

# SUSY Lepton Flavor Violation: Radiative Decays and Collider Searches

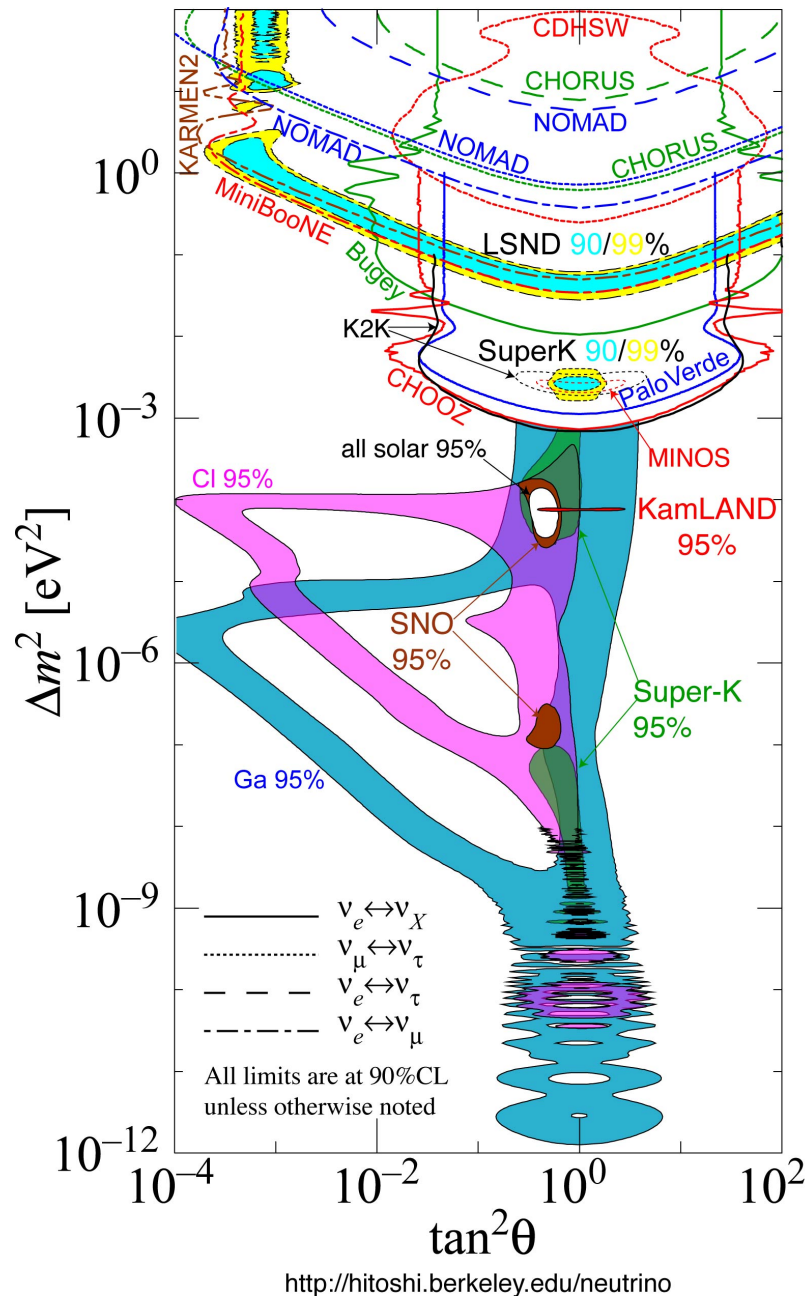
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neutrino masses and mixing  
give hints towards BSM physics

- lepton flavor violation
- leptonic CP violation
- Majorana mass and GUT scales
- lepton number violation

low-energy observables suppressed  
by light neutrino masses unless  
there is other new physics

e.g. **SUPERSYMMETRY**

## minimal supersymmetric scenario

- MSSM + 3 families of right-handed neutrino singlet fields  $\nu_R$
- Majorana mass term:  $\frac{1}{2}\nu_R^{cT} M \nu_R^c$
- Yukawa coupling to Higgs field (hypercharge  $+\frac{1}{2}$ ):  $\nu_R^{cT} Y_\nu L H_2$
- EWSB  $\rightarrow$  Dirac mass term:  $m_D = Y_\nu \langle H_2 \rangle$

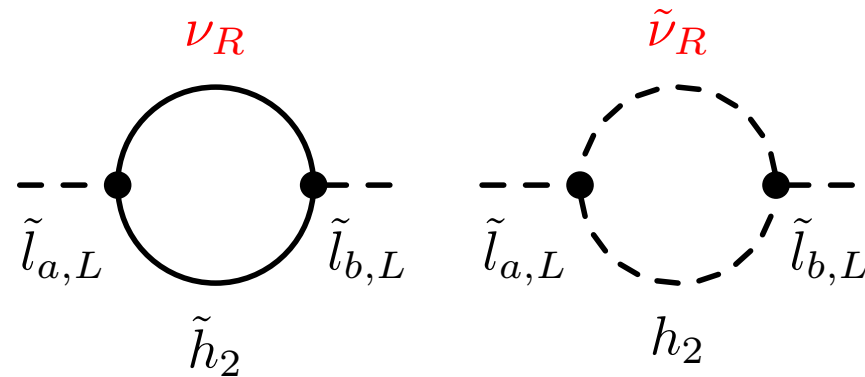
diagonalization of neutrino mass matrix for  $\langle H_2 \rangle \ll M_R$ :

$$\frac{1}{2} \begin{pmatrix} \nu_L^T & \nu_R^{cT} \end{pmatrix} \begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

**light neutrinos:**  $M_\nu = m_D^T M^{-1} m_D \sim \frac{\langle H_2 \rangle^2}{M_R}$

**heavy neutrinos:**  $M_N \sim M_R$

## virtual effects of heavy (s)neutrinos



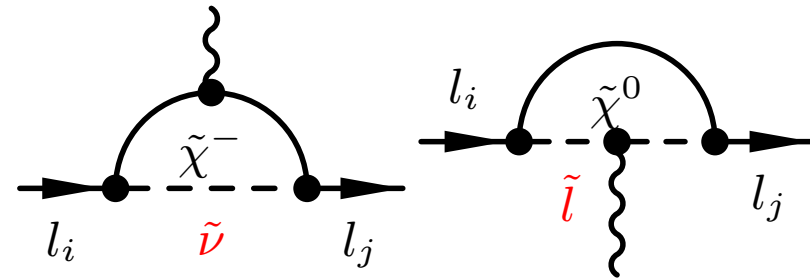
generate, via renormalization, **flavor non-diagonal terms** in

$$m_{\tilde{l}}^2 = \begin{pmatrix} m_{\tilde{l}_L}^2 & m_{\tilde{l}_{LR}}^{2\dagger} \\ m_{\tilde{l}_{LR}}^2 & m_{\tilde{l}_R}^2 \end{pmatrix} = \tilde{m}_{MSSM}^2 + \begin{pmatrix} \delta m_L^2 & \delta m_{LR}^{2\dagger} \\ \delta m_{LR}^2 & \delta m_R^2 \end{pmatrix}$$

e.g. in MSUGRA [ $L = D(\ln(M_{GUT}/M_{N_i}))$ ]:

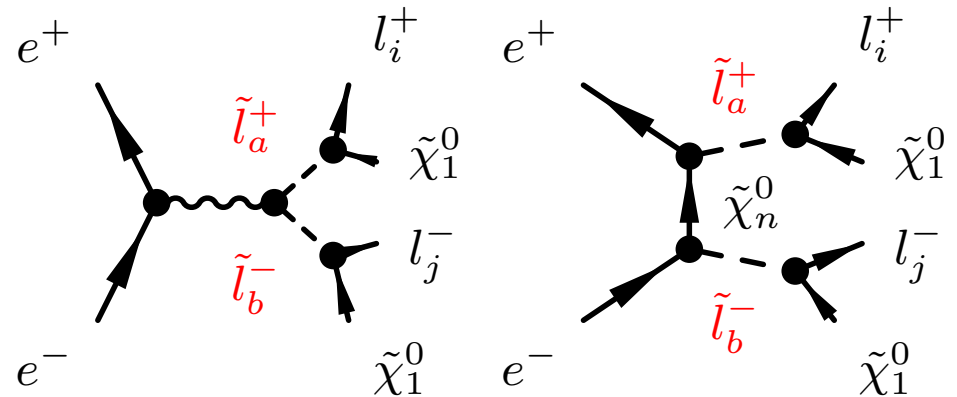
$$\begin{aligned} \delta m_L^2 &\simeq -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) Y_\nu^\dagger L Y_\nu \\ \delta m_R^2 &\simeq 0 \\ \delta m_{LR}^2 &\simeq -\frac{3}{16\pi^2} Y_l v \cos \beta A_0 Y_\nu^\dagger L Y_\nu \end{aligned}$$

$$\mu \rightarrow e\gamma, \tau \rightarrow e\gamma, \tau \rightarrow \mu\gamma$$



$$\Gamma(l_i \rightarrow l_j \gamma) \propto \frac{|\delta m_{Lij}^2|^2}{\tilde{m}^4} \frac{\alpha^3 m_{l_i}^5}{\tilde{m}^4} \tan^2 \beta, \quad \frac{Br(l_i \rightarrow 3l_j)}{Br(l_i \rightarrow l_j \gamma)} \simeq O(10^{-3})$$

$$e^+ e^- \rightarrow (\mu e / \tau e / \tau \mu) + 2\tilde{\chi}_1^0$$



$$\sigma(l_i^+ l_j^-) \propto \frac{|\delta m_{Lij}^2|^2}{\tilde{m}^2 \Gamma_{\tilde{l}}^2} \sigma(e^+ e^- \rightarrow \tilde{l}^+ \tilde{l}^-) Br(\tilde{l}^+ \rightarrow l_i^+ \tilde{\chi}_1^0) Br(\tilde{l}^- \rightarrow l_j^- \tilde{\chi}_1^0)$$

Casas, Ibarra, hep-ph/0103065 (NPB618)

$$\delta m_L^2 \propto Y_\nu^\dagger L Y_\nu, \quad Y_\nu = \frac{1}{v \sin \beta} D(\sqrt{M_i}) \cdot R \cdot D(\sqrt{m_j}) \cdot U^\dagger$$

- light and **heavy** neutrino **eigenmasses**:  $m_i$  (from experiment) and  $M_i$
- mass diagonalization and **mixing**:  $\theta_{ij}$  (from experiment) and  $\phi_i, \delta$

$$\begin{aligned} U^T M_\nu U &= D(m_1, m_2, m_3) \\ U &= D(e^{i\phi_1}, e^{i\phi_2}, 1) V(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \end{aligned}$$

- **arbitrary complex matrix,  $R^T R = \mathbf{1}$** :

$$R = \begin{pmatrix} c_2 c_3 & -c_1 s_3 - s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\ c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & -s_1 c_3 - c_1 s_2 s_3 \\ s_2 & s_1 c_2 & c_1 c_2 \end{pmatrix}$$

$$c(s)_i = \cos(\sin)\theta_i, \quad \text{complex angles } \theta_i = x_i + I y_i$$

$$\Delta m_{12}^2 = 6.9_{-0.36}^{+0.36} \cdot 10^{-5} \text{ eV}^2$$

$$\Delta m_{13}^2 = 2.6_{-1.2}^{+1.2} \cdot 10^{-3} \text{ eV}^2$$

$$\tan^2 \theta_{12} = 0.43_{-0.22}^{+0.47}$$

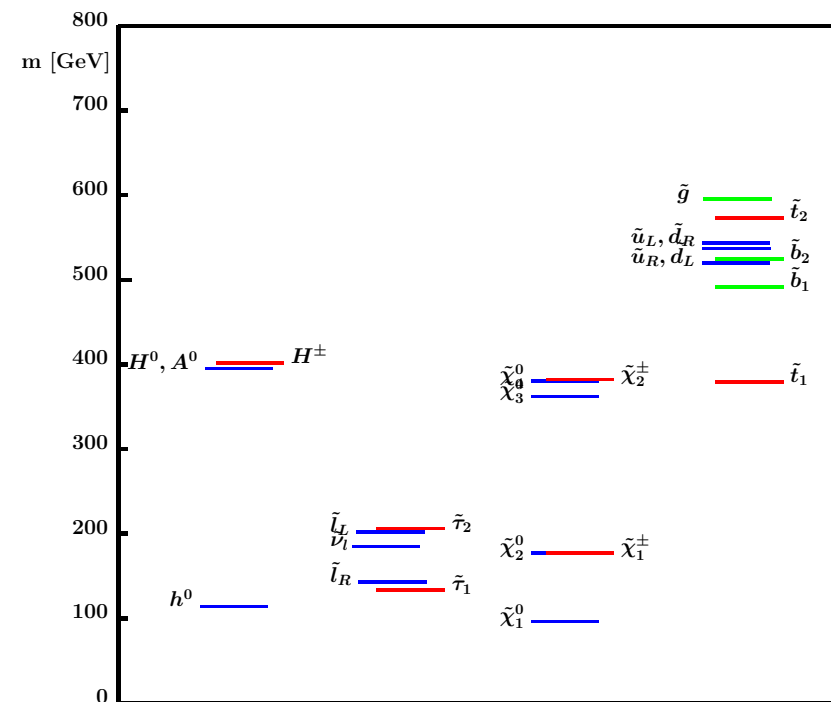
$$\tan^2 \theta_{23} = 1.10_{-0.60}^{+1.39}$$

$$\tan^2 \theta_{13} = 0.006_{-0.006}^{+0.001}$$

- central values from [Maltoni et al., hep-ph/0309130 \(PRD68\)](#)
- 90% C.L. errors as anticipated for running/proposed experiments
- Dirac and Majorana phases unconstrained
- hierarchical ( $m_1 \leq 0.03 \text{ eV}$ ) or degenerate ( $m_1 \approx 0.3 \text{ eV}$ ) spectrum

Scenario	$m_{1/2}/\text{GeV}$	$m_0/\text{GeV}$	$\tan \beta$	$A_0/\text{GeV}$	$\text{sign}\mu$
B'	250	60	10	0	+
C'	400	85	10	0	+
G'	375	115	20	0	+
I'	350	175	35	0	+
<b>SPS1a</b>	<b>250</b>	<b>100</b>	<b>10</b>	<b>-100</b>	<b>+</b>

- MSUGRA benchmark models  
B',...I' (*Battaglia et al., hep-ph/0306219*)  
**SPS1a** (<http://spa.desy.de/spa>)  
consistent with experiment and CDM
- universal scalar masses keep LFV small

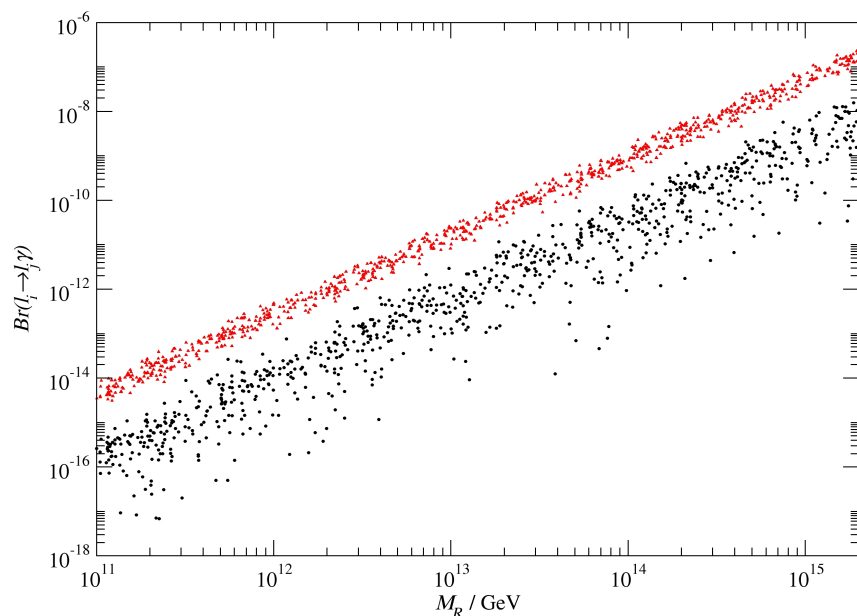




$$Br(\mu \rightarrow e\gamma) \text{ and } Br(\tau \rightarrow \mu\gamma)$$

degenerate  $M_i = M_R$ , real R:  $Y_\nu^\dagger L Y_\nu = \frac{M_R}{v^2 \sin^2 \beta} V \cdot D(m_i) \cdot V^\dagger L$ ,  $L = \ln \frac{M_{GUT}}{M_R}$

degenerate  $m_i$ , SUSY point SPS1a, scatter: uncertainties from neutrino data



related LFV decays:

$$\frac{Br(\tau \rightarrow 3\mu)}{Br(\tau \rightarrow \mu\gamma)} \approx 2 \cdot 10^{-3}$$

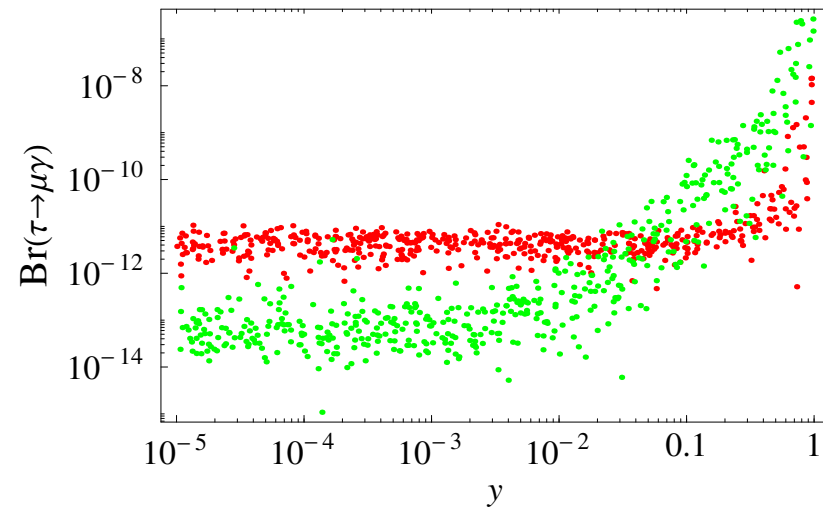
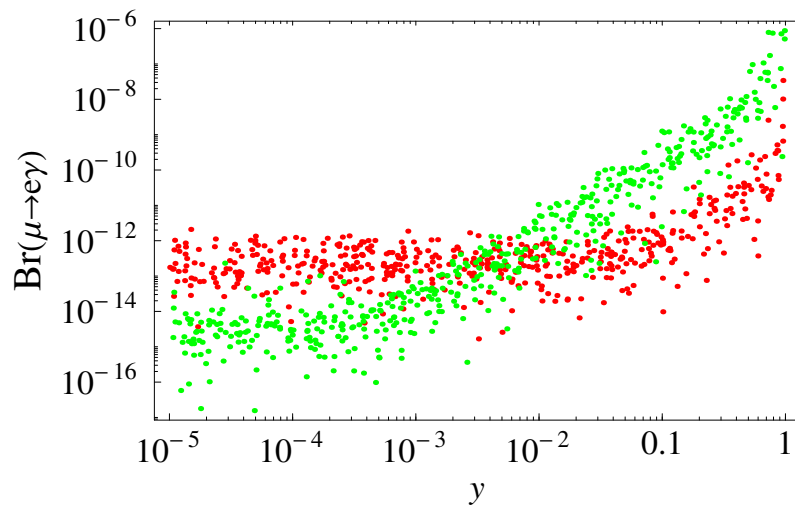
$$\frac{Br(\mu \rightarrow 3e)}{Br(\mu \rightarrow e\gamma)} \approx 7 \cdot 10^{-3}$$

$$Br(\tau \rightarrow \mu\gamma) < 6.8 \cdot 10^{-8} \quad (90 \% \text{ C.L., BABAR 2005})$$

$$Br(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11} \quad (90 \% \text{ C.L., PDG 2004})$$

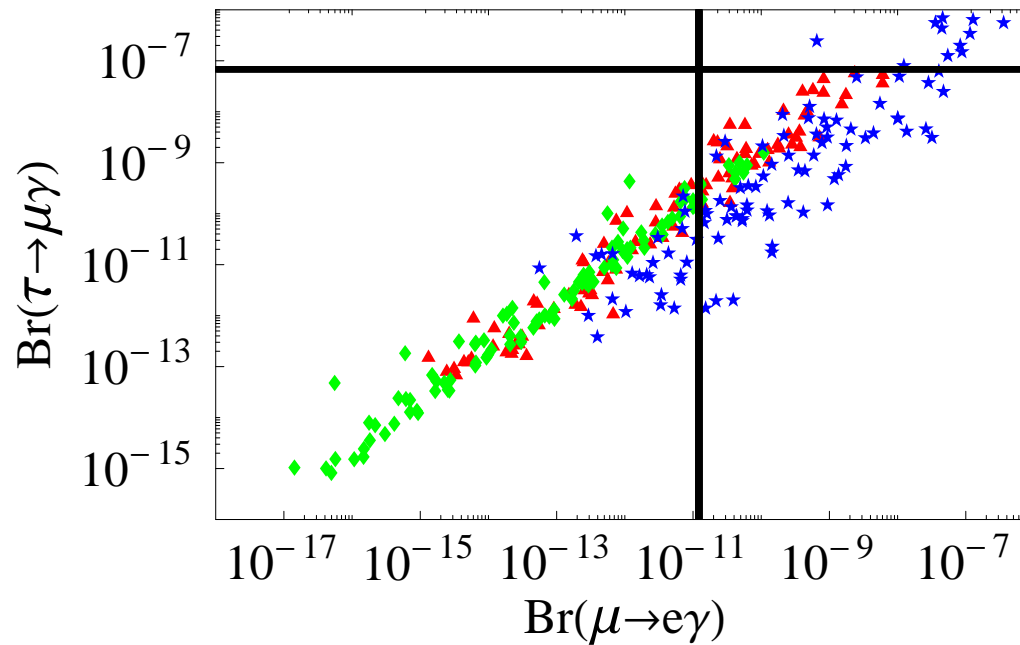
$$R(\cos \theta_i, \sin \theta_i) = R(\cos x_i \cosh y_i - I \sin x_i \sinh y_i, \sin x_i \cosh y_i + I \cos x_i \sinh y_i)$$

- $0 \leq x_i \leq 2\pi, y_i = y$
- degenerate heavy Majorana neutrinos  $M_i = M_R = 10^{12}$  GeV
- **hierarchical** or **degenerate** light neutrinos and their mixing
- SUSY point SPS1a



complex  $R$  enhances LFV and generates lepton number violation in N decays

seesaw parameters in preferred ranges ( $M_i = 10^{10}$  to  $10^{13}$  GeV), SUSY point SPS1a



deg. N, hier.  $\nu$ , R real

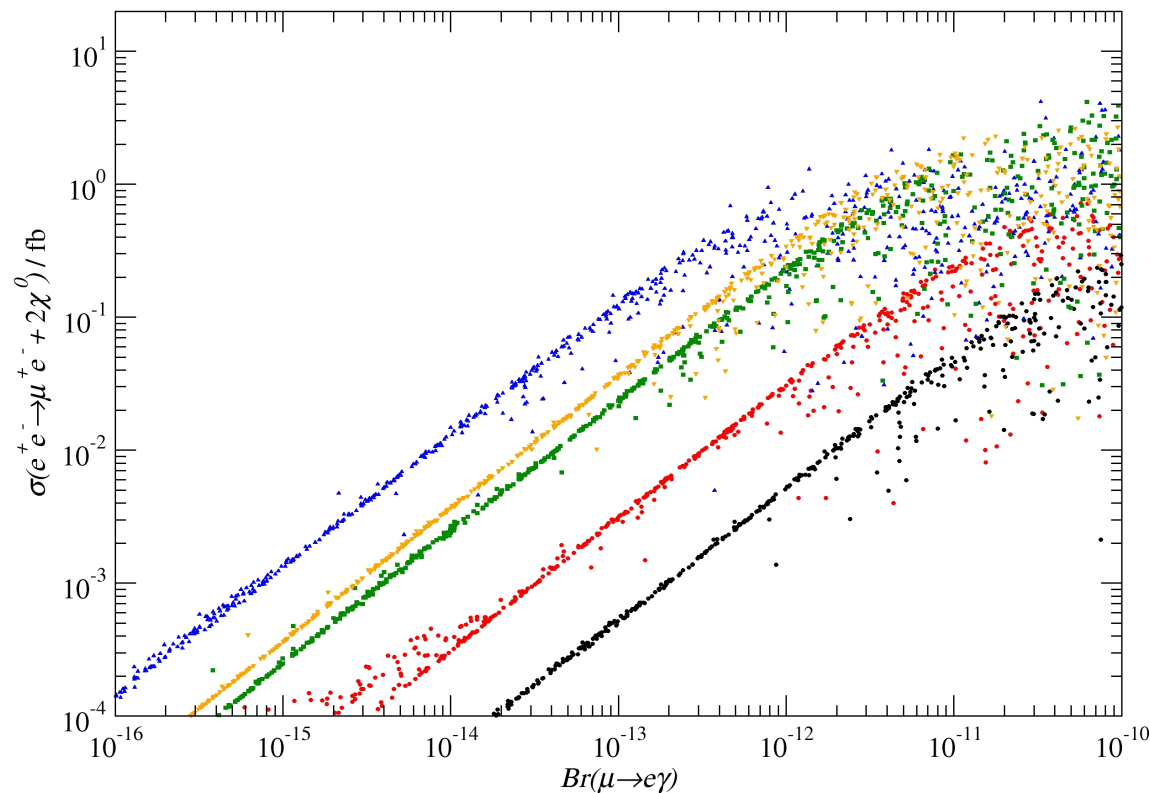
deg. N, deg.  $\nu$ , R real

hier. N, hier.  $\nu$ , R complex

yields model-dependent bound  $Br(\tau \rightarrow \mu\gamma) < 10^{-9}$

$$\sigma(e^+e^- \rightarrow \mu^+e^- + 2\tilde{\chi}_1^0), \sqrt{s} = 800 \text{ GeV}$$

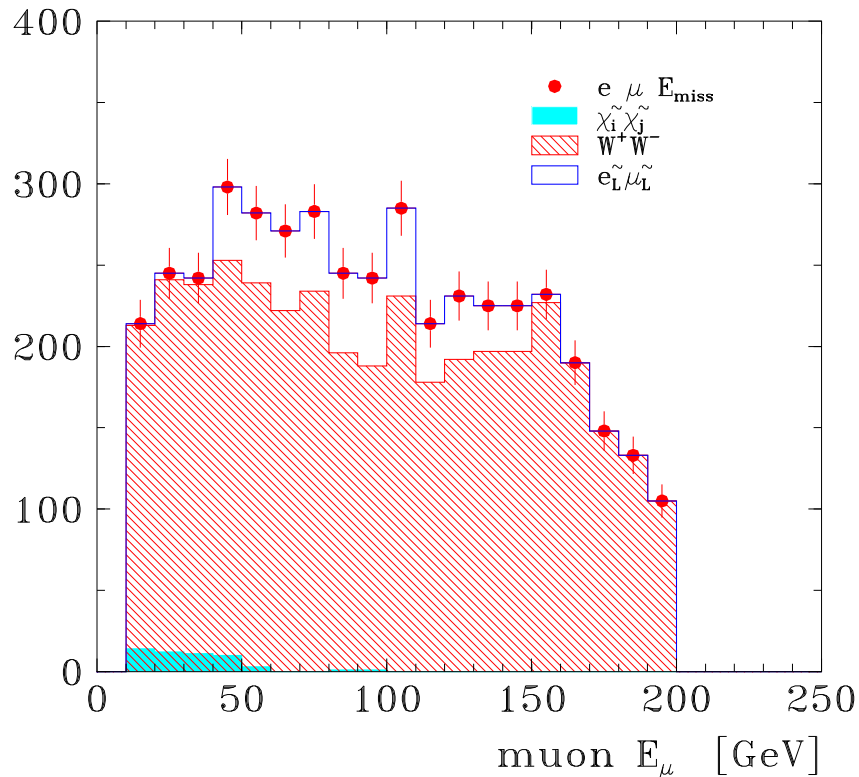
degenerate  $M_i$  and  $m_i$ , real R, SUSY points C', G', B', SPS1a, I'



$Br(\mu \rightarrow e\gamma) \approx 10^{-12}$  implies  $\sigma(e^+e^- \rightarrow \mu e + 2\tilde{\chi}_1^0) \approx 0.01$  to  $1 \text{ fb}$

## $e\mu E_{miss}$ final states

MSUGRA point SPS1a,  $\sqrt{s} = 500$  GeV, unpolarized,  $500 \text{ fb}^{-1}$

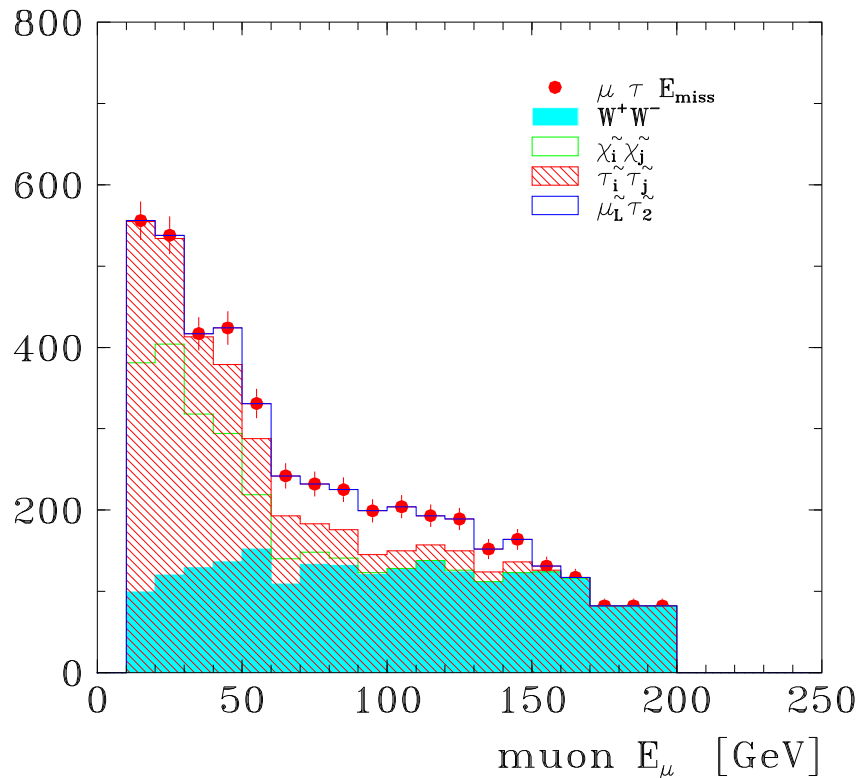


- 2 fb signal cross section (flat lepton energy spectrum)
- SM+MSSM background
- standard selection criteria (50% efficiency)
- $\sigma(\tilde{e}_L \tilde{\mu}_L) = 1 \text{ fb} \rightarrow 5\sigma$  effect
- improvements possible ( $E_e$  spectrum, polarization)

*Deppisch, Martyn, Päs, Redelbach, RR, hep-ph/0408140*

## $\tau\mu E_{miss}$ final states

MSUGRA point SPS1a,  $\sqrt{s} = 500$  GeV, unpolarized,  $500 \text{ fb}^{-1}$

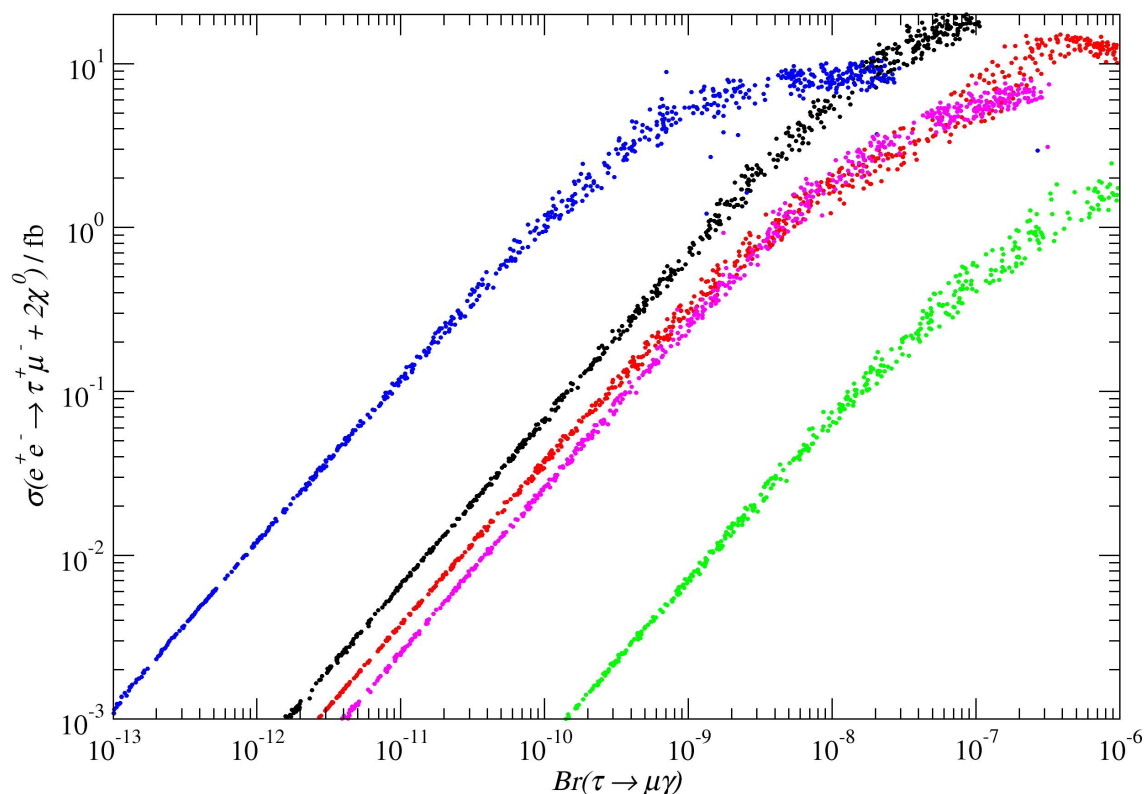


- 4 fb signal cross section (flat lepton energy spectrum)
- SM+MSSM background (soft  $E_\mu$  spectrum)
- standard selection criteria ( $\tau$  identification via hadronic decays, 25% efficiency)
- $\sigma(\tilde{\tau}_2 \tilde{\mu}_L) = 2 \text{ fb} \rightarrow 5\sigma$  effect

*Deppisch, Martyn, Päs, Redelbach, RR, hep-ph/0408140*

$$\sigma(e^+e^- \rightarrow \tau^+\mu^- + 2\tilde{\chi}_1^0), \sqrt{s} = 800 \text{ GeV}$$

degenerate  $M_i$  and  $m_i$ , real R, SUSY points C', B', SPS1a, G', I'



$$\sigma(e^+e^- \rightarrow \tau\mu + 2\tilde{\chi}_1^0) \approx 2 \text{ fb} \quad \text{implies} \quad Br(\tau \rightarrow \mu\gamma) \approx 10^{-6} \text{ to } 10^{-10}$$

$\tau$  radiative decay:  $W \rightarrow \tau\nu, \tau \rightarrow \mu\gamma$

*Serin, Stroynowski, ATLAS Internal Note (1997)*

- signal:  $M(\mu\gamma) = m_\tau$
- background: QED radiation in production and decay
- reach ( $30 \text{ fb}^{-1}$ ):  $Br(\tau \rightarrow \mu\gamma) \simeq 6 \cdot 10^{-7}$

direct DY production of sleptons:  $\tilde{l}_i \tilde{l}_i \rightarrow l_j l_k 2\tilde{\chi}_1^0$

*Bityukov, Krasnirov, hep-ph/9712358*

*Agashe, Graesser, hep-ph/9904422*

- signal: dileptons of different flavour and large missing  $p_T$
- background:  $t\bar{t}, W^+W^-, \tilde{\chi}^+\tilde{\chi}^-$
- generally very difficult



LFV neutralino and slepton decays:  $\tilde{g}, \tilde{q} \rightarrow \tilde{\chi}_2^0 \rightarrow \tilde{l}_a l_j \rightarrow l_i l_j \tilde{\chi}_1^0$

*Agashe, Graesser, hep-ph/ 9904422*

*Hinchliffe, Paige, hep-ph/0010086*

*Hisano et al., PRD D65(2002)116002*

*Carvalho et al., hep-ph/0206148*

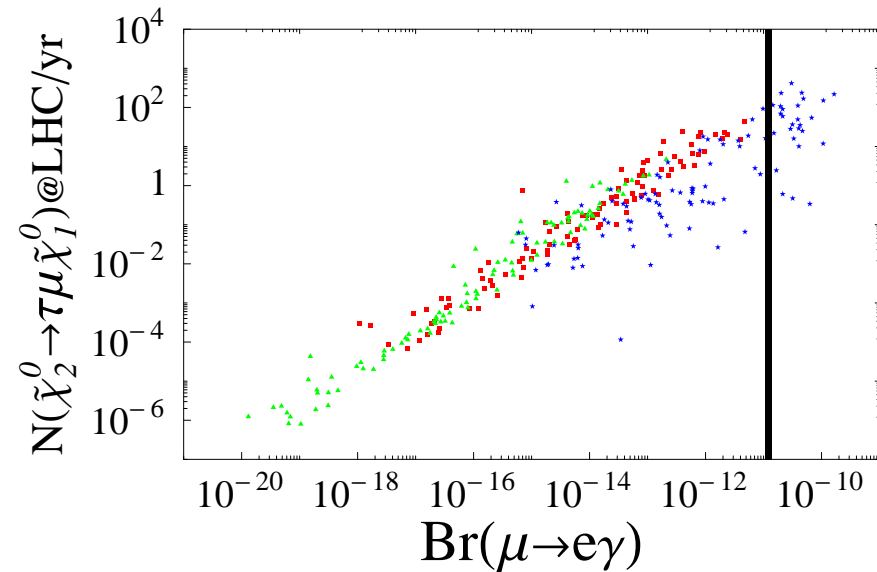
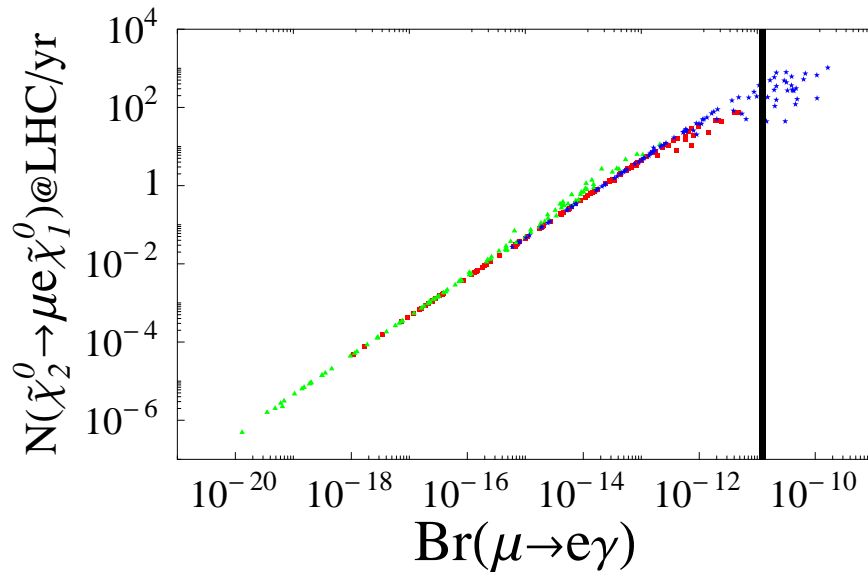
*Bartl et al., hep-ph/0510074*

- signal: dilepton mass distribution
- background:  $t\bar{t}$ , SUSY channels
- reach ( $30 \text{ fb}^{-1}$ ):  $Br(\tilde{\chi}_2^0 \rightarrow l_i l_j \tilde{\chi}_1^0) \simeq 2 \text{ to } 4 \%$

$$\tilde{\chi}_2^0 \rightarrow \mu^+ e^- (\tau^+ \mu^-) \tilde{\chi}_1^0 \quad \text{vs.} \quad BR(\mu \rightarrow e\gamma)$$

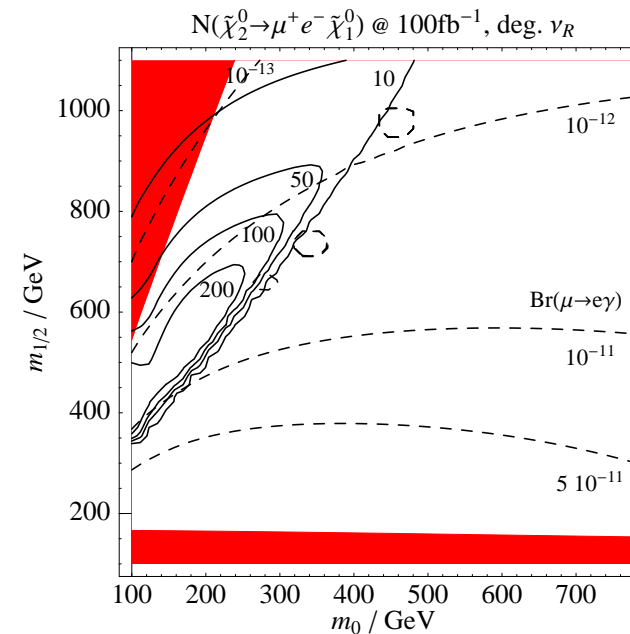
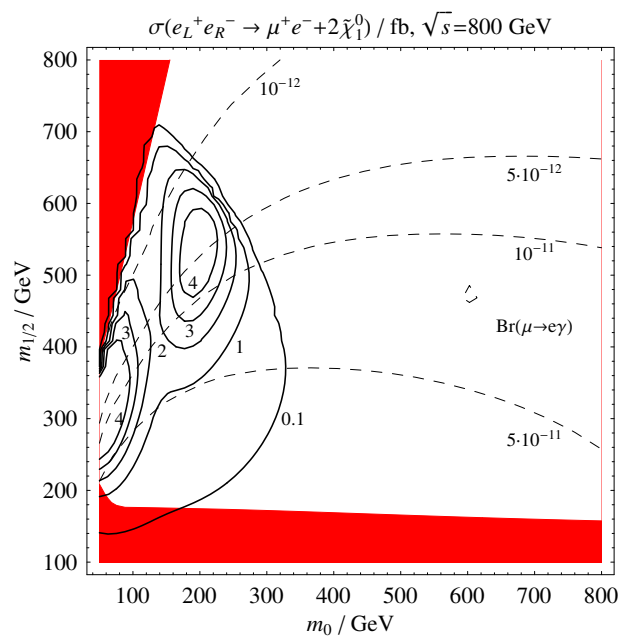
$$M_i = 10^{10} \text{ to } 10^{13} \text{ and } 100 \text{ fb}^{-1}$$

MSUGRA point C' ( $m_0 = 85 \text{ GeV}$ ,  $m_{1/2} = 400 \text{ GeV}$ ,  $A_0 = 0 \text{ GeV}$ ,  $\tan \beta = 10 \text{ GeV}$ ,  $\text{sign} \mu = +$ )



hierarch.  $N$ , hierarch.  $\nu$   
 deg.  $N$ , hierarch.  $\nu$   
 deg.  $N$ , deg.  $\nu$

- SUSY seesaw with  $M_R = 10^{10}$  to  $10^{14}$  GeV suggests **sizeable charged LFV**
- details **model dependent**, generically **strong correlations** in  $\mu e, \tau e, \tau \mu$  channels
- for sufficiently small  $m_0$  **radiative decays and collider searches complementary**
- BAU via  $CP$  violating out-of-equilibrium decays of heavy  $N_1$  and sphaleron processes, i.e. **leptogenesis**



$A_0 = 0, \tan \beta = 5, \text{sign } \mu = +,$   
 $M_R = 10^{14} \text{ GeV},$  zero complex phases, central values of neutrino data,  $m_1 = 0$

- **evolution of neutrino mass matrix:  $m_Z \rightarrow M_1$**   
input: light neutrino data

$$16\pi^2 \frac{d}{dt} M_\nu = \left( -6g_2^2 - \frac{6}{5}g_1^2 + \text{Tr}(6Y_U^\dagger Y_U) \right) M_\nu + (Y_l^\dagger Y_l) M_\nu + M_\nu (Y_l^\dagger Y_l)^T$$

- inversion of  $M_\nu = Y_\nu^T M^{-1} Y_\nu (v \sin \beta)^2$  and  
**evolution of Yukawa coupling matrix:  $M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow M_{GUT}$**

$$16\pi^2 \frac{d}{dt} Y_\nu = Y_\nu \left( -3g_2^2 - \frac{3}{5}g_1^2 + \text{Tr}(3Y_U^\dagger Y_U + Y_\nu^\dagger Y_\nu) + Y_l^\dagger Y_l + 3Y_\nu^\dagger Y_\nu \right)$$

- **evolution of slepton mass matrix:  $M_{GUT} \rightarrow m_Z$**   
input: neutrino Yukawa couplings at  $M_{GUT}$

$$16\pi^2 \frac{d\delta m_L^2}{d \ln \mu} = m_L^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_L^2 + 2 \left( Y_\nu^\dagger m_{\tilde{\nu}}^2 Y_\nu + m_{h_2}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu \right)$$

$$16\pi^2 \frac{d\delta m_R^2}{d \ln \mu} = 0$$

$$16\pi^2 \frac{d\delta A_e}{d \ln \mu} = 2Y_e Y_\nu^\dagger A_\nu + A_e Y_\nu^\dagger Y_\nu$$

general parametrization of Yukawa matrix:  $Y_\nu = \frac{1}{v \sin \beta} D(\sqrt{M_i}) R D(\sqrt{m_i}) U^\dagger$

**for degenerate Majorana masses  $M_i = M_R$  and real  $R$ :**

$$Y_\nu^\dagger L Y_\nu = \frac{M_R}{v^2 \sin^2 \beta} V \cdot D(m_i) \cdot V^\dagger L, \quad L = \ln \frac{M_{GUT}}{M_R}$$

- for hierarchical light  $\nu$  spectrum:

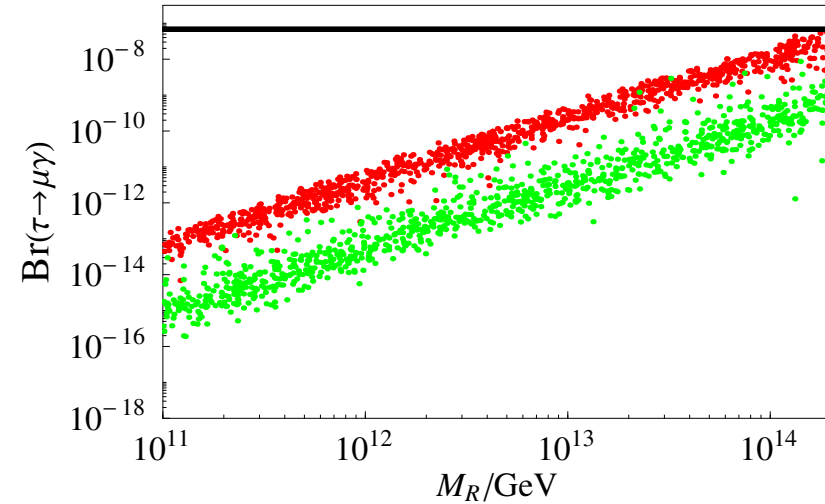
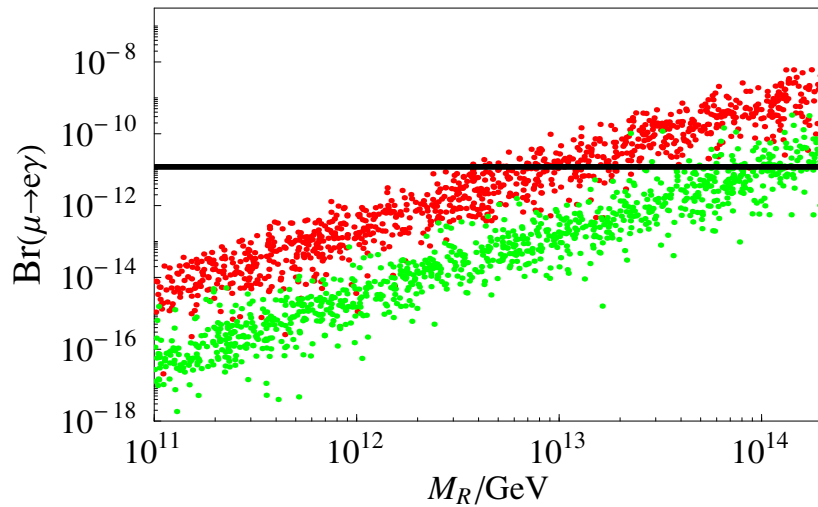
$$(Y_\nu^\dagger Y_\nu)_{ab} \approx \frac{M_R}{v^2 \sin^2 \beta} \left( \sqrt{\Delta m_{12}^2} V_{a2} V_{b2}^* + \sqrt{\Delta m_{23}^2} V_{a3} V_{b3}^* \right)$$

- for degenerate light  $\nu$  spectrum:

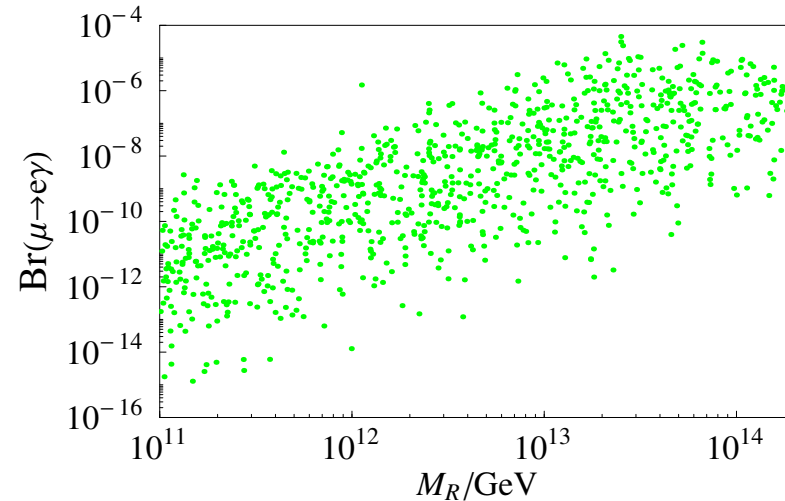
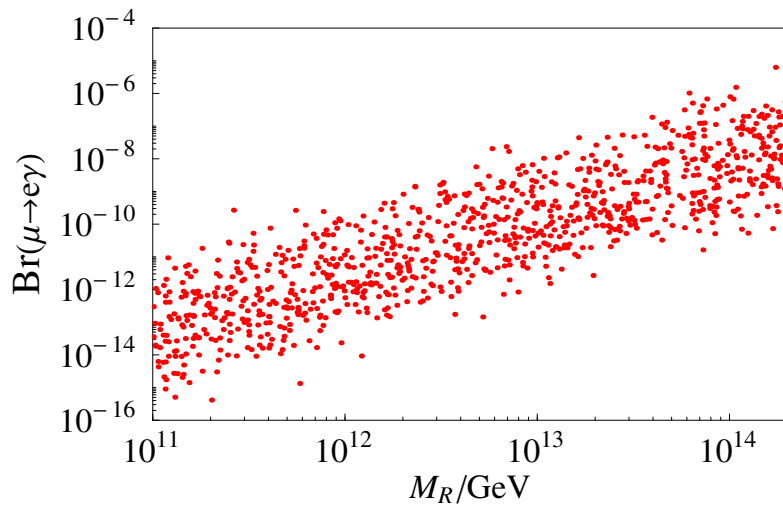
$$(Y_\nu^\dagger Y_\nu)_{ab} \approx \frac{M_R}{v^2 \sin^2 \beta} \left( m_1 \delta_{ab} + \frac{1}{2m_1} (\Delta m_{12}^2 V_{a2} V_{b2}^* + \Delta m_{23}^2 V_{a3} V_{b3}^*) \right)$$

degenerate  $M_{N_i} = M_R$ , **hierarch./degen.**  $m_{\nu_i}$ , SUSY point SPS1a

**Real  $R$ :**  $\delta m_L^2 \sim Y_\nu^\dagger L Y_\nu \sim M_R \ln \frac{M_{\text{GUT}}}{M_R} V \cdot D(m_j) \cdot V^\dagger$



**Complex  $R$ :**  $\delta m_L^2 \sim Y_\nu^\dagger L Y_\nu$ ,  $Y_\nu \sim \sqrt{M_R} R \cdot D(\sqrt{m_j}) \cdot U^\dagger$



## ratios of branching ratios

$$\frac{Br(l_i \rightarrow l_j \gamma)}{Br(l_{i'} \rightarrow l_{j'} \gamma)} \sim \frac{m_{l_i}^5 \Gamma_{i'}}{m_{l_{i'}}^5 \Gamma_i} \frac{|(Y_\nu^\dagger L Y_\nu)_{ij}|^2}{|(Y_\nu^\dagger L Y_\nu)_{i'j'}|^2}$$

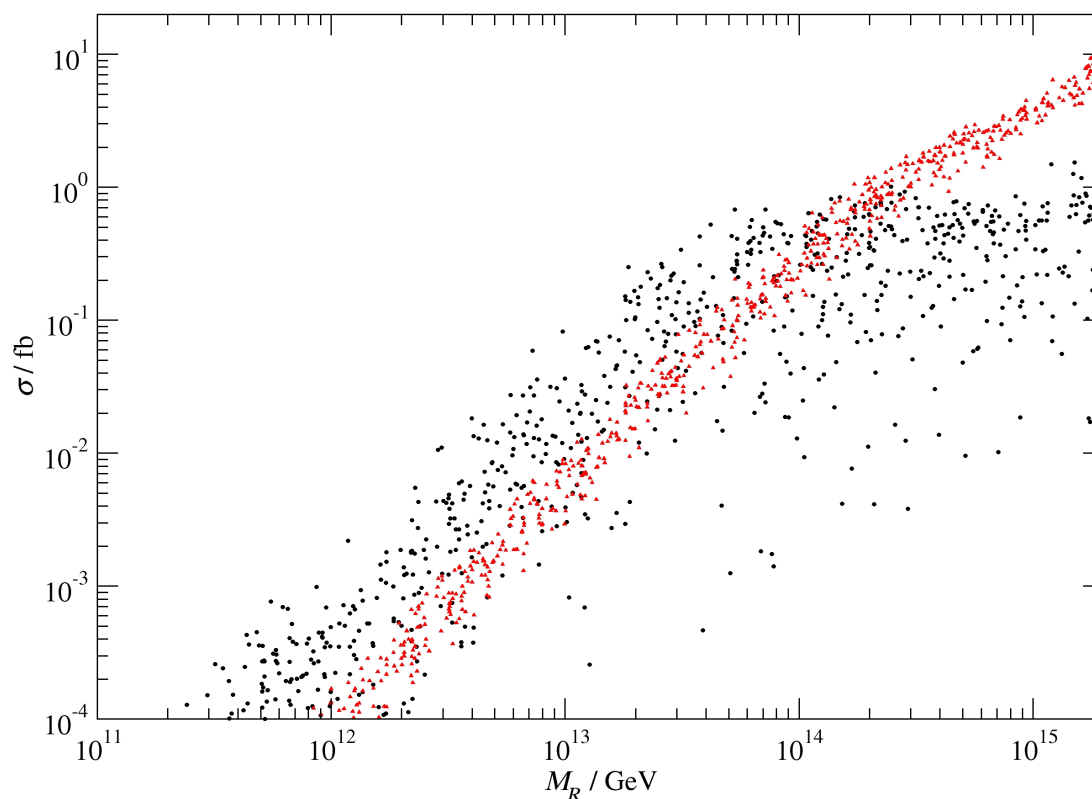
## example:

- hierarchical light neutrinos, central best-fit values for neutrino parameters
- vanishing Dirac/Majorana phases
- SUSY scenario C

Majorana masses		
Ratios	$M_i = M_R$	$M_1 : M_2 : M_3 = 1 : 10 : 100$
$\tau \rightarrow \mu \gamma / \mu \rightarrow e \gamma$	4	12
$\tau \rightarrow \mu \gamma / \tau \rightarrow e \gamma$	2500	160
$\mu \rightarrow e \gamma / \tau \rightarrow e \gamma$	640	13

$$\sigma(e^+e^- \rightarrow \mu^+e^-(\tau^+\mu^-) + 2\tilde{\chi}_1^0)$$

degenerate Majorana masses, real R, SUSY point SPS1a



direct production at  $\sqrt{s} = 500$  GeV, neutralino and chargino cascades not included



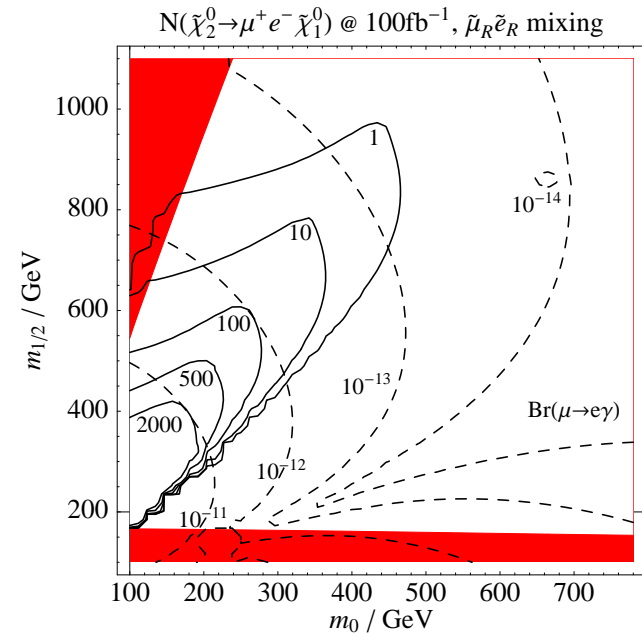
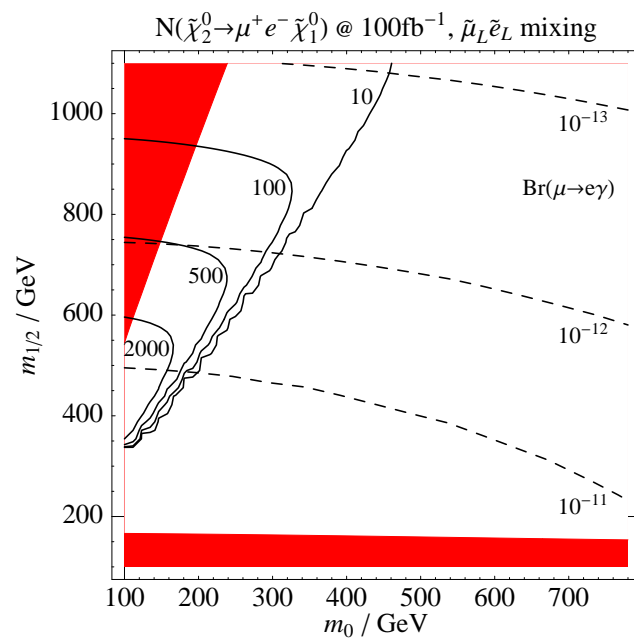
# LHC: maximal flavor mixing in L- and R-slepton sector

- two-flavor model with parameters  $\theta_{L,R}$  and  $\Delta\tilde{m}_{L,R}$

- LFV branching ratio:

$$\text{Br}(\tilde{\chi}_2^0 \rightarrow \mu^+ e^- \tilde{\chi}_1^0) = 2 \sin^2 \theta_{L,R} \cos^2 \theta_{L,R} \frac{\Delta\tilde{m}_{L,R}^2}{\Delta\tilde{m}_{L,R}^2 + \Gamma_{\tilde{l}}^2} \text{Br}(\tilde{\chi}_2^0 \rightarrow e^+ e^- \tilde{\chi}_1^0)$$

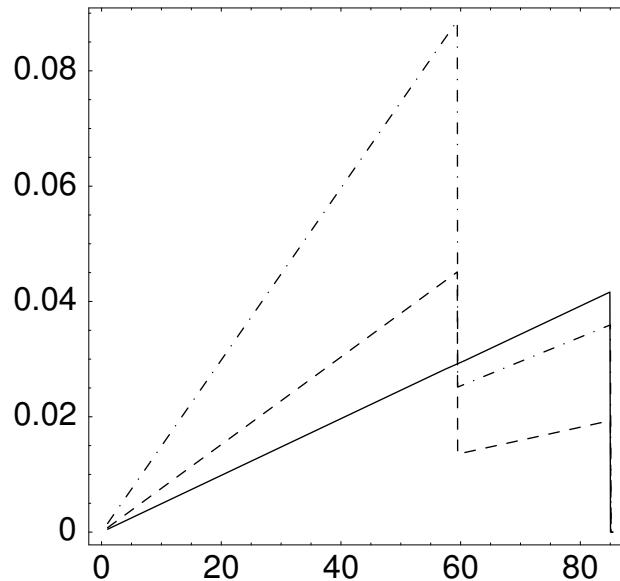
- maximal mixing:  $\theta_{L,R} = \pi/4$ ,  $\Gamma_{\tilde{l}} \ll \Delta\tilde{m}_{L,R} = 0.5 \text{ GeV}$



$$\tilde{\chi}_2^0 \rightarrow \tilde{l}_a l_j \rightarrow l_i l_j \tilde{\chi}_1^0: \quad m_{edge}^2(\ell\ell) = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_a}^2)(m_{\tilde{l}_a}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_a}^2}$$

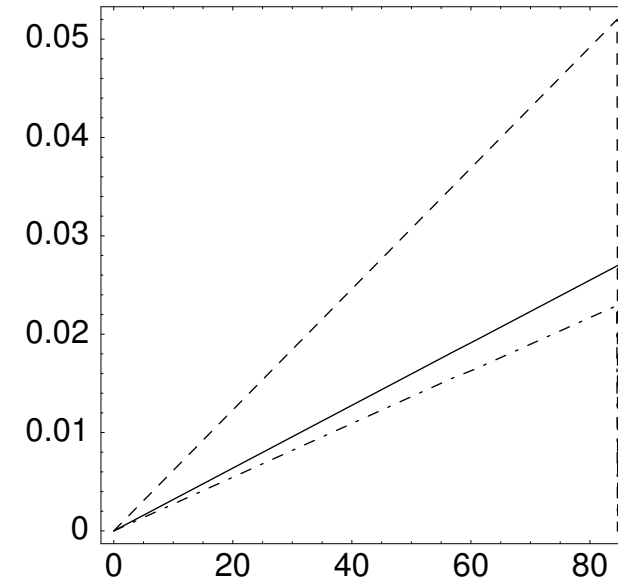
MSUGRA point SPS1a', mixing in the R-slepton sector

$100\Gamma_{tot}^{-1} d\Gamma(\tilde{\chi}_2^0 \rightarrow l_i l_j \tilde{\chi}_1^0)/dm(l_i l_j)$  [GeV<sup>-1</sup>]



LFV:  $m(e\tau, \mu\tau, e\mu)$

$100\Gamma_{tot}^{-1} d\Gamma(\tilde{\chi}_2^0 \rightarrow \ell\ell\tilde{\chi}_1^0)/dm(\ell\ell)$  [GeV<sup>-1</sup>]



LFC:  $m(\mu\mu) = m(ee)$ , LFV:  $m(\mu\mu, ee)$

*Bartl et al., hep-ph/0510074*: double edge structure due to  $m_{\tilde{l}_1} < m_{\tilde{l}_{2,3}}$

## baryon asymmetry in the universe

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10} \quad \text{from CMB}$$

generation of lepton asymmetry in **out-of-equilibrium decays of  $N_1$**   
 later on transformation to baryon asymmetry via **sphaleron processes**

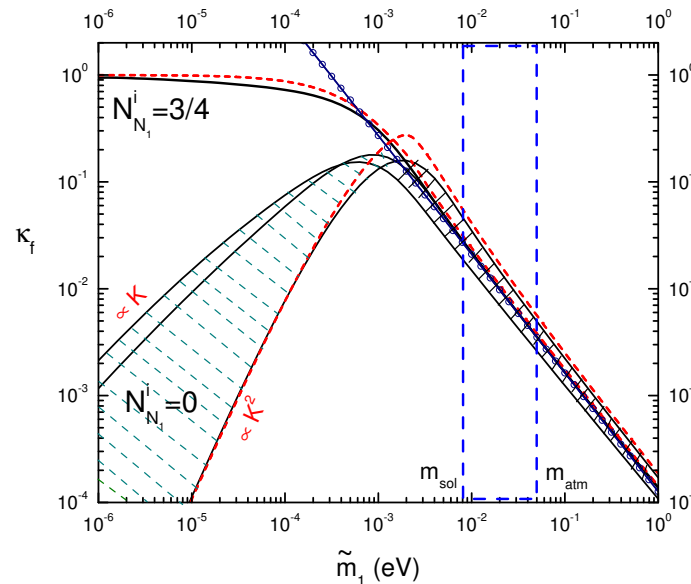
$$\eta_B = \kappa_f d a_{sph} \epsilon_1$$

- $\epsilon_1 = \text{CP asymmetry} = \frac{\Gamma(N_1 \rightarrow h_2 + l) - \Gamma(N_1 \rightarrow \bar{h}_2 + \bar{l})}{\Gamma(N_1 \rightarrow h_2 + l) + \Gamma(N_1 \rightarrow \bar{h}_2 + \bar{l})}$
- $a_{sph} = \text{fraction of } L\text{-asymmetry converted to } B\text{-asymmetry} = \frac{8}{23}$
- $d = \text{dilution factor due to } \gamma \text{ production } T_{\cancel{L}} \rightarrow T_{\text{rec}} = \frac{1}{78}$
- $\kappa_f = \text{efficiency factor (washout processes, Boltzmann equations)}$

*Buchmüller, Di Bari, Plümacher, hep-ph/0406014*

hierarchical neutrino spectra:  $\epsilon_1 \simeq -\frac{3}{8\pi} \frac{M_1}{v_2^2} \frac{\sum_i m_i^2 \text{Im}(R_{1i}^2)}{\sum_i m_i |R_{1i}|^2} < \frac{3}{8\pi} \frac{M_1}{v_2^2} m_3$

independence of initial conditions:  $\sqrt{\Delta m_{12}^2} < \tilde{m}_1 = v_2 \frac{(Y_\nu Y_\nu^\dagger)_{11}}{M_1} < \sqrt{\Delta m_{23}^2}$



no gravitino problem for BBN:  $M_1 \lesssim 10 T_R \lesssim 10^{10} \text{ GeV}$  for  $m_{3/2} = 1 \text{ TeV}$

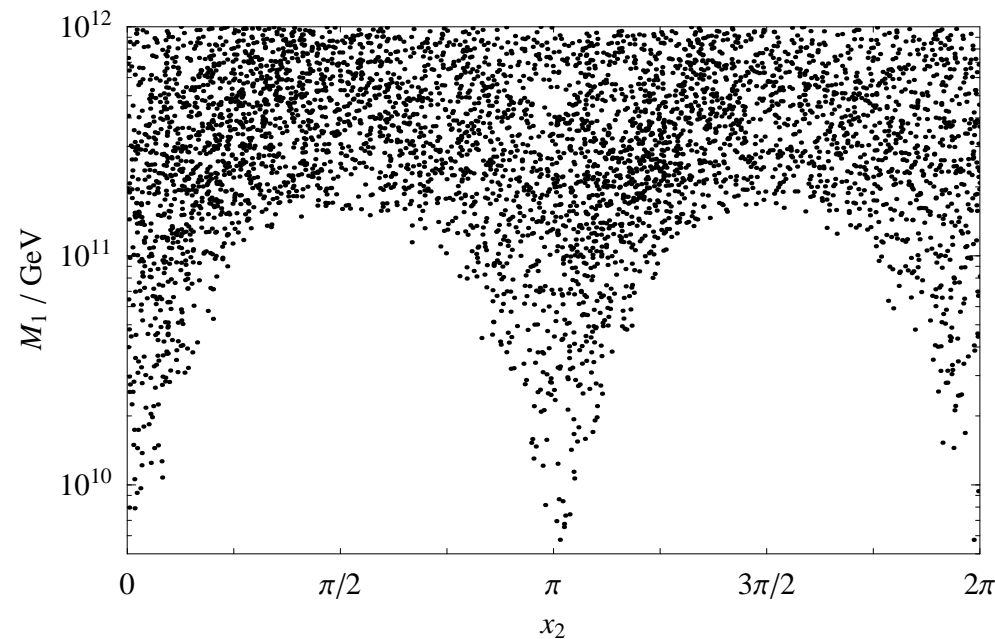
*Abada et al., hep-ph/0605281*: washout factors are flavor-dependent for  $M_1 \lesssim 10^{12} \text{ GeV}$

$$Y_\nu = \frac{1}{v \sin \beta} D \left( \sqrt{M_i} \right) R(x_i + I y_i) D \left( \sqrt{m_j} \right) U^\dagger$$

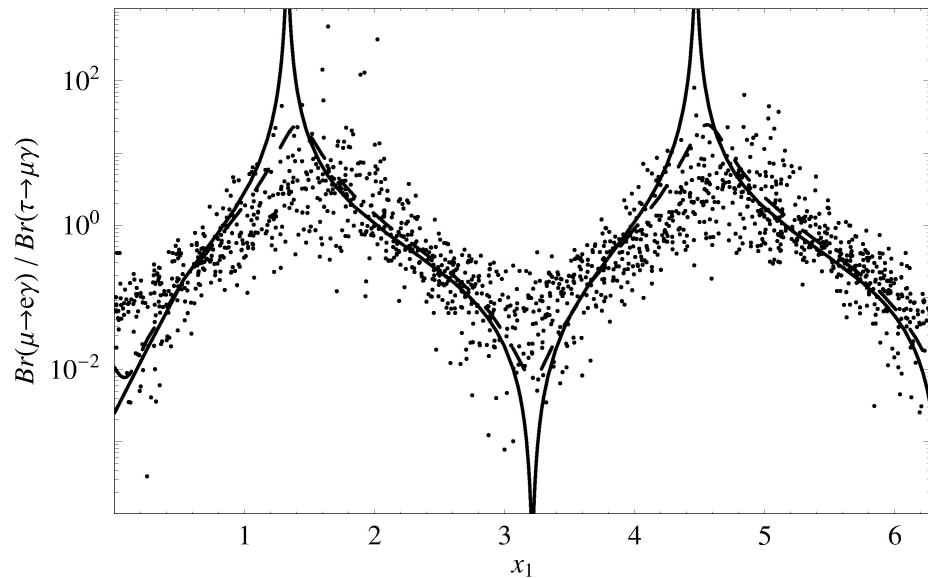
hierarchical heavy and light neutrino masses

$$0 \leq x_{1,3} \leq 2\pi, \quad 10^{-3} < y_i < \mathcal{O}(1)$$

MSUGRA scenario SPS1a



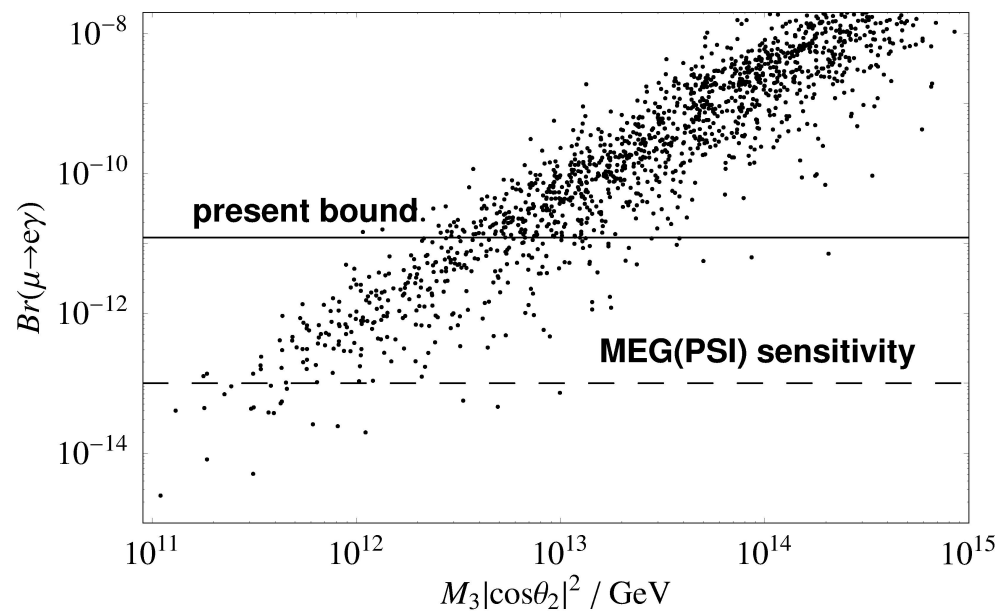
$$\eta_B = 6.3 \cdot 10^{-10}, \quad M_1 < 10^{11} \text{ GeV} \Rightarrow \sin x_2 \simeq 0$$



$\Rightarrow x_1$

- $x_{2,3} \simeq n\pi$
- $0 \leq x_1 \leq 2\pi$
- $10^{-3} < y_i < \mathcal{O}(1)$

$y_i = 0.01$  (solid),  $0.1$  (dashed)



$\Rightarrow M_3 \lesssim 10^{13} \text{ GeV}$