Radiative Neutralino Decay—21 Years Later

Howard E. Haber

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The New, the Rare and the Beautiful

Celebrating Daniel Wyler's 60th Birthday

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H.E. Haber and D. Wyler, "Radiative Neutralino Decay," Nucl. Phys. **B323** (1989) 267.

Phenomenological consequences of the latter have appeared in:

S. Ambrosanio and B. Mele, Phys. Rev. **D55** (1997) 1399; M.A. Diaz, B. Panes and Pedro Urrejola, arXiv:0910.1554 [hep-ph].

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RADIATIVE NEUTRALINO DECAY

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We compute the partial width for neutralino radiative decay $\tilde{\chi}_j^0 \to \tilde{\chi}_i^0 \gamma$ in the minimal supersymmetric extension of the Standard Model, for arbitrary neutralino mixing and mass parameters. We identify regions of the supersymmetric parameter space in which the branching ratio can be appreciable. The advantages of using the nonlinear R-gauge in the calculation are emphasized.

1. Introduction

The search for supersymmetric particles remains one of the most important tasks for existing and future experimental facilities. Indeed, much theoretical work has been devoted to the hypothesis that "low-energy" supersymmetry is responsible for the scale of electroweak physics, which would imply that supersymmetric particles should exist with masses less than 1 TeV. Beyond this general expectation, there are no firm theoretical constraints on supersymmetric masses. Experimental bounds from current experiments imply that squarks and gluinos must be heavier than about 60 GeV [1], and (charged) sleptons are heavier than about 20 GeV [2]. Other limits exist, although they are rather complicated, since they usually depend on other assumptions, such as the existence of a very light LSP (lightest supersymmetric particle).

In the development of search strategies for supersymmetric particles, it is crucial to have a thorough understanding of both the production mechanisms and the decay properties. At hadron colliders, it is the production of the colored squarks and

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Outline

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  - fermion-fermion-boson interactions
  - rules for invariant amplitudes
- Conventions for fermion names and fields
- Radiative neutralino decays revisited
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Motivation for two-component spinors

- The fundamental irreducible spin-1/2 degrees of freedom are two-component fermions.

- In chiral theories, each two-component fermion possesses different quantum numbers under the Standard Model gauge group.

- In practical computations involving two-component fermions, Dirac and Majorana fermions are treated in a universal framework.

- Theories of massive Majorana neutrinos and supersymmetric theories (which contain many Majorana fermions) are especially well suited for two-component spinor techniques.

I assume that you are well familiar with the basics of undotted and dotted two-component spinor indices. We follow the usual convention for suppressing spinor indices by adopting a summation convention where we contract indices as follows:

$$ {}^{\alpha}{}_{\alpha} \qquad \text{and} \qquad {}_{\dot\alpha}{}^{\dot\alpha}\,. $$

For example,

$$ \xi\eta \equiv \xi^{\alpha}\eta_{\alpha}, \qquad \xi^{\dagger}\eta^{\dagger} \equiv \xi^{\dagger}_{\dot\alpha}\eta^{\dagger\,\dot\alpha}, $$

$$ \xi^{\dagger}\overline{\sigma}^{\mu}\eta \equiv \xi^{\dagger}_{\dot\alpha}\overline{\sigma}^{\mu\dot\alpha\beta}\eta_{\beta}, \qquad \xi\sigma^{\mu}\eta^{\dagger} \equiv \xi^{\alpha}\sigma^{\mu}_{\alpha\dot\beta}\eta^{\dagger\,\dot\beta} $$

$$ \xi\sigma^{\mu\nu}\eta \equiv \xi^{\alpha}(\sigma^{\mu\nu})_{\alpha}{}^{\beta}\eta_{\beta}\,, \qquad \xi^{\dagger}\overline{\sigma}^{\mu\nu}\eta^{\dagger} \equiv \xi^{\dagger}_{\dot\alpha}(\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}\eta^{\dagger\,\dot\beta}\,, $$

where

$$ \sigma^{\mu} = (\mathbb{1}\,,\,\vec{\boldsymbol{\sigma}})\,, \qquad \overline{\sigma}^{\mu} = (\mathbb{1}\,,\,-\vec{\boldsymbol{\sigma}})\,, $$

and

$$ \sigma^{\mu\nu} = \frac{i}{4}\left(\sigma^{\mu}\overline{\sigma}^{\nu} - \sigma^{\nu}\overline{\sigma}^{\mu}\right)\,, \qquad \overline{\sigma}^{\mu\nu} = \frac{i}{4}\left(\overline{\sigma}^{\mu}\sigma^{\nu} - \overline{\sigma}^{\nu}\sigma^{\mu}\right)\,. $$

Useful identities and Fierz relations

$$\epsilon_{\alpha\beta}\epsilon^{\gamma\delta} = -\delta_\alpha^\gamma\delta_\beta^\delta + \delta_\alpha^\delta\delta_\beta^\gamma, \qquad \epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\gamma\dot\delta} = -\delta_{\dot\alpha}^{\dot\gamma}\delta_{\dot\beta}^{\dot\delta} + \delta_{\dot\alpha}^{\dot\delta}\delta_{\dot\beta}^{\dot\gamma}\,,$$

$$\sigma^\mu_{\alpha\dot\alpha}\overline{\sigma}_\mu^{\dot\beta\beta} = 2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta}\,,$$

$$\sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\,, \qquad \overline{\sigma}^{\mu\dot\alpha\alpha}\overline{\sigma}_\mu^{\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\,,$$

$$[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu]_\alpha{}^\beta = 2g^{\mu\nu}\delta_\alpha^\beta\,,$$

$$[\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu]^{\dot\alpha}{}_{\dot\beta} = 2g^{\mu\nu}\delta_{\dot\beta}^{\dot\alpha}\,,$$

$$\sigma^\mu\overline{\sigma}^\nu\sigma^\rho = g^{\mu\nu}\sigma^\rho - g^{\mu\rho}\sigma^\nu + g^{\nu\rho}\sigma^\mu + i\epsilon^{\mu\nu\rho\kappa}\sigma_\kappa\,,$$

$$\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho = g^{\mu\nu}\overline{\sigma}^\rho - g^{\mu\rho}\overline{\sigma}^\nu + g^{\nu\rho}\overline{\sigma}^\mu - i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_\kappa\,,$$

where $g_{\mu\nu} = \mathrm{diag}(1\,,\,-1\,,\,-1\,,\,-1)$ in our conventions. Computations of cross sections and decay rates generally require traces of alternating products of σ and $\overline{\sigma}$ matrices:

$$\mathrm{Tr}[\sigma^\mu\overline{\sigma}^\nu] = \mathrm{Tr}[\overline{\sigma}^\mu\sigma^\nu] = 2g^{\mu\nu}\,,$$

$$\mathrm{Tr}[\sigma^\mu\overline{\sigma}^\nu\sigma^\rho\overline{\sigma}^\kappa] = 2\left(g^{\mu\nu}g^{\rho\kappa} - g^{\mu\rho}g^{\nu\kappa} + g^{\mu\kappa}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\kappa}\right)\,,$$

$$\mathrm{Tr}[\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho\sigma^\kappa] = 2\left(g^{\mu\nu}g^{\rho\kappa} - g^{\mu\rho}g^{\nu\kappa} + g^{\mu\kappa}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\kappa}\right)\,,$$

where $\epsilon^{0123} = -\epsilon_{0123} = +1$ in our conventions. Traces involving an odd number of σ and $\overline{\sigma}$ matrices cannot arise, since there is no way to connect the spinor indices consistently.

We shall deal with both commuting and anticommuting spinors, which we shall denote generically by z_i. Then, the following identities hold

$$
\begin{aligned}
z_1 z_2 &= -(-1)^A z_2 z_1 \\
z_1^\dagger z_2^\dagger &= -(-1)^A z_2^\dagger z_1^\dagger \\
z_1 \sigma^\mu z_2^\dagger &= (-1)^A z_2^\dagger \overline{\sigma}^\mu z_1 \\
z_1 \sigma^\mu \overline{\sigma}^\nu z_2 &= -(-1)^A z_2 \sigma^\nu \overline{\sigma}^\mu z_1 \\
z_1^\dagger \overline{\sigma}^\mu \sigma^\nu z_2^\dagger &= -(-1)^A z_2^\dagger \overline{\sigma}^\nu \sigma^\mu z_1^\dagger \\
z_1^\dagger \overline{\sigma}^\mu \sigma^\rho \overline{\sigma}^\nu z_2 &= (-1)^A z_2 \sigma^\nu \overline{\sigma}^\rho \sigma^\mu z_1^\dagger \,,
\end{aligned}
$$

where $(-1)^A = +1[-1]$ for commuting [anticommuting] spinors. Finally, the Fierz identities are given by:

$$
\begin{aligned}
(z_1 z_2)(z_3 z_4) &= -(z_1 z_3)(z_4 z_2) - (z_1 z_4)(z_2 z_3) \,, \\
(z_1^\dagger z_2^\dagger)(z_3^\dagger z_4^\dagger) &= -(z_1^\dagger z_3^\dagger)(z_4^\dagger z_2^\dagger) - (z_1^\dagger z_4^\dagger)(z_2^\dagger z_3^\dagger) \,, \\
(z_1 \sigma^\mu z_2^\dagger)(z_3^\dagger \overline{\sigma}_\mu z_4) &= -2(z_1 z_4)(z_2^\dagger z_3^\dagger) \,, \\
(z_1^\dagger \overline{\sigma}^\mu z_2)(z_3^\dagger \overline{\sigma}_\mu z_4) &= 2(z_1^\dagger z_3^\dagger)(z_4 z_2) \,, \\
(z_1 \sigma^\mu z_2^\dagger)(z_3 \sigma_\mu z_4^\dagger) &= 2(z_1 z_3)(z_4^\dagger z_2^\dagger) \,.
\end{aligned}
$$

Two-component spinor wave functions

The $(\frac{1}{2},0)$ spinor field $\xi_\alpha(x)$ describes a neutral Majorana fermion. The free-field Lagrangian is:

$$\mathscr{L} = i\xi^\dagger\overline{\sigma}^\mu\partial_\mu\xi - \tfrac{1}{2}m(\xi\xi + \xi^\dagger\xi^\dagger)\,.$$

On-shell, ξ satisfies the free-field Dirac equation, $i\overline{\sigma}^{\mu\dot{\alpha}\beta}\partial_\mu\xi_\beta = m\xi^{\dagger\,\dot{\alpha}}$. The solution is:

$$\xi_\alpha(x) = \sum_s \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^{3/2}(2E_{\boldsymbol{p}})^{1/2}}\left[x_\alpha(\vec{\boldsymbol{p}},s)a(\vec{\boldsymbol{p}},s)e^{-ip\cdot x} + y_\alpha(\vec{\boldsymbol{p}},s)a^\dagger(\vec{\boldsymbol{p}},s)e^{ip\cdot x}\right]\,,$$

and $\xi^\dagger_{\dot{\alpha}} = (\xi_\alpha)^\dagger$. The two-component fermion wave functions, x and y are *commuting* spinors that satisfy the momentum-space Dirac equation:

$$(p\cdot\overline{\sigma})^{\dot{\alpha}\beta}x_\beta = my^{\dagger\,\dot{\alpha}}\,, \qquad (p\cdot\sigma)_{\alpha\dot{\beta}}y^{\dagger\,\dot{\beta}} = mx_\alpha\,,$$

$$(p\cdot\sigma)_{\alpha\dot{\beta}}x^{\dagger\,\dot{\beta}} = -my_\alpha\,, \qquad (p\cdot\overline{\sigma})^{\dot{\alpha}\beta}y_\beta = -mx^{\dagger\,\dot{\alpha}}\,.$$

The spin or helicity is labeled by $s=\pm\frac{1}{2}$. For spin, we quantize in the rest frame along a fixed axis $\hat{\boldsymbol{s}} \equiv (\sin\theta\cos\phi\,,\,\sin\theta\sin\phi\,,\,\cos\theta)$. Eigenstates of $\frac{1}{2}\vec{\boldsymbol{\sigma}}\boldsymbol{\cdot}\hat{\boldsymbol{s}}$ are denoted by χ_s, *i.e.*, $\frac{1}{2}\vec{\boldsymbol{\sigma}}\boldsymbol{\cdot}\hat{\boldsymbol{s}}\,\chi_s = s\chi_s$,. Explicitly,

$$\chi_{1/2}(\hat{\boldsymbol{s}}) = \begin{pmatrix} e^{-i\phi/2}\cos(\theta/2) \\ e^{i\phi/2}\sin(\theta/2) \end{pmatrix}, \quad \chi_{-1/2}(\hat{\boldsymbol{s}}) = \begin{pmatrix} -e^{-i\phi/2}\sin(\theta/2) \\ e^{i\phi/2}\cos(\theta/2) \end{pmatrix}.$$

Introduce the spin 4-vector for massive fermions. For fixed-axis spin states, $S^\mu \equiv (0\,;\,\hat{\boldsymbol{s}})$ in the rest frame, boosting to the frame where $p = (E_{\boldsymbol{p}}\,;\,\vec{\boldsymbol{p}})$,

$$S^\mu = \left(\frac{\vec{\boldsymbol{p}}\boldsymbol{\cdot}\hat{\boldsymbol{s}}}{m}\,;\,\hat{\boldsymbol{s}} + \frac{(\vec{\boldsymbol{p}}\boldsymbol{\cdot}\hat{\boldsymbol{s}})\,\vec{\boldsymbol{p}}}{m(E+m)}\right).$$

Helicity states are defined to be eigenstates of $\frac{1}{2}\vec{\boldsymbol{\sigma}}\boldsymbol{\cdot}\hat{\boldsymbol{p}}$, *i.e.*, $\frac{1}{2}\vec{\boldsymbol{\sigma}}\boldsymbol{\cdot}\hat{\boldsymbol{p}}\,\chi_\lambda = \lambda\chi_\lambda$ $(\lambda=\pm\frac{1}{2})$. The explicit forms for χ_λ are the same as above, with θ and ϕ the polar and azimuthal angles of $\hat{\boldsymbol{p}}$. The spin 4-vector is defined by taking $\hat{\boldsymbol{s}} = \hat{\boldsymbol{p}}$. Thus, $S^\mu = \frac{1}{m}\,(|\vec{\boldsymbol{p}}|\,;\,E\hat{\boldsymbol{p}})$. In the high energy limit, $S^\mu = p^\mu/m + \mathcal{O}(m/E)$.

Explicit construction of the x and y wave functions

The Dirac equation implies that in the rest frame $x_1 = y^{\dagger\,1}$ and $x_2 = y^{\dagger\,2}$. That is, $x_\alpha(\vec{\boldsymbol{p}}=0) = y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}=0)$ are linear combinations of the χ_s ($s=\pm\frac{1}{2}$). Choose $x_\alpha(\vec{\boldsymbol{p}}=0,s) = y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}=0,s) = \sqrt{m}\chi_s$, and boost to $\vec{\boldsymbol{p}}\neq 0$:

$$\boxed{\begin{array}{ll} x_\alpha(\vec{\boldsymbol{p}},s) = \sqrt{p\cdot\sigma}\,\chi_s\,, & y_\alpha(\vec{\boldsymbol{p}},s) = 2s\sqrt{p\cdot\sigma}\,\chi_{-s}\,, \\ x^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}},s) = -2s\sqrt{p\cdot\overline{\sigma}}\,\chi_{-s}\,, & y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}},s) = \sqrt{p\cdot\overline{\sigma}}\,\chi_s\,. \end{array}}$$

For helicity spinors, replace s with λ. For massless fermions, we must use helicity spinors. Putting $E=|\vec{\boldsymbol{p}}|$ and $m=0$,

$$x_\alpha(\vec{\boldsymbol{p}},\lambda) = \sqrt{E/2}\;(1-2\lambda)\,\chi_\lambda\,,$$

$$y_\alpha(\vec{\boldsymbol{p}},\lambda) = \sqrt{E/2}\;(1+2\lambda)\,\chi_{-\lambda}\,,$$

$$x^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}},\lambda) = \sqrt{E/2}\;(1-2\lambda)\,\chi_{-\lambda}\,,$$

$$y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}},\lambda) = \sqrt{E/2}\;(1+2\lambda)\,\chi_\lambda\,.$$

For a given λ, only one helicity component of x and y survives.

Projection operators

$$x_\alpha(\vec{\boldsymbol{p}}, s)x^\dagger_{\dot\beta}(\vec{\boldsymbol{p}}, s) = \tfrac{1}{2}(p_\mu - 2smS_\mu)\sigma^\mu_{\alpha\dot\beta}\,,$$

$$y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}, s)y^\beta(\vec{\boldsymbol{p}}, s) = \tfrac{1}{2}(p^\mu + 2smS^\mu)\overline{\sigma}^{\dot\alpha\beta}_\mu\,,$$

$$x_\alpha(\vec{\boldsymbol{p}}, s)y^\beta(\vec{\boldsymbol{p}}, s) = \tfrac{1}{2}\left(m\delta_\alpha{}^\beta - 2s[S\!\cdot\!\sigma\, p\!\cdot\!\overline{\sigma}]_\alpha{}^\beta\right)\,,$$

$$y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}, s)x^\dagger_{\dot\beta}(\vec{\boldsymbol{p}}, s) = \tfrac{1}{2}\left(m\delta^{\dot\alpha}{}_{\dot\beta} + 2s[S\!\cdot\!\overline{\sigma}\, p\!\cdot\!\sigma]^{\dot\alpha}{}_{\dot\beta}\right)\,.$$

For massless spinors, the helicity projection operators are:

$$x_\alpha(\vec{\boldsymbol{p}}, \lambda)x^\dagger_{\dot\beta}(\vec{\boldsymbol{p}}, \lambda) = (\tfrac{1}{2} - \lambda)p\!\cdot\!\sigma_{\alpha\dot\beta}\,,$$

$$y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}, \lambda)y^\beta(\vec{\boldsymbol{p}}, \lambda) = (\tfrac{1}{2} + \lambda)p\!\cdot\!\overline{\sigma}^{\dot\alpha\beta}\,,$$

$$x_\alpha(\vec{\boldsymbol{p}}, \lambda)y^\beta(\vec{\boldsymbol{p}}, \lambda) = y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}, \lambda)x^\dagger_{\dot\beta}(\vec{\boldsymbol{p}}, \lambda) = 0\,.$$

Summing over s (or λ) yields:

$$\sum_s x_\alpha(\vec{\boldsymbol{p}}, s)x^\dagger_{\dot\beta}(\vec{\boldsymbol{p}}, s) = p\!\cdot\!\sigma_{\alpha\dot\beta}\,, \qquad \sum_s y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}, s)y^\beta(\vec{\boldsymbol{p}}, s) = p\!\cdot\!\overline{\sigma}^{\dot\alpha\beta}\,,$$

$$\sum_s x_\alpha(\vec{\boldsymbol{p}}, s)y^\beta(\vec{\boldsymbol{p}}, s) = m\delta_\alpha{}^\beta\,, \qquad \sum_s y^{\dagger\,\dot\alpha}(\vec{\boldsymbol{p}}, s)x^\dagger_{\dot\beta}(\vec{\boldsymbol{p}}, s) = m\delta^{\dot\alpha}{}_{\dot\beta}\,.$$

Fermion mass diagonalization

The Lagrangian of a collection of free anti-commuting spin-1/2 "interaction-eigenstate" fields $\hat{\xi}_{\alpha i}(x)$, labeled by flavor index i:

$$\mathscr{L} = i\hat{\xi}^{\dagger\, i}\overline{\sigma}^{\mu}\partial_{\mu}\hat{\xi}_i - \tfrac{1}{2}M^{ij}\hat{\xi}_i\hat{\xi}_j - \tfrac{1}{2}M_{ij}\hat{\xi}^{\dagger, i}\hat{\xi}^{\dagger, j}\,,$$

where $M_{ij} \equiv (M^{ij})^*$ is a *complex symmetric* matrix. We shall rewrite this in terms of mass-eigenstate fields $\xi(x) = \Omega^{-1}\hat{\xi}(x)$, where Ω is unitary and chosen such that

$$\Omega^T M\,\Omega = m = \mathrm{diag}(m_1, m_2, \ldots).$$

In linear algebra, this is called the Takagi-diagonalization of a complex symmetric matrix M. To compute the values of the diagonal elements of m, one may simply note that

$$\Omega^T M M^{\dagger}\Omega^* = m^2.$$

$MM^{\dagger}$ is hermitian, and thus it can be diagonalized by a unitary matrix. Thus, the m_i of the Takagi diagonalization are the non-negative square-roots of the eigenvalues of $MM^{\dagger}$. In terms of the mass eigenstates,

$$\mathscr{L} = i\xi^{\dagger\, i}\overline{\sigma}^{\mu}\partial_{\mu}\xi_i - \tfrac{1}{2}m_i(\xi_i\xi_i + \xi^{\dagger\, i}\xi^{\dagger\, i})\,.$$

The Dirac fermion

A charged fermion has twice the number of degrees of freedom as the neutral fermion. If χ and η are oppositely charged and degenerate in mass, then the corresponding free-field Lagrangian is:

$$\mathscr{L} = i\chi^\dagger\overline{\sigma}^\mu\partial_\mu\chi + i\eta^\dagger\overline{\sigma}^\mu\partial_\mu\eta - m(\chi\eta + \chi^\dagger\eta^\dagger)\,.$$

Together, χ and $\eta^\dagger$ constitute a single Dirac fermion. The corresponding mass matrix is $\left(\begin{smallmatrix} 0 & m \\ m & 0 \end{smallmatrix}\right)$. One could Takagi-diagonalize this matrix, although the corresponding mass eigenstates would not be eigenstates of charge.

The solutions to the corresponding Dirac field equations are:

$$\chi_\alpha(x) = \sum_s \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^{3/2}(2E_{\boldsymbol{p}})^{1/2}}\left[x_\alpha(\vec{\boldsymbol{p}},s)a(\vec{\boldsymbol{p}},s)e^{-ip\cdot x} + y_\alpha(\vec{\boldsymbol{p}},s)b^\dagger(\vec{\boldsymbol{p}},s)e^{ip\cdot x}\right]\,,$$

$$\eta_\alpha(x) = \sum_s \int \frac{d^3\vec{\boldsymbol{p}}}{(2\pi)^{3/2}(2E_{\boldsymbol{p}})^{1/2}}\left[x_\alpha(\vec{\boldsymbol{p}},s)b(\vec{\boldsymbol{p}},s)e^{-ip\cdot x} + y_\alpha(\vec{\boldsymbol{p}},s)a^\dagger(\vec{\boldsymbol{p}},s)e^{ip\cdot x}\right]\,.$$

More generally, for a collection of interaction-eigenstate charged fermion pairs $\hat{\chi}_{\alpha i}(x)$, $\hat{\eta}^i_\alpha(x)$, the free-field Lagrangian is:

$$\mathscr{L} = i\hat{\chi}^{\dagger\, i}\overline{\sigma}^\mu \partial_\mu \hat{\chi}_i + i\hat{\eta}^\dagger_i \overline{\sigma}^\mu \partial_\mu \hat{\eta}^i - M^i{}_j \hat{\chi}_i \hat{\eta}^j - M_i{}^j \hat{\chi}^{\dagger\, i} \hat{\eta}^\dagger_j \,,$$

where $M^i{}_j$ is an arbitrary complex matrix, and $M_i{}^j \equiv (M^i{}_j)^*$. We diagonalize the mass matrix by introducing mass-eigenstates $\chi(x) = L^{-1}\hat{\chi}(x)$ and $\eta(x) = R^{-1}\hat{\eta}(x)$ where L and R are unitary matrices that are chosen such that:

$$L^T M R = m = \mathrm{diag}(m_1, m_2, \ldots),$$

with the m_i real and non-negative. This is the singular-value decomposition of linear algebra, which states that for any complex matrix M, the unitary matrices L and R above exist. Due to

$$L^T(MM^\dagger)L^* \;=\; R^\dagger(M^\dagger M)R \;=\; m^2 \,,$$

the m_i are the non-negative square roots of the eigenvalues of $MM^\dagger$ (or equivalently, $M^\dagger M$). In terms of the mass eigenstates,

$$\mathscr{L} = i\chi^{\dagger\, i}\overline{\sigma}^\mu \partial_\mu \chi_i + i\eta^\dagger_i \overline{\sigma}^\mu \partial_\mu \eta^i - m_i(\chi_i \eta^i + \chi^{\dagger\, i}\eta^\dagger_i) \,.$$

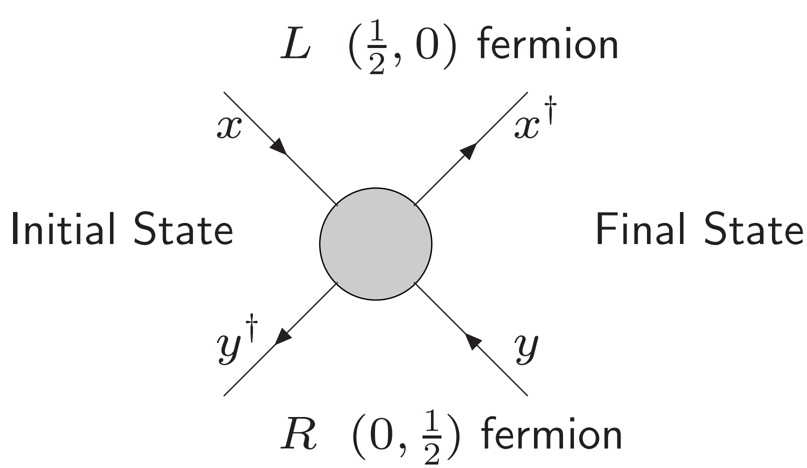
The mass matrix now consists of 2×2 blocks $\left(\begin{smallmatrix} 0 & m_i \\ m_i & 0 \end{smallmatrix}\right)$ along the diagonal.

Feynman rules for two-component fermions

The rules for assigning two-component external state spinors are then as follows.

- For an initial-state left-handed $(\frac{1}{2}, 0)$ fermion: x.
- For an initial-state right-handed $(0, \frac{1}{2})$ fermion: $y^\dagger$.
- For a final-state left-handed $(\frac{1}{2}, 0)$ fermion: $x^\dagger$.
- For a final-state right-handed $(0, \frac{1}{2})$ fermion: y.

The two-component external state fermion wave functions are distinguished by their Lorentz group transformation properties, rather than by their particle or antiparticle status as in four-component Feynman rules. These rules are summarized in the mnemonic diagram:



Propagators

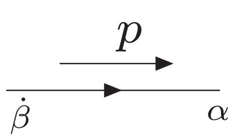
$$\langle 0|\, T\xi_\alpha(x)\bar{\xi}_{\dot\beta}(y)\,|0\rangle_{\rm FT} = \frac{i}{p^2-m^2+i\epsilon}\sum_s x_\alpha(\boldsymbol{\vec p},s)\bar{x}_{\dot\beta}(\boldsymbol{\vec p},s)$$

$$\langle 0|\, T\bar{\xi}^{\dot\alpha}(x)\xi^{\beta}(y)\,|0\rangle_{\rm FT} = \frac{i}{p^2-m^2+i\epsilon}\sum_s \bar{y}^{\dot\alpha}(\boldsymbol{\vec p},s)y^{\beta}(\boldsymbol{\vec p},s)$$

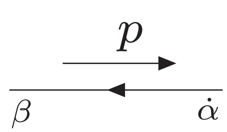
$$\langle 0|\, T\bar{\xi}^{\dot\alpha}(x)\bar{\xi}_{\dot\beta}(y)\,|0\rangle_{\rm FT} = \frac{i}{p^2-m^2+i\epsilon}\sum_s \bar{y}^{\dot\alpha}(\boldsymbol{\vec p},s)\bar{x}_{\dot\beta}(\boldsymbol{\vec p},s)$$

$$\langle 0|\, T\xi_\alpha(x)\xi^{\beta}(y)\,|0\rangle_{\rm FT} = \frac{i}{p^2-m^2+i\epsilon}\sum_s x_\alpha(\boldsymbol{\vec p},s)y^{\beta}(\boldsymbol{\vec p},s)$$

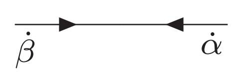
where FT indicates the Fourier transform from position to momentum space. These results have an obvious diagrammatic representation:

(a) 

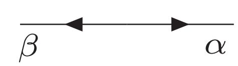
$$\frac{ip\cdot\sigma_{\alpha\dot\beta}}{p^2-m^2}$$

(b) 

$$\frac{ip\cdot\overline{\sigma}^{\dot\alpha\beta}}{p^2-m^2}$$

(c) 

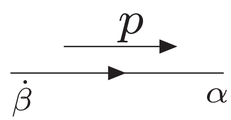
$$\frac{im}{p^2-m^2}\delta^{\dot\alpha}{}_{\dot\beta}$$

(d) 

$$\frac{im}{p^2-m^2}\delta_\alpha{}^\beta$$

Arrows on two-component fermion lines always run away from dotted indices at a vertex and toward undotted indices at a vertex. Arrows *do not* represent the flow of fermion number!

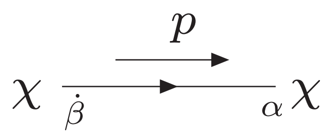
The arrow-preserving propagators can be described by one diagram:



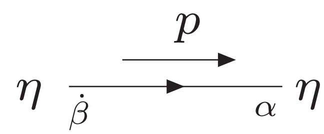
$$\frac{ip\cdot\sigma_{\alpha\dot{\beta}}}{p^2 - m^2} \quad \underline{\text{or}} \quad \frac{-ip\cdot\overline{\sigma}^{\dot{\beta}\alpha}}{p^2 - m^2}$$

Here the choice of the σ or the $\overline{\sigma}$ version of the rule is uniquely determined by the height of the indices on the vertex to which the propagator is connected.

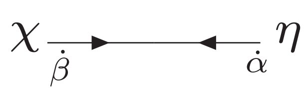
For the case of charged fermions, we write down the rules for propagators involving the charged pair χ and η:



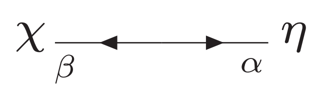
$$\frac{ip\cdot\sigma_{\alpha\dot{\beta}}}{p^2 - m^2} \quad \underline{\text{or}} \quad \frac{-ip\cdot\overline{\sigma}^{\dot{\beta}\alpha}}{p^2 - m^2}$$



$$\frac{ip\cdot\sigma_{\alpha\dot{\beta}}}{p^2 - m^2} \quad \underline{\text{or}} \quad \frac{-ip\cdot\overline{\sigma}^{\dot{\beta}\alpha}}{p^2 - m^2}$$



$$\frac{im}{p^2 - m^2}\delta^{\dot{\alpha}}{}_{\dot{\beta}}$$



$$\frac{im}{p^2 - m^2}\delta_{\alpha}{}^{\beta}$$

Fermion interactions

The mass-eigenstate basis ψ is related to the interaction-eigenstate basis $\hat{\psi}$ by a unitary rotation:

$$\hat{\psi} \equiv \begin{pmatrix} \hat{\xi} \\ \hat{\chi} \\ \hat{\eta} \end{pmatrix} = U\psi \equiv \begin{pmatrix} \Omega & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & R \end{pmatrix} \begin{pmatrix} \xi \\ \chi \\ \eta \end{pmatrix} ,$$

where Ω, L, and R are constructed as described previously. Thus, in terms of mass-eigenstate fields, the fermion–scalar boson interactions are:

$$\mathscr{L}_{\rm int} = -\tfrac{1}{2} Y^{Ijk} \phi_I \psi_j \psi_k - \tfrac{1}{2} Y_{Ijk} \phi^I \psi^{\dagger\, j} \psi^{\dagger\, k} .$$

In the gauge-interaction basis for the left-handed two-component fermions the corresponding interaction Lagrangian is given by

$$\mathscr{L}_{\rm int} = -g_a A_a^\mu \hat{\psi}^{\dagger\, i} \overline{\sigma}_\mu (\boldsymbol{T}^a)_i{}^j \hat{\psi}_j ,$$

If the gauge symmetry is unbroken, then the index a runs over the adjoint representation of the gauge group, and the $(\boldsymbol{T}^a)_i{}^j$ are hermitian representation matrices of the gauge group acting on the left-handed fermions.

In terms of mass-eigenstate fermion fields,

$$\mathscr{L}_{\rm int} = -A^\mu_a \psi^{\dagger\, i}\, \overline{\sigma}_\mu (G^a)_i{}^j \psi_j \,,$$

where $G^a = g_a U^\dagger \boldsymbol{T}^a U$ (no sum over a).

Consider separately the case of gauge interactions of charged Dirac fermions. Consider pairs of left-handed $(\frac{1}{2},0)$ interaction-eigenstate fermions $\hat{\chi}_i$ and $\hat{\eta}^i$ that transform as conjugate representations of the gauge group (hence the difference in the flavor index heights). The Lagrangian for the gauge interactions of Dirac fermions can be written in the form:

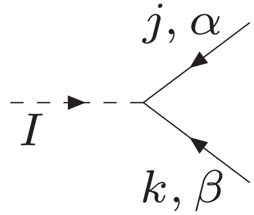
$$\mathscr{L}_{\rm int} = -g_a A^\mu_a \hat{\chi}^{\dagger\, i}\, \overline{\sigma}_\mu (\boldsymbol{T}^a)_i{}^j \hat{\chi}_j + g_a A^\mu_a \hat{\eta}^\dagger_i\, \overline{\sigma}_\mu (\boldsymbol{T}^a)_j{}^i \hat{\eta}^j \,,$$

where the A^a_μ are gauge boson mass-eigenstate fields. Here we have used the fact that if $(\boldsymbol{T}^a)_i{}^j$ are the representation matrices for the $\hat{\chi}_i$, then the $\hat{\eta}_i$ transform in the complex conjugate representation with generator matrices $-(\boldsymbol{T}^a)^* = -(\boldsymbol{T}^a)^T$. In terms of mass-eigenstate fermion fields,

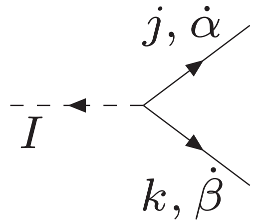
$$\mathscr{L}_{\rm int} = -A^\mu_a \left[\chi^{\dagger\, i}\, \overline{\sigma}_\mu (G^a_L)_i{}^j \chi_j - \eta^\dagger_i\, \overline{\sigma}_\mu (G^a_R)_j{}^i \eta^j\right] \,,$$

where $G^a_L = g_a L^\dagger \boldsymbol{T}^a L$ and $G^a_R = g_a R^\dagger \boldsymbol{T}^a R$ (no sum over a).

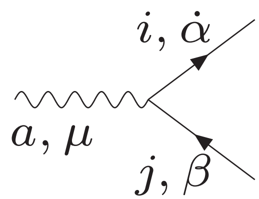
Feynman rules for fermion interactions



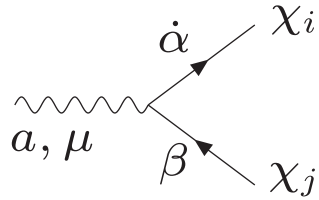
$-iY^{Ijk}\delta_{\alpha}{}^{\beta}$ or $-iY^{Ijk}\delta_{\beta}{}^{\alpha}$



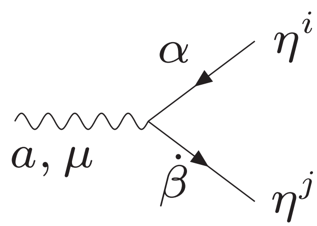
$-iY_{Ijk}\delta^{\dot{\alpha}}{}_{\dot{\beta}}$ or $-iY_{Ijk}\delta^{\dot{\beta}}{}_{\dot{\alpha}}$



$-i(G^a)_i{}^j\,\overline{\sigma}_{\mu}^{\dot{\alpha}\beta}$ or $i(G^a)_i{}^j\,\sigma_{\mu\beta\dot{\alpha}}$



$-i(G^a_L)_i{}^j\,\overline{\sigma}_{\mu}^{\dot{\alpha}\beta}$ or $ig(G^a_L)_i{}^j\,\sigma_{\mu\beta\dot{\alpha}}$



$i(G^a_R)_i{}^j\,\overline{\sigma}_{\mu}^{\dot{\beta}\alpha}$ or $-ig(G^a_R)_i{}^j\,\sigma_{\mu\alpha\dot{\beta}}$

Rules for invariant amplitudes

- When computing an amplitude for a given process, all possible diagrams should be drawn that conform with the rules for external wave functions, propagators, and interactions.
- Starting from any external wave function spinor, or from any vertex on a fermion loop, factors corresponding to each propagator and vertex should be written down from left to right, following the line until it ends at another external state wave function or at the original point on the fermion loop.
- If one starts a fermion line at an x or y external state spinor, it should have a raised undotted index in accord with our summation conventions. Or, if one starts with an $x^\dagger$ or $y^\dagger$, it should have a lowered dotted spinor index. If one ends with an x or y external state spinor, it will have a lowered undotted index, while if one ends with an $x^\dagger$ or $y^\dagger$ spinor, it will have a raised dotted index. The preceding determines whether a σ or $\overline{\sigma}$ rule should be used.
- A relative minus sign is imposed between terms contributing to a given amplitude whenever the ordering of external state spinors (written left-to-right) differs by an odd permutation.
- Each closed fermion loop gets a factor of -1.

Conventions for fermion names and fields

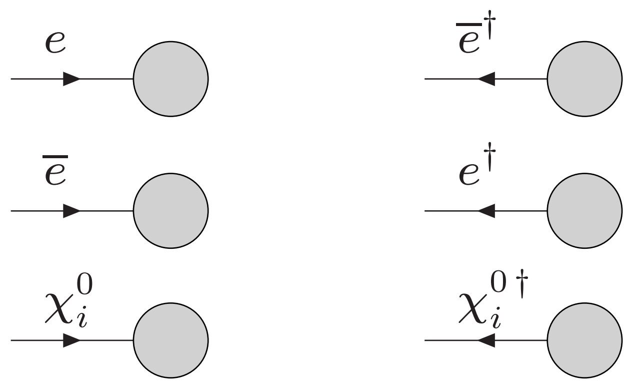
There is a one-to-one correspondence between the Majorana fermion particle names and the left-handed $(\frac{1}{2}, 0)$ fields, but for Dirac fermions there are always two distinct two-component fields that correspond to each particle name. We shall always label fermion lines with the two-component fields, rather than the particle names, with the following conventions:

- Initial-state external fermion lines (which always have physical four-momenta going into the vertex) in Feynman diagrams are labeled by the corresponding unbarred (left-handed) field if the arrow is into the vertex, and by the barred (right-handed) field if the arrow is away from the vertex.

Initial-state e^-:

Initial-state e^+:

Initial-state $\widetilde{N}_i$:

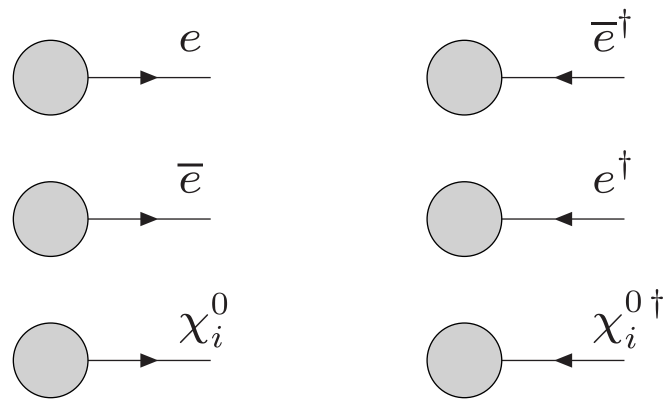


- Final-state external fermion lines in complete Feynman diagrams (which always have physical four-momenta going out of the vertex) are labeled by the corresponding barred (right-handed) field if the arrow is into the vertex, and by the unbarred (left-handed) field if the arrow is away from the vertex.

Final-state e^-:

Final-state e^+:

Final-state $\widetilde{N}_i$:



- Internal fermion lines in Feynman diagrams are also always labeled by the unbarred, left-handed field(s). Internal lines containing a propagator with opposing arrows can carry two labels.

- In the Feynman rules for interaction vertices, the external lines are always labeled by the unbarred left-handed field, regardless of its arrow direction.

Fermion name	Two-component fields
ℓ^- (lepton)	$\ell\,,\ \overline{\ell}^{\,\dagger}$
ℓ^+ (anti-lepton)	$\overline{\ell}\,,\ \ell^\dagger$
ν (neutrino)	$\nu\,,\ \overline{\nu}^{\,\dagger}$
$\bar\nu$ (antineutrino)	$\overline{\nu}\,,\ \nu^\dagger$
q (quark)	$q\,,\ \overline{q}^{\,\dagger}$
$\bar q$ (anti-quark)	$\overline{q}\,,\ q^\dagger$
f (quark or lepton)	$f\,,\ \overline{f}^{\,\dagger}$
$\bar f$ (anti-quark or anti-lepton)	$\overline{f}\,,\ f^\dagger$
$\widetilde{N}_i$ (neutralino)	$\chi_i^0\,,\ \chi_i^{0\,\dagger}$
$\widetilde{C}_i^+$ (chargino)	$\chi_i^+\,,\ \chi_i^{-\,\dagger}$
$\widetilde{C}_i^-$ (anti-chargino)	$\chi_i^-\,,\ \chi_i^{+\,\dagger}$
$\widetilde{g}$ (gluino)	$\widetilde{g}\,,\ \widetilde{g}^\dagger$

Fermion and anti-fermion names and two-component fields in the Standard Model and the MSSM. (To incorporate massive neutrinos, one should add $\overline{\nu}$ and $\overline{\nu}^{\,\dagger}$.)

Gauge interactions of charginos and neutralinos

Introduce U and V, the unitary matrices that diagonalize the chargino mass matrix via the singular value decomposition:

$$U^* \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix} V^{-1} = \mathrm{diag}(m_{\widetilde{C}_1}, m_{\widetilde{C}_2}) \,.$$

Similarly, N is a unitary matrix that Takagi-diagonalizes the neutralino mass matrix,

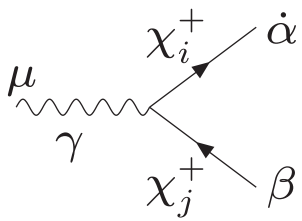
$$N^* \begin{pmatrix} M_1 & 0 & -g'v_d/\sqrt{2} & g'v_u/\sqrt{2} \\ 0 & M_2 & gv_d/\sqrt{2} & -gv_u/\sqrt{2} \\ -g'v_d/\sqrt{2} & gv_d/\sqrt{2} & 0 & -\mu \\ g'v_u/\sqrt{2} & -gv_u/\sqrt{2} & -\mu & 0 \end{pmatrix} N^{-1} = \mathrm{diag}(m_{\widetilde{N}_1}, m_{\widetilde{N}_2}, m_{\widetilde{N}_3}, m_{\widetilde{N}_4}) \,.$$

Here, v_u and v_d are the neutral Higgs vacuum expectation values, M_1, M_2 are the gaugino Majorana mass parameters and μ is the higgsino mass parameter.

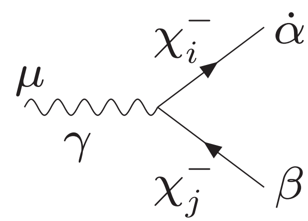
We define:

$$
\begin{aligned}
O^L_{ij} &= -\tfrac{1}{\sqrt{2}} N_{i4} V^*_{j2} + N_{i2} V^*_{j1}\,, \\
O^R_{ij} &= \quad \tfrac{1}{\sqrt{2}} N^*_{i3} U_{j2} + N^*_{i2} U_{j1}\,.
\end{aligned}
$$

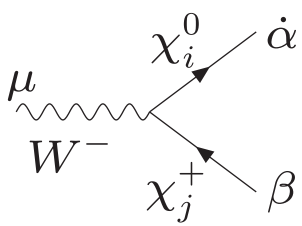
The Feynman rules for the interactions of the W and the photon with the neutralinos and charginos are displayed below.



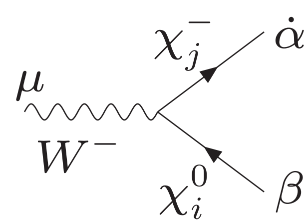
$$-ie\,\delta_{ij}\overline{\sigma}_\mu^{\dot\alpha\beta}$$



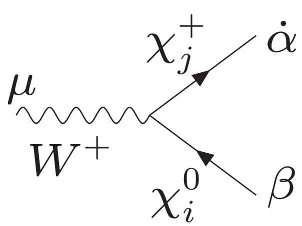
$$ie\,\delta_{ij}\overline{\sigma}_\mu^{\dot\alpha\beta}$$



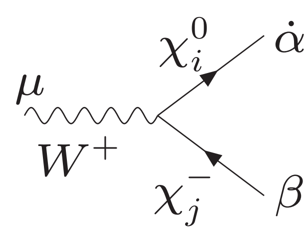
$$ig\,O^L_{ij}\,\overline{\sigma}_\mu^{\dot\alpha\beta}$$



$$-ig\,O^R_{ij}\,\overline{\sigma}_\mu^{\dot\alpha\beta}$$



$$ig\,O^{L*}_{ij}\,\overline{\sigma}_\mu^{\dot\alpha\beta}$$



$$-ig\,O^{R*}_{ij}\,\overline{\sigma}_\mu^{\dot\alpha\beta}$$

For each rule, one has a version with lowered spinor indices by replacing $\overline{\sigma}_\mu^{\dot\alpha\beta} \to -\sigma_{\mu\beta\dot\alpha}$. Fermion lines are labeled by the two-component fermion field names previously given.

Neutralino radiative decay

The dominant neutralino decays are expected to be:

- Two body decays: $\widetilde{N}_j \to \widetilde{N}_i Z^0\,, \widetilde{N}_i h^0\,, \widetilde{C}_i^{\pm} W^{\mp}\,,$ $\qquad \Gamma \sim \mathcal{O}(g^2 M_{\tilde{N}_j})$,
- Three body decays: $\widetilde{N}_j \to \widetilde{N}_i f \bar{f}\,, \widetilde{N}_j \to \widetilde{C}_i^{\pm} f \bar{f}\,'$, $\qquad \Gamma \sim \mathcal{O}(g^4 M_{\tilde{N}_j})$,
- Radiative one-loop decays: $\widetilde{N}_j \to \widetilde{N}_i \gamma\,,$ $\qquad \Gamma \sim \mathcal{O}(g^4 e^2 M_{\tilde{N}_j})$.

Often, the two-body decay mode is kinematically closed. It is possible for the radiative one-loop branching ratio to be enhanced as compared to the three-body tree-level decay branching ratio, in certain special regions of the SUSY parameter space. This would lead to SUSY events with missing transverse energy that contain hard photons.

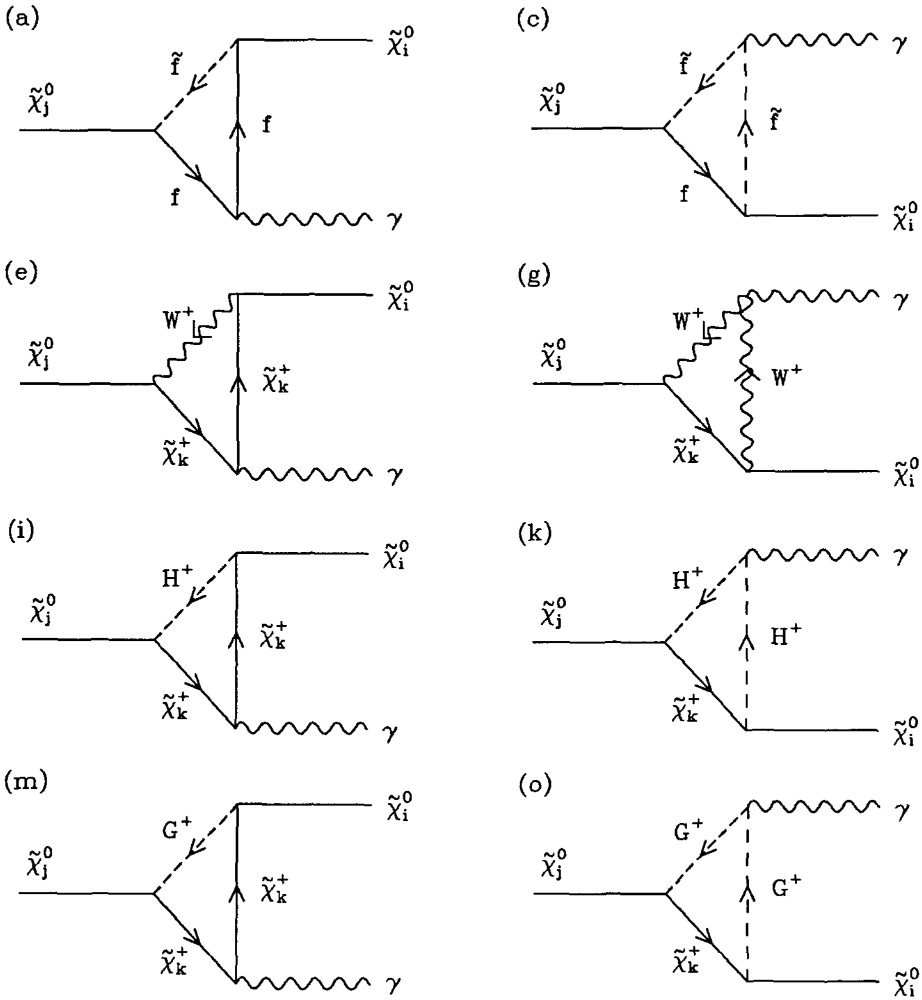
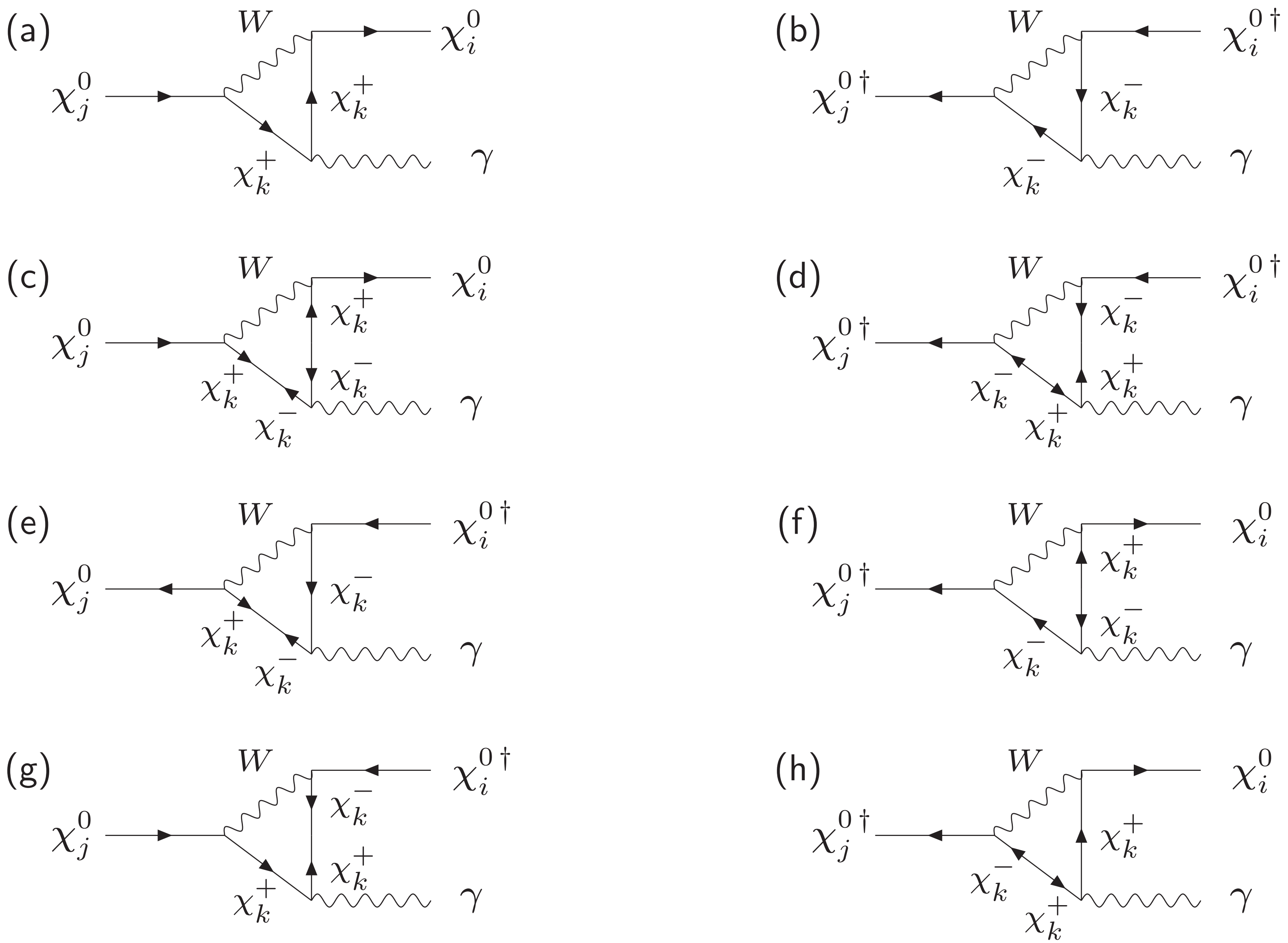


Fig. 1. Triangle graphs contributing to $\tilde{\chi}_j^0 \to \tilde{\chi}_i^0\gamma$. Only half the graphs are shown, with internal particles: (a) $\tilde{f}ff$; (c) $\tilde{f}\tilde{f}f$; (e) $\tilde{\chi}^+\tilde{\chi}^+W$; (g) $\tilde{\chi}^+WW$; (i) $\tilde{\chi}^+\tilde{\chi}^+H^-$; (k) $\tilde{\chi}^+H^-H^-$; (m) $\tilde{\chi}^+\tilde{\chi}^+G^-$; (o) $\tilde{\chi}^+G^-G^-$. The graphs not shown (b, d, ..., p) differ from their "partners" (a, c, ..., o) only in that the momentum routing of the loop is clockwise instead of counterclockwise.

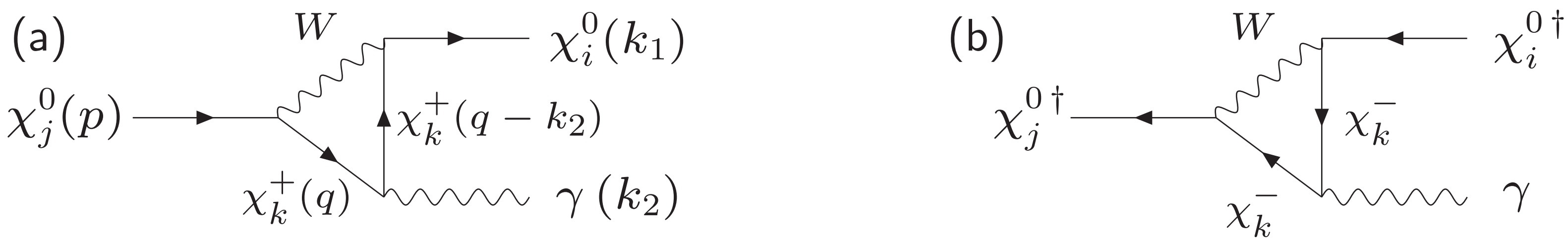
where M_j is the mass of $\tilde{\chi}_j^0$ and ϵ_i is the sign of the mass eigenvalue of $\tilde{\chi}_i^0$ which arises when the neutralino mass matrix is diagonalized. The above form of the matrix element also indicates the physical relevance of ϵ_i; it is related to the *CP* quantum number of $\tilde{\chi}_i^0$. That is, depending on the relative *CP* quantum numbers of $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$, the effective $\tilde{\chi}_i^0\tilde{\chi}_j^0\gamma$ interaction is either proportional to $\sigma^{\mu\nu}k_\mu\epsilon_\nu^*$ or $\gamma_5\sigma^{\mu\nu}k_\mu\epsilon_\nu^*$. The radiative decay width of $\tilde{\chi}_j^0$ is then easily calculated:

$$\Gamma\left(\tilde{\chi}_j^0 \to \tilde{\chi}_i^0\gamma\right) = \frac{g^2_{\tilde{\chi}_j^0\tilde{\chi}_i^0\gamma}\left(M_j^2 - M_i^2\right)^3}{8\pi M_j^5}. \tag{4}$$

$W^\pm \widetilde{C}^\pm \widetilde{C}^\pm$ triangles using two-component spinor techniques



One graph from the computation that employs four-component spinor techniques becomes eight separate graphs in the two-component spinor formalism.



$$i\mathscr{M}_a = \sum_k \int \frac{d^4q}{(2\pi)^4}\, x_i^\dagger [igO^L_{jk}\overline{\sigma}^\mu] \frac{i\sigma\cdot(q-k_2)}{(q-k_2)^2-m_k^2} [-ie\overline{\sigma}\cdot\varepsilon^*] \frac{i\sigma\cdot q}{q^2-m_k^2} [igO^{L\,*}_{ik}\overline{\sigma}^\nu] x_j D^W_{\mu\nu}\,,$$

$$i\mathscr{M}_b = \sum_k \int \frac{d^4q}{(2\pi)^4}\, y_i [igO^R_{jk}\sigma^\mu] \frac{i\overline{\sigma}\cdot(q-k_2)}{(q-k_2)^2-m_k^2} [-ie\sigma\cdot\varepsilon^*] \frac{i\overline{\sigma}\cdot q}{q^2-m_k^2} [igO^{R\,*}_{ik}\sigma^\nu] y_j^\dagger D^W_{\mu\nu}\,,$$

where $m_k \equiv M_{\tilde{C}_k}$ and

$$D^W_{\mu\nu} \equiv \frac{-ig_{\mu\nu}}{(q-p)^2-m_W^2}\,.$$

The choice of the σ version or $\overline{\sigma}$ version of the propagator and vertex rule is dictated by the order you choose to circulate the graph. The other six graphs are similarly evaluated, with appropriate vertex and propagator factors. There are no extra minus signs if we keep the same ordering of the spinors for all graphs.

Note that in all the graphs shown, positive electric charge circulates in a counterclockwise direction. In addition, we must add the corresponding eight graphs in which positive electric charge circulates in a clockwise direction. This can be done by flipping the electric charge of all internal chargino labels, without altering any other feature of the graphs. The corresponding amplitudes are simply obtained by the replacements:

$$O^L \to -O^{R\,*}\,, \qquad O^R \to -O^{L\,*}\,, \qquad e \to -e\,.$$

Adding up all graphs yields:

$$\begin{aligned}
\mathscr{M} = ig^2 e \sum_k \int \frac{d^4q}{(2\pi)^4} & \frac{1}{(q^2-m_k^2)[(q-k_2)^2-m_k^2][(q-p)^2-m_W^2]} \\
\times \Big\{ & (O^L_{ik}O^{L\,*}_{jk} - O^{R\,*}_{ik}O^R_{jk})x_i^\dagger \overline{\sigma}^\mu[\sigma\cdot(q-k_2)\overline{\sigma}\cdot\varepsilon^*\sigma\cdot q + m^2\sigma\cdot\varepsilon^*]\overline{\sigma}_\mu x_j \\
& (O^R_{ik}O^{R\,*}_{jk} - O^{L\,*}_{ik}O^L_{jk})y_i \sigma^\mu[\overline{\sigma}\cdot(q-k_2)\sigma\cdot\varepsilon^*\overline{\sigma}\cdot q + m^2\overline{\sigma}\cdot\varepsilon^*]\sigma_\mu y_j^\dagger \\
& m(O^L_{ik}O^{R\,*}_{jk} - O^{R\,*}_{ik}O^L_{jk})x_i^\dagger \overline{\sigma}^\mu[\sigma\cdot(q-k_2)\overline{\sigma}\cdot\varepsilon^* + \sigma\cdot\varepsilon^*\overline{\sigma}\cdot q]\sigma_\mu y_j^\dagger \\
& m(O^R_{ik}O^{L\,*}_{jk} - O^{L\,*}_{ik}O^R_{jk})y_i \sigma^\mu[\overline{\sigma}\cdot(q-k_2)\sigma\cdot\varepsilon^* + \overline{\sigma}\cdot\varepsilon^*\sigma\cdot q]\overline{\sigma}_\mu x_j \Big\}\,.
\end{aligned}$$

In four-component notation [where $P_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)$], the loop amplitude is:

$$\mathscr{M} = ig^2 e \sum_k \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_k^2)[(q-k_2)^2 - m_k^2][(q-p)^2 - m_W^2]}$$

$$\times \Big\{ \overline{u}_i \gamma^\mu (O_{ik}^L P_L + O_{ik}^R P_R)(\not{q} - \not{k}_2 + m)\not{\epsilon}^*(\not{q} + m)\gamma_\mu (O_{jk}^{L\,*} P_L + O_{jk}^{R\,*} P_R) u_j$$

$$-(O^L \to -O^{R\,*}\,,\, O^R \to -O^{L\,*}) \Big\}\,,$$

which reproduces the result obtained by Haber and Wyler. The first term above arises from the four-component spinor Feynman graph with the positive charge circulating counterclockwise in the loop. The second term, denoted above by $-(O^L \to -O^{R\,*}\,,\, O^R \to -O^{L\,*})$ corresponds to a clockwise circulation of positive charge. This second term requires careful treatment in the four-component spinor formalism (which has been systematized by the four-component Majorana Feynman rules of Denner and collaborators* and rederived by Dreiner, Haber and Martin).

*A. Denner, H. Eck, O. Hahn and J. Kublbeck, "Compact Feynman rules for Majorana fermions," Phys. Lett. **B291** (1992) 278; "Feynman rules for fermion number violating interactions," Nucl. Phys. **B387** (1992) 467.

and the couplings of the outgoing neutralino to the particles in the loop by

$$G = g_{\mathrm{L}} P_{\mathrm{L}} + g_{\mathrm{R}} P_{\mathrm{R}}, \tag{6}$$

where $P_{\mathrm{L,R}} = \frac{1}{2}(1 \mp \gamma_5)$ and factors of $i\gamma_\mu$ have been removed. Then, the matrix element corresponding to graph e, for a fixed chargino state $\tilde{\chi}_k^+$ (with mass M_k) in the triangle, is given by:

$$\mathscr{M}_{\mathrm{e}} = ieg^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \bar{u}(k_1)\gamma^\mu G(\not{q} - \not{k}_2 + M_k)\not{\epsilon}^*(k_2)(\not{q} + M_k)\gamma_\mu F u(p)\frac{1}{D_1}, \tag{7}$$

where

$$D_1 = \left(q^2 - M_k^2\right)\left[(q-k_2)^2 - M_k^2\right]\left[(q-p)^2 - m_{\tilde{\mathrm{W}}}^2\right]. \tag{8}$$

Graph f is obtained from graph e by reversing the direction of the flow of charge in the triangle. By convention, the arrow on a fermion line denotes the direction of the $\tilde{\chi}^+$, and in general indicates the flow of positive fermion number. (In this language, electrons and quarks have positive fermion number, while the corresponding antiparticles have negative fermion number.) Thus, graph f will involve vertices with clashing arrows. In appendix A, we discuss how one deals with such a situation. Following the procedure described there, we find

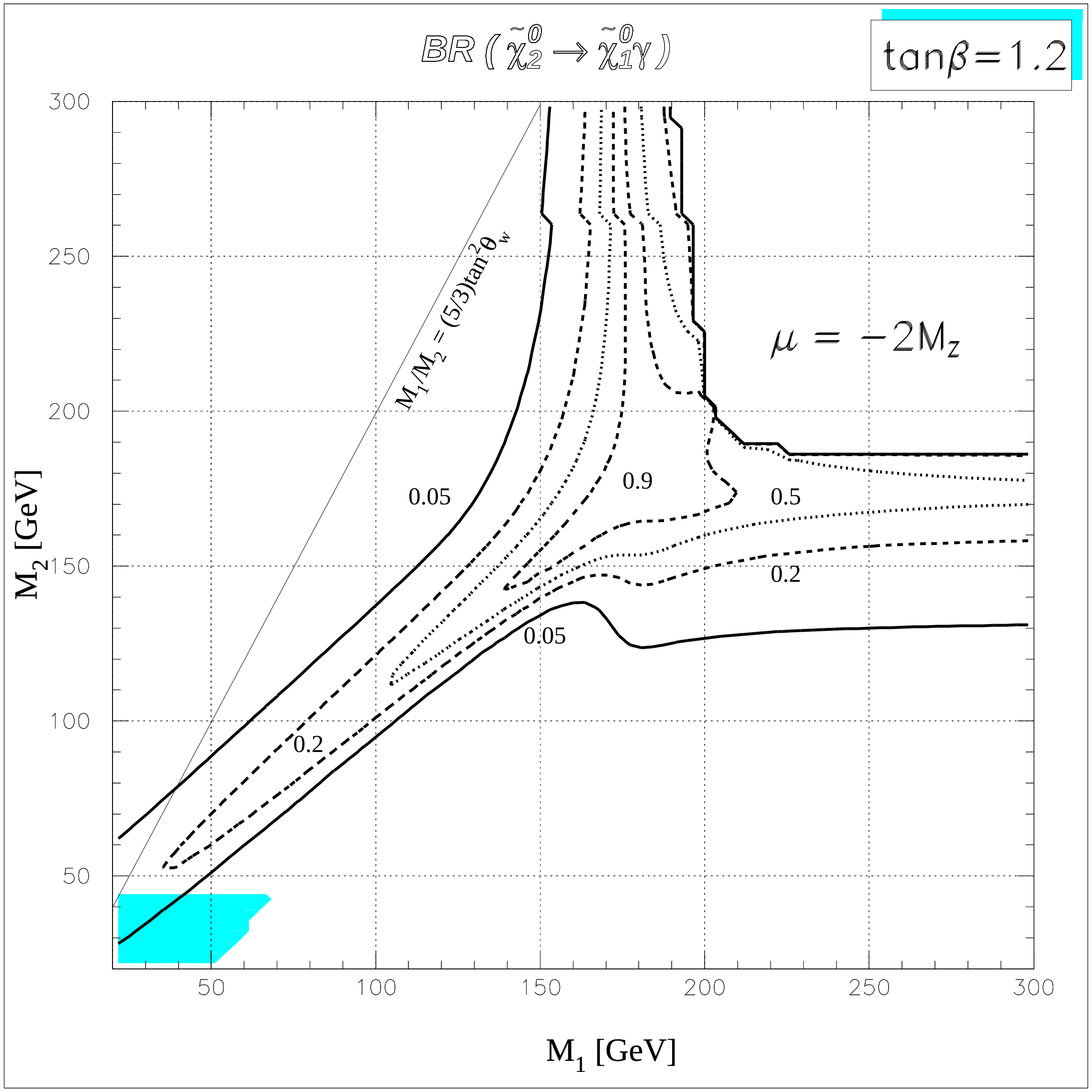
$$\mathscr{M}_{\mathrm{f}} = -ieg^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \bar{u}(k_1)[\gamma^\mu G^* C]^{\mathrm{T}}(\not{k}_2 - \not{q} + M_k)^{\mathrm{T}}\not{\epsilon}^{*\mathrm{T}}(k_2)$$
$$\times(-\not{q} + M_k)^{\mathrm{T}}\left[-C^{-1}\gamma_\mu F^*\right]^{\mathrm{T}} u(p)(1/D_1), \tag{9}$$

where F^* and G^* are defined by

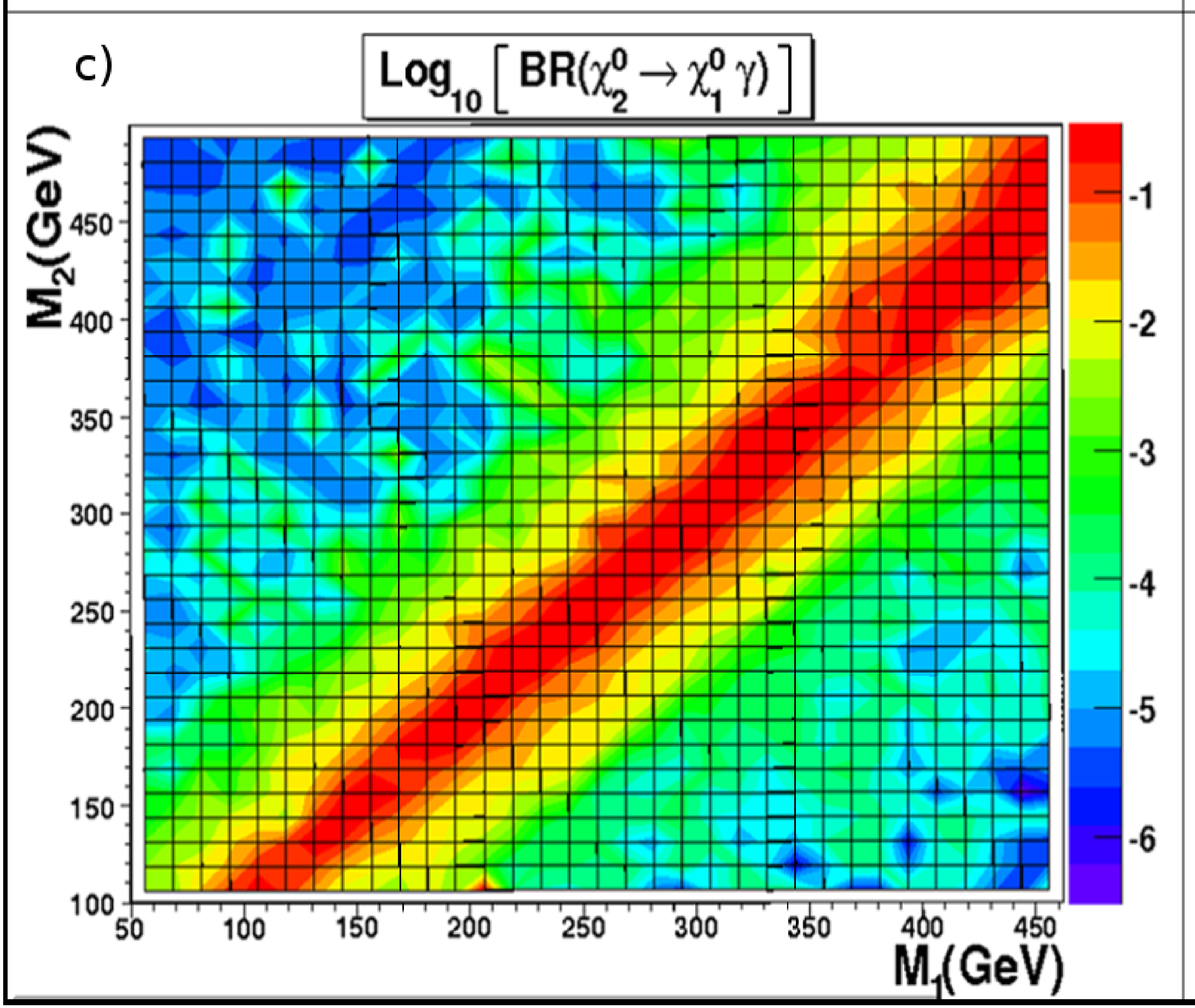
$$F^* \equiv (f_{\mathrm{L}}^* P_{\mathrm{L}} + f_{\mathrm{R}}^* P_{\mathrm{R}})\epsilon_j, \qquad G^* \equiv (g_{\mathrm{L}}^* P_{\mathrm{L}} + g_{\mathrm{R}}^* P_{\mathrm{R}})\epsilon_i. \tag{10,11}$$

An extra minus sign has been inserted due to Pauli statistics, since the two diagrams differ by the exchange of two (anticommuting) external fermions. (This is most easily understood by examining the crossed reaction $\gamma \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$.) Note that in obtaining eq. (9), we have reversed the direction of the loop momentum q as compared with graph e. Using the properties of the charge conjugation matrix C: (i) $C^{\mathrm{T}} = -C$, (ii) $C^{-1}\gamma_\mu C = -\gamma_\mu^{\mathrm{T}}$, and (iii) $C^{-1}\gamma_5 C = \gamma_5^{\mathrm{T}}$, we may rewrite eq. (9) as follows:

$$\mathscr{M}_{\mathrm{f}} = -ieg^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \bar{u}(k_1)\gamma^\mu \tilde{G}^*(\not{q} - \not{k}_2 + M_k)\not{\epsilon}^*(k_2)(\not{q} + M_k)\gamma_\mu \tilde{F}^* u(p)\frac{1}{D_1}, \tag{12}$$



Contour plot for the branching ratio for the radiative neutralino decay, $\widetilde{N}_2 \rightarrow \widetilde{N}_1 + \gamma$, for $\tan\beta = 1.2$ and $\mu = -2m_Z$. The squark and slepton masses are all taken degenerate and equal to 1 TeV, and $m_A = 300$ GeV. Taken from S. Ambrosanio and B. Mele, Phys. Rev. **D55** (1997) 1399.



Branching ratio for the radiative neutralino decay, $\widetilde{N}_2 \to \widetilde{N}_1 + \gamma$, for different choices of gaugino mass parameters M_1 and M_2 in a model of split supersymmetry. Values of the higgsino mass parameter μ and $\tan\beta$ are varied randomly. This figure is taken from M.A. Diaz, B. Panes and P. Urrejola, arXiv:0910.1554 [hep-ph].

I have very fond memories of my one month visit to Zurich in June, 1987 (with special thanks to the hospitality of Zoltan Kunszt). My resulting collaboration with Daniel was especially enjoyable. I am very happy to have the opportunity to participate in this celebration. With great pleasure, I contribute these greetings from myself and Daniel's many friends in California.

HAPPY 60th Birthday, Daniel !!!

May you have many more years of rare and beautiful particle physics.