The old, the abundant and the ugly: why we still work hard on QCD

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"The new, the rare and the beautiful"
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Why bother with old/plentyful/messy stuff a.k.a. QCD ?

Tentative answer #1

... because it's there! [British alpinist George Leigh Mallory in 1923]

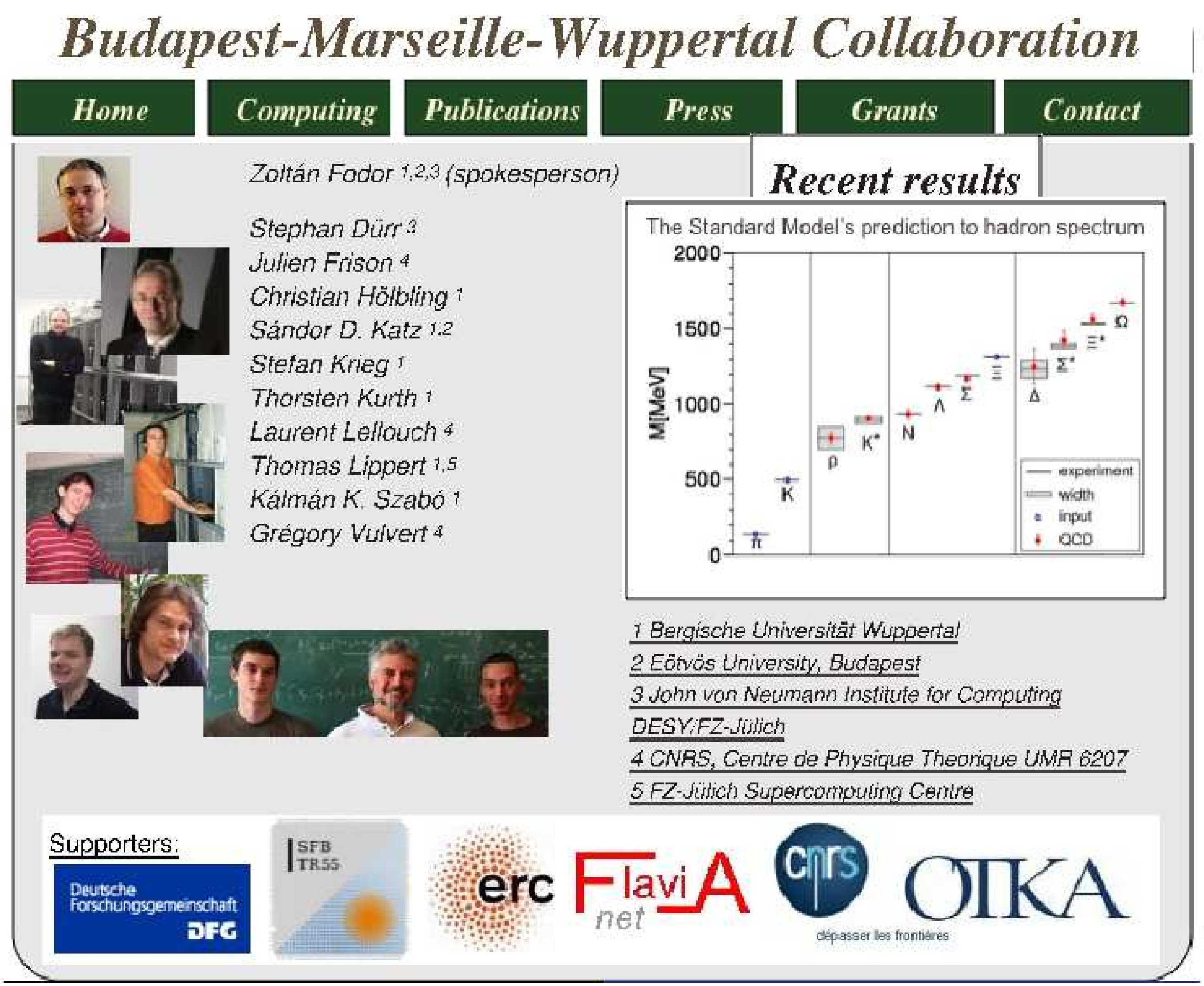
Tentative answer #2

... because it helps us to tell the old from the new !

Tentative answer #3

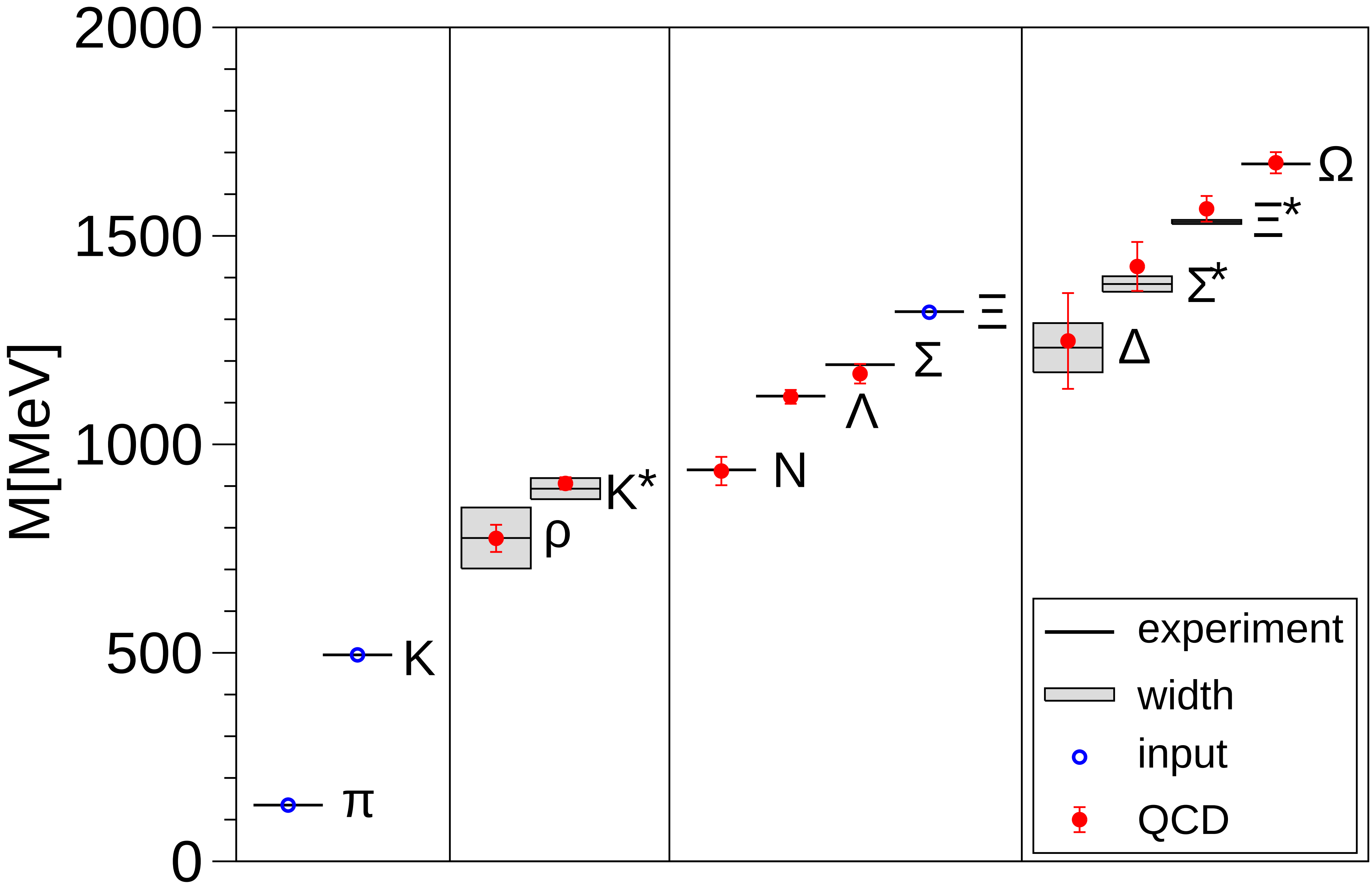
... because it gets us ready for technicolor/whatever_it_is !

Budapest-Marseille-Wuppertal collaboration



S. Dürr, Z. Fodor (spokesperson), C. Hoelbling, S.D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, A. Ramos, K.K. Szabo, G. Vulvert

QCD spectrum: BMW collaboration, Science 322, 1224 (2008)



Lattice QCD basics (1)

Elementary degrees of freedom are quarks and gluons, transforming in the fundamental representation of $SU(3)$ [Fritzsch, Gell-Mann and Leutwyler (1973)]. In euclidean space:

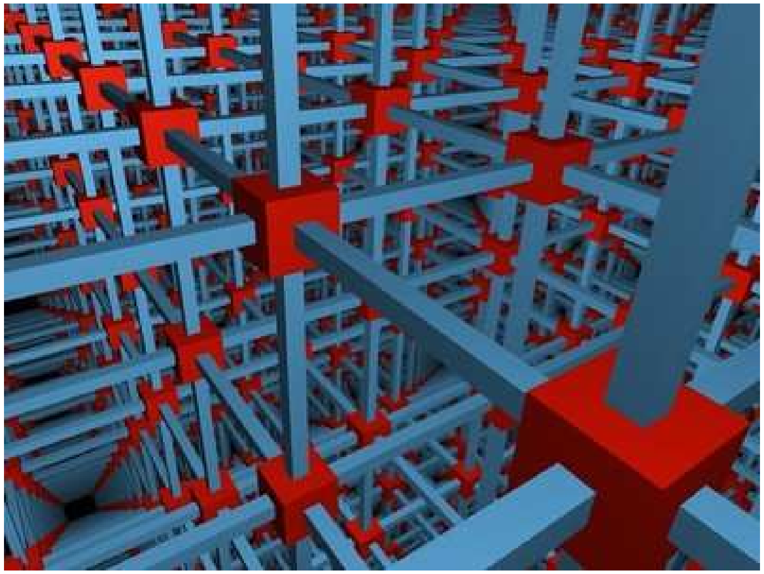
$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_{i=1}^{N_f} \bar{q}^{(i)}(\not{D}+m^{(i)})q^{(i)} + \mathrm{i}\theta\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}(F_{\mu\nu}F_{\rho\sigma})$$

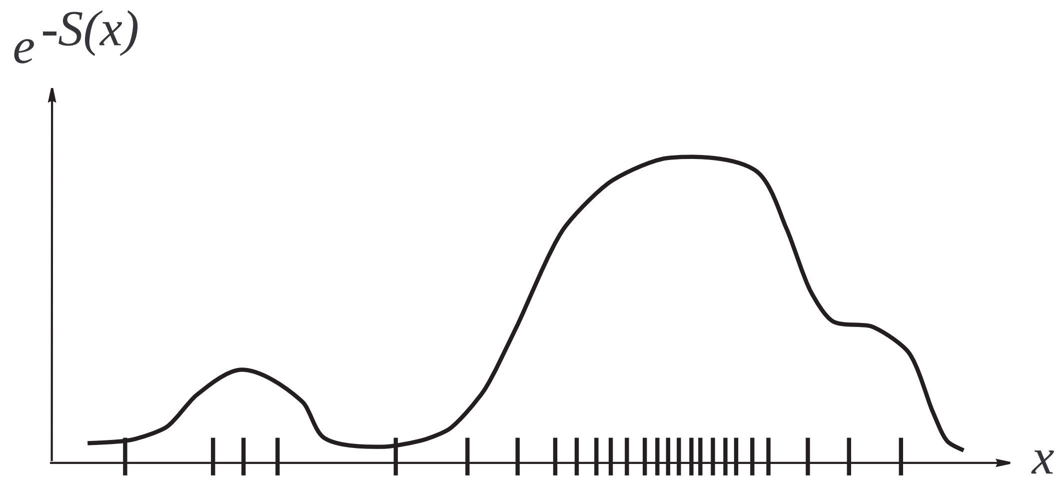
- QCD must be regulated both in the UV and in the IR.
- The lattice does this by $a>0$ and $V=L^4<\infty$, but other options are (in principle) possible. In fact, different gauge/fermion actions represent such options.
- The extrapolations $a\to 0$ and $V\to\infty$ are performed in the resulting observables.
- The result is independent of the action, thanks to universality (spin sys., RG, FP).

$\Longrightarrow$ Lattice discretization is not an approximation to continuous space-time, but (generically) an *unavoidable interim part* of the definition of QCD !

$\Longrightarrow$ Does this Lagrangian-regulator-extrapolation *package* explain confinement, chiral/conformal symmetry breaking, hadron spectrum, ... ?

Lattice QCD basics (2)





- Define space-time as regular 4D grid (spacing a) with periodic boundary conditions.
- Put matter fields on sites: scalar $\phi(x)$ or spinor $\psi(x)$ with $x=(an_1,..,an_4)$.
- Put gauge fields on links: photon or gluon within $U_\mu(x)=\exp(\mathrm{i}\int_x^{x+\hat{\mu}} A_\mu(x')dx')$.
- Define gluon and fermion action with correct weak-coupling limit and $S=S_G+S_F$.
- Define $Z=\int DU D\bar{\psi}D\psi\ \exp(-S[U,\bar{\psi},\psi])$ via integration over *all* field variables.
- Use methods from statistical mechanics to sample *relevant* field configurations.

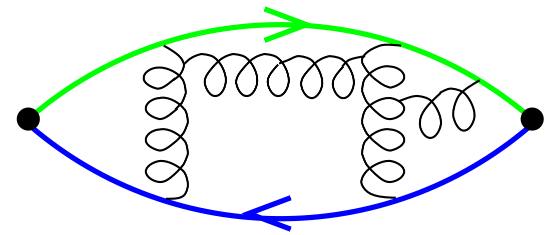
Lattice QCD spectroscopy (1)

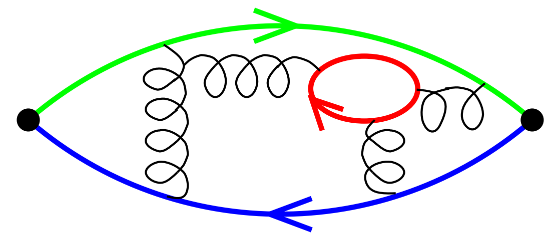
Hadronic correlator in $N_f \geq 2$ QCD: $C(t) = \int d^4x\, C(t,\mathbf{x})\, e^{i\mathbf{px}}$ with

$$C(x) = \langle O(x)\, O(0)^\dagger \rangle = \frac{1}{Z} \int DU\, D\bar{q} Dq\;\; O(x)\, O(0)^\dagger\;\; e^{-S_G - S_F}$$

where $O(x) = \bar{d}(x)\Gamma u(x)$ and $\Gamma = \gamma_5, \gamma_4\gamma_5$ for $\pi^\pm$ and
$S_G = \beta \sum (1 - \frac{1}{3}\mathrm{Re}\mathrm{Tr}\, U_{\mu\nu}(x))$, $S_F = \sum \bar{q}(D+m)q$

$$\langle \bar{d}(x)\Gamma_1 u(x)\; \bar{u}(0)\Gamma_2 d(0) \rangle = \frac{1}{Z}\int DU\;\det(D+m)^{N_f}\; e^{-S_G}$$
$$\times\, \mathrm{Tr}\left\{ \Gamma_1 (D+m)^{-1}_{x0}\, \Gamma_2\, \underbrace{(D+m)^{-1}_{0x}}_{\gamma_5[(D+m)^{-1}_{x0}]^\dagger\gamma_5} \right\}$$

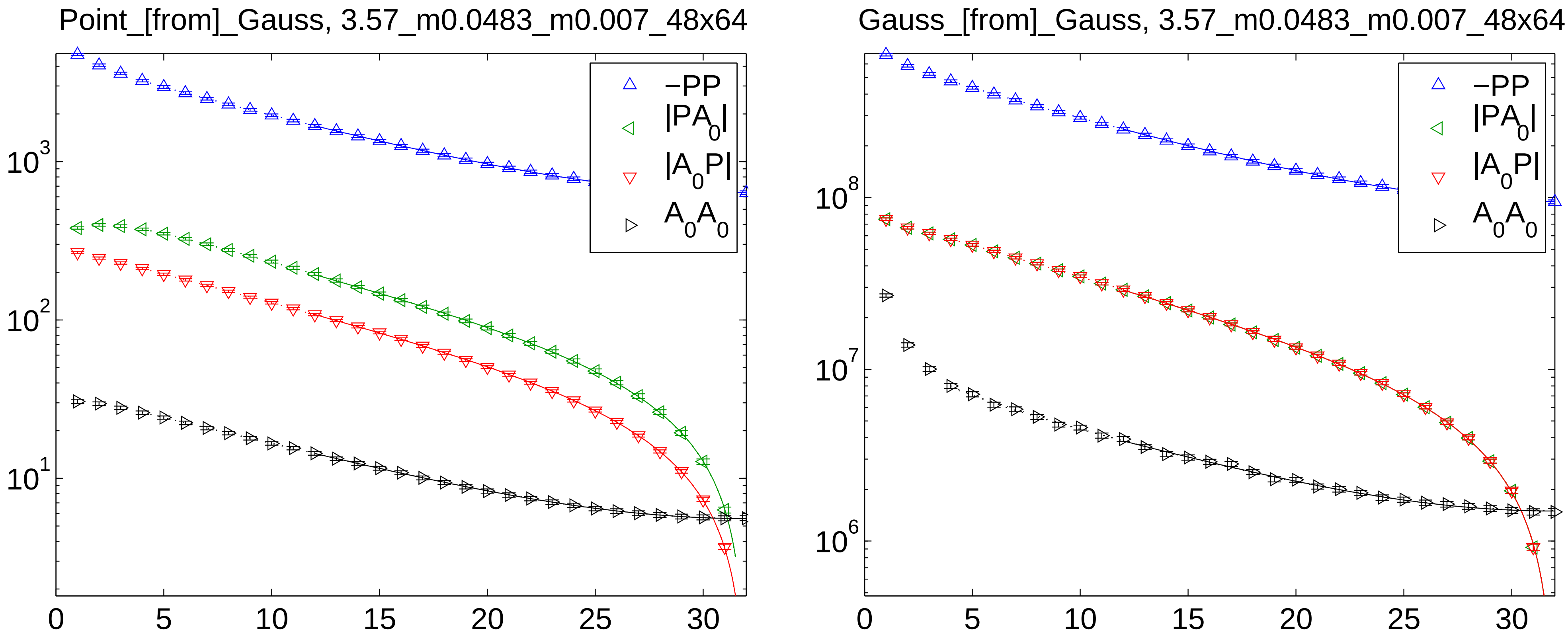

(A) Quenched QCD: quark loops neglected


(B) Full QCD

- Choose $m_u = m_d$ to save CPU time, since isospin $SU(2)$ is a good symmetry.
- In principle $m_{\mathrm{valence}} = m_{\mathrm{sea}}$, but often additional valence quark masses to broaden data base. Note that "partially quenched QCD" is an *extension* of "full QCD".
- $(D+m)^{-1}_{x0}$ for all x amounts to 12 *columns* (with spinor and color) of the inverse.

Lattice QCD spectroscopy (2)

Excellent data quality even on our lightest ensemble ($M_\pi \simeq 190\,\mathrm{MeV}$ and $L \simeq 4.0\,\mathrm{fm}$):



$\cosh(.)/\sinh(.)$ for $-PP, |PA_0|, |A_0P|, A_0A_0$ with Gauss source and local/Gauss sink

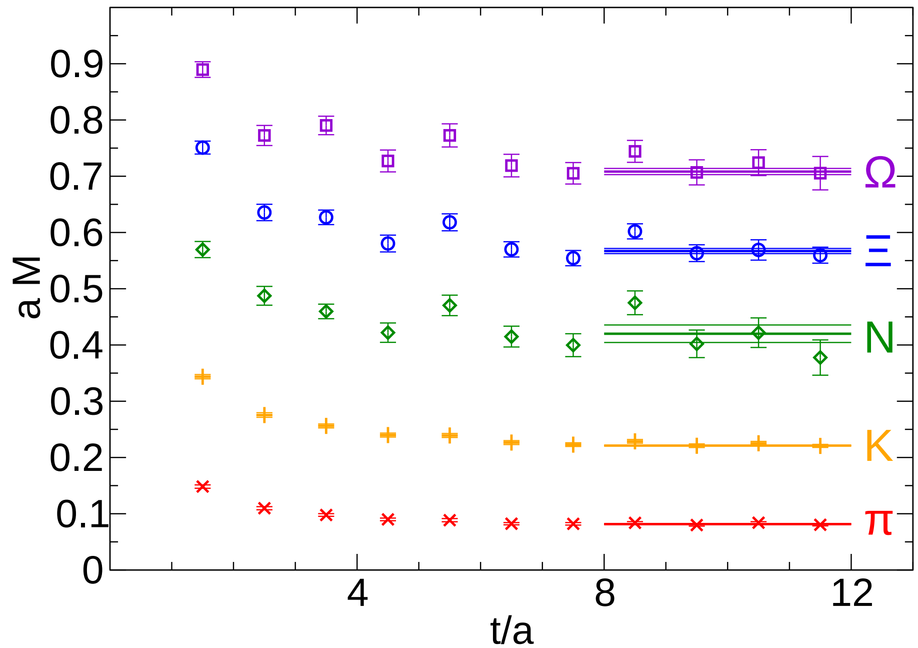
$C_{Xx,Yy}(t) = c_0 e^{-M_0 t} \pm c_0 e^{-M_0(T-t)} + \ldots$ with $X, Y \in \{P, A_0\}$ and $x, y \in \{\mathrm{loc}, \mathrm{gau}\}$

$\longrightarrow$ $c_0 = G\tilde{G}/M_0, G\tilde{F}, F\tilde{G}, F\tilde{F}M_0$ (left) and $c_0 = \tilde{G}\tilde{G}/M_0, \tilde{G}\tilde{F}, \tilde{F}\tilde{G}, \tilde{F}\tilde{F}M_0$ (right)

$\longrightarrow$ combined 1-state fit of 8 correlators with 5 parameters yields $M_\pi, F_\pi, m_{\mathrm{PCAC}}$

Lattice QCD spectroscopy (3)

With similar techniques for other channels we find in each run
aM_π, aM_K, aM_ρ, aM_{K^*}, aM_N, aM_Σ, aM_Ξ, aM_Λ, aM_Δ, aM_{Σ^*}, aM_{Ξ^*}, aM_Ω.



Cost growth (Lattice 2001, "Berlin wall phenomenon") recently tamed [in two parts]:

$a \to 0$	"continuum limit"	cost $\propto (1/a)^{4-6}$
$V \to \infty$	"infinite volume limit"	cost $\propto V^{5/4}$ with HMC
$m_{ud} \to m_{ud}^{\mathrm{phys}}$	"chiral limit"	cost $\propto (1/m)^{1-2}$ with tricks
δ(observable) $\to 0$	"reduce statistical error"	cost $\propto \delta^{-2}$

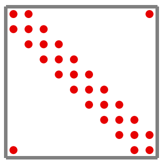
See review by K. Jansen, Lattice 2008 [arXiv:0810.5634].

Technicalities (1): sparse matrix inversion

$$D_{\mathrm{st}}(x,y) = \frac{1}{2}\sum_\mu \eta_\mu(x)\Big\{U_\mu(x)\delta_{x+\hat\mu,y} - U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}\Big\} + m\delta_{x,y}$$

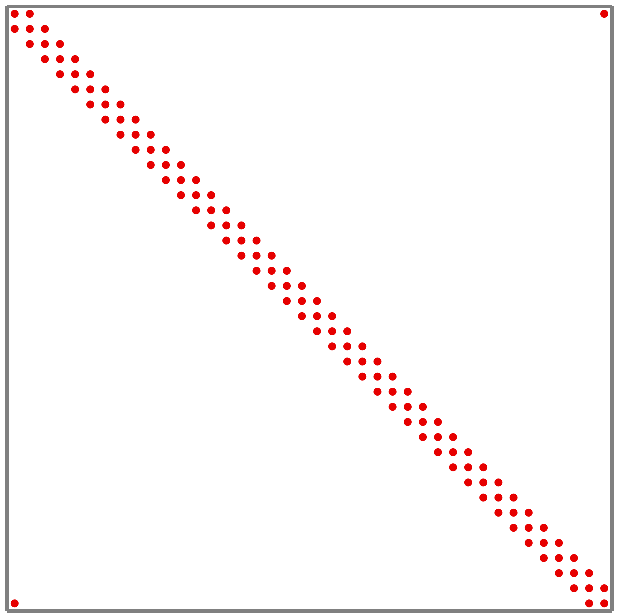
$$D_{\mathrm{W}}(x,y) = \frac{1}{2}\sum_\mu \Big\{(\gamma_\mu - I)U_\mu(x)\delta_{x+\hat\mu,y} - (\gamma_\mu + I)U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}\Big\} + (4+m_0)\delta_{x,y}$$

staggered:

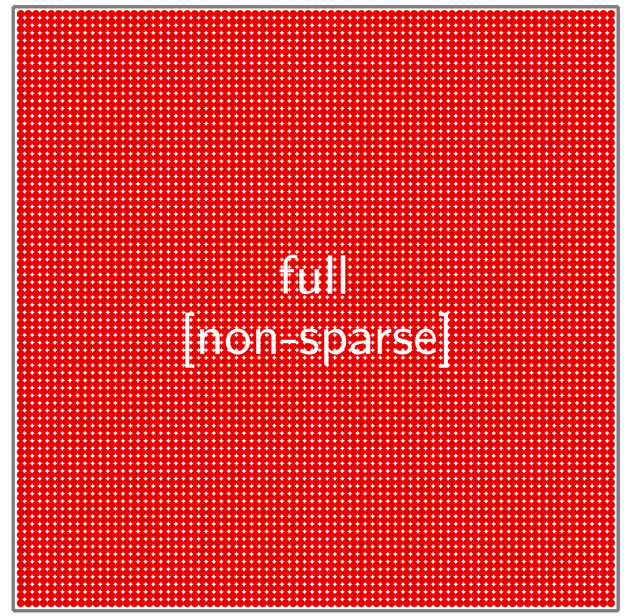


$\eta_\mu(x) = (-)^{\sum_{\nu<\mu} x_\nu}$

Wilson:



overlap:



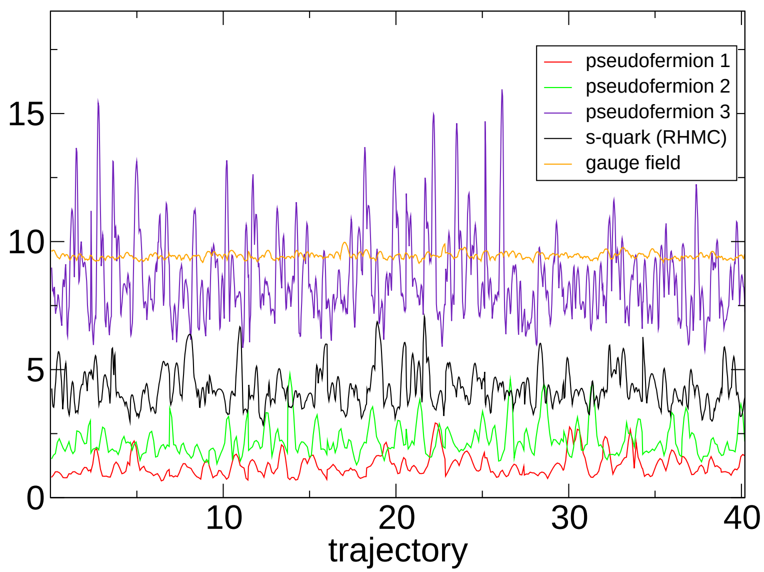
- Wilson: $D \equiv \not{D}$ is $12N\times 12N$ complex sparse matrix, since (in chiral representation) any line/column contains only $3\cdot(1+2\cdot 8) = 51$ non-zero entries.
- Any inverse is full [non-sparse].
- CG solver yields $D^{-1}\eta \simeq c_0\eta + c_1 D\eta + \ldots + c_n D^n\eta$ with $n^2 \propto \mathrm{cond}(D^\dagger D) = \frac{\lambda_{\max}}{\lambda_{\min}}$.

Technicalities (2): stochastic determinant evaluation

Full QCD requires (frequent) evaluations of $\det(D)$, but:

- state-of-the-art lattices have $L/a=64$ and thus $N=64^4=16'777'216$ sites
- D for Wilson-like fermions is $12N\times12N=201'326'592\times201'326'592$ matrix
- storing $4\cdot10^{16}$ complex numbers in single precision takes $32\cdot10^{16}$ bytes
- complete 16-rack BG/P at Jülich has $32\,\mathrm{TB}$ memory, i.e. $32\cdot10^{12}$ bytes

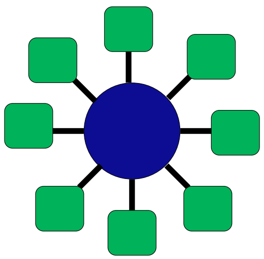
$N_f=2$ part: $$\det^2(D)=\det(D^\dagger D)=\frac{1}{\det((D^\dagger D)^{-1})}=\int D\phi^\dagger D\phi\; e^{-\phi^\dagger(D^\dagger D)^{-1}\phi}$$

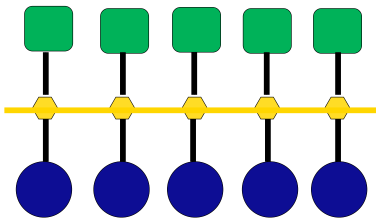


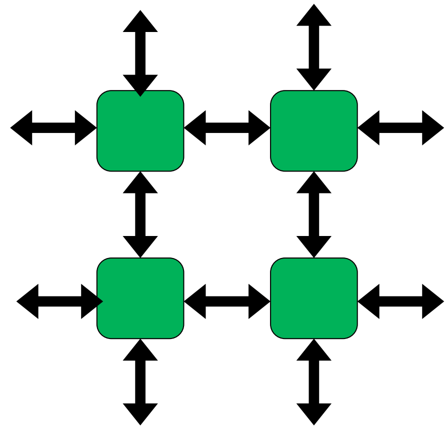
BMW uses battery of tricks:

- even-odd preconditioning
- multiple time-scale integration (“Sexton-Weingarten scheme”)
- mass preconditioning (“Hasenbusch trick”)
- Omelyan integrator
- RHMC acceleration with multiple pseudofermions
- mixed-precision solver
- direct SPI (as opposed to MPI) implementation: 37% sustained performance and perfect weak scaling [problem size grows] up to full 16 racks

Technicalities (3): machine details



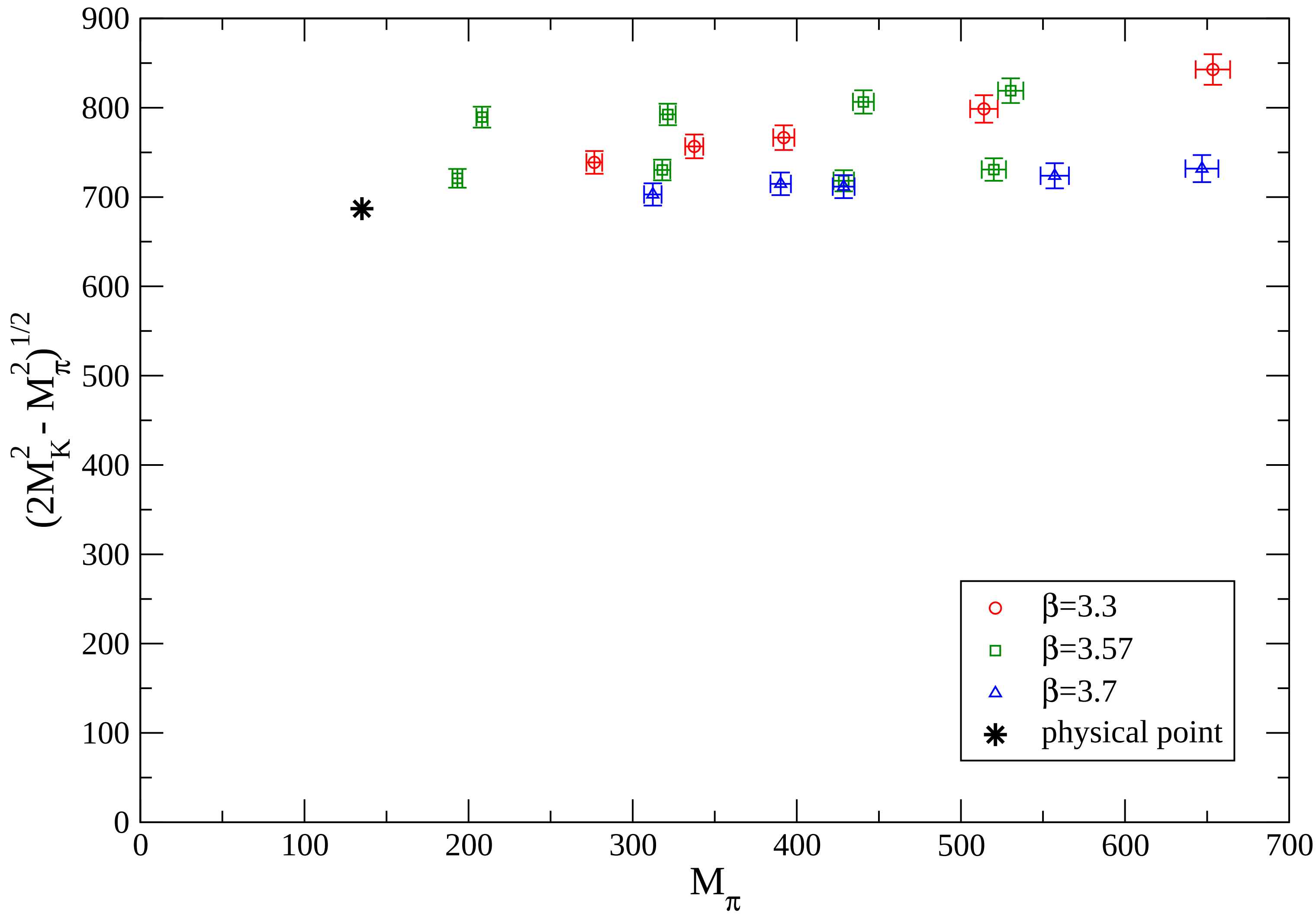




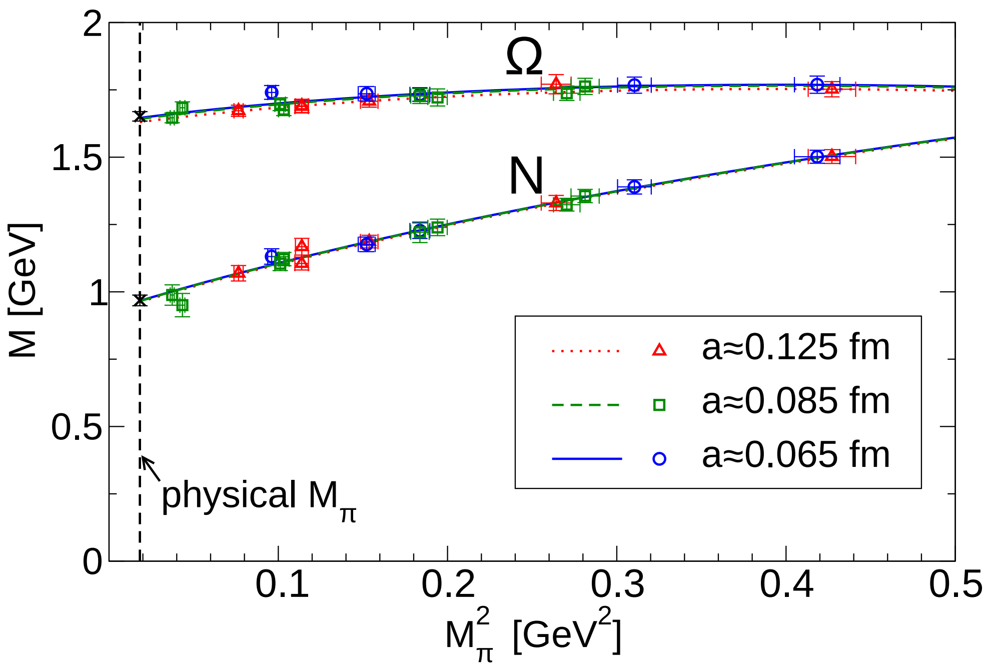
"JUGENE" [IBM BG/P]	02/2008 - 02/2009	06/2009 - ...
processor type	32-bit PowerPC 450 core 850 MHz	(3.4 Gflops each)
compute node	4-way SMP processor	
racks, nodes, processors	16, 16'384, 65'536	72, 73'728, 294'912
memory	2 GB per node, aggregate 32 TB	aggregate 144 TB
performance (peak/Lapack)	223/180 Teraflops [double prec.]	1/0.825 Petaflops
power consumption	$<$40 kW/rack, aggregate 0.5 MW	2.2 Megawatt
network topology	3D torus among compute nodes (plus global tree collective network, plus ethernet admin network)	
network latency	160 nsec (light travels 48 meters)	
network bandwidth	5.1 Gigabyte/s	

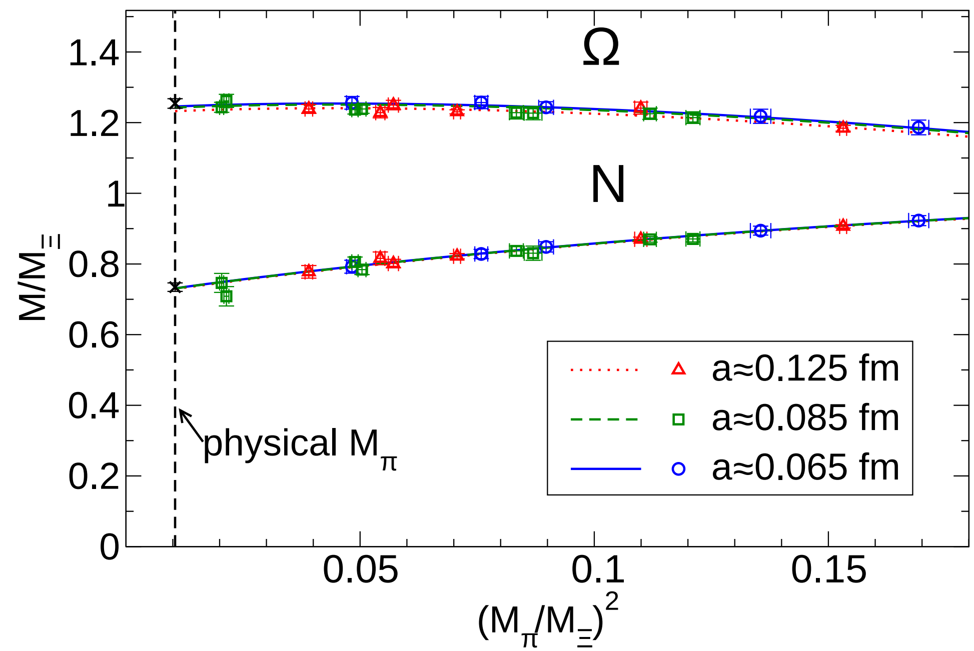
Full QCD spectrum (1): simulation landscape

We simulate $N_f = 2+1$ QCD and set m_{ud}, m_s, a^{-1} through M_π, M_K, M_Ξ (or M_Ω).
We have in total 18 ensembles at 3 lattice spacings: $a \sim 0.124/0.083/0.065\,\mathrm{fm}$.



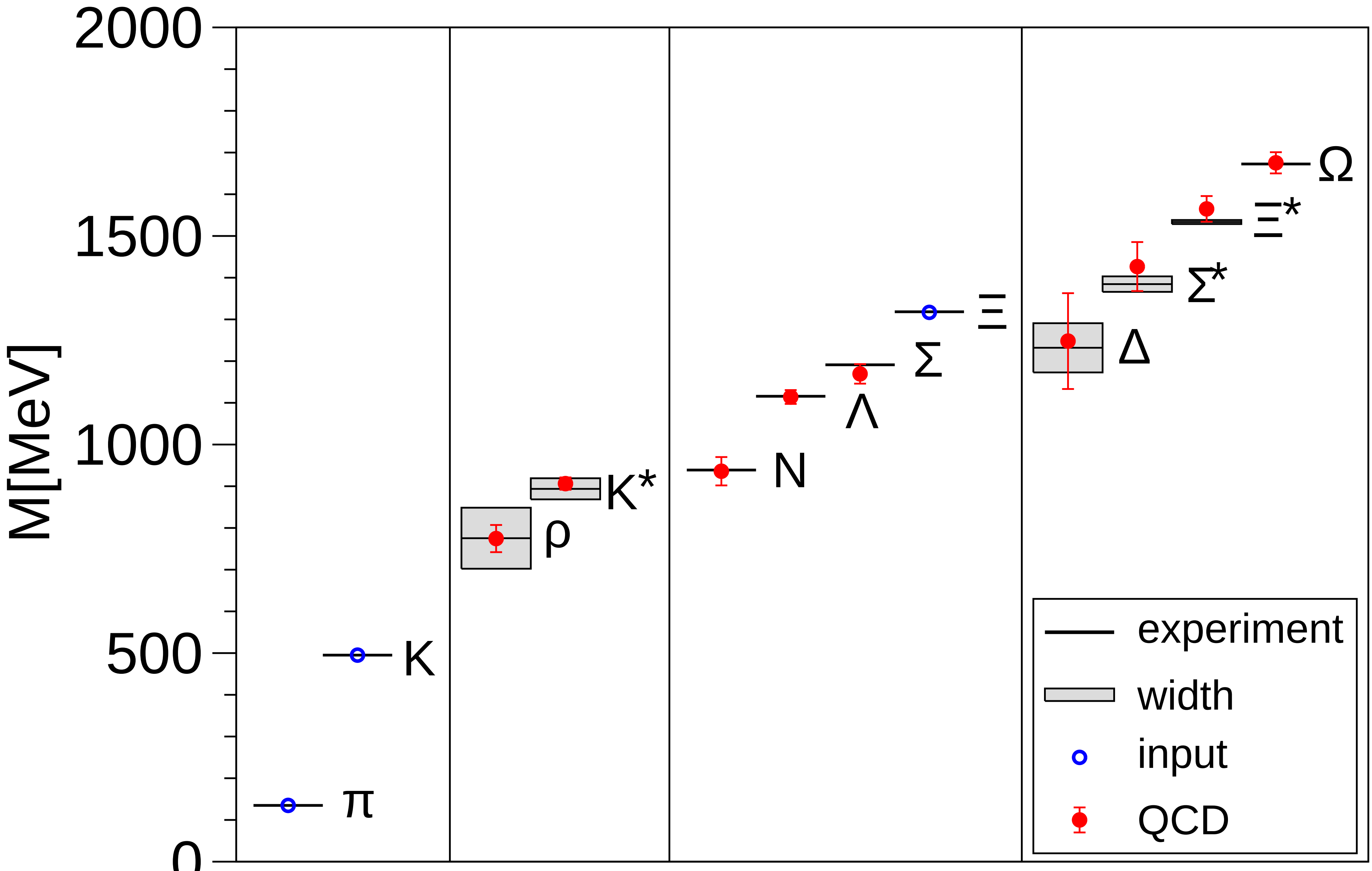
Full QCD spectrum (2): chiral extrapolation, i.e. $m_q \to m_q^{\mathrm{phys}}$





- We use two scale-setting schemes, one where a depends only on the coupling $\beta = 6/g_0^2$ with the latter defined via aM_Ξ at the physical mass point ("standard"), and another one where a is determined from the simulated Ξ ("ratio method"). The physical mass point is defined, at each coupling, as the point where $M_\pi/M_K/M_\Xi$ assumes its PDG value. Just as a check, we use Ω instead of Ξ.
- Having the scale set, we extrapolate linearly in M_π^2 and mimic chiral logs through M_π^3 or M_π^4 terms, with free coefficients.
- Our action seems to entail rather small scaling violations for hadron masses.

Final result: BMW collaboration, Science 322, 1224 (2008)



f_K/f_π: self-consistency of the SM

W.J. Marciano, PRL 93 231803 (2004) [hep-ph/0402299]:

- $|V_{\rm ud}|$ is known, from super-allowed nuclear β-decays, with 0.03% precision [HT].

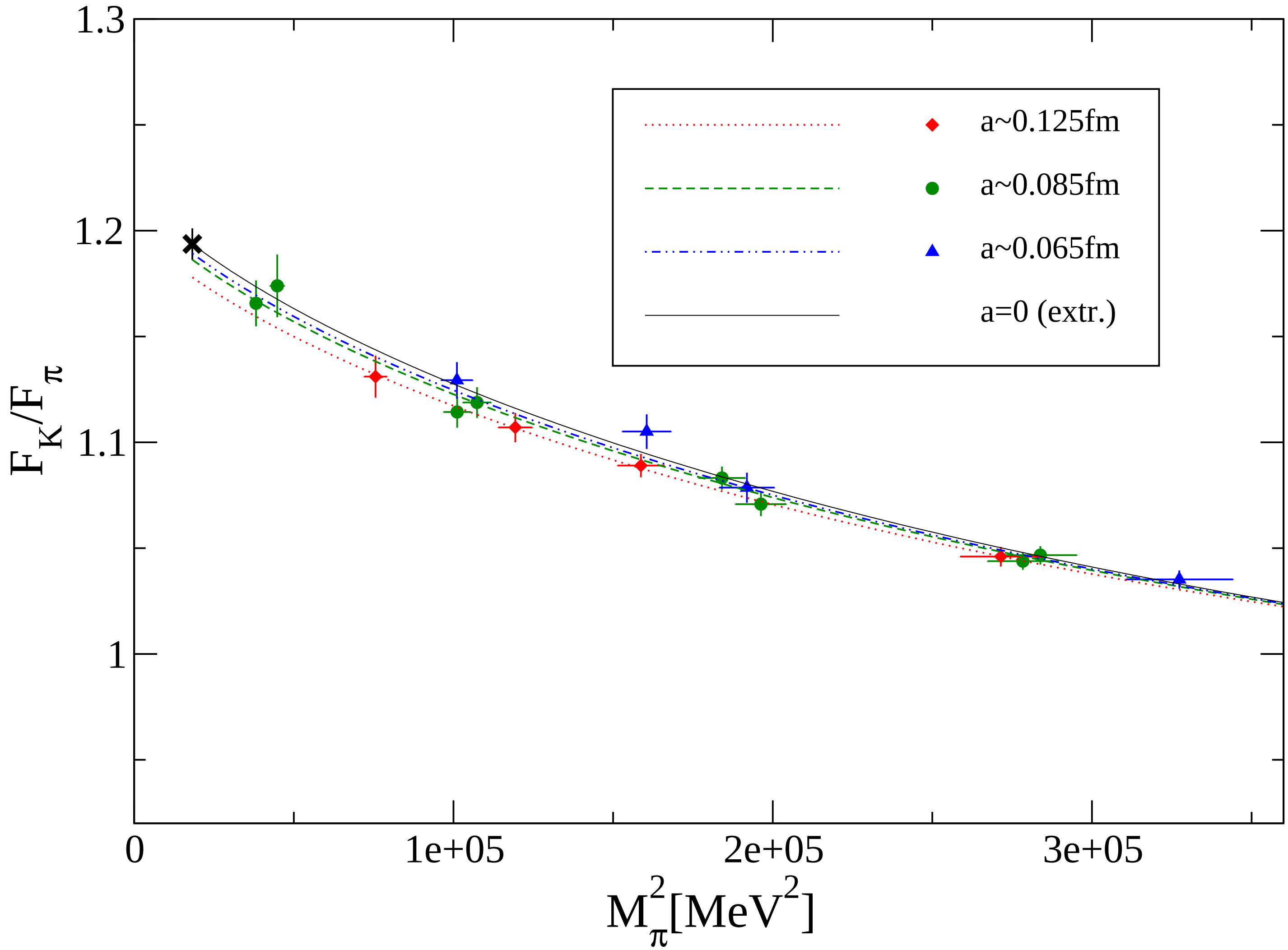
- $|V_{\rm us}|$ is much less precisely known, but can be linked to $|V_{\rm ud}|$ via a relation involving f_K/f_π, with everything else known rather accurately:

$$\frac{\Gamma(K\to l\bar{\nu}_l)}{\Gamma(\pi\to l\bar{\nu}_l)}=\frac{|V_{\rm us}|^2}{|V_{\rm ud}|^2}\frac{f_K^2}{f_\pi^2}\,\frac{M_K(1-m_l^2/M_K^2)^2}{M_\pi(1-m_l^2/M_\pi^2)^2}\{1+\frac{\alpha}{\pi}(C_K-C_\pi)\}$$

- CKM unitarity $|V_{\rm ud}|^2+|V_{\rm us}|^2+|V_{\rm ub}|^2=1$ (with $|V_{\rm ub}|$ being negligibly small) is genuine to the SM; any deviation is a *unambiguous* signal of BSM physics.

$\Longrightarrow$ calculate f_K/f_π in $N_f=2\!+\!1$ QCD (with quark masses extrapolated to the physical point) on the lattice; the precision attained gives the precision of $|V_{\rm us}|$.

f_K/f_π: combined $m \to m^{\rm phys}, a \to 0, V \to \infty$ fits



$\longrightarrow$ plot shows $\mathrm{data}(M_\pi^2, 2M_K^2{-}M_\pi^2) - \mathrm{fit}(M_\pi^2, 2M_K^2{-}M_\pi^2) + \mathrm{fit}(M_\pi^2, [2M_K^2{-}M_\pi^2]_{\rm phys})$

$\longrightarrow$ f_K/f_π scales rather nicely [note $a^2/\mathrm{fm}^2 = 0.0042, 0.0072, 0.0156$]

$\Longrightarrow$ $f_K/f_\pi = 1.192(7)(6)$ (prelim) at physical m_{ud}, in continuum, infinite volume

f_K/f_π: update on $|V_{\rm us}|$ and CKM unitarity

- Latest nuclear structure calculations [Hardy Towner'09] give

$$|V_{\rm ud}| = 0.97425(22)\ .$$

- Plug experimental information $\Gamma(K\to\mu\bar\nu)/\Gamma(\pi\to\mu\bar\nu) = 1.3363(37)$ [PDG'08] and $C_K - C_\pi = -3.0 \pm 1.5$ [Marciano] into Marciano's equation (-2 pages); this yields

$$\frac{|V_{\rm us}|}{|V_{\rm ud}|}\frac{f_K}{f_\pi} = 0.27599(59)\ .$$

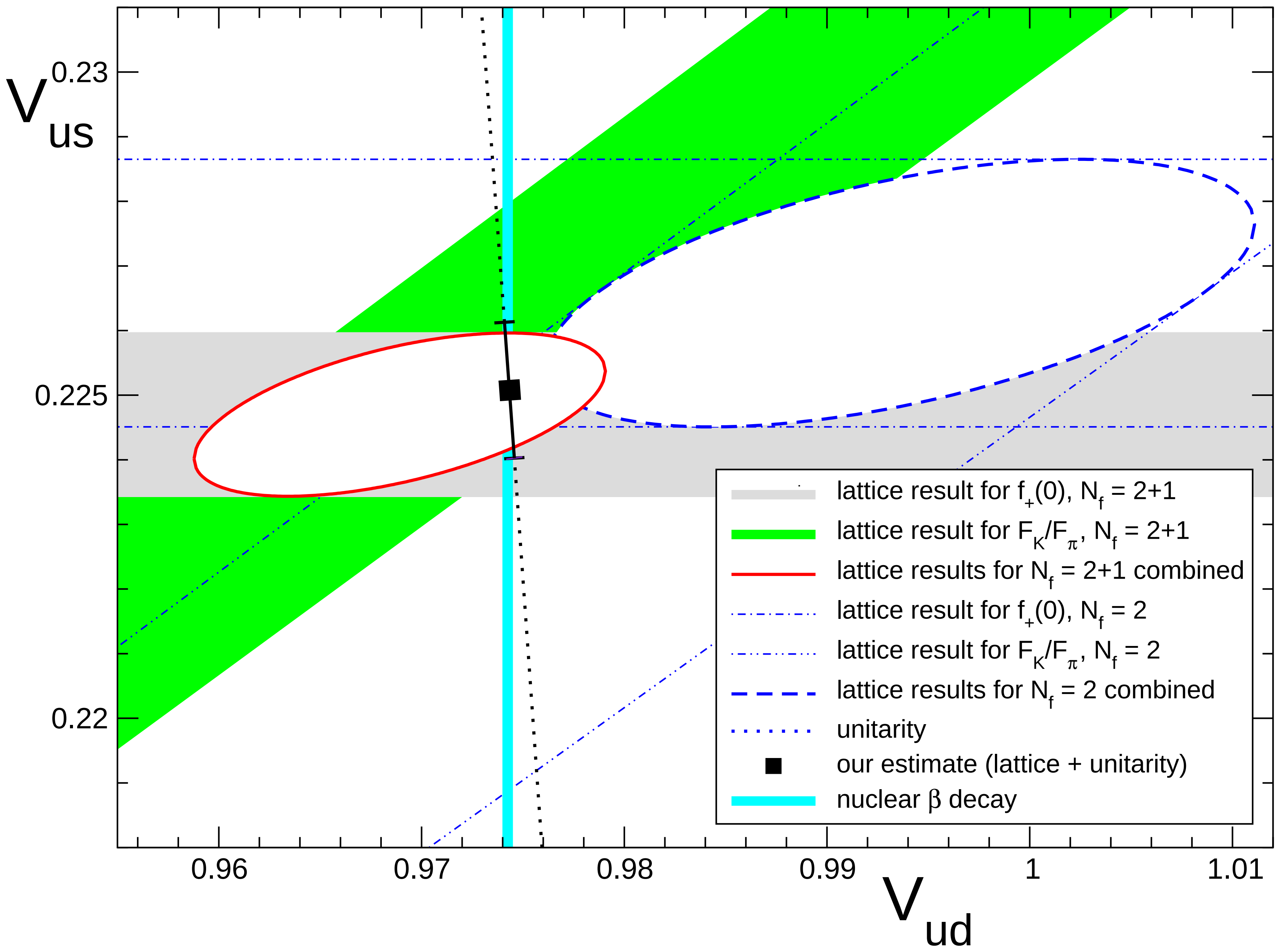
- Upon combining the previous one/two points and our value for f_K/f_π we obtain

$$|V_{\rm us}|/|V_{\rm ud}| = 0.2315(19) \quad \text{and} \quad |V_{\rm us}| = 0.2256(18)\ .$$

- Upon including $|V_{\rm ub}| = 3.39(36)10^{-3}$ [PDG'08] we end up with [BMW, preliminary]

$$|V_{\rm ud}|^2 + |V_{\rm us}|^2 + |V_{\rm ub}|^2 = 1.0001(9)\ .$$

FLAG preview: information landscape



Outlook: more strong dynamics

QCD computations:

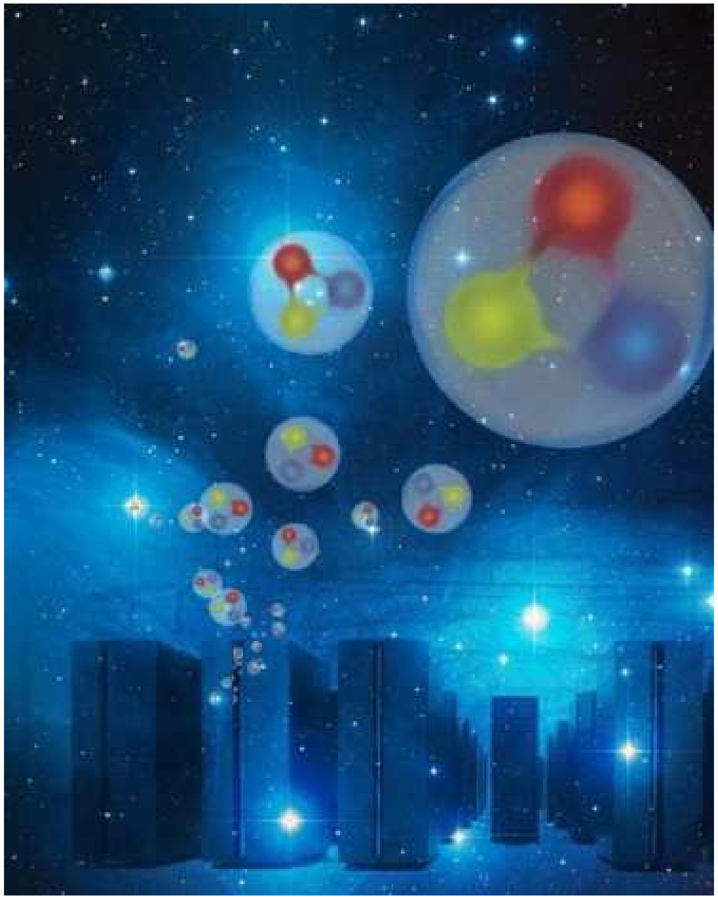
- f_K/f_π, f_{D_s}/f_D, B_K, $\langle N|\bar{u}u+\bar{d}d|N\rangle$, ...
- $\Delta I=1/2$, ϵ'/ϵ, resonances, flavor-singlets, ...
- critical endpoint in (T,μ) plane, ...
- non-equilibrium dynamics, ...

SM/BSM problems:

- Higgs dynamics (both SM/BSM)
- technicolor theories (QCD-type theories with bosons/fermions in higher reps)
- generation of scale hierarchies (beyond strong coupling)
- construction of chiral gauge theories on the lattice
- construction of SUSY on the lattice

Summary

LQCD as a first-principles based approach for solving QCD has come of age:



locally known as "JUMP under water"

- quenched spectroscopy calculations since 20 years [GF-11 to CP-PACS]
- nowadays determinant of light quarks included [$M_\pi \simeq 140\,\mathrm{MeV}$ to come]
- all systematics controlled [excited states, $a \to 0$, $V \to \infty$, $m_q \to m_q^{\mathrm{phys}}$]
- important physics applications: f_K/f_π, f_{D_s}/f_D, B_K, $\langle N|\bar{u}u+\bar{d}d|N\rangle$, ...
- hard problems remain: $\Delta I = 1/2$, ϵ'/ϵ, resonances, ...