Minimal flavour violations in SUSY GUTs

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Outline

Introduction

Minimal flavour violation

Running of MFV models

Summary and conclusions

Work done in collaboration with
Lorenzo Mercolli, Emanuel Nikolidakis and Christopher Smith

Outline

The SM as an effective theory

The SM as the renormalizable part of a low-energy effective theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM}^{d\leq 4} + \sum_{d=5}^{\infty} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} O_i^{(d)}$$

Assuming $c_i^{(d)} \sim \mathcal{O}(1)$, low–energy measurements
[particularly those of effects suppressed in the SM, like FCNC]
impose bounds on Λ:

$$\begin{array}{lcl} K^0 - \bar{K}^0 & \Rightarrow & \Lambda > 10^4 \text{ TeV} \\ B_d^0 - \bar{B}_d^0 & \Rightarrow & \Lambda > 10^3 \text{ TeV} \\ B_s^0 - \bar{B}_s^0 & \Rightarrow & \Lambda > 10^2 \text{ TeV} \end{array}$$

SUSY and flavour

- the MSSM has many more sources of flavour violation than the SM

  Masses:

  $$-\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q} - \tilde{U}\mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{D}\mathbf{m}_D^2 \tilde{D}^\dagger - \tilde{L}^\dagger \mathbf{m}_L^2 \cdot \tilde{L} - \tilde{E}\mathbf{m}_E^2 \tilde{E}^\dagger$$

  Trilinear terms:

  $$-\tilde{U}\mathbf{A}_u(\tilde{Q})_a(H_u)^a + \tilde{D}\mathbf{A}_d(\tilde{Q})_a(H_d)^a + \tilde{E}\mathbf{A}_\ell(\tilde{L})_a(H_d)^a + h.c.$$

- precise measurements of flavour violations at low energy still compatible with the SM
- unless $M_{\mathrm{SUSY}} \geq 10^2$ TeV
  $\Rightarrow$ flavour violating couplings are strongly constrained...

SUSY and flavour

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  Masses:

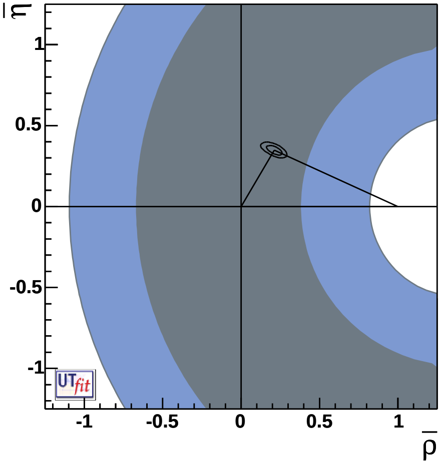
  $$-\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q} - \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{D} \mathbf{m}_D^2 \tilde{D}^\dagger - \tilde{L}^\dagger \mathbf{m}_L^2 \cdot \tilde{L} - \tilde{E} \mathbf{m}_E^2 \tilde{E}^\dagger$$

  Trilinear terms:

  $$-\tilde{U} \mathbf{A}_u (\tilde{Q})_a (H_u)^a + \tilde{D} \mathbf{A}_d (\tilde{Q})_a (H_d)^a + \tilde{E} \mathbf{A}_\ell (\tilde{L})_a (H_d)^a + h.c.$$

- precise measurements of flavour violations at low energy still compatible with the SM
- unless $M_{\mathrm{SUSY}} \geq 10^2$ TeV
  $\Rightarrow$ flavour violating couplings are strongly constrained...
- ...but there is still room for discovering new effects!

SUSY and flavour



Constraints from $K^+ \to \pi^+ \nu\bar{\nu}$

Minimal flavour violation in SUSY

- Minimal flavour violation solves the flavour problem by transforming it into a symmetry principle Hall, Randall (90)

  Ciuchini, Degrassi, Gambino, Giudice (98) Buras et al., Ali, London (00)

  D'Ambrosio, Isidori, Giudice, Strumia (02)
- it drastically reduces the number of free parameters in the MSSM $\Rightarrow$ although MFV defines a large class of models it is still predictive
- makes the agreement with the known phenomenology easy to achieve
- deviations from the SM rather mild
  $\Rightarrow$ look hard if you want to find them

MFV as a substitute for *R* parity?

- In the following we will discuss soft SUSY breaking terms respecting *R*-parity
- Nikolidakis and Smith (07) have analyzed how MFV constrains the *R*-parity violating terms
- They have shown that, surprisingly:
  MFV alone is sufficient to forbid a too fast proton decay

SUSY and unification

- the MSSM provides a successful unification of the coupling constants $\Rightarrow$ strong point in favour of SUSY
- Interplay with the flavour problem?
  - imposing that flavour violations be small at one scale can make them large at another scale
  - “solutions” of the flavour problem at one scale may break down at a different scale or appear fine tuned
  - in particular a symmetry solution like MFV should better be scale invariant
  - how does an MFV-model change as you change the scale? RGE’s for MFV models?

Outline

Flavour symmetry in the SM

Gauge interactions are flavour-blind. Flavour mixing in the SM occurs in the Yukawa terms:

$$\mathcal{L}_Y = \bar{U}_R \, \mathbf{Y}_u \, Q_L H + \bar{D}_R \, \mathbf{Y}_d \, Q_L H_c + E_R \, \mathbf{Y}_e \, L_L H_c + \text{h.c.} \quad ,$$

if $\mathbf{Y}_u = \mathbf{Y}_d = \mathbf{Y}_e = 0$ the SM acquires a large global flavour symmetry:

$$G_F \equiv G_q \otimes G_\ell \otimes U(1)_B \otimes U(1)_L \otimes U(1)_Y \otimes U(1)_{PQ} \otimes U(1)_{E_R}$$

where

$$G_q \equiv SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} \ , \quad G_\ell \equiv SU(3)_{L_L} \otimes SU(3)_{E_R}$$

Chivukula, Georgi (87)

Hall, Randall (90)

Spurions

The SM remains (formally) invariant under $G_q \otimes G_\ell$ if the Yukawas are promoted to spurion fields transforming as

$$\mathbf{Y}_u \sim (\bar{3}, 3, 1)\,, \quad \mathbf{Y}_d \sim (\bar{3}, 1, 3) \quad \text{under } G_q$$
$$\mathbf{Y}_e \sim (\bar{3}, 3) \quad \text{under } G_\ell \ .$$

Symmetry breaking occurs if the Yukawa's take a specific value

VEV of truly dynamical fields?
Dynamical explanation of flavour violations?

Minimal flavour violation

An extension of the SM is defined to respect minimal flavour violation (MFV) if it is symmetric under $G_q \otimes G_\ell$ in the presence of:

D'Ambrosio, Giudice, Isidori, Strumia (02)

- new matter fields with definite transformation properties under G_F
- the Yukawa spurions
- no other spurion fields

AND if coupling constants in front of the Yukawa's are $\mathcal{O}(1)$

SUSY:

- the superpotential is automatically MFV ✓
- soft SUSY-breaking terms are not MFV unless one can express the matrices in terms of the Yukawas

MFV in the MSSM

D'Ambrosio et al. (02) wrote the soft SUSY-breaking terms in the MSSM (almost) as follows

$$\begin{aligned}
\mathbf{m}_Q^2 &= m_0^2 \left(a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + b_3 (\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d) \right) \\
\mathbf{m}_U^2 &= m_0^2 \left(a_2 \mathbf{1} + b_5 \mathbf{Y}_u \mathbf{Y}_u^\dagger \right) \\
\mathbf{m}_D^2 &= m_0^2 \left(a_3 \mathbf{1} + b_6 \mathbf{Y}_d \mathbf{Y}_d^\dagger \right) \\
\mathbf{A}_u &= A_0 \mathbf{Y}_u (a_4 + b_7 \mathbf{Y}_d^\dagger \mathbf{Y}_d) \\
\mathbf{A}_d &= A_0 \mathbf{Y}_d (a_5 + b_8 \mathbf{Y}_u^\dagger \mathbf{Y}_u)
\end{aligned}$$

- number of free parameters is drastically reduced
- flavour violations are kept under control

Revisiting the construction of the MSSM with MFV

Consider the term $\mathcal{L}_{\mathbf{m}^2_Q} \equiv -\tilde{Q}^\dagger \mathbf{m}^2_Q \cdot \tilde{Q}$

this respects MFV if $\mathbf{m}^2_Q$ transforms like $(8, 1, 1)$:

$$\mathbf{m}^2_Q = a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + c_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u + c_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d + \ldots$$

with in principle an infinite sum of admissible terms.

Revisiting the construction of the MSSM with MFV

Consider the term $\mathcal{L}_{\mathbf{m}_Q^2} \equiv -\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q}$

this respects MFV if $\mathbf{m}_Q^2$ transforms like $(8,1,1)$:

$$\mathbf{m}_Q^2 = a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + c_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_u^\dagger \mathbf{Y}_u + c_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d + \dots$$

with in principle an infinite sum of admissible terms.
However, $\mathbf{m}_Q^2$ is a 3×3 hermitian matrix and Cayley-Hamilton identities

$$\mathbf{X}^3 - \langle \mathbf{X} \rangle \mathbf{X}^2 + \frac{1}{2} \mathbf{X} \left(\langle \mathbf{X} \rangle^2 - \left\langle \mathbf{X}^2 \right\rangle \right) - \det \mathbf{X} = 0$$

constrain the number of independent terms

Revisiting the construction of the MSSM with MFV

Consider the term $\mathcal{L}_{\mathbf{m}_Q^2} \equiv -\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q}$

$$
\begin{aligned}
& \text{Cayley–Hamilton} \Rightarrow \\
\mathbf{m}_Q^2 \;=\; & z_1 \mathbf{1} + z_2 \mathbf{Y}_u^\dagger \mathbf{Y}_u + z_3 \mathbf{Y}_d^\dagger \mathbf{Y}_d + z_4 (\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 + z_5 (\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 \\
+ \; & z_6 \left(\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \text{h.c.} \right) + z_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \\
+ \; & z_8 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d + z_9 \left((\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 (\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 + \text{h.c.} \right) \\
+ \; & i z_{10} (\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u - \text{h.c.}) + i z_{11} \left((\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d - \text{h.c.} \right) \\
+ \; & i z_{12} \left((\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u - \text{h.c.} \right) + i z_{13} \left((\mathbf{Y}_u^\dagger \mathbf{Y}_u)^2 (\mathbf{Y}_d^\dagger \mathbf{Y}_d)^2 - \text{h.c.} \right) \\
+ \; & \text{four more terms}
\end{aligned}
$$

MFV can be viewed as a reparametrization!

Revisiting the construction of the MSSM with MFV

Consider the term $\mathcal{L}_{\mathbf{m}_Q^2} \equiv -\tilde{Q}^\dagger \mathbf{m}_Q^2 \cdot \tilde{Q}$

Side remark: Nikolidakis (08), Ellis et al. (09)

if Jarlskog's determinant $\neq 0$

$\Rightarrow$ one can choose 9 linearly independent MFV terms

$\Rightarrow$ MFV provides a basis for $\mathbf{m}_Q^2$

An almost singular parametrization

MFV provides a very special parametrization: the Yukawa's are very far from generic matrices. Choose, e.g.

$$\mathbf{Y}_u = \lambda_u V \ , \quad \mathbf{Y}_d = \lambda_d \ , \quad \mathbf{Y}_e = \lambda_e$$

where

$$\lambda_u = \mathrm{diag}(y_u, y_c, y_t) \ , \lambda_d = \mathrm{diag}(y_d, y_s, y_b) \ , \quad V = V_{\mathrm{CKM}}$$

then

$$\left(\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)^2 - y_t^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \sim \mathcal{O}(y_c^2) \qquad \left(\mathbf{Y}_d^\dagger \mathbf{Y}_d\right)^2 - y_b^2 \mathbf{Y}_d^\dagger \mathbf{Y}_d \sim \mathcal{O}(y_s^2)$$

$\Rightarrow$ if the coefficients in front of MFV monomials are $\mathcal{O}(1)$, one can dispose of terms containing squares of $\mathbf{Y}_u^\dagger \mathbf{Y}_u$ and $\mathbf{Y}_d^\dagger \mathbf{Y}_d$.
$\Rightarrow$ strong reduction of the number of free parameters

A more convenient parametrization

$\Rightarrow$ work in a basis where the elements are almost "orthogonal" to each other, rather than almost parallel:

$$\left(\mathbf{Y}_u^\dagger \mathbf{Y}_u\right)^2 - y_t^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u = y_t^2 y_c^2 V_{2i}^* V_{2j} + \mathcal{O}(y_c^4)$$

Enlarge the basis until it closes under multiplication

$$\begin{array}{llll} X_1 = \delta_{3i}\delta_{3j} & X_5 = \delta_{3i}V_{3j} & X_9 = V_{3i}^*\delta_{3j} & X_{13} = V_{3i}^*V_{3j} \\ X_2 = \delta_{2i}\delta_{2j} & X_6 = \delta_{2i}V_{2j} & X_{10} = V_{2i}^*\delta_{2j} & X_{14} = V_{2i}^*V_{2j} \\ X_3 = \delta_{3i}\delta_{2j} & X_7 = \delta_{3i}V_{2j} & X_{11} = V_{3i}^*\delta_{2j} & X_{15} = V_{3i}^*V_{2j} \\ X_4 = \delta_{2i}\delta_{3j} & X_8 = \delta_{2i}V_{3j} & X_{12} = V_{2i}^*\delta_{3j} & X_{16} = V_{2i}^*V_{3j} \end{array}$$

all basis vectors are of order one

$\Rightarrow$ the coefficients are expected to have very different sizes

$\Rightarrow$ need to keep track of that systematically

A more convenient parametrization

Introduce a counting scheme:

$$V = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-(\rho+i\eta)) & -A\lambda^2 & 1 \end{pmatrix}$$

where $\lambda = V_{us} \sim 0.23$

$$\frac{m_u}{m_t} \sim \mathcal{O}(\lambda^7), \quad \frac{m_c}{m_t} \sim \mathcal{O}(\lambda^4), \quad y_t \sim \mathcal{O}(1)$$
$$\frac{m_d}{m_t} \sim \mathcal{O}(\lambda^7), \quad \frac{m_s}{m_t} \sim \mathcal{O}(\lambda^5), \quad \frac{m_b}{m_t} \sim \mathcal{O}(\lambda^3),$$

A more convenient parametrization

Systematic analysis:

- write down all independent terms
- drop terms of $\mathcal{O}(\lambda^n)$ with n of your choice

For $n = 6$:

$$\begin{aligned}
\mathbf{m}_Q^2 &= \tilde{a}_1 + x_1 X_{13} + y_1 X_1 + y_2 X_5 + y_2^* X_9 \\
\mathbf{m}_U^2 &= \tilde{a}_2 + x_2 X_1 \\
\mathbf{m}_D^2 &= \tilde{a}_3 + y_3 X_1 + w_1 X_3 + w_1^* X_4 \\
\mathbf{A}^U &= \tilde{a}_4 X_5 + y_4 X_1 + w_2 X_6 \\
\mathbf{A}^D &= \tilde{a}_5 X_1 + y_5 X_5 + w_3 X_2 + w_4 X_4
\end{aligned}$$

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\mathbf{A}^U &= \tilde{a}_4 X_5 + y_4 X_1 + w_2 X_6 \\
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\end{aligned}$$

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\mathbf{m}_D^2 &= \tilde{a}_3 + y_3 X_1 + w_1 X_3 + w_1^* X_4 \\
\mathbf{A}^U &= \tilde{a}_4 X_5 + y_4 X_1 + w_2 X_6 \\
\mathbf{A}^D &= \tilde{a}_5 X_1 + y_5 X_5 + w_3 X_2 + w_4 X_4
\end{aligned}$$

$$\frac{\tilde{a}_5}{\tilde{a}_4} \sim \frac{y_5}{\tilde{a}_4} \sim \mathcal{O}(\lambda^3 t_\beta) \qquad \frac{y_{1,2}}{\tilde{a}_1} \sim \frac{y_3}{\tilde{a}_3} \sim \frac{y_4}{\tilde{a}_4} \sim \mathcal{O}(\lambda^6 t_\beta^2)$$

$$\frac{w_1}{\tilde{a}_3} \sim \mathcal{O}(\lambda^{10} t_\beta^2) \quad \frac{w_2}{\tilde{a}_3} \sim \mathcal{O}(\lambda^4) \quad \frac{w_3}{\tilde{a}_5} \sim \mathcal{O}(\lambda^2) \quad \frac{w_4}{\tilde{a}_5} \sim \mathcal{O}(\lambda^4)$$

Advantages of this parametrization

- in this basis it is easier and more transparent to decide what should be kept and what not
  $+$ it is easy to keep track of how things change with $\tan\beta$
- the relation between the x_i coefficients and the mass insertions is particularly simple
- the study of the RGE of MFV model is more convenient in the X_i basis, as we will discuss in the following

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Running of MFV models

Summary and conclusions

RGE's of MFV parameters?

- viewing MFV as a reparametrization, it is clear that one can derive the RGE for the MFV parameters exactly
- MFV is useful only because one can throw away some terms and reduce the number of free parameters
- the scheme is RGE invariant only if terms which can be neglected at one scale do not become important at another scale (and viceversa)
- $\Rightarrow$ apply systematically our counting rule also to the β-functions

RGE's for the Yukawa's

In MFV the basis is expressed in terms of the Yukawa's, which, however, also run, according to:

$$\beta_{\mathbf{Y}_u} = \mathbf{Y}_u \left[3\mathrm{Tr}\left(\mathbf{Y}_u \mathbf{Y}_u^\dagger\right) + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \right]$$

where $$\frac{d}{dt}\mathbf{Y}_u = \frac{1}{N}\beta^{(1)}_{\mathbf{Y}_u} + \dots \qquad N \equiv 16\pi^2$$

Use the running Yukawa's to define a "running" MFV?

⇒ Keep the CKM matrix at $\mu = M_Z$ fixed and express everything – including the Yukawa's – in terms of this:

$$\begin{aligned} \mathbf{Y}_u(M_Z) &= y_c(M_Z) X_6 + y_t(M_Z) X_5 \\ \mathbf{Y}_d(M_Z) &= y_s(M_Z) X_2 + y_b(M_Z) X_1 \end{aligned}$$

At a different scale, the Yukawa's will look different and contain new structures

RGE's for the Yukawa's

If we apply our counting rule also to the Yukawa's, however, only one new structure is generated by the running:

$$\mathbf{Y}_u(\mu) = y_c(\mu)X_6 + y_t(\mu)X_5 + c_t(\mu)X_1$$
$$\mathbf{Y}_d(\mu) = y_s(\mu)X_2 + y_b(\mu)X_1 + c_b(\mu)X_5$$

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and the β-functions of the coefficients read

$$\begin{aligned} \beta_{y_c} &= y_c\left(3y_t^2 - K_u\right) \\ \beta_{y_t} &= y_t\left(6y_t^2 - K_u\right) \\ \beta_{c_t} &= y_t y_b^2 + c_t\left(6y_t^2 + y_b^2 - K_u\right) \end{aligned}$$

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and the β-functions of the coefficients read

$$\begin{aligned} \beta_{y_s} &= y_s\left(3y_b^2 + y_\tau^2 - K_d\right) \\ \beta_{y_b} &= y_b\left(6y_b^2 + y_\tau^2 - K_d\right) \\ \beta_{c_b} &= y_b y_t^2 + c_b\left(6y_b^2 + y_t^2 + y_\tau^2 - K_d\right) \end{aligned}$$

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$$\mathbf{Y}_d(\mu) = y_s(\mu)X_2 + y_b(\mu)X_1 + c_b(\mu)X_5$$

and the β-functions of the coefficients read

where $K_u = \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2$, $K_d = K_u - \frac{2}{5}g_1^2$

Running of the MFV coefficients

Applying the same counting rules as above (*i.e.* dropping any correction of order λ^2 in the β-functions) we get

$$\begin{aligned}
\beta_{\tilde{a}1} &= -\tfrac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \tfrac{2}{15}g_1^2|M_1|^2 + \tfrac{1}{5}g_1^2 S \\
\beta_{x_1} &= 2y_t^2\left(m_{Hu}^2 + \tilde{a}_1 + \tilde{a}_2 + x_1 + x_2 + \mathrm{Re}\,y_2\right) + 2\left(|\tilde{a}_4|^2 + |y_5|^2\right) \\
\beta_{y_1} &= 2y_b^2\left(m_{Hd}^2 + \tilde{a}_1 + \tilde{a}_3 + y_1 + \mathrm{Re}\,y_2 + y_3\right) + 2\left(|\tilde{a}_5|^2 + |y_4|^2\right) \\
\beta_{y_2} &= y_b^2(x_1 + y_2) + y_t^2\,(y_1 + y_2) + 2\left(\tilde{a}_5^* y_5 + \tilde{a}_4 y_4^*\right)
\end{aligned}$$

$$\begin{aligned}
\beta_{\tilde{a}2} &= -\tfrac{32}{3}g_3^2|M_3|^2 - \tfrac{32}{15}g_1^2|M_1|^2 - \tfrac{4}{5}g_1^2 S \\
\beta_{x_2} &= 4y_t^2\left(m_{Hu}^2 + \tilde{a}_1 + \tilde{a}_2 + x_1 + x_2 + y_1 + 2\,\mathrm{Re}\,y_2\right) + 4|\tilde{a}_4 + y_4|^2 \\
\ldots &\quad \ldots
\end{aligned}$$

Running of the MFV coefficients

Applying the same counting rules as above (*i.e.* dropping any correction of order λ^2 in the β-functions) we get

where

$$
\begin{aligned}
S \;=\; & m_{H_u}^2 - m_{H_d}^2 + 3(\tilde{a}_1 - 2\tilde{a}_2 + \tilde{a}_3) + x_1 - 2x_2 \\
& + y_1 + 2y_2 + y_3 \\
& - 3(\tilde{a}_6 - \tilde{a}_7) - y_6 + y_7
\end{aligned}
$$

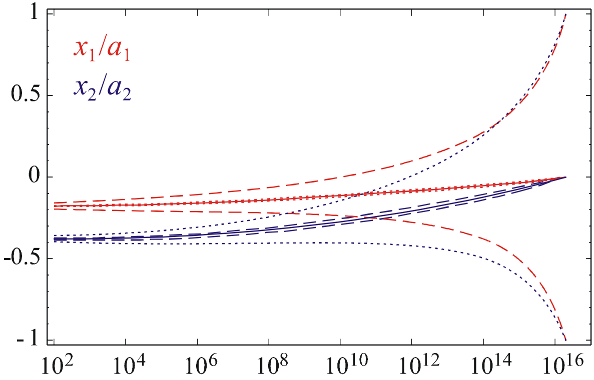
Remarks

The RGE's for the soft-SUSY breaking terms written in this form are simple and transparent

- it is possible to provide (approximate) analytical solutions
- these allow one to understand the running behaviour
- MFV is explicitly scale invariant (but see below!):
  - no new terms are generated in the running (according to our counting rules)
  - the β-functions of the coefficients are of the same order (or lower) as the coefficients themselves
    [this is only a necessary condition – not a sufficient one]

Running of x_1 and x_2

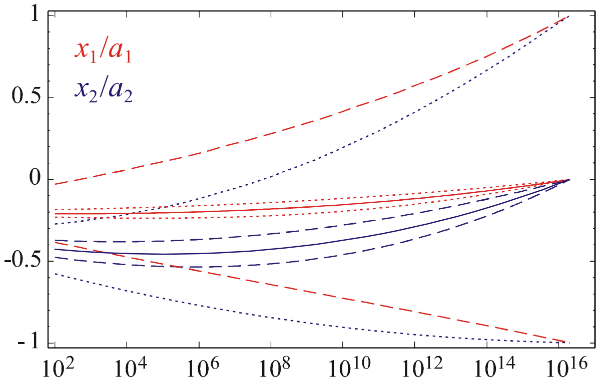
SPS-1a point



Fixed-point like behaviour first observed by Paradisi et al. (08)

Running of x_1 and x_2

SPS-4 point



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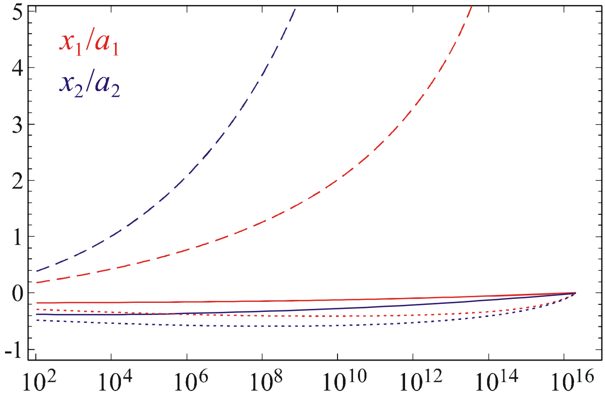
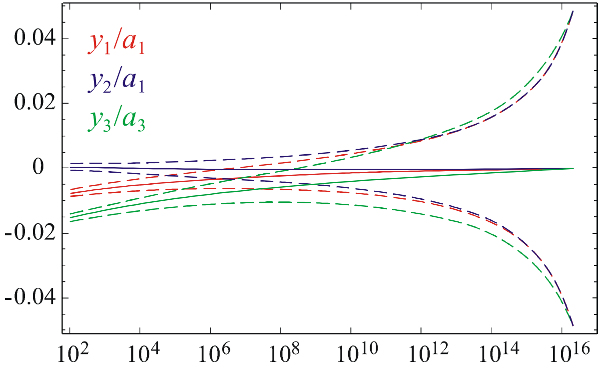


Flipping the sign of $\frac{x_1}{a_1}$ and $\frac{x_2}{a_2}$ at $Q = M_{\rm SUSY}$

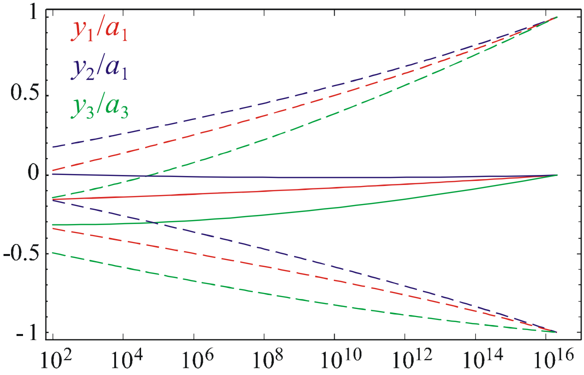
Running of y_1, y_2 and y_3

SPS-1a point



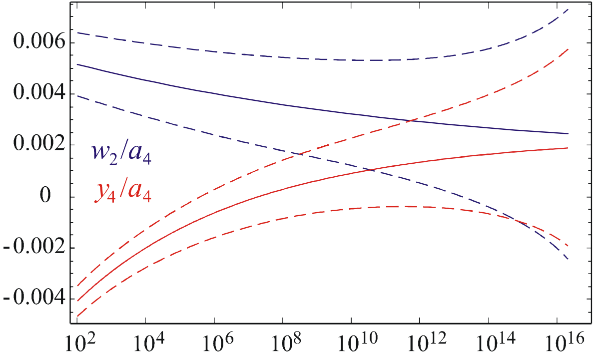
Running of y_1, y_2 and y_3

SPS-4 point



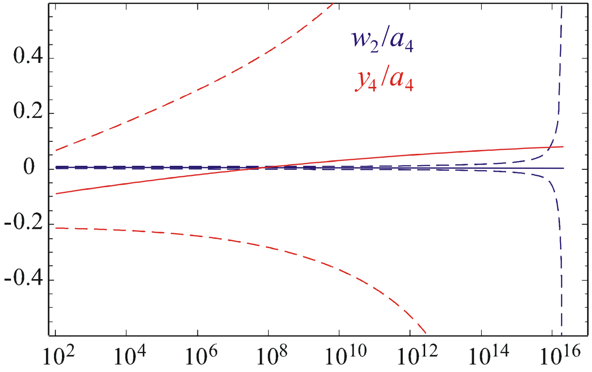
Running of y_4 and w_2

SPS-1a point



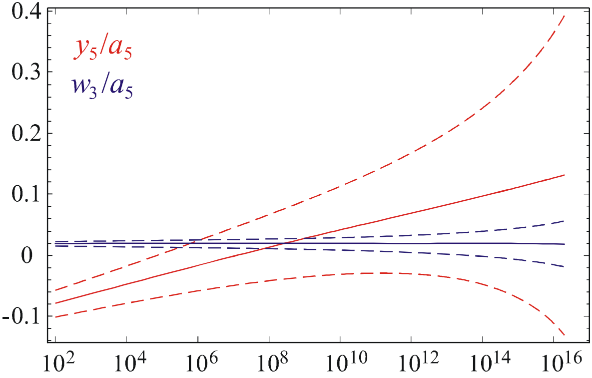
Running of y_4 and w_2

SPS-4 point



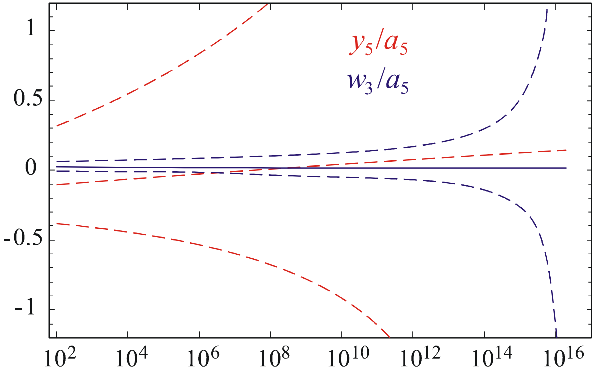
Running of y_5 and w_3

SPS-1a point



Running of y_5 and w_3

SPS-4 point



Simplified analytical solutions

Can one understand the fixed-point behaviour?

Simplified analytical solutions

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For an mSUGRA point $S(M_{\rm GUT}) = 0 \Rightarrow$ neglect S everywhere
$\Rightarrow$ the RGE's can be solved analytically

$$\tilde{a}_1(t) = \tilde{a}_1(t_0) + \frac{8}{9}\left(|M_3(t)|^2 - |M_3(t_0)|^2\right) - \frac{3}{2}\left(|M_2(t)|^2 - |M_2(t_0)|^2\right) - \frac{1}{198}\left(|M_1(t)|^2 - |M_1(t_0)|^2\right)$$

$$\tilde{a}_2(t) = \tilde{a}_2(t_0) + \frac{8}{9}\left(|M_3(t)|^2 - |M_3(t_0)|^2\right) - \frac{8}{99}\left(|M_1(t)|^2 - |M_1(t_0)|^2\right)$$

$$\tilde{a}_3(t) = \tilde{a}_3(t_0) + \frac{8}{9}\left(|M_3(t)|^2 - |M_3(t_0)|^2\right) - \frac{2}{99}\left(|M_1(t)|^2 - |M_1(t_0)|^2\right)$$

M_3 grows fast from the GUT to the EW scale
$\Rightarrow$ good fraction of the squark masses generated by the running

Simplified analytical solutions

Can one understand the fixed-point behaviour?

For an mSUGRA point $S(M_{\mathrm{GUT}}) = 0 \Rightarrow$ neglect S everywhere
$\Rightarrow$ the RGE's can be solved analytically

Analytical solutions can be derived also for the other MFV coefficients (somewhat more complicated)

Dependence on the initial conditions

$$x_1(t) = x_1^{RG}(t) + \frac{x_1(t_0)}{6}\left[G_{n_1}(t) + 5\right] + \frac{m^2_{H_u}(t_0) + x_2(t_0)}{6}\left[G_{n_1}(t) - 1\right]$$

G_{n_1} is almost a linear function: $G_{n_1}(t_0) = 1$, $G_{n_1}(t_{\rm ew}) \sim 1/3$

The pure RGE contributions are

$$x_1^{RG}(t) = -\frac{G_{n_1}(t)}{3N}\int_t^{t_0} dt' \frac{f_{n_1}(t') - f_{n_2}(t')}{G_{n_1}(t')} + \frac{1}{3N}\int_t^{t_0} dt' f_{n_2}(t')\left[1 - \frac{G_{n_1}(t)}{G_{n_1}(t')}\right]$$

and give respectively (SPS-1a)

$$x_1^{RG}(t_{\rm ew}) = -(53.2 + 2.7)\cdot 10^3 = -56\cdot 10^3\ \mathrm{GeV}^2$$

Dependence on the initial conditions

$$x_1(t) = x_1^{RG}(t) + \frac{x_1(t_0)}{6}\left[G_{n_1}(t) + 5\right] + \frac{m_{H_u}^2(t_0) + x_2(t_0)}{6}\left[G_{n_1}(t) - 1\right]$$

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and give respectively (SPS-1a)

$$\frac{x_1^{RG}(t_{\rm ew})}{\tilde{a}_1(t_{\rm ew})} = -0.17$$

Dependence on the initial conditions

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G_{n_1} is almost a linear function: $G_{n_1}(t_0) = 1$, $G_{n_1}(t_{\rm ew}) \sim 1/3$

Sensitivity to the initial conditions $[\delta_1 \equiv x_1/\tilde{a}_1(t_0)]$ (SPS-1a)

$$\frac{\Delta x_1}{\tilde{a}_1} = \frac{\tilde{a}_1(t_0)}{\tilde{a}_1(t_{\rm ew})}\frac{G_{n_1}(t_{\rm ew}) + 5}{6}\delta_1 = 0.02 \cdot 0.9 \cdot \delta_1 = 0.02 \cdot \delta_1$$

the (small) sensitivity to the IC (fixed-point behaviour) can be understood on the basis of these analytical formulae

x_1 is generated almost completely by the RGE, by the $\tilde{a}_i$'s!

Dependence on the initial conditions

$$x_1(t) = x_1^{RG}(t) + \frac{x_1(t_0)}{6}\left[G_{n_1}(t) + 5\right] + \frac{m^2_{H_u}(t_0) + x_2(t_0)}{6}\left[G_{n_1}(t) - 1\right]$$

G_{n_1} is almost a linear function: $G_{n_1}(t_0) = 1$, $G_{n_1}(t_{\rm ew}) \sim 1/3$

Sensitivity to the initial conditions $[\delta_1 \equiv x_1/\tilde{a}_1(t_0)]$ (SPS-4)

$$\frac{\Delta x_1}{\tilde{a}_1} = \frac{\tilde{a}_1(t_0)}{\tilde{a}_1(t_{\rm ew})}\frac{G_{n_1}(t_{\rm ew}) + 5}{6}\delta_1 = 0.2 \cdot 0.9 \cdot \delta_1 = 0.18 \cdot \delta_1$$

the increased sensitivity to the IC for the SPS-4 point has nothing to do with $\tan\beta$, but rather with how much of $\tilde{a}_1$ is generated radiatively

CP-violating phases

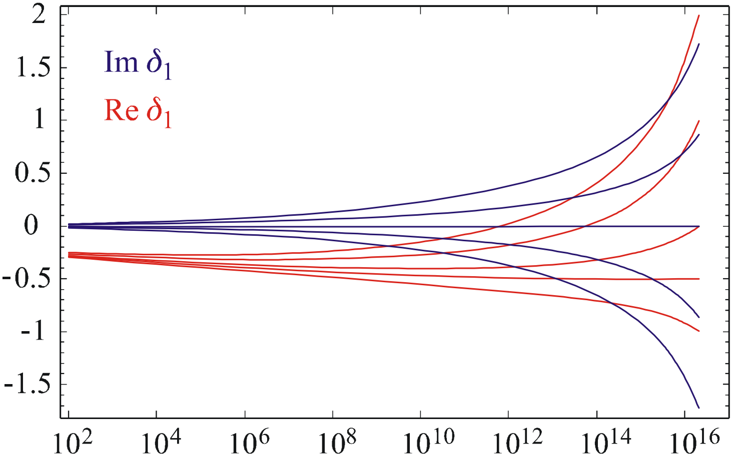
Define

$$\delta_1 \equiv \frac{(\delta^U_{RL})^{32}}{V_{ts}} = \frac{(\delta^U_{RL})^{31}}{V_{td}} = \frac{v_u \tilde{a}_4}{|\tilde{a}_1|^{1/2}|\tilde{a}_2 + x_2|^{1/2}}$$

$$\delta_2 \equiv \frac{(\delta^D_{RL})^{32}}{V_{ts}} = \frac{(\delta^D_{RL})^{31}}{V_{td}} = \frac{v_d y_5}{|\tilde{a}_1|^{1/2}|\tilde{a}_3 + y_3|^{1/2}}$$

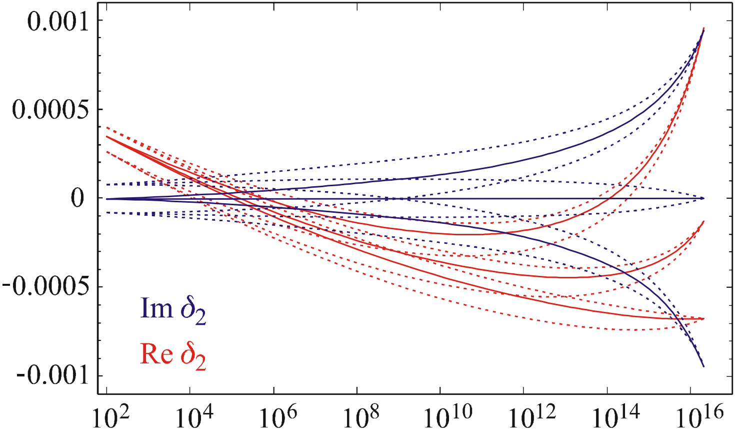
CP-violating phases

SPS-1a point



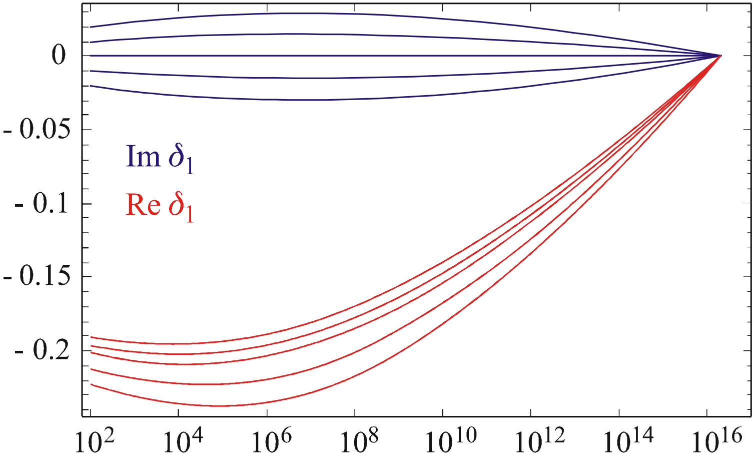
CP-violating phases

SPS-1a point



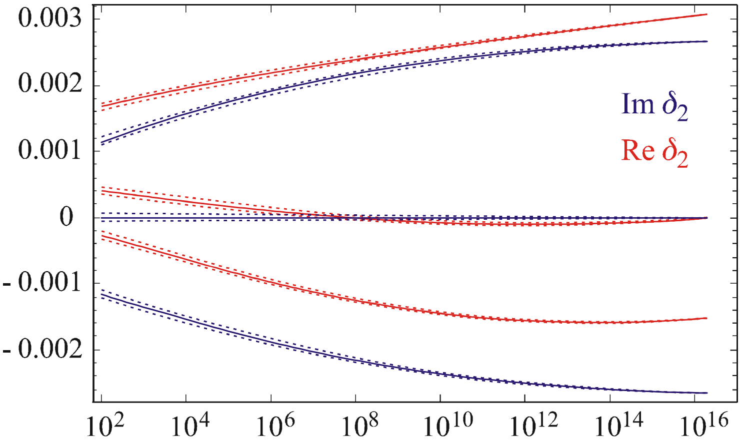
CP-violating phases

SPS-4 point



CP-violating phases

SPS-4 point



Mass insertions

Definition of mass insertions:

$$(\delta_{LL})^{IJ} = \frac{(\mathbf{m}^2_Q)^{IJ}}{\sqrt{|(\mathbf{m}^2_Q)^{II}||(\mathbf{m}^2_Q)^{JJ}|}} \,, \quad (\delta^F_{RR})^{IJ} = \frac{(\mathbf{m}^2_F)^{IJ}}{\sqrt{|(\mathbf{m}^2_F)^{II}||(\mathbf{m}^2_F)^{JJ}|}},$$

with $F = U, D$.

The $SU(2)_L$-breaking RL mass-insertions are defined as

$$(\delta^U_{RL})^{IJ} = \frac{v_u(\mathbf{A}^U)^{IJ}}{\sqrt{|(\mathbf{m}^2_U)^{II}||(\mathbf{m}^2_Q)^{JJ}|}} \,, \quad (\delta^D_{RL})^{IJ} = \frac{v_d(\mathbf{A}^D)^{IJ}}{\sqrt{|(\mathbf{m}^2_D)^{II}||(\mathbf{m}^2_Q)^{JJ}|}}$$

Mass insertions

In terms of the MFV coefficients, they are
(just a couple of examples)

$$(\delta_{LL})^{23} = V_{tb}V_{ts}^* \frac{x_1 + y_2^*}{|\tilde{a}_1|^{1/2}|\tilde{a}_1 + x_1 + y_1 + 2\,\mathrm{Re}\,y_2|^{1/2}} = \frac{V_{tb}V_{ts}^*}{V_{tb}V_{td}^*}(\delta_{LL})^{13} \,,$$

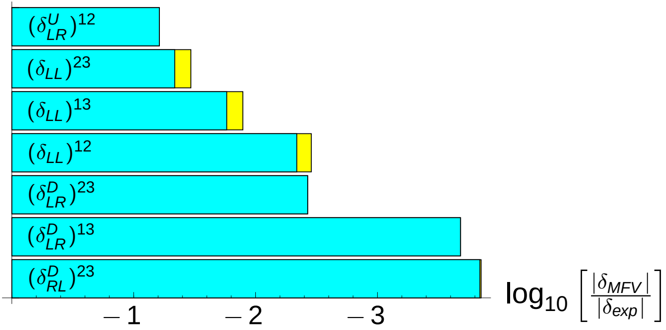
$$(\delta_{LL})^{12} = V_{ts}V_{td}^* \frac{x_1}{|\tilde{a}_1|} \,,$$

$$(\delta_{RL}^U)^{21} = V_{cd} \frac{v_u w_2}{|\tilde{a}_1\tilde{a}_2|^{1/2}} \,,$$

$$(\delta_{RL}^D)^{32} = V_{ts} \frac{v_d y_5}{|\tilde{a}_1|^{1/2}|\tilde{a}_3 + y_3|^{1/2}} = \frac{V_{ts}}{V_{td}}(\delta_{RL}^D)^{31} \,,$$

$$(\delta_{RL}^D)^{23} = \frac{v_d w_4}{|\tilde{a}_3|^{1/2}|\tilde{a}_1 + x_1 + y_1 + 2\,\mathrm{Re}\,y_2|^{1/2}} \,,$$

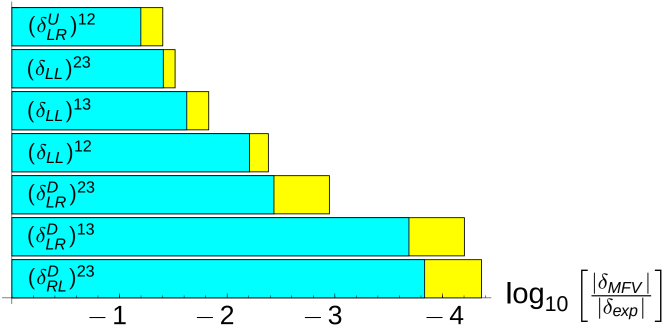
Mass insertions – comparison to exp. bounds



Logarithmic distance of the MI from the experimental bounds when the running is started around SPS-1a.

GC, Mercolli, Nikolidakis, work in progress, cf. Altmannshofer et al. (09)

Mass insertions – comparison to exp. bounds



Logarithmic distance of the MI from the experimental bounds when the running is started around SPS-4.

GC, Mercolli, Nikolidakis, work in progress, cf. Altmannshofer et al. (09)

Outline

Introduction

Minimal flavour violation

Running of MFV models

Summary and conclusions

Summary and conclusions

- I have reviewed the definition of minimal flavour violation in SUSY extensions of the standard model and bound it to power counting rules
- I have shown how the application of the same counting rules leads to a scale-invariant definition of MFV
- we have derived simple RGE's for the MFV parameters and discussed their behaviour with a couple of examples
- nontrivial consequences of the running of the MFV parameters emerge:
  if MFV originates at a higher scale, at low energy it is a lot more constraining than believed so far
- we have derived simplified analytical formulae which allow us to understand this behaviour in detail