(z)/720 + \ldots$$

$$R \equiv \int_0^{M^2}\frac{dq^2}{M^2}\left(1 - 3\frac{q^4}{M^4} + 2\frac{q^6}{M^6}\right)24\pi\,\mathrm{Im}\,\Pi(q^2)$$

$$= 1 - \frac{\lambda^2}{M^2} + \frac{1}{30}\frac{\lambda^8}{M^8} \qquad - \frac{1}{3\sqrt{12}}\frac{\lambda^6}{M^6}\sin\left(2\pi\frac{M^2}{\lambda^2}\right)$$

OPE — duality violation



$$\frac{d\Gamma(l_1 \to l_2 e^+ e^-)}{ds} = \frac{G^2 \alpha^2 m_1^5}{27\pi}(1-s)^2(1+2s)\left|\Pi(q^2)\right|^2$$

$$\boxed{s = q^2 / m_1^2}$$

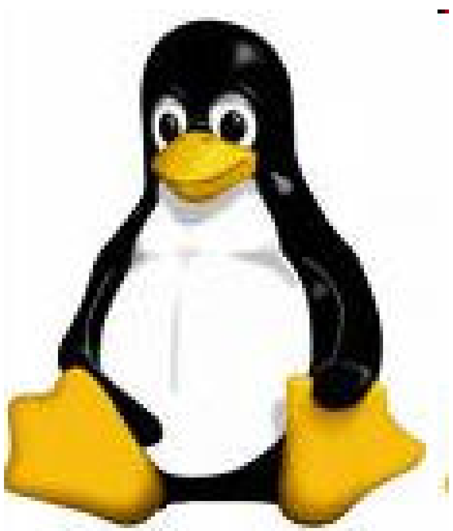
contribution from resonance $\quad \Pi(q^2) = -\frac{f_\psi^2}{q^2 - M_\psi^2 + i M_\psi \Gamma_\psi}$

$$\Rightarrow \quad \left|\Pi(q^2)\right|^2 = \frac{f_\psi^2}{M_\psi \Gamma_\psi} \mathrm{Im}\,\Pi(q^2)$$

$|\Pi|^2$ more singular than $\mathrm{Im}\,\Pi$ for small Γ_ψ

$$\frac{f_\psi^2}{M_\psi \Gamma_\psi} \approx 560 \qquad \frac{f_\psi^2}{M_\psi \Gamma_\psi} \overset{HQL}{\approx} \frac{243}{40(\pi^2 - 9)\alpha_s^3(M_\psi)} \approx 450$$

Quark-Hadron Duality (QHD)



?

$\leftrightarrow$

$$\mathrm{Im}\,\Pi(M_\psi^2) = \frac{f_\psi^2}{M_\psi \Gamma_\psi} >> \mathrm{Im}\,\Pi_{quark}(M_\psi^2) = O(1)$$

local QHD violated, but…

$$\int_0^{m_1^2} dq^2\, \mathrm{Im}\,\Pi(q^2) \approx \pi\, f_\psi^2 << m_1^2$$

consistent with global QHD for $\mathrm{Im}\,\Pi$

$$\int_0^{m_1^2} dq^2 \left|\Pi(q^2)\right|^2 \approx \pi\, f_\psi^2 \times \frac{f_\psi^2}{M_\psi \Gamma_\psi}$$

arbitrarily large!

- global duality violated for $|\Pi|^2$ between hadronic and fixed-order partonic result

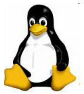
$$\int_0^1 ds \frac{d\Gamma_{res}(l_1 \to l_2 e^+ e^-)}{ds} = \Gamma(l_1 \to l_2 \psi) \frac{\Gamma(\psi \to e^+ e^-)}{\Gamma_\psi}$$

resonance enhancement in toy model:

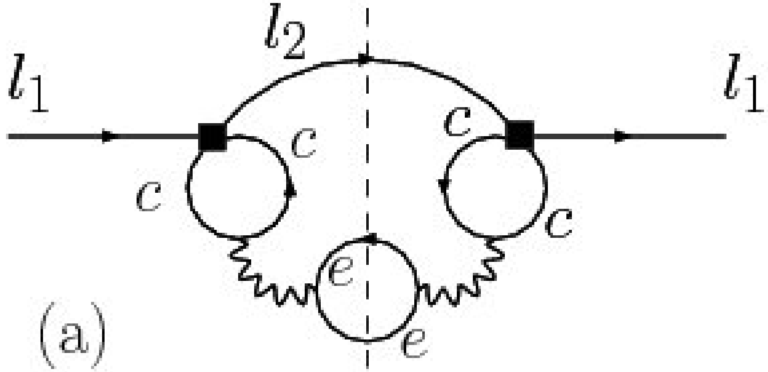
$$\frac{\Gamma_{res}(l_1 \to l_2 \psi \to l_2 e^+ e^-)}{\Gamma(l_1 \to l_2 e^+ e^-)_{SD}} = \frac{8\pi^5 (1-r)^2 (1+2r)}{[\ln(m_t/m_c)+1/2]^2} \times \frac{f_\psi^2}{m_1^2} \times \frac{f_\psi^2}{M_\psi \Gamma_\psi} \approx 210$$

55 0.007 560

>>



$$r = M_\psi^2 / m_1^2$$

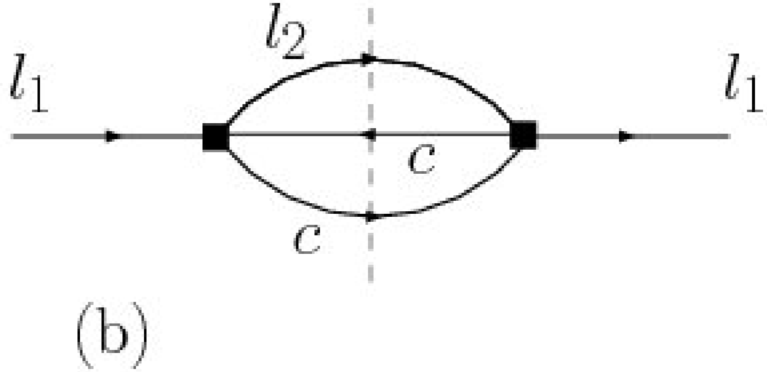


QHD violated

$\leftrightarrow$ no OPE for $\Gamma(l_1 \to l_2 e^+ e^-)$

restricted set of cuts

„exclusive" final state
even for integrated rate



$$\Gamma(l_1 \to l_2 X) = \frac{G^2 m_1^5}{16\pi^2} \int_0^1 ds\,(1-s)^2(1+2s)\left[\mathrm{Im}\,\Pi + \mathrm{Im}\,\Pi_A\right]$$

„tau-decay"

duality holds in the
usual sense

Charm Resonances in $B \to X_s l^+ l^-$

$$R_\psi = \frac{B(B \to X_s \psi \to X_s l^+ l^-)}{B(B \to X_s l^+ l^-)_{SD}} =$$

$$\frac{512\pi^5 \kappa^2 a_2^2 (1-r)^2 (1+2r)}{9\left[\left\langle |C_9|^2 \right\rangle + |C_{10}|^2\right]} \times \frac{f_\psi^2}{m_b^2} \times \frac{f_\psi^2}{M_\psi \Gamma_\psi} \approx 90$$

(23) 0.007 560

Coulomb limit

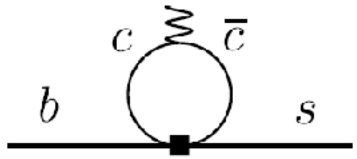
$$R_\psi = [23] \times \frac{54}{5\pi(\pi^2 - 9)} \left(\frac{\alpha_s(m_c \mathrm{v})}{\alpha_s(M_\psi)} \right)^3 \times \frac{m_c^2}{m_b^2} \approx 60$$

light resonance

$$R_\rho = [10-50] \times \frac{f_\rho^2}{m_b^2} \times \frac{f_\rho^2}{M_\rho \Gamma_\rho} \approx [0.007 - 0.036]$$

Discussion

- high- q^2 region in $b \to s l^+ l^-$:



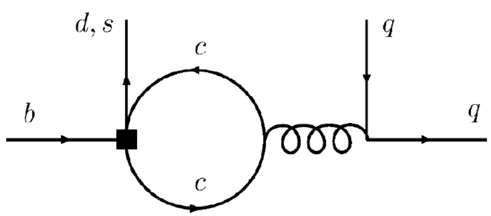
OPE $\langle T\, H_{eff}(x)\, j_\mu(0) \rangle$

G.B., Isidori; Grinstein, Pirjol

$$\frac{d\Gamma}{dq^2} \sim \left|C_9 + \Delta\right|^2 = \left|C_9\right|^2 + 2\,\mathrm{Re}\, C_9^* \Delta + \left|\Delta\right|^2$$

duality ↑ small

- charm-penguin in $B \to M_1 M_2$



$$\Delta a_{10}^p = \frac{C_1 + N C_2}{N} \frac{\alpha}{9\pi} \left[-\frac{4}{3} \ln \frac{\mu}{m_b} + \frac{2}{3} - G_\pi(s_p) \right]$$

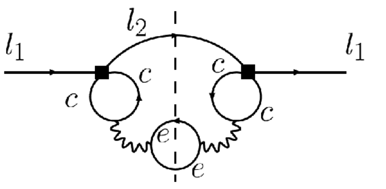
$$G_\pi^\psi = -\frac{8\pi^2 f_\psi^2}{m_b^2} \left[2r_\psi (1 - r_\psi) \left(\ln \frac{1 - r_\psi}{r_\psi} - i\pi \right) + 1 - 2r_\psi \right]$$

$$\frac{A_{c\bar{c}}}{A_{\rm LO}} \sim \alpha_s(2m_c) f\left(\frac{2m_c}{m_b}\right) v \times \frac{4m_c^2 v^2}{m_b^2}$$

Conclusions

► integrated $b \to s l^+ l^-$ rate:

no OPE for charm contribution,



enters as $\sim |\Pi|^2$

► $f_\psi^2 / M_\psi \Gamma_\psi$, very small Γ_ψ

► huge violation of global duality between hadronic and fixed-order partonic result

► high- q^2 : global duality o.k.

Happy Birthday Daniel !