NNLO QCD corrections to non-leptonic charmless B decays

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In collaboration with Martin Beneke and Xin-Qiang Li

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Daniel Wyler's academic genealogical tree

[Courtesy of John Donoghue]

Jakob Bernoulli
b. 6.1.1655

Johann Bernoulli
Univ. Basel, c.1691

Leonhard Euler
Univ. Basel, 1727

Joseph Louis Lagrange
c. 1755

Simeon D. Poisson
Ecole Polytechnique

Jean B. J. Fourier
Ecole Normale

Johann Peter G. L. Dirichlet
PhD: Rheinische Fr.-W.-Univ. Bonn, 1827

Rudolf O. S. Lipschitz
PhD: Univ. Berlin, 1853

Julius Plücker
PhD: Univ. Marburg, 1823

Felix Christian Klein
PhD: Rheinische Fr.-W.-Univ. Bonn, 1868

Carl Louis F. von Lindemann
PhD: Univ. Erlangen-Nürnberg, 1873

Arnold Sommerfeld
PhD: Univ. Königsberg, 1891

Werner Heisenberg
PhD: LMU München, 1908

Edward Teller
PhD: Univ. Leipzig, 1930

Lincoln Wolfenstein
PhD: Univ. of Chicago, 1949

Barry R. Holstein
PhD: Carnegie Mellon, 1969

Daniel Wyler
PhD: Carnegie Mellon, 1977

John F. Donoghue
PhD: UMass, 1976

A. Äppli (1991), H. Simma (1992), S. Dürr (1997), R. Zwicky (2003), S. Schilling (2005), TH (2006), ...

Outline

- Introduction and theoretical framework of non-leptonic B decays
- Motivation for NNLO calculation
- Two-loop techniques in a nut-shell
- Results on tree-dominated $B \to \pi\pi$, $\pi\rho$, $\rho\rho$ decays
- Conclusion

Introduction

- Non-leptonic B decays offer a rich and interesting phenomenology
  - Large data sets from B-factories, in the future from LHCb, possibly SuperB
  - $\mathcal{O}(100)$ final states. Numerous observables: BR, CP asymetries, polarisations ...
  - Test of CKM mechanism (CP violation), New Physics?

Theory (here QCDF)

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.5 \pm 1.0) \times 10^{-6}$$
$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.0 \pm 1.2) \times 10^{-6}$$
$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (0.73 \pm 0.54) \times 10^{-6}$$

[Beneke, Jäger'05]

$$\mathcal{B}(\bar{B}^0 \to \rho^0\rho^0) = (0.9 \pm 1.4) \times 10^{-6}$$

[Beneke, Rohrer, Yang'06]

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^+\pi^-) = 0.103$$
$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^0\pi^0) = -0.190$$

[Beneke, Neubert'03]

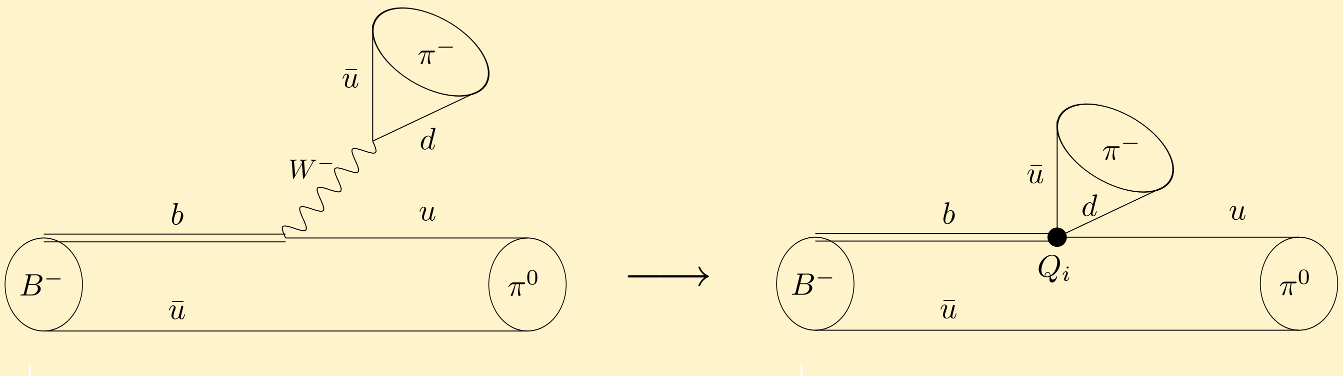
Experiment

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.59^{+0.41}_{-0.40}) \times 10^{-6}$$
$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6}$$
$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (1.55 \pm 0.19) \times 10^{-6}$$
$$\mathcal{B}(\bar{B}^0 \to \rho^0\rho^0) = (0.73^{+0.27}_{-0.28}) \times 10^{-6}$$
$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^+\pi^-) = 0.38 \pm 0.06$$
$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^0\pi^0) = 0.43^{+0.25}_{-0.24}$$

[PDG'08, HFAG'09]

- Problems with "colour-suppressed" tree-dominated decays (e. g. $\bar{B}^0 \to \pi^0\pi^0$).

Effective theory for B decays



- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

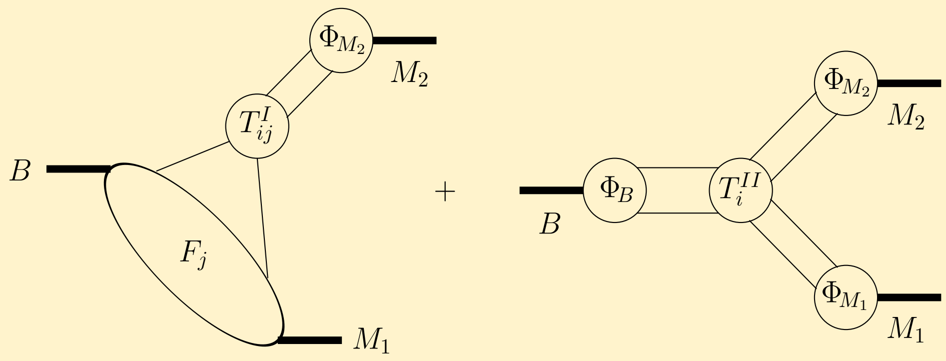
$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2} m_b\, \bar{d}_L\, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

- To be supplemented by evanescent operators (vanish in 4 dim., but not in D dim.)
  - Required to make the system closed under renormalisation
- Can use naïvely anticommuting γ_5 in dim. reg. in CMM basis

QCD factorisation



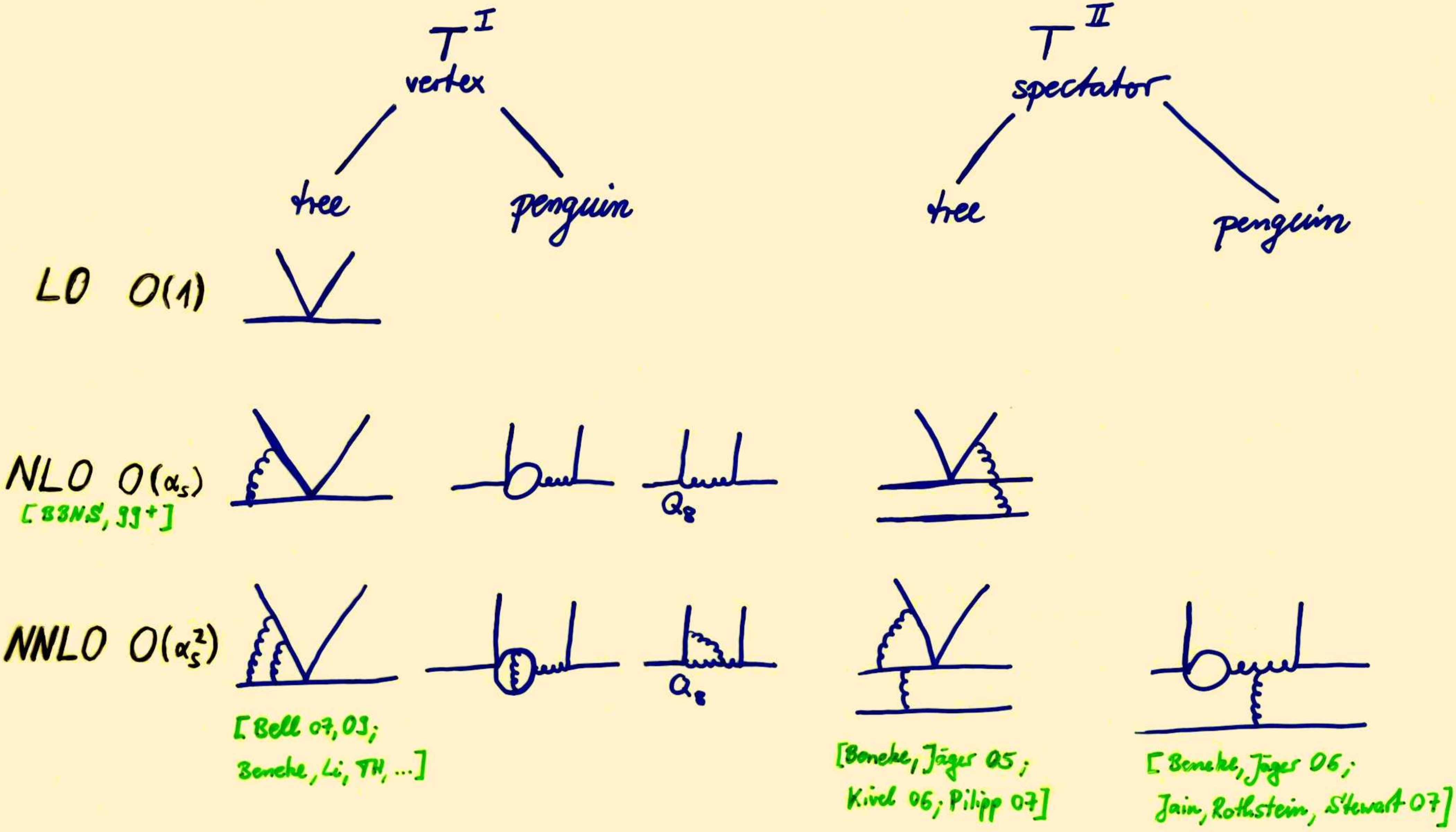
- Theoretical description of non-leptonic B decays difficult due to complicated QCD effects in the purely hadronic final state
- Simplification in the limit $m_b \gg \Lambda_{\rm QCD}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \; F_+^{B\to M_1}(0) \; f_{M_2} \int_0^1 du \;\; T_i^I(u) \, \phi_{M_2}(u)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^1 d\omega dv du \;\; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable. $T^{II} = \mathcal{O}(\alpha_s)$
- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes

QCD factorisation



moreover: „right" vs. „wrong" insertion

QCD factorisation, motivation for NNLO

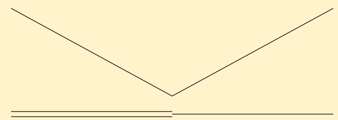
$$\sqrt{2}\,\langle \pi^- \pi^0 | \,\mathcal{H}_{eff}\, | B^- \rangle = \lambda_u \big[\alpha_1(\pi\pi) + \alpha_2(\pi\pi) \big] \; A_{\pi\pi}$$

$$\langle \pi^+ \pi^- | \,\mathcal{H}_{eff}\, | \bar{B}^0 \rangle = \big\{ \lambda_u \big[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi) \big] + \lambda_c\, \alpha_4^c(\pi\pi) \big\} \; A_{\pi\pi}$$

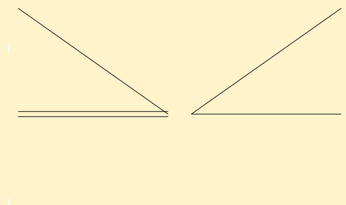
$$-\,\langle \pi^0 \pi^0 | \,\mathcal{H}_{eff}\, | \bar{B}^0 \rangle = \big\{ \lambda_u \big[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi) \big] - \lambda_c\, \alpha_4^c(\pi\pi) \big\} \; A_{\pi\pi}$$

[Beneke,Neubert'03]

- α_1 : colour-allowed tree amplitude, "right insertion"



- α_2 : colour-suppressed tree amplitude, "wrong insertion"



- NLO results

$$\alpha_1(\pi\pi) \;=\; 1.009 + [0.023 + 0.010\, i]_{\rm NLO} - \left[\tfrac{r_{\rm sp}}{0.445}\right] \Big\{ [0.014]_{\rm LOsp} + [0.008]_{\rm tw3} \Big\} = 1.010 + 0.010 i$$

$$\alpha_2(\pi\pi) \;=\; 0.220 - [0.179 + 0.077\, i]_{\rm NLO} + \left[\tfrac{r_{\rm sp}}{0.445}\right] \Big\{ [0.114]_{\rm LOsp} + [0.067]_{\rm tw3} \Big\} = 0.222 - 0.077 i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

QCD factorisation, motivation for NNLO

- NLO results

$$\alpha_1(\pi\pi) \;=\; 1.009 + [0.023 + 0.010\,i]_{\rm NLO} - \left[\tfrac{r_{\rm sp}}{0.445}\right] \Big\{[0.014]_{\rm LOsp} + [0.008]_{\rm tw3}\Big\} = 1.010 + 0.010i$$

$$\alpha_2(\pi\pi) \;=\; 0.220 - [0.179 + 0.077\,i]_{\rm NLO} + \left[\tfrac{r_{\rm sp}}{0.445}\right] \Big\{[0.114]_{\rm LOsp} + [0.067]_{\rm tw3}\Big\} = 0.222 - 0.077i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

- Large cancellation in LO + NLO in α_2, particularly sensitive to NNLO
- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$, NNLO is only the first correction
- Q: Does factorization hold? Does NNLO QCDF tend toward the right direction?

- Goal: $\mathcal{O}(\alpha_s^2)$ vertex corrections to α_1 and α_2 $\Leftrightarrow$ 2-loop matrix elements of Q_1, Q_2

Matching of QCD onto SCET

- Consider matrix element of $Q_1 = (\bar{d}_L \gamma^\mu T^a u_L)(\bar{u}_L \gamma_\mu T^a b_L)$. One needs

$$\langle Q_1 \rangle = \sum_a H_{1a}\, O_a \quad \text{if} \quad M_2 = [\bar{d}u] \qquad \text{right insertion}$$

$$\langle Q_1 \rangle = \sum_a \tilde{H}_{1a}\, \tilde{O}_a \quad \text{if} \quad M_2 = [\bar{u}u] \qquad \text{wrong insertion}$$

- O_1 is the only physical SCET operator. It factorizes into form factor times LCDA
- Fierz$(\tilde{O}_1) = O_1$ in $D = 4$
- $O_{2,3}$, $\tilde{O}_{2,3}$ and $\tilde{O}_1 - O_1$ are evanscent and must be renormalized to zero

Right insertion

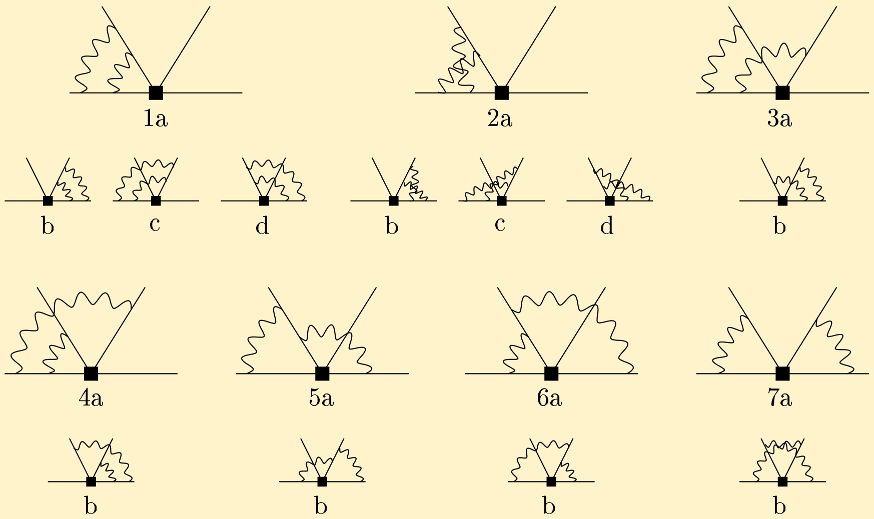
$$\begin{aligned}
O_1 &= \left[\bar{\chi}\, \frac{\not{n}_-}{2} (1-\gamma_5)\chi\right] \left[\bar{\xi}\, \not{n}_+ (1-\gamma_5) h_v\right] \\
O_2 &= \left[\bar{\chi}\, \frac{\not{n}_-}{2} (1-\gamma_5)\gamma_\perp^\alpha \gamma_\perp^\beta \chi\right] \left[\bar{\xi}\, \not{n}_+ (1-\gamma_5)\gamma^\perp_\beta \gamma^\perp_\alpha h_v\right] \\
O_3 &= \left[\bar{\chi}\, \frac{\not{n}_-}{2} (1-\gamma_5)\gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi\right] \left[\bar{\xi}\, \not{n}_+ (1-\gamma_5)\gamma^\perp_\delta \gamma^\perp_\gamma \gamma^\perp_\beta \gamma^\perp_\alpha h_v\right]
\end{aligned}$$

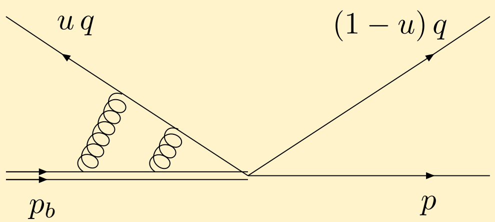
Wrong insertion

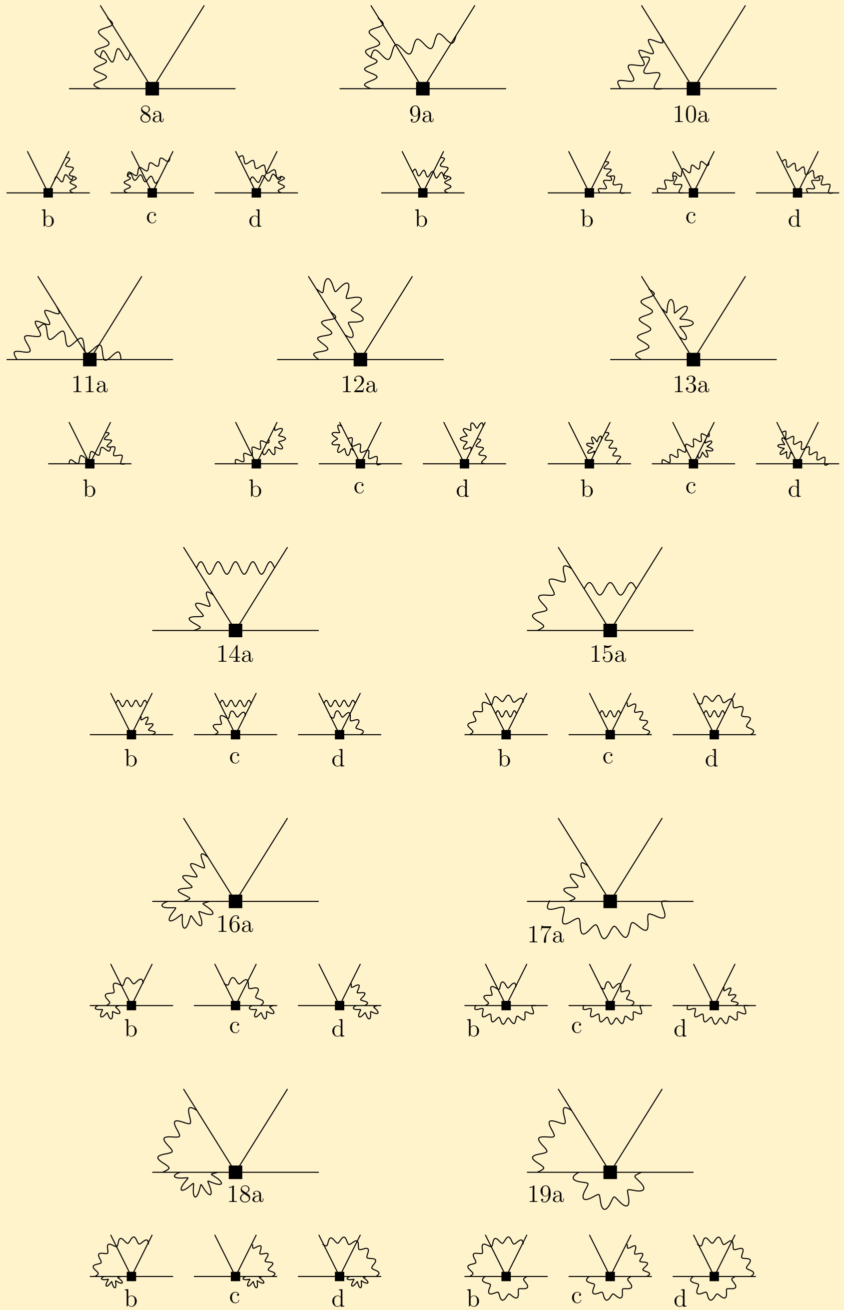
$$\begin{aligned}
\tilde{O}_1 &= \left[\bar{\xi}\, \gamma_\perp^\alpha (1-\gamma_5)\chi\right] \left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha h_v\right] \\
\tilde{O}_2 &= \left[\bar{\xi}\, \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma (1-\gamma_5)\chi\right] \left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha \gamma^\perp_\gamma \gamma^\perp_\beta h_v\right] \\
\tilde{O}_3 &= \left[\bar{\xi}\, \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \gamma_\perp^\epsilon (1-\gamma_5)\chi\right] \left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha \gamma^\perp_\epsilon \gamma^\perp_\delta \gamma^\perp_\gamma \gamma^\perp_\beta h_v\right]
\end{aligned}$$

Two-loop diagrams

- Non-factorizable two-loop diagrams for non-leptonic B-decays *[Beneke,Buchalla,Neubert,Sachrajda'00]*



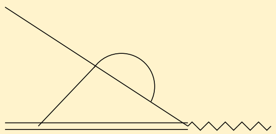




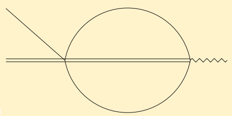
- Kinematics: $p_b^2 = m_b^2$, $q^2 = 0$, $p^2 = 0$ or $p^2 = m_c^2$

Multi-loop techniques in a nut-shell

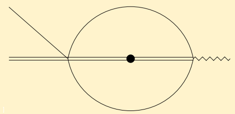
- Dimensional regularisation with $D = 4 - 2\epsilon$ regulates UV and IR. Poles up to $1/\epsilon^4$.
- Passarino-Veltman reduction of tensor integrals to scalar integrals *[Passarino,Veltman'79]*
- Reduction of scalar integrals to a small set of master integrals
  - Integration-by-parts and Lorentz-invariance identities *[Tkachov'81; Chetyrkin,Tkachov'81; Gehrmann,Remiddi'99]*
  - System of equations solved by Laporta algorithm *[Laporta'01;Anastasiou,Lazopoulos'04;Smirnov'08]*



$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$



$$-\frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$

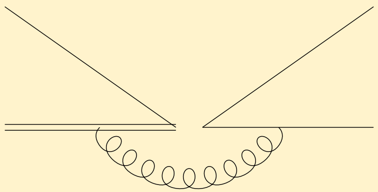


- Techniques for the evaluation of the 42 master integrals
  - Hypergeometric functions, ϵ-expansion in Mathematica or Form *[Moch,Uwer'05; Maitre,TH'05,'07]*
  - Differential equations *[Kotikov'91; Remiddi'97]*
  - Mellin-Barnes representations *[Smirnov'99; Tausk'99; Czakon'05; Gluza,Kajda,Riemann'07]*

Master formula, hard scattering kernel for α_2

$$
\begin{aligned}
\widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} \\
&\quad + \underbrace{\widetilde{A}_{i1}^{(1)\mathrm{f}} - A^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} \\
&\quad + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}} + Z_{ext}^{(1)}\big[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}\big] \\
&\quad - \widetilde{T}_i^{(1)}\big[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}\big] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
&\quad + [\widetilde{A}_{i1}^{(2)\mathrm{f}} - A^{(2)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
&\quad + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\mathrm{f}} - A^{\prime(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
&\quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)})[\widetilde{A}_{i1}^{(1)\mathrm{f}} - A^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
&\quad - (C_{FF}^{(1)} - \xi_{45}^{(1)})[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} \\
&\quad - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)}
\end{aligned}
$$



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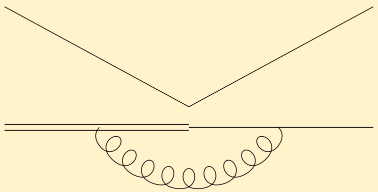


$$
\begin{aligned}
T_1^{(2),re} &= \frac{(47u^5 - 278u^4 + 1223u^3 - 2316u^2 + 2036u - 652)\ln^4(1-u)}{162(u-1)^2u^3} \\
&- \frac{(2u^3 + 4u^2 + 173u + 16)\ln^3(1-u)}{81u} - \frac{(4u^3 - 61u^2 - 436u + 16)\ln^2(1-u)}{54u^2} \\
&+ \frac{2\,(73u^5 + 38u^4 - 1103u^3 + 2316u^2 - 2036u + 652)\ln(u)\ln^3(1-u)}{81(u-1)^2u^3} \\
&- \frac{(17u^3 + 300u^2 - 1098u + 978)\ln^2(u)\ln^2(1-u)}{27u^3} \\
&- \frac{\pi^2\,(9u^5 + 166u^4 - 1167u^3 + 2316u^2 - 2036u + 652)\ln^2(1-u)}{81(u-1)^2u^3} \\
&+ \frac{(2u^5 - 20u^3 + 125u^2 - 76u - 52)\ln(u)\ln^2(1-u)}{27(u-1)^2u} + \frac{2}{9}\ln^3(u)\ln(1-u) \\
&+ \frac{7(u-2)^2\ln(2-u)\ln^2(1-u)}{9(u-1)^2} + \frac{16}{9}\mathrm{Li}_2(u)\ln^2(1-u) \\
&+ \frac{(2u^6 + 4u^5 - 191u^4 - 167u^3 + 1022u^2 - 646u - 6)\ln^2(u)\ln(1-u)}{27(u-1)u^3} \\
&- \frac{\pi^2\,(2u^5 + 355u^3 - 623u^2 + 385u - 140)\ln(1-u)}{81(u-1)^2u} \\
&- \frac{(4u^4 - 638u^3 + 1487u^2 - 1597u + 664)\ln(u)\ln(1-u)}{27(u-1)u^2} \\
&+ \frac{14(u-2)^2\mathrm{Li}_2(u-1)\ln(1-u)}{9(u-1)^2} + \frac{16\,(6u^2 - 16u - 5)\,\mathrm{Li}_3(u)\ln(1-u)}{27(u-1)^2} \\
&- \frac{2\,(94u^3 - 271u^2 + 166u + 32)\,\mathrm{Li}_2(u)\ln(1-u)}{27(u-1)^2u} + \frac{(1601u - 1172)\ln(1-u)}{54u} \\
&+ \frac{4\,(4u^3 - 50u^2 + 183u - 163)\ln(u)\mathrm{Li}_2(u)\ln(1-u)}{27u^3} \\
&+ \frac{(2u^3 - 436u^2 + 657u - 332)\ln^2(u)}{27(u-1)u} - \frac{8\,(3u^2 - 14u - 19)\,\zeta(3)\ln(1-u)}{27(u-1)^2} \\
&+ \frac{2\,(20u^5 - 94u^4 + 292u^3 - 579u^2 + 509u - 163)\,\mathrm{Li}_2(u)^2}{27(u-1)^2u^3} \\
&+ \frac{\pi^2\,(4u^4 - 435u^3 + 3174u^2 - 5346u + 2688)}{162(u-1)u^2} + \frac{64}{9}\mathrm{Li}_3(1-u)\ln(1-u)
\end{aligned}
$$

+ … (five pages)

Numerical Results

- Convolution of hard scattering kernels with pion LCDA yields topological tree amplitudes $\alpha_1(\pi\pi)$ and $\alpha_2(\pi\pi)$ to NNLO

- Have expressions for $\alpha_1(\pi\pi)$ and $\alpha_2(\pi\pi)$ completely analytically, including m_c dependence

$$\alpha_1(\pi\pi) \supset \ldots + 8194\zeta_5 - 2028\pi^2\zeta_3 - \ln^3\left(\frac{4z}{\left(\sqrt{4z+1}+1\right)^2}\right)$$
$$-12\,\mathrm{Li}_3\left(\frac{4z}{\left(\sqrt{4z+1}+1\right)^2}\right) + 2\,\mathrm{Li}_3\left(\frac{2\sqrt{z}}{\sqrt{z}+1}\right) + \ldots (3\,\text{pages})$$

- We find complete agreement (numerically) with G. Bell *[G. Bell'09]*

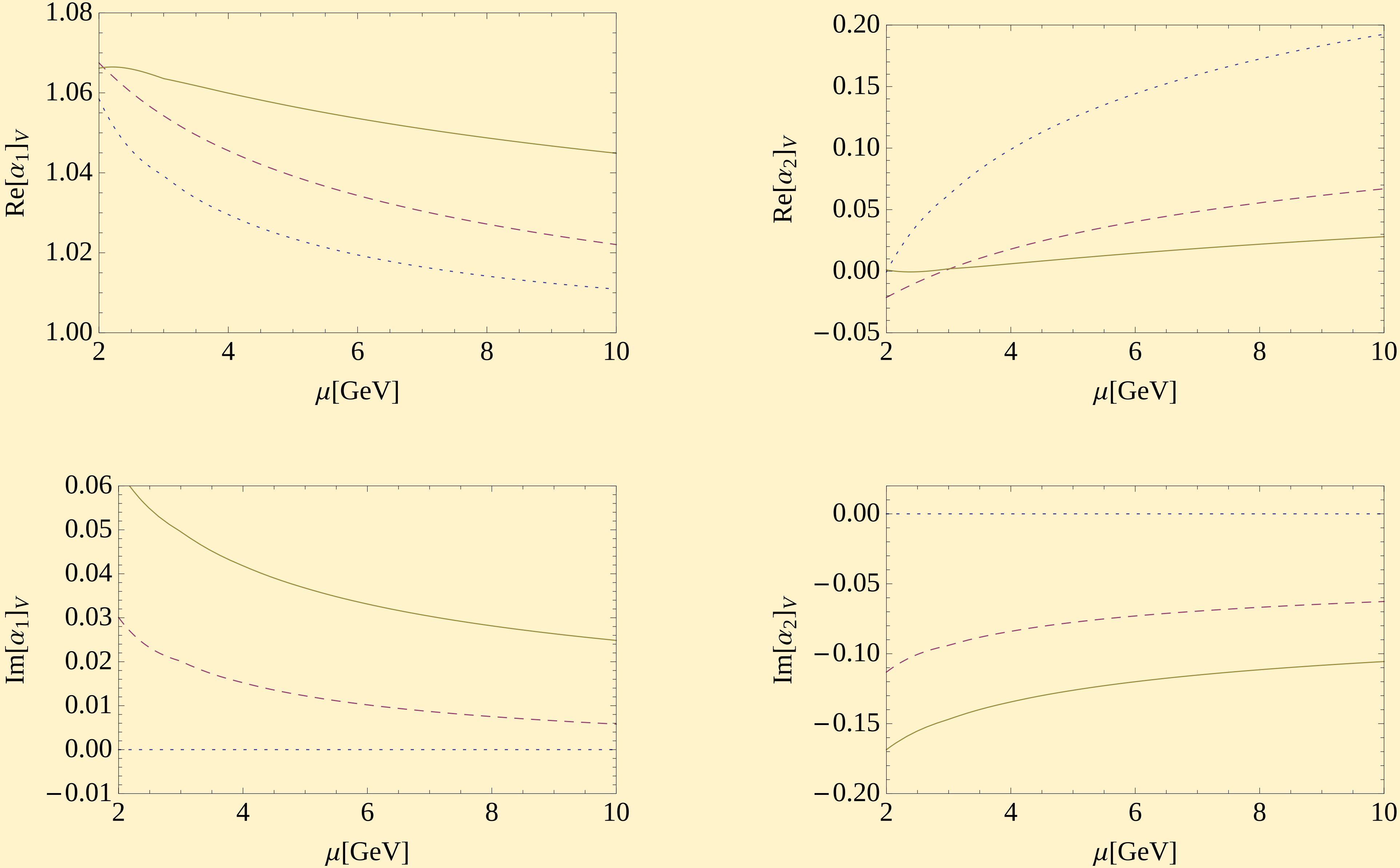
$$\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\text{NLO}} + [0.026 + 0.028\,i]_{\text{NNLO}} \\
&\quad - \left[\frac{r_{\text{sp}}}{0.445}\right]\left\{[0.014]_{\text{LOsp}} + [0.034 + 0.027\,i]_{\text{NLOsp}} + [0.008]_{\text{tw3}}\right\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})\,i
\end{aligned}$$

$$r_{\text{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b \lambda_B f_+^{B\pi}(0)}$$

$$\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\text{NLO}} - [0.031 + 0.050\,i]_{\text{NNLO}} \\
&\quad + \left[\frac{r_{\text{sp}}}{0.445}\right]\left\{[0.114]_{\text{LOsp}} + [0.049 + 0.051\,i]_{\text{NLOsp}} + [0.067]_{\text{tw3}}\right\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})\,i
\end{aligned}$$

- NNLO corrections to vertex and spectator terms significant but tend to cancel! ☹

Renormalization scale dependence



Factorisation test

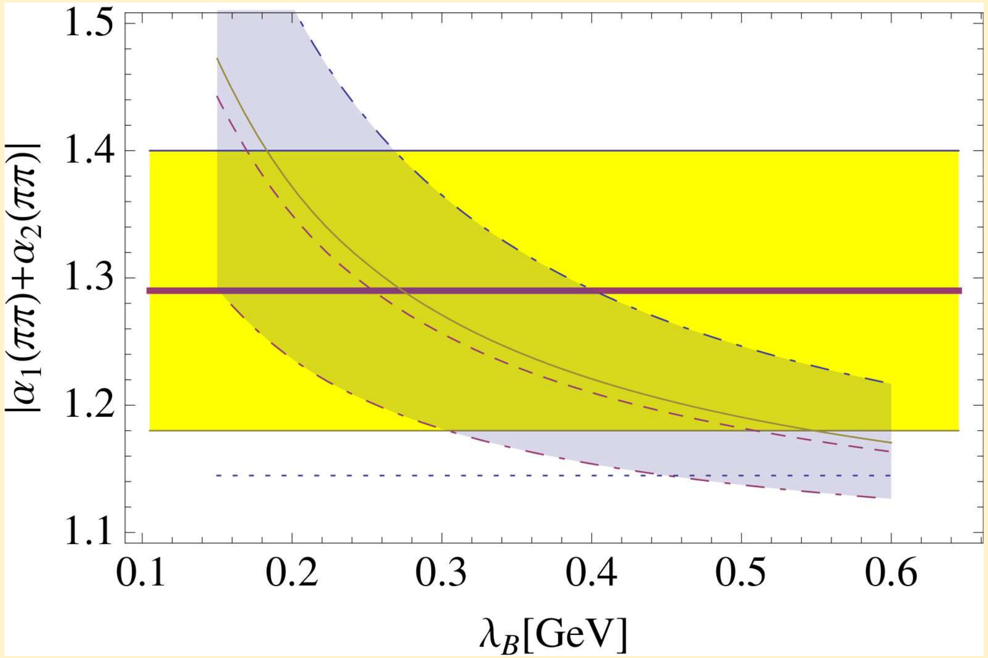
$$R \equiv \frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+\ell^-\bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2\,|V_{ud}|^2\,|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|^2$$

- From semi-leptonic data
  [cf. Becher,Hill'05; Ball'06; BaBar'06]

  $$|V_{ub}|f_+^{B\pi}(0) = (9.1 \pm 0.7)\times 10^{-4}$$

  equivalent to

  $$|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|_{\rm exp} = 1.29 \pm 0.11$$



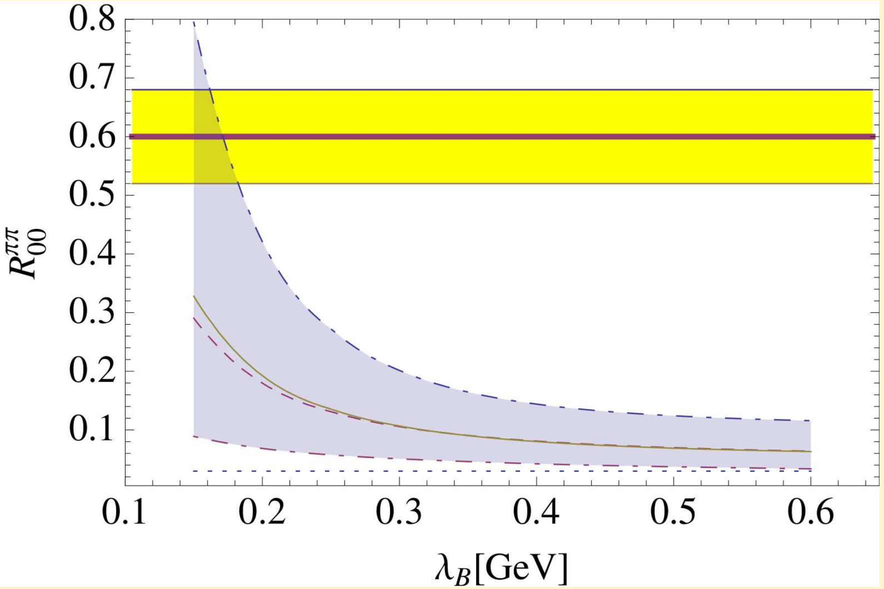
- Good agreement with theory supports QCDF approach

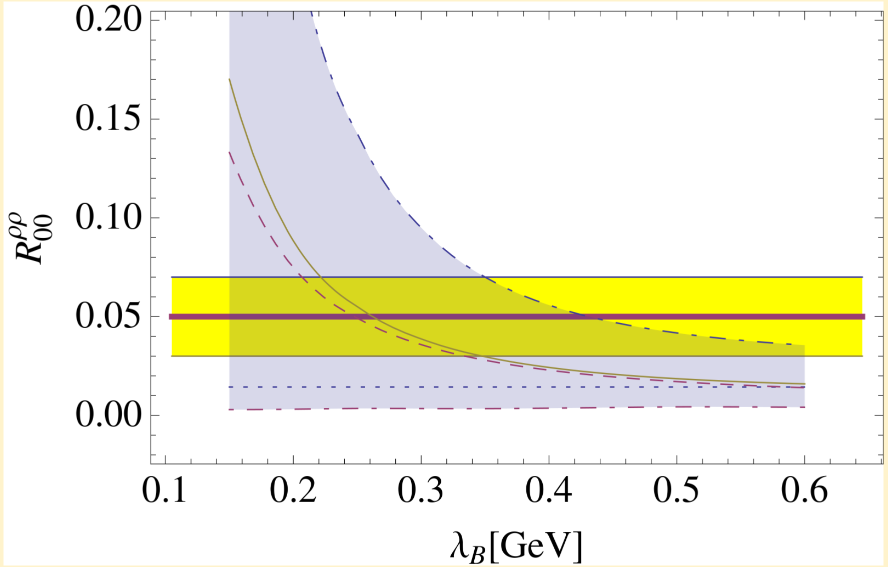
  $$|\alpha_1(\pi\pi)+\alpha_2(\pi\pi)|_{\rm th.} = 1.24^{+0.16}_{-0.10}$$

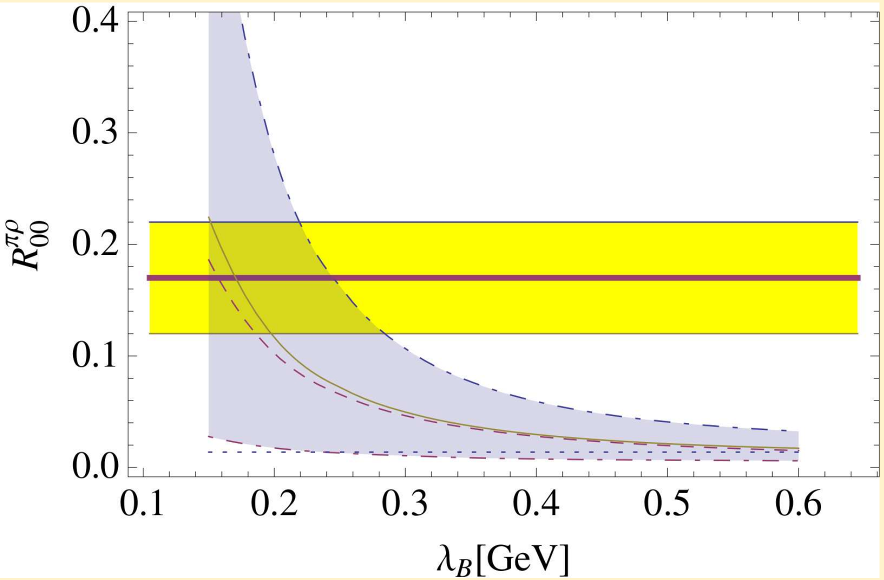
- Central exptl. value allows $\lambda_B \in [150, 400]$ MeV (on lower side of expectations).

[for phenomenologcal applications, see also Bell,Pilipp'09]

Ratios involving colour-suppressed decays







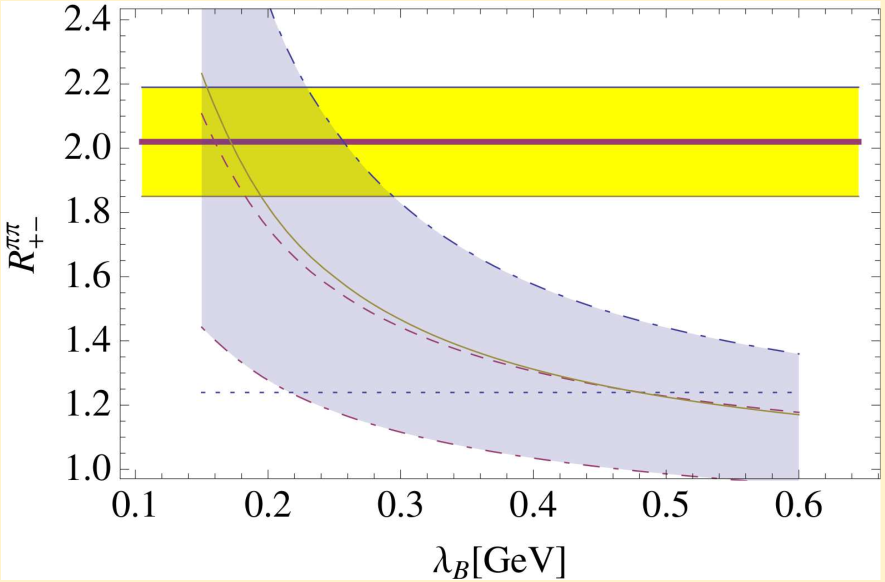
$$R_{00}^{\pi\pi} = 2\,\frac{\Gamma(B^0 \to \pi^0\pi^0)}{\Gamma(B^0 \to \pi^+\pi^-)}$$

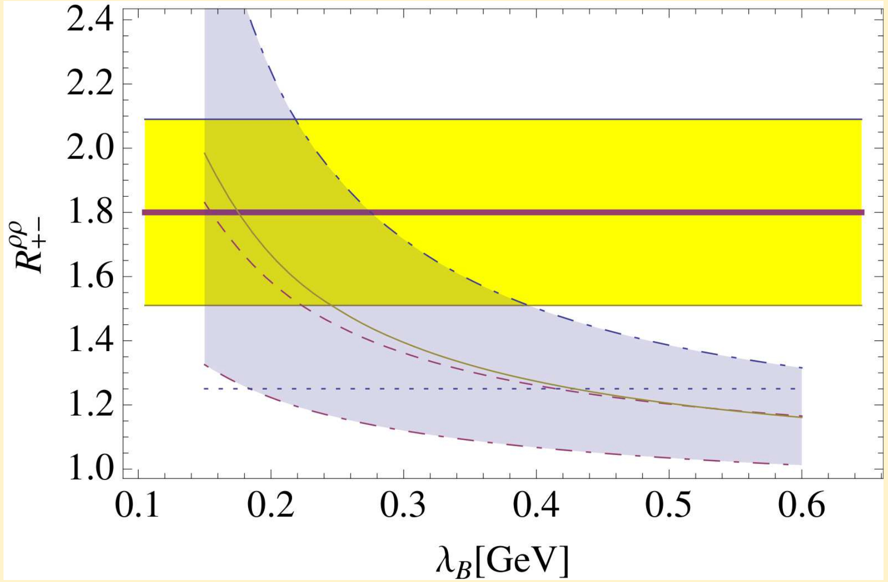
$$R_{00}^{\rho\rho} = 2\,\frac{\Gamma(B^0 \to \rho_L^0\rho_L^0)}{\Gamma(B^0 \to \rho_L^+\rho_L^-)}$$

$$R_{00}^{\pi\rho} = \frac{2\,\Gamma(B^0 \to \pi^0\rho^0)}{\Gamma(B^0 \to \pi^+\rho^-) + \Gamma(B^0 \to \pi^-\rho^+)}$$

Preference for small λ_B, i.e. strong spectator scattering, as already found at NLO in *[Beneke,Neubert'03]*

More ratios



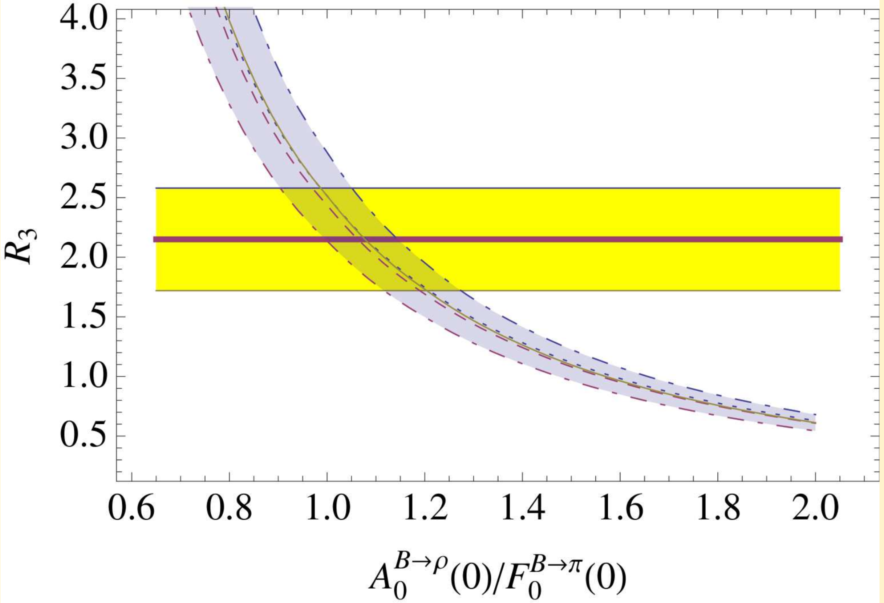


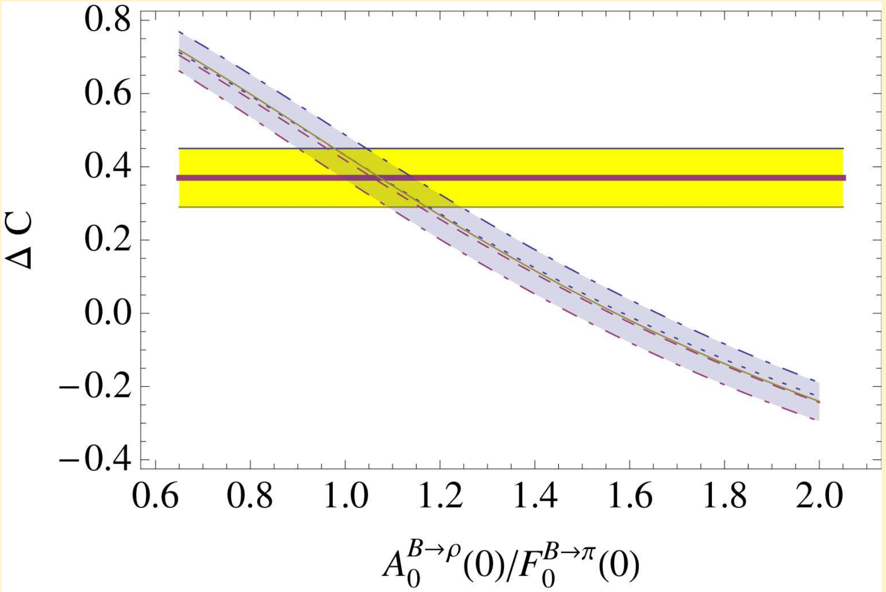
$$R_{+-}^{\pi\pi} = 2\,\frac{\Gamma(B^{\pm} \to \pi^{\pm}\pi^{0})}{\Gamma(B^{0} \to \pi^{+}\pi^{-})}$$

$$R_{+-}^{\rho\rho} = 2\,\frac{\Gamma(B^{\pm} \to \rho_L^{\pm}\rho_L^{0})}{\Gamma(B^{0} \to \rho_L^{+}\rho_L^{-})}$$

Also here: Preference for small $\lambda_B \simeq 200$ MeV

Dependence on form factors





$$R_3 = \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-)}{\Gamma(\bar{B}^0 \to \pi^-\rho^+)}$$

$$\Delta C = \frac{1}{2}\left[C(\pi^-\rho^+) - C(\pi^+\rho^-)\right]$$

- Default values
  - $A_0^{B\to\rho}(0)/F_0^{B\to\pi}(0) = 1.2$
  - $F_0^{B\to\pi}(0) = 0.25$ (fixed)
- Agreement excellent for $A_0^{B\to\rho}(0)/F_0^{B\to\pi}(0) \in [1.0, 1.2]$

Final numerical results

[See also Bell,Pilipp'09]

	Theory I	Theory II	Exp.
$B^- \to \pi^-\pi^0$	$5.43\,^{+0.06\,+1.45}_{-0.06\,-0.84}$	$5.82\,^{+0.07\,+1.42}_{-0.06\,-1.35}$	$5.59^{+0.41}_{-0.40}$
$\bar{B}_d^0 \to \pi^+\pi^-$	$7.37\,^{+0.86\,+1.22}_{-0.69\,-0.97}$	$5.70\,^{+0.70\,+1.16}_{-0.55\,-0.97}$	5.16 ± 0.22
$\bar{B}_d^0 \to \pi^0\pi^0$	$0.33\,^{+0.11\,+0.42}_{-0.08\,-0.17}$	$0.63\,^{+0.12\,+0.64}_{-0.10\,-0.42}$	1.55 ± 0.19
$B^- \to \pi^-\rho^0$	$8.68\,^{+0.42\,+2.71}_{-0.41\,-1.56}$	$9.84\,^{+0.41\,+2.54}_{-0.40\,-2.52}$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38\,^{+0.90\,+2.18}_{-0.77\,-1.41}$	$12.13\,^{+0.85\,+2.23}_{-0.73\,-2.17}$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08\,^{+0.27\,+3.82}_{-0.19\,-3.50}$	$21.90\,^{+0.20\,+3.06}_{-0.12\,-3.55}$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52\,^{+0.04\,+1.11}_{-0.03\,-0.43}$	$1.49\,^{+0.07\,+1.77}_{-0.07\,-1.29}$	2.0 ± 0.5
$B^- \to \rho_L^-\rho_L^0$	$18.42^{+0.23+3.92}_{-0.21-2.55}$	$19.06^{+0.24+4.59}_{-0.22-4.22}$	$22.8^{+1.8}_{-1.9}$
$\bar{B}_d^0 \to \rho_L^+\rho_L^-$	$25.98^{+0.85+2.93}_{-0.77-3.43}$	$20.66^{+0.68+2.99}_{-0.62-3.75}$	$23.7^{+3.1}_{-3.2}$
$\bar{B}_d^0 \to \rho_L^0\rho_L^0$	$0.39^{+0.03+0.83}_{-0.03-0.36}$	$1.05^{+0.05+1.62}_{-0.04-1.04}$	$0.55^{+0.22}_{-0.24}$
$R_{+-}^{\pi\pi}$	$1.38^{+0.12+0.53}_{-0.13-0.32}$	$1.91^{+0.18+0.72}_{-0.20-0.64}$	2.02 ± 0.17
$R_{00}^{\pi\pi}$	$0.09^{+0.03+0.12}_{-0.02-0.04}$	$0.22^{+0.06+0.28}_{-0.05-0.16}$	0.60 ± 0.08
$R_{+-}^{\rho\rho}$	$1.32^{+0.02+0.44}_{-0.03-0.27}$	$1.72^{+0.03+0.64}_{-0.03-0.53}$	$1.80^{+0.28}_{-0.29}$
$R_{00}^{\rho\rho}$	$0.03^{+0.00+0.07}_{-0.00-0.03}$	$0.10^{+0.01+0.19}_{-0.01-0.11}$	0.05 ± 0.02
$R_{00}^{\pi\rho}$	$0.04^{+0.00+0.09}_{-0.00-0.03}$	$0.14^{+0.01+0.20}_{-0.01-0.13}$	0.17 ± 0.05
R_3	$1.73^{+0.13+1.12}_{-0.12-0.82}$	$1.69^{+0.13+0.72}_{-0.12-0.59}$	2.15 ± 0.43

- Theory II: With lower λ_B and form factors. Our preferred scenario.

Conclusion

- The colour-allowed and colour-supressed tree amplitudes have been computed completely analytically to NNLO
- The two-loop computation requires sophisticated computational techniques
- The NNLO corrections are very small. Accidental cancellation between vertex and spectator term
- QCDF beyond naive factorization describes data well, especially for low λ_B and form factors. Exceptions are observables with $\pi^0\pi^0$ final state
- To do: Two-loop penguin amplitudes, CP asymmetries at NLO

On Sechseläuten day...

Famous people at the Sechseläuten Parade

Daniel as guest of honour at Zunft Höngg

“Prof. Dr. Daniel Wyler nahm am diesjährigen Sechseläuten quasi in Doppelfunktion teil: Einerseits - und aus Höngger Sicht primär - als Ehrengast der Zunft, andererseits aber in seiner Funktion als Dekan der MNF auch als Mitglied der Delegation der Universitt Zürich anlässlich derer 175-Jahr-Jubiläums. Der Ehrengast erschien daher nicht ohne Grund in historischer Kleidung: Im Zug der Zünfte stellte er später als Leiter des ITP seinen Vorgänger Prof. Erwin Schrödinger dar, welcher 1933 den Nobelpreis für seine Forschungen in der Quantenphysik erhalten hatte.”

“Prof Dr. Daniel Wyler alias Prof. Erwin Schrödinger mit alchimistischer Versuchsanordnung zur Herstellung von alkoholischem flüssigem Gold”

Famous people at the Sechseläuten Parade

[Pics courtesy of John Donoghue]

- Daniel as guest of honour at Zunft Höngg 2008

Other famous people at the Sechseläuten Parade

- At Zunft Oberstrass, 2004

Other famous people at the Sechseläuten Parade

- At Zunft St. Niklaus, 2005 and 2006

Happy Birthday, Daniel!

Cheers!

Backup slides

Penguin amplitudes to NLO

- Penguin amplitudes to NLO

$$\alpha_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [??\ +\ ??\ i]_{\mathcal{O}(\alpha_s^2)}$$
$$+ \left[\frac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.024^{+0.004}_{-0.002} + (-0.012^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_V - [0.001 + 0.007i]_P + [??\ +\ ??\ i]_{\mathcal{O}(\alpha_s^2)}$$
$$+ \left[\frac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.028^{+0.005}_{-0.003} + (-0.006^{+0.003}_{-0.002})i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

Some definitions

$$
\begin{aligned}
A_{\pi\pi} &= i\,\frac{G_F}{\sqrt{2}}\, m_B^2 F_+^{B\to\pi}(0) f_\pi \\
r_{\rm sp} &= \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)} \\
\lambda_B^{-1} &= \int_0^\infty \frac{d\omega}{\omega}\, \phi_B(\omega,\mu)
\end{aligned}
$$