From atomic alchemy to incl. rare $B-$decays

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FlaviA net

Before going to my talk on inclusive rare $B-$decays, I would like to mention that even Daniel managed to publish a paper in PRD with 0 citations:

PHYSICAL REVIEW D VOLUME 52, NUMBER 7 1 OCTOBER 1995

Atomic alchemy: Weak decays of muonic and pionic atoms into other atoms

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(Received 9 May 1994)

The rates of weak transitions between electromagnetic bound states, for example, $(\pi^+e^-) \to (\mu^+e^-)\nu_\mu$, and the exclusive weak decay of a muonic atom into an electronic atom, $(Z\mu^-) \to (Ze^-)\nu_\mu\bar{\nu}_e$, are calculated. For $Z = 80$, relativistic effects are shown to increase the latter rate by a factor of 50 compared to the results of a nonrelativistic calculation. It is argued that the conditions for producing the muonic decay in neon gas ($Z = 10$), where the branching ratio for the decay per captured muon is 1.7×10^{-9}, can be realized using cyclotron traps, though the prospect for a practical experiment seems remote. In lead the same ratio would be approximately $\sim 1 \times 10^{-6}$. In addition to providing detailed information on the high momentum tail of the wave functions in atomic physics, these decays of QED bound states provide a simple toy model for investigating kinematically analogous situations in exclusive heavy hadronic decays in quantum chromodynamics, such as $B \to K^*\gamma$ or $B \to \pi e\nu$.

PACS number(s): 36.10.Dr, 11.10.St, 13.20.He, 13.35.−r

8) **Atomic alchemy.**
C. Greub, D. Wyler, (Zurich U.) , S.J. Brodsky, C.T. Munger, (SLAC) .
SLAC-PUB-6487, ZU-TH-5-94, Apr 1994. 21pp.
Published in **Phys.Rev.D52:4028-4037,1995**.
e-Print: **hep-ph/9405230**

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords 0 cit.
Abstract and Postscript and PDF from arXiv.org (mirrors: au br cn de es fr il in it jp kr ru tw uk za aps lanl)
Journal Server [doi:10.1103/PhysRevD.52.4028]
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The reason certainly is, that Daniel had a bad collaborator in this work!

Let me briefly tell why we worked on this alchemy-topics.

During my postdoc time in Zurich, around 1992, we formulated a covariant constituent-quark model to describe the exclusive transitions

$$B \to \pi e \nu \qquad B \to \rho e \nu \,.$$

The $B-$ meson was modelled as a state consisting of $b-$quark and a massless spectator q.

Rest-frame of the $B-$meson: q: 3-momentum $\vec{p}_q$ b: 3-momentum $(-\vec{p}_q)$

We required that the energies of $b-$quark and of the spec. add up to the mass of the $B-$meson:

$$p + \sqrt{m_b^2 + p^2} = m_B \,; \qquad (p = |\vec{p}_q|) \,.$$

This only works when we consider $b-$quark mass to be dependent on p:

$$m_b(p) = \sqrt{m_B^2 - 2m_B p}$$

The essential feature is that the four-momenta of $b-$quark and of the spect. add up to the four-momentum of the B, however at the cost of a momentum dependent $b-$quark mass.

For the $B-$ meson state $|\Psi_B\rangle$ we used the representation

$$|\Psi_B\rangle = \sqrt{2m_B}\int \frac{d^3p_q}{(2\pi)^3}\frac{f(p)}{\sqrt{2p_b^0\,2p_q^0}}\frac{1}{\sqrt{2N_c}}\left[a_\uparrow^+(p_b,c)b_\downarrow^+(p_q,c)-a_\downarrow^+(p_b,c)b_\uparrow^+(p_q,c)\right]|0\rangle$$

where the wave-function $f(p)$ describes the momentum distribution of the $b-$quark. For $f(p)$ suitable Ansätze were chosen.

The quasi-free consituents can be boosted in a straightforward way, leading to a description of a moving $B-$meson.

For the light mesons in the final state we used a picture of two parallel light (anti)quarks sharing the four-momentum of the meson ($\phi = 6x(1-x)$).

This construction allowed us to calculate the decay form-factors and the branching ratios in a manifestly covariant way.

When doing this work, Stan Brodsky visited Zurich. He was unhappy with our concept of variable $b-$quark mass: “This $B-$meson has no binding and falls apart”!

In order to learn how to describe excl. $B-$decays properly, Stan suggested to first discuss transitions of electromagnetically bounded systems, as e.g.

$$(\pi^+ e^-) \to (\mu^+ e^-) + \nu_\mu \qquad \text{or} \qquad (Z\mu^-) \to (Ze^-) + \nu_\mu + \bar{\nu}_e \,.$$

We referred to such processes as “atomic alchemy”, because ’atoms’ are transformed into other atoms.

The description of these decays, however, soon became a science for itself.

I don’t want to go into this now, this easily would cost me 1/2 hour. But please read our paper in PRD 52:4028,1995.

I am sure, on the 70th birthday of Daniel it will have $\geq N$ citations (N is the number of participants of this meeting).

Outline (of the remainder of this talk)

- Introduction to incl. rare B-decays
- Theoretical framework to calculate BR$(B \to X_s\gamma)$
- Comment on the NLL results
- Ingredients for the NNLL calculation
- NNLL results
- Comment on non-local power corrections
- Comment on the photon energy cut-off effects
- Summary on $B \to X_s\gamma$

Introduction to incl. rare B-decays

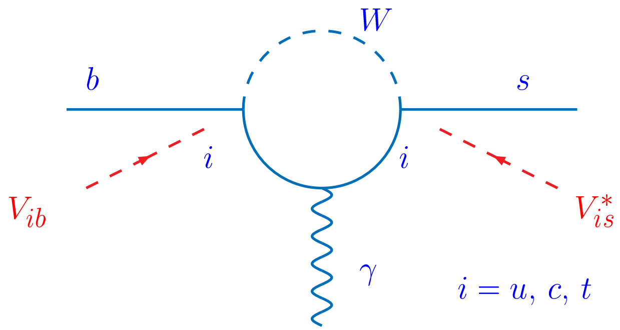
By definition, rare decays are loop-induced in the SM.

The decay $b \longrightarrow s\gamma$ is a specific example of such a decay.

$b \longrightarrow s\gamma$ does not exist at tree level.

However, at the one-loop level, $b \longrightarrow s\gamma$ does occur in SM:

typical diagram (e.m. penguin)



-tests SM at the QT level

-sensitive to certain CKM matrix elements

-sentistive to new physics (new particles in the loop)

The loop-induction naturally suppresses the BR. As we know to a value compatible with the exp!!

Theoretical framework to calculate BR$(B \to X_s\gamma)$

$B \to X_s\gamma$ is an inclusive decay. $\to$ theoretically clean.

HQE: $\Gamma[B \to X_s\gamma] = \Gamma[b \to s\gamma(g)] +$ corr. in Λ_{QCD}/m_b.

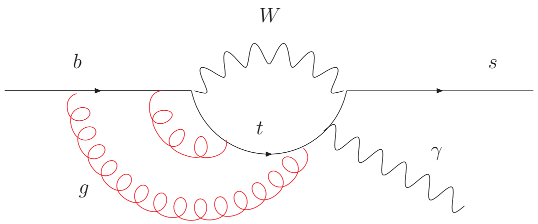
- no linear corrections in Λ_{QCD}/m_b when restricting to local operators
- Corr. start at $\mathcal{O}(\Lambda_{QCD}^2/m_b^2)$; they are related to the motion of the b-quark inside the meson

There are, however, contributions which scale like $\alpha_s(m_b)\Lambda/m_b$, induced by (non-local) light-cone operators (Lee,Neubert,Paz 2006)

But let us first discuss the main contribution: the free b-quark decay $\Gamma[b \to X_s\gamma]$.

Well-known: This partonic decay rate is significantly enhanced by QCD-effects.

When exchanging n gluons, there are large logs:



$$\left(\frac{\alpha_s}{\pi}\right)^n \log^n \frac{m_b^2}{M^2} \qquad M = m_t, m_W: \text{ leading logs (LL)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n-1} \frac{m_b^2}{M^2} \qquad \text{next-to-leading logs (NLL)}$$

$$\left(\frac{\alpha_s}{\pi}\right)^n \log^{n-2} \frac{m_b^2}{M^2} \qquad \text{NNLL}$$

To get a theoretical branching ratio is of similar precision as the present measurements, on has to resum LL, NLL and NNLL terms.

To achieve this resummation, one first constructs an effective Hamiltonian and then resums the logs using RGE techniques.

The effective Ham. obtained by integrating out heavy particles: In SM: top-quark, W, Z.

For $b \to s\gamma$ ($b \to s\gamma g$) one gets the following Hamiltonian $\mathcal{H}_{eff}$:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu) \quad .$$

The operators are:

$O_1 = (\bar{c}_{L\beta} \gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha} \gamma_\mu c_{L\beta})$ current-current operator

$O_2 = (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{s}_{L\beta} \gamma_\mu c_{L\beta})$ current-current operator

$O_3 - O_6$ Gluonic penguin operators (also 4-Fermi operators)

$O_7 = \frac{e}{16\pi^2} m_b(\mu) \, (\bar{s} \sigma_{\mu\nu} R b) \, F^{\mu\nu}$ phot. dipole

$O_8 = \frac{g_s}{16\pi^2} m_b(\mu) \, (\bar{s_\alpha} \sigma_{\mu\nu} T^A_{\alpha\beta} R b_\beta) \, G^{\mu\nu,A}$ gluonic dipole

$C_i(\mu)$ are determined through the matching procedure, i.e. one requires that certain amplitudes in the full theory are id. to those in the effective theory.

Let's look at the structure of the eff. Hamiltonian:

$$\mathcal{H}_{eff} \quad \sim \quad \sum_i C_i(\mu) O_i(\mu)$$

$\mathcal{H}_{eff}$ independent of μ, while C_i and O_i depend on μ:

$\longrightarrow$ RGE for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^T C_j(\mu)\,; \qquad \gamma_{ij} \text{ : anomalous dim. matrix}$$

Matching usually done at high scale μ_W, i.e. $\mu_W \sim O(m_W)$:

μ_W: full theory and mat. el. of op. have same large log's:

Corr. to $C_i(\mu_W)$ rel. small.

$\downarrow$ RGE

$\mu_b = O(m_b)$: mat. el. of op. don't have large log's: They are contained in the $C_i(\mu_b)$.

Calculation of BR$(B \to X_s \gamma)$ consists of three steps:

	LL	NLL	NNLL
-matching at $\mu = \mu_W$: $\to C_i(\mu_W)$	α_s^0	α_s^1	α_s^2
-RGE: $\to C_i(\mu_b)$ [with $\mu_b = O(m_b)$]	α_s^1	α_s^2	α_s^3
-calc. of matrix element $\langle X_s \gamma \| O_i(\mu_b) \| b \rangle$	α_s^0	α_s^1	α_s^2

Comment on the NLL results

The calculation of the branching ratio at NLL order was already a complicated enterprise where many groups were involved. Also Daniel was involved in the calc. of the two-loop matrix elements associated with $O_{1,2}$ [For this work we did get citations]

These NLL QCD calc. were completed in 1998.

Also certain classes of electro-weak corrections were calculated (Czarnecki, Marciano; Neubert, Kagan; Baranowski, Misiak; Gambino, Haisch)

In 2001, Gambino and Misiak realized that the NLL BR has a rather large theor. uncertainty ($\sim 11\%$) related to the renormalization scheme/scale which is used for m_c.

It became clear that a NNLL calc. is necessary to remove/milder this scheme dependence.

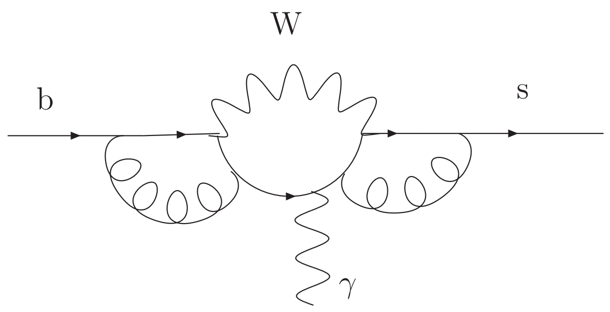
Recently, the most important contributions at NNLL were finalized!!

Contributions which are expected to be less important numerically, are in progress now.

Ingredients for the NNLL calculation

Matching: needed to α_s^2 precision.

This means in particular a 3-loop calculation in the full theory to fix $C_7(\mu_W)$ and $C_8(\mu_W)$ [$O(10^3)$ diagrams]:



→ done by Misiak and Steinhauser, hep-ph/0401041.

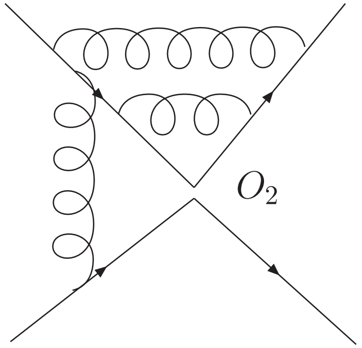
For other operators $O(\alpha_s^2)$ means two-loop. Done some time ago.

→ matching complete for NNLL $b \to s\gamma$!

<u>Anomalous dimensions:</u> needed up to α_s^3 precision.

• $(O_1 - O_6)$-sector

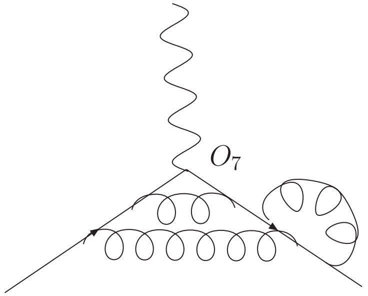
done by Gorbahn and Haisch, hep-ph/0411071.



• (O_7, O_8)-sector

was finished in 2005 by

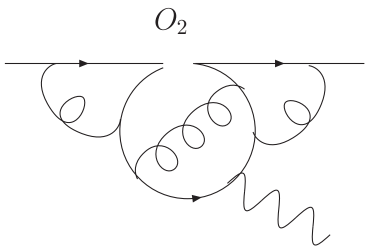
Gorbahn, Haisch and Misiak, hep-ph/0504194.



• most difficult: mixing $O_2 \to O_7$, $O_2 \to O_8$:

about 22'000 4-loop diags!.

Done in 2006 by Czakon, Haisch and Misiak, hep-ph/0612329



$\to 8 \times 8$ anomalous dimension matrix complete for NNLL precision!

Matrix elements of O_i: needed up to α_s^2

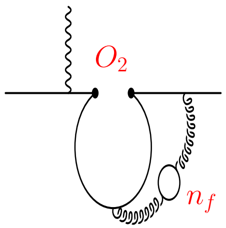
At order α_s^2 the following subprocesses are involved:

$$b \to s\gamma \; ; \; b \to s\gamma g \; ; \; b \to s\gamma gg \; ; \; b \to s\gamma q\bar{q}$$

The decay width can be decomposed into various interferences of the form (O_i, O_j). Let's look at a few of them:

- (O_2, O_7)-interference is numerically very crucial.

Only the fermionic contributions are known exactly at NNLL order:

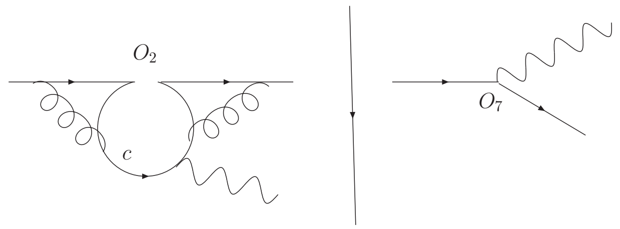


The virtual corrections were calculated by Bieri,Greub, Steinhauser, 2003 for massless quarks in the bubble.

The bremsstahlung corrections in this approx. are also available exactly (Ligeti, Luke, Manohar, Wise 1999.)

Later, also massive quarks in the bubble were taken into account (Boughezal, Czakon, Schutzmeier 2007.)

The non-fermionic corrections are extremely difficult to calculate:



m_c-dependence extremly hard to get.

Misiak and Steinhauser obtained a result for the unphys. case $m_c \gg m_b$, using HME techniques.

They then formulated an extrapolation procedure to the physical m_c, which they tested at the NLL level hep-ph/0609241.

By comparing different versions of the extrapolation procedure, they conclude that their result should be accurate within $\pm 3\%$ (at level of BR).

At the moment exact calculations are in progress for $m_c = 0$ (Czakon et al.), which will improve the extrapolation procedure.

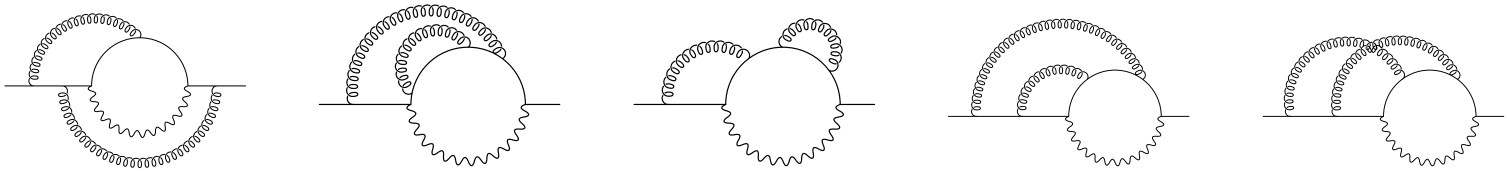
I hope that even exact calc. will come for physical m_c and make the extrapolation obsolete!

• (O_7, O_7)-interference

The first paper on this was published by Blokland et al., hep-ph/0506055.

According to the optical theorem, the (O_7, O_7)-contribution to the decay width can be obtained by taking the absorptive (imaginary) part of b-quark self energy. Partial list of such diagrams:

1



(k) (l) (m) (n) (o)

Blokland et al. calculated the imaginary parts of these diagrams by loop techniques.

We calculated independently the individual cuts of the (O_7, O_7) interference, i.e. the subprocesses $b \to s\gamma$, $b \to s\gamma g$, $b \to s\gamma gg$ and $b \to s\gamma q\bar{q}$ (Asatrian et al, 2006).

The results are identical.

- (O_7, O_8)-interference

So far, only the fermionic corrections with massless bubbles inserted in the gluon-propagator entered the prediction for the NNLL BR.

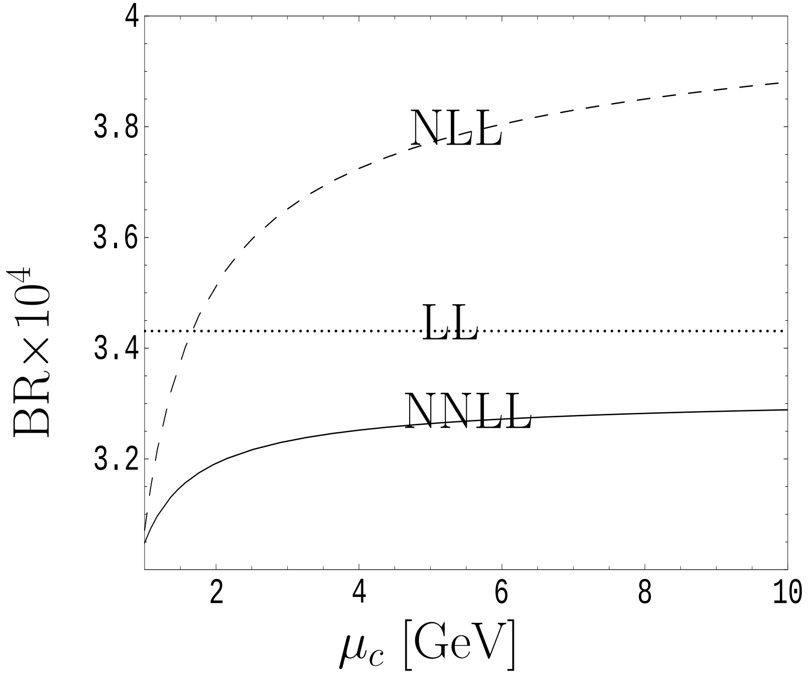
Recently, T. Ewerth, hep-ph/0805.3911 published a paper on the fermionic contributions, including the mass effects in the $c-$ and $b-$bubbles.

The full $O(\alpha_s^2)$ corrections are not known yet. But they are in progress and only a few MI have to be double-checked (Asatrian,Ewerth,Ferroglia,Greub).

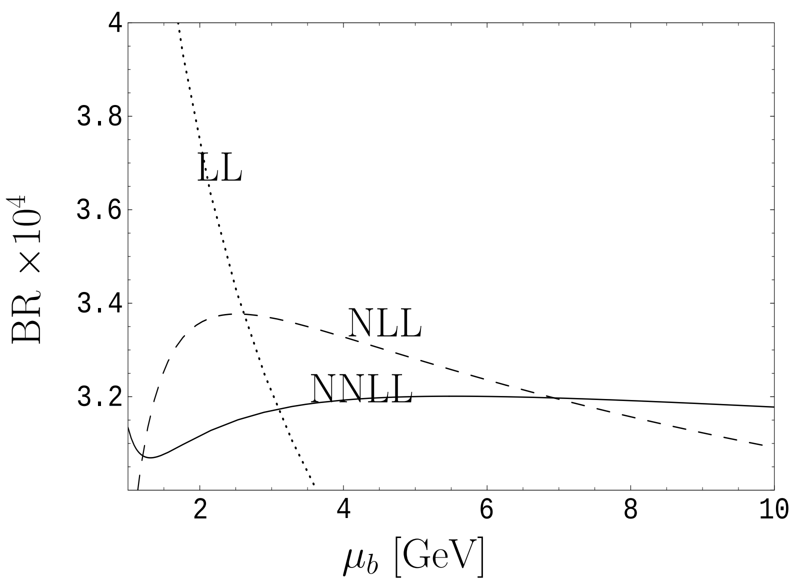
NNLL results

Recently the known individual NNLL-pieces were combined. A phenom. paper was published in PRL 98:022002, 2007, (Misiak+16 authors!).

As expected, the large uncertainty due to the renorm. scale μ_c (at which m_c is renormalized), gets drastically reduced at NNLL.

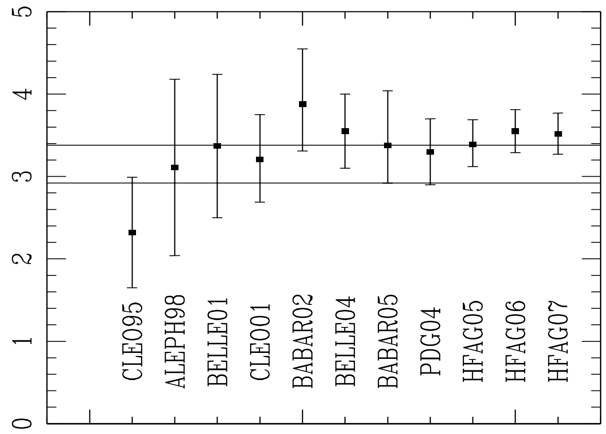


Also the dependence on the scale μ_b gets drastically reduced at NNLL.



At NNLL we obtain (PRL, 2007, (Misiak+16 authors!))

$$BR(B \to X_s\gamma)|^{\rm th}_{E_\gamma>1.6\ {\rm GeV}} = (3.15 \pm 0.23)\cdot 10^{-4}$$



$$BR(B \to X_s\gamma)|^{\rm exp}_{E_\gamma>1.6\ {\rm GeV}} = (3.52 \pm 0.23 \pm 0.09)\cdot 10^{-4}$$

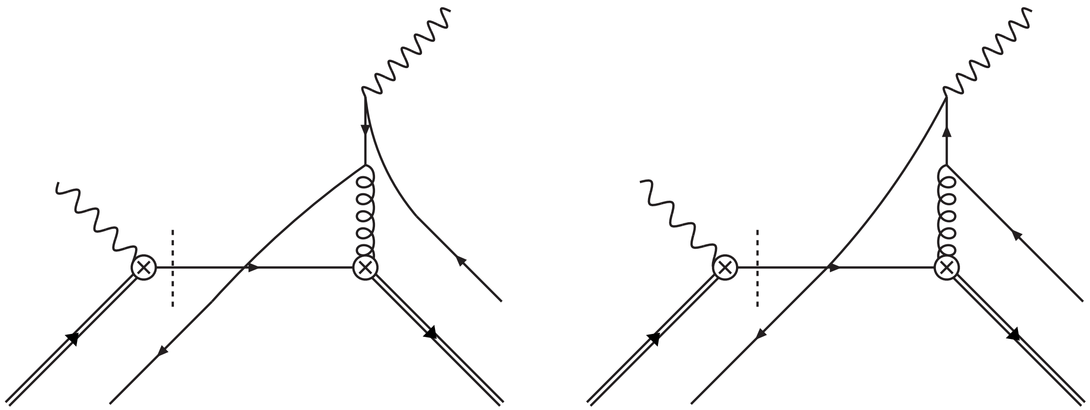
HFAG, ArXiv:0808.1297

In the theory result various errors were added in quadrature, viz.

1. 3% higher orders (scale dependences)
2. 3% m_c extrapolation
3. 3% parametric: from $\alpha_s(m_Z)$, $m_c(m_c)$, BR_{sl} etc.
4. 5% due to a new class of non-perturbative corr. which scale like $\alpha_s \frac{\Lambda}{m_b}$
   Lee,Neubert,Paz Sept. 2006

Comment on the non-local power corrections

There are power corrections which scale like $\alpha_s \frac{\Lambda}{m_b}$. They arise for example in the (O_7, O_8)-interference through the mechanism shown below (Lee,Neubert,Paz Sept. 2006)



O_7-side: energetic photon is directly emitted from the operator

O_8-side: a gluon is emitted from the operator. It goes into an energetic photon by emitting two soft quarks.

The two 'vertical' propagators have virtualities of order $m_b\Lambda$

This mechanism leads to tri-local four-quark operators like

$$O_1 = \sum q e_q \bar{h}_v(0)\Gamma_R q(t\bar{n})\,\bar{q}(s\bar{n})\Gamma_R h_v(0) \quad \text{tri-local operator}\,.$$

h_v: b-field in HQET; $q, \bar{q}$: SCET fields located on the light-cone defined by the direction $\bar{n}$ of the emitted photon.

If these 4-quark operators were local, their matrix elements would scale like $(\Lambda/m_b)^3$. In the present non-local situation, the two 'vertical' propagators introduce two powers of the soft scale Λ in the denominator, leading to a scaling like Λ/m_b.

The matrix elements $\langle B|O_i|B\rangle$ are very difficult to calc. Naive model estimates point to a small red. of the total decay rate. The uncertainty (at the level of the decay rate) is estimated to be of $O(\sim 5\%)$. The authors (Lee,Neubert,Paz Sept. 2006) stress that it is very difficult to really substantiate this number!

Comment on the photon energy cut-off effects

The NNLL result

$$BR(B \to X_s\gamma)|_{E_\gamma > 1.6\ \mathrm{GeV}} = (3.15 \pm 0.23) \cdot 10^{-4}$$

given above was derived in fixed-order perturbation theory.

In 2004 Neubert pointed out that the photon energy cut E_0 induces an additional scale Δ

$$\Delta = m_b - 2E_0 \qquad \text{twice the width of the observed energy window}$$

Numerically, $\Delta \approx 1.4$ GeV (for $E_0 = 1.6$ GeV).

Accounting for the photon-energy cut properly, requires to disentangle contributions associated with the hard-scale $\mu_h \sim m_b$, the soft scale $\mu_0 \sim \Delta$ and the intermediate scale $\mu_i \sim \sqrt{\Delta m_b}$, set by the typical invariant mass of the hadronic final state.

In the present application $\Delta \sim 1.4$ GeV can be treated as a perturbative scale, allowing to construct a multiple scale Operator Product Expansion (MSOPE).

In 2006, Becher and Neubert used this framework to resum those leading and next-to-leading logs of $\delta \equiv \Delta/m_b$ which are not power-suppressed (in δ).

Power-suppressed terms on the other hand were retained in fixed order perturbation theory $[O(\alpha_s^2)]$ (resummation of power-suppressed log's not yet available)

Combining this calculation with our fixed order results, they got a BR which somewhat smaller (but it lies in the error bars of our result):

$$BR(B \to X_s\gamma)|_{E_\gamma > 1.6\ \mathrm{GeV}} = (2.98 \pm 0.26) \cdot 10^{-4} \qquad \text{Becher-Neubert 2006}$$

Note: Misiak (arXiv:0808.3134) pointed out that only resumming the non power-suppressed δ-logs is misleading, because these terms are not really very dominant numerically. Therefore, before the resummation of the power-suppressed terms can also done, the fixed order result is probably more reliable.

Summary on $B \to X_s\gamma$

At NLL order the BR for $B \to X_s\gamma$ has a large uncertainty related to the def. of m_c.

Recently a first NNLL estimate of this BR was published, where this uncertainty gets drastically reduced.

The matching calc. for the Wilson coefficents are complete at NNLL.

Also the anomalous dimensions are completely known.

There are missing matrix elements, e.g the (O_7, O_8) and (O_8, O_8) interferences.

The three-loop results for the matrix elements $\langle s\gamma|O_2|b\rangle$ for $m_c = 0$ are awaited. They are expected to improve the extrapolation in m_c.

I would like to thank Daniel for all the wonderful collaborations we had in the last 19 years!!!

Äs het uhuere gfägt!

$b \to s\gamma$ in type-II 2HDM [the one realized in MSSM]

A second Higgs-doublet is added to the SM. As a consequence, there are 3 neutral Higgses, h^0, H^0, A^0 and 2 charged Higgses, $H^\pm$.

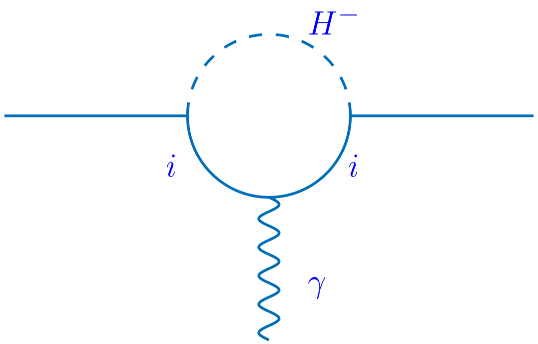
The two doublets pick up no-zero vevs: v_1 and v_2.

In the type-II model, the quark masses are as follows:

$$m_{\rm down} \propto v_1 \; ; \quad m_{\rm up} \propto v_2 \; .$$

As in the SM, flavor-changing neutral currents are absent at tree-level.

There are, however, additional contr. to $b \to s\gamma$ due to charged Higgs (H^-) exchange:



+ QCD corrections

In the type-II 2HDM the operator basis is the same as in the SM.

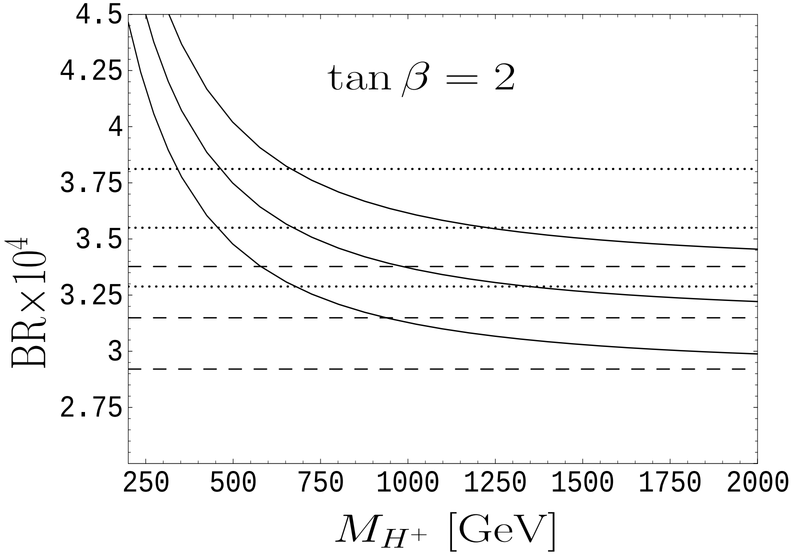
$\rightarrow$ Therefore, the additional contr. only modify Wilson coeffs. at the matching scale.

The new contr. contributions are only known to NLL precision (Borzumati, Greub 1998; Giudice et al. 1998)

New contr. characterized by 2 parameters: m_{H^-}, $\tan\beta = v_2/v_1$.

$m_{H^-} \geq 295$ GeV @ 95% CL $\quad (\tan\beta \rightarrow \infty) \qquad$ most stringent bound!!

Stays basically unchanged for $\tan\beta > 2$. For $\tan\beta < 2$, BR and bound increase (PRL, 2007, (Misiak+16 authors!)).



solid: type-II 2HDM

dashed: SM-theory

dotted: measurements

Recently Misiak, arXiv:0808.3134 pointed out that this “partial resummation” is unreliable.

To illustrate this, consider the (O_7, O_7) contribution to $F(E_0)$ in perturbation theory: [$F =$ fraction of events which passes the photon energy cut]

$$F_{77}(E_0) = 1 + \frac{\alpha_s}{\pi}\phi^{(1)}(\delta) + \frac{\alpha_s^2}{\pi^2}\phi^{(2)}(\delta) + \ldots$$

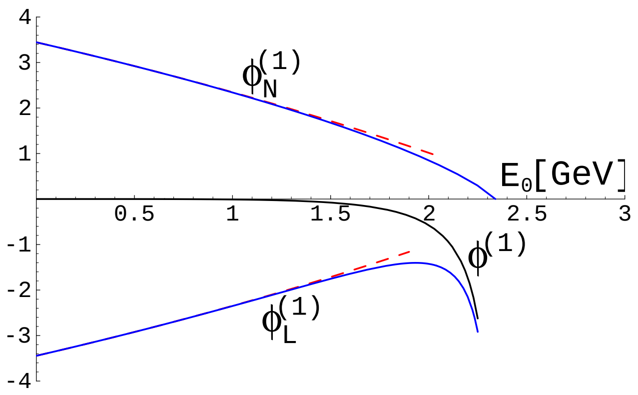
Split each $\phi^{(k)}$ into two parts:

$$\phi^{(k)} = \phi_L^{(k)} + \phi_N^{(k)}$$

$\phi_L^{(k)}$ is a polynomial in $\log(\delta)$

$\phi_N^{(k)}$ contains powers of δ (vanishing at the endpoint)

Concretely, for $\phi^{(1)}$ the splitting reads:



$$\phi_L^{(1)}(\delta) = -\frac{2}{3}\ln^2\delta - \frac{7}{3}\ln\delta - \frac{31}{9}$$

$$\phi_N^{(1)}(\delta) = \frac{10}{3}\delta+\frac{1}{3}\delta^2-\frac{2}{9}\delta^3+\frac{1}{3}\delta(\delta-4)\ln\delta$$

Only at very large values of E_0 $\phi_L^{(1)}$ dominates.

However, at the relevant $E_0 = 1.6$ GeV large cancellations between $\phi_L^{(1)}$ and $\phi_N^{(1)}$!!

Same situation for $\phi_L^{(2)}$ and $\phi_N^{(2)}$, which are expl. known.

General arguments imply the same situation for all the other $\phi^{(k)}$.

$\Longrightarrow$ When resumming the leading power pieces and leaving the power-suppressed pieces unresummed at $O(\alpha_s^2)$-level, the necessary cancellations do not happen at the $O(\alpha_s^3)$-level.

As a consequence, the $O(\alpha_s^3)$-terms get highly overestimated.

Would be nice if one could resum logs in power-suppressed contributions as well.

Until this can be done, the fixed order result for the BR seems more reliable.

Comment on the normalization factor C

In the result above, the branching ratio was written as

$$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma)_{\mathrm{E}_\gamma > \mathrm{E}_0} = \mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_c} e\bar{\nu})_{\mathrm{exp}} \left| \frac{\mathrm{V}^*_{\mathrm{ts}}\mathrm{V}_{\mathrm{tb}}}{\mathrm{V}_{\mathrm{cb}}} \right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi \mathrm{C}} \left[\mathrm{P}(\mathrm{E}_0) + \mathrm{N}(\mathrm{E}_0)\right].$$

The perturbative part $P(E_0)$ is

$$\frac{\Gamma(b \to X_s\gamma)_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\,\Gamma(b \to X_u e\bar{\nu})} = \left| \frac{V^*_{ts}V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi} P(E_0)\,.$$

C is the so-called semileptonic phase-space factor:

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \to X_c e\bar{\nu})}{\Gamma(\bar{B} \to X_u e\bar{\nu})}$$

The expression for C is a function of m_c/m_b and of non-perturbative OPE-parameters.

All the occuring quantities in C are determined in a single global fit from the measured decay spectra of $B \to X_c e\bar{\nu}$.

For C one obtains

$$C = \begin{cases} 0.582 \pm 0.016 & \text{C. Bauer et al., hep-ph/0408002} \quad \text{1S scheme} \\ 0.546^{+0.023}_{-0.033} & \text{P. Gambino and P. Giordano, arXiv:0805.0271} \quad \text{kinetic scheme} \end{cases}$$

and m_c (after converting it to the MS-bar scheme)

$$\overline{m}_c(\overline{m}_c) = \begin{cases} 1.224 \pm 0.057 & \text{1S scheme} \\ 1.267 \pm 0.056 & \text{kinetic scheme} \end{cases}$$

The differences cancel to some extent in the radiative BR, leading to

$$\mathrm{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma)^{\mathrm{NNLL}}_{\mathrm{E}_\gamma > 1.6\mathrm{GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}\,, & \text{using 1S scheme hep-ph/0609232} \\ (3.25 \pm 0.24) \times 10^{-4}\,, & \text{following the kin. scheme analysis of Gambino,Giordano, see also Misiak, arXiv:0808.3134} \end{cases}$$