Collider Physics Applications of SCET

fairly new, by no means rare, increasingly beautiful

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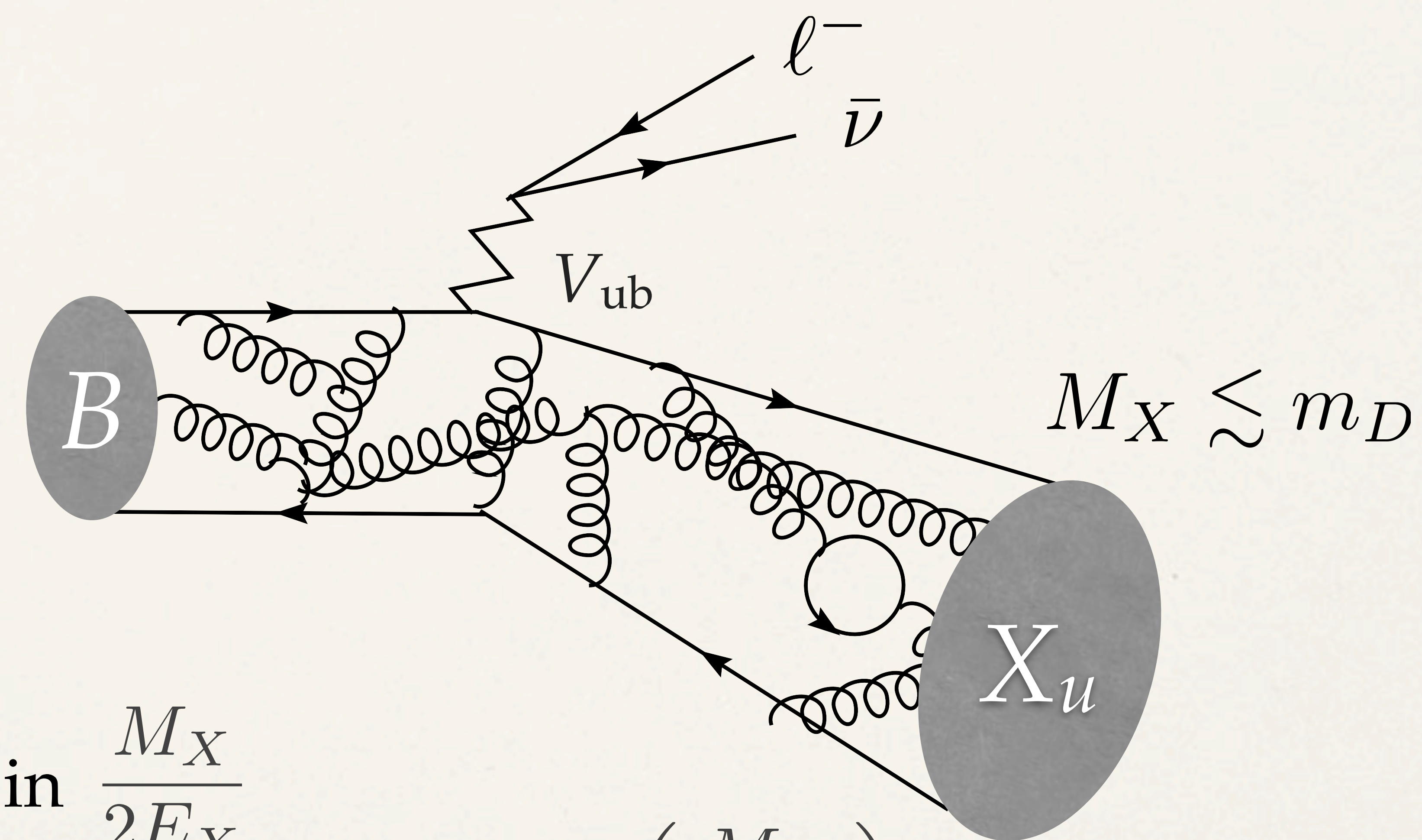
The New, the Rare and the Beautiful, University of Zürich, January 2010

Soft-Collinear Effective Theory

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke et al. 2002

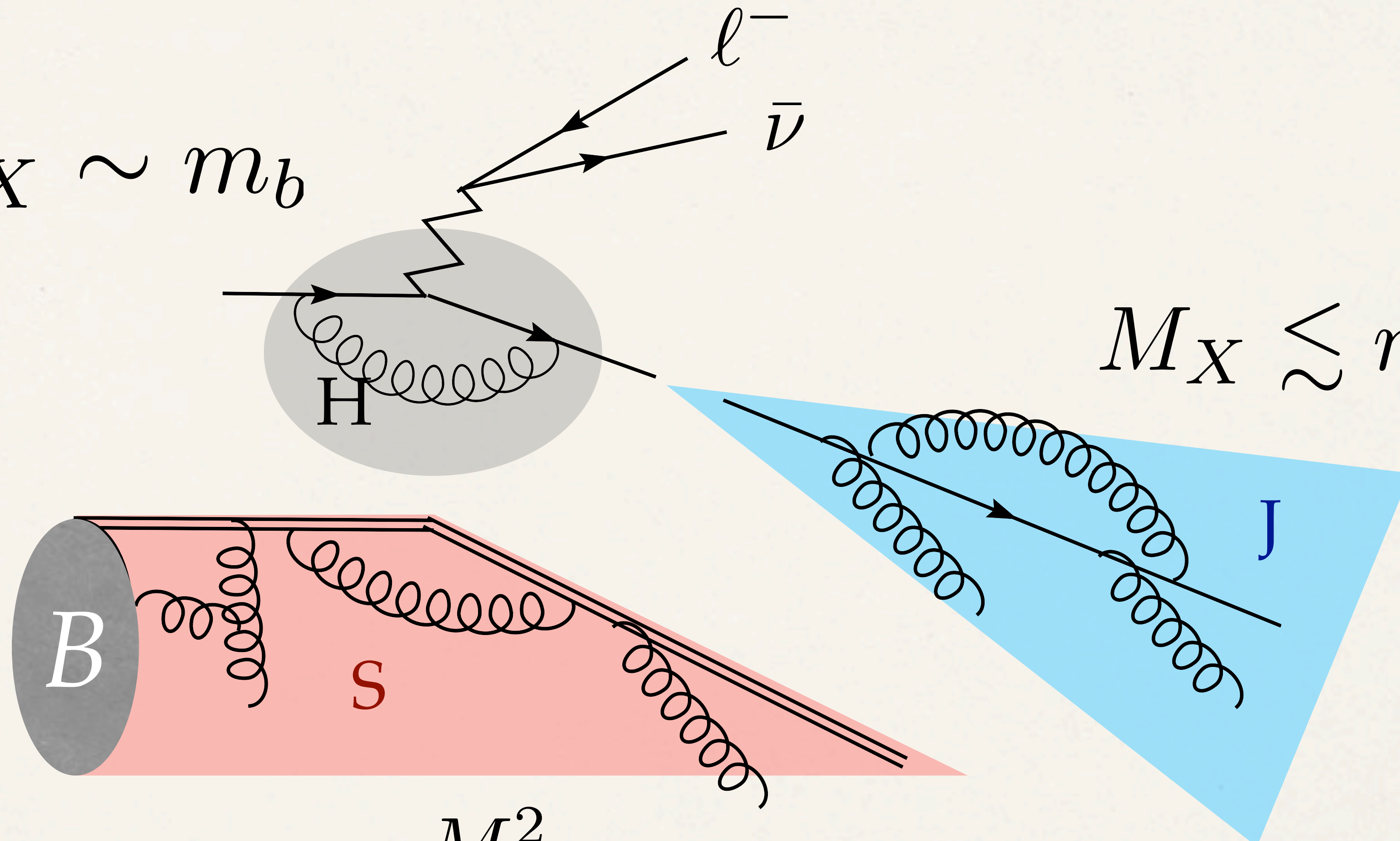
- An effective theory for processes for processes with energetic particles.



- Expansion in $\frac{M_X}{2E_X}$
- Sudakov resummation $\alpha_s^n \ln^{2n}\left(\frac{M_X}{2\,E_X}\right)$

Soft-Collinear Factorization

$$2E_X \sim m_b$$



$$M_X \lesssim m_D$$

$$\Lambda_S = \frac{M_X^2}{2E_X}$$

$$d\Gamma = H \cdot J \otimes S$$

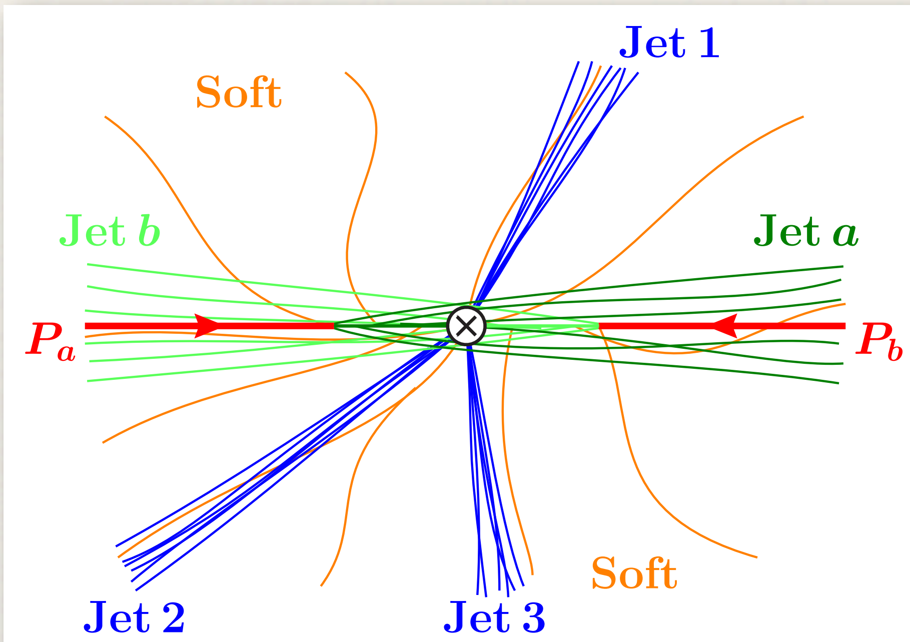
From B-decays to collider applications

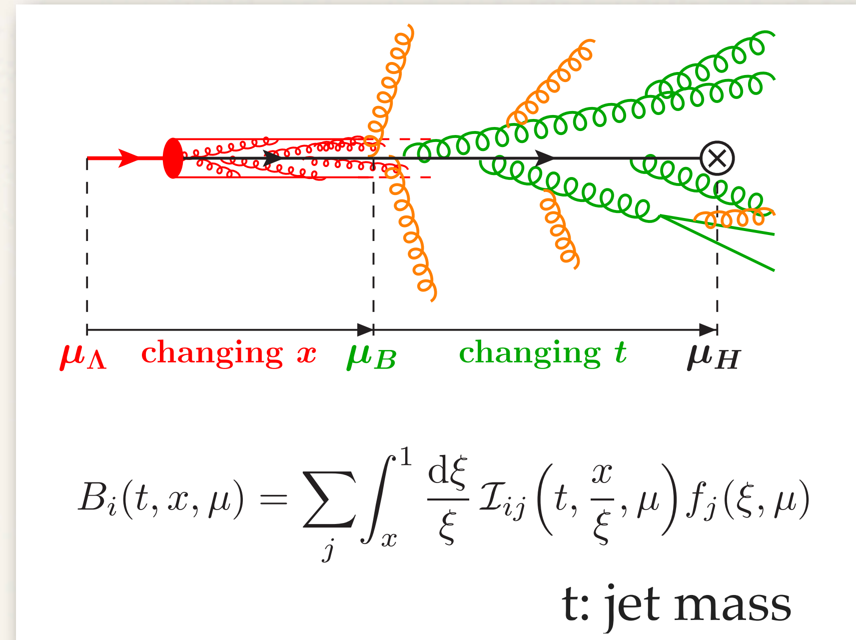
B-decays	Collider processes
1-jet processes processes: $B \to X_u\, l\, \nu$, $B \to X_s\, \gamma$, $B \to \gamma\, l\, \nu$ al. et Wyler	several jets, i.e. directions of large momentum flow
hadronically inclusive rates	jet algorithms, event shapes, treatment of the beam in hadronic collisions
power corrections, hadronic input	large energies, power corrections less of an issue

Jets in SCET

- Several papers in the last few months on jet observables in $e^+ e^-$
  - Cheung, Luke and Zuberi 0910.2479: evaluation of $e^+ e^- \to qqg$ in SCET for JADE, Sterman-Weinberg and k_T-algortihms
  - Jouttenus 0912.5509: one-loop jet function for Sterman-Weinberg jet definition
  - S. Ellis et al. 0912.0262, 1001.0014: one-loop jet and soft functions for cone and recombination algorithms. NLL calculation of 3-jet shapes.
- Issues:
  - Care needs to be taken to avoid double counting: a collinear particle becomes soft if the energy become small.
  - Non-global log's: for some observables soft function may contain large log's.

Beam jets Stewart, Tackmann, Waalewijn 0910.0467





- In hadronic collisions the incoming hadrons (and associated outgoing remants) should be treated as jets.
  - corresponds to initial state shower
  - soft emissions from initial state are calculable part of what's usually called the underlying event
- Stewart et al. factorize hadron beam jet function B_i into PDFs times a perturbatively calculable coefficient I_{ij}.
- Application: p_T resummation for Higgs Mantry and Petriello '09

SCET for n-jet processes

- n different types of collinear quark and gluon fields (jet functions J_i), interacting only via soft gluons (soft function S)
- Hard contributions (Q ~ √s) are integrated out and absorbed into Wilson coefficients:

$$\mathcal{H}_n = \sum_i \mathcal{C}_{n,i}(\mu)\, O_{n,i}^{\text{ren}}(\mu)$$

- Scale dependence controlled by RGE:

$$\frac{d}{d\ln\mu}\,|\mathcal{C}_n(\{\underline{p}\},\mu)\rangle = \boldsymbol{\Gamma}(\mu,\{\underline{p}\})\,|\mathcal{C}_n(\{\underline{p}\},\mu)\rangle$$

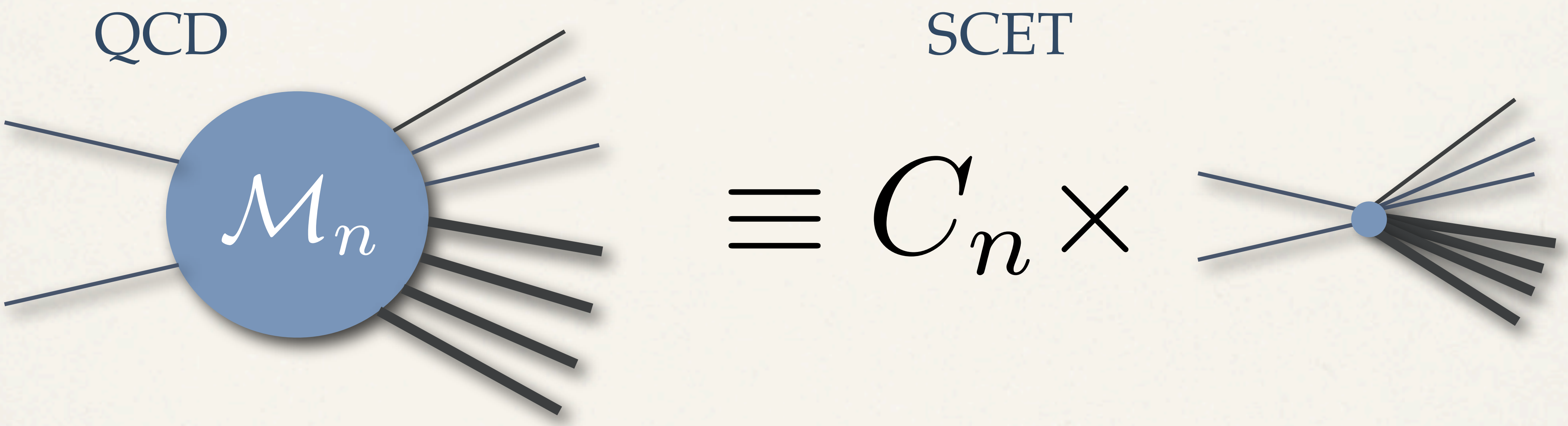
anomalous-dimension matrix of n-jet SCET operators

- Same anomalous-dimension matrix governs IR poles of dimensionally regularized, on-shell parton scattering amplitudes.

TB, Neubert 2009

On-shell matching

- To determine hard function, calculate on-shell amplitudes in QCD and effective theory



- In effective theory all loop corrections vanish on-shell, because integrals are scaleless.

$$\lim_{\epsilon \to 0} \boldsymbol{Z}^{-1}(\epsilon, \{\underline{p}\}, \mu) \, |\mathcal{M}_n(\epsilon, \{\underline{p}\})\rangle = |C_n(\{\underline{p}\}, \mu)\rangle$$

- IR poles in QCD map onto UV poles of n-jet operators in SCET

UV-IR connection

- implies that IR singularities of QCD amplitudes can be understood with renormalization group methods.
- Soft-collinear factorization implies a constraints on the hard anomalous dimension TB, Neubert ‘09; Gardi, Magnea ‘09;

trivial color structure

$$\boldsymbol{\Gamma}(s_{ij}) = \boldsymbol{\Gamma}_s(\Lambda_{ij}^2) + \sum_i \Gamma_c^i(M_i^2)\,\mathbf{1}\,, \text{ with } \Lambda_{ij}^2 = \frac{M_i^2 M_j^2}{s_{ij}}$$

soft scale

M_i dependence must cancel!

  - Soft function is matrix element of Wilson lines. Due to non-abelian exponentiation only a small set of color structures can appear in $\boldsymbol{\Gamma}_s$.
- An additional strong constraint is provided by the factorization of amplitudes in the collinear limit. TB, Neubert ’09

All-order proposal for $\mathbf{\Gamma}$ (massless case)

- Anomalous dimension is conjectured to be extremely simple:

TB, Neubert 2009; Gardi, Magnea 2009; Bern et al. 2008

color charges

anom. dimensions, known to three-loop order

$$\mathbf{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\text{cusp}}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

sum over pairs i≠j of partons

$(p_i + p_j)^2$

- minimal structure, reminiscent of QED
- IR poles determined by color charges and momenta of external partons
- color dipole correlations, like at one-loop order

Order-by-order analysis TB, Neubert '09

- Up to two loops, the constraints do not allow for any additional terms beyond the conjecture
  - Explains earlier two-loop result for $\mathbf{\Gamma}_s$. Dixon, Mert Aybat and Sterman '06
- At three loops a *single* additional structure can appear

$$\mathbf{\Delta\Gamma}_3(\{\underline{p}\},\mu) = \sum_{(i,j,k,l)} f^{ade} f^{bce} \boldsymbol{T}_i^a \, \boldsymbol{T}_j^b \, \boldsymbol{T}_k^c \, \boldsymbol{T}_l^d \, F(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk})$$

sum over legs ↗

- F must depend on conformal ratios $\beta_{ijkl} = \ln \frac{(-s_{ij})(-s_{kl})}{(-s_{ik})(-s_{jl})}$ and must vanish in all collinear limits.
- Dixon, Gardi and Magnea '09 have constructed candidate functions which could arise at 3 loops. Simplest example is

$$F(x,y) = x^3(x^2 - y^2)$$

Applications

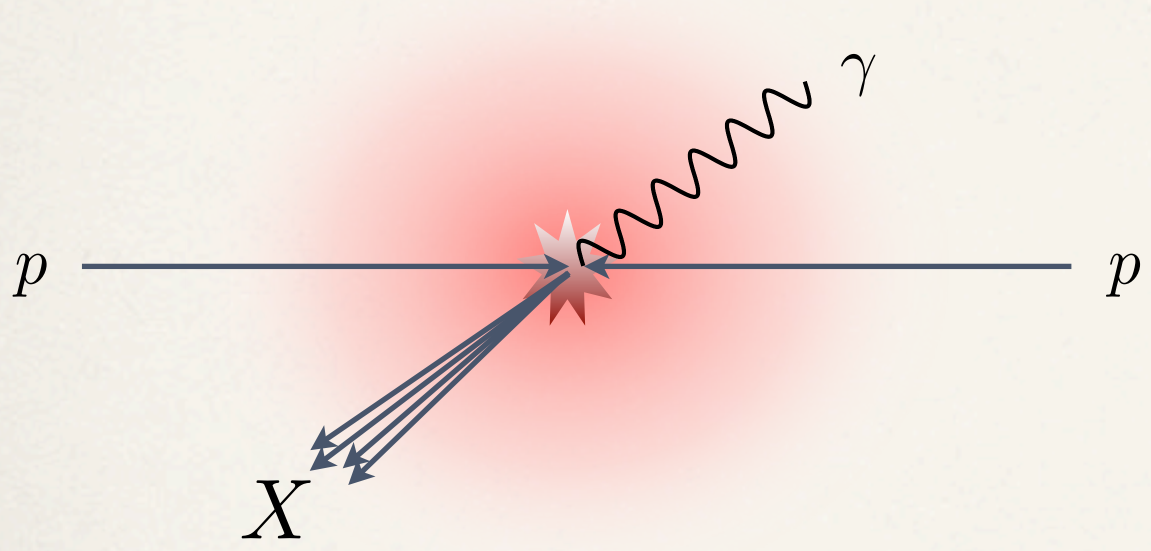
- Determination of F would need three-loop computation of four point-function.
- Independent of whether all-order ansatz holds, we have the anomalous dimension $\mathbf{\Gamma}$ relevant for NNLL resummations of n-jet processes.
  - Logarithmic part of $\mathbf{\Gamma}$ to three loops, non-logarithmic part to two.
  - $\rightarrow$ Enough for resummations of one-loop processes.
- So far, resummations in SCET were performed for two jet processes
  - DIS, $e^+ e^-$ event shapes, Higgs production, Drell-Yan, top production, sparticle production...
  - With only two directions of large energy, we cannot have any energetic partons in the final state at a hadron collider!

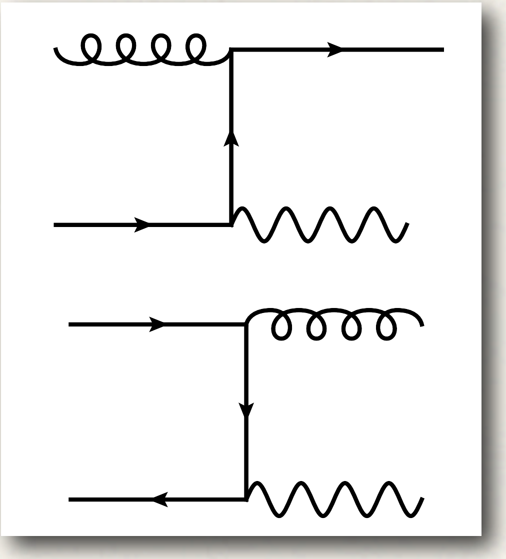
Photon production $pp \longrightarrow \gamma + X$ at large p_T

TB, M. Schwartz 0911.0681

- First SCET calculation of a physical cross section with energetic particles in three directions.
- Perform NNLL resummation of $\alpha_s^n \log^{2n}(M_X/p_T)$ corrections arising for at large p_T.
    - NLL was known Laenen et al. '98, Catani et al. '98, Kidonakis and Owens '99
- At large p_T fragmentation contribution is suppressed. Resum the prompt photon contribution.

$pp \to \gamma + X$ at large p_T





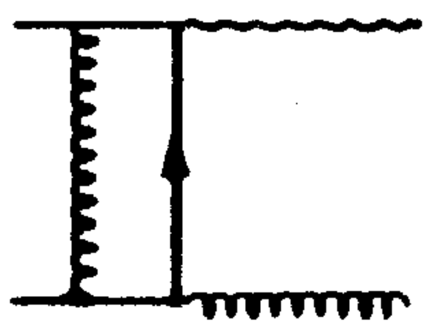
- Have derived factorization theorem for prompt photon production at large $p_T \gg M_X$

$$\frac{d^2\sigma}{dy\,dp_T} = H \otimes J \otimes S \otimes f_1 \otimes f_2$$

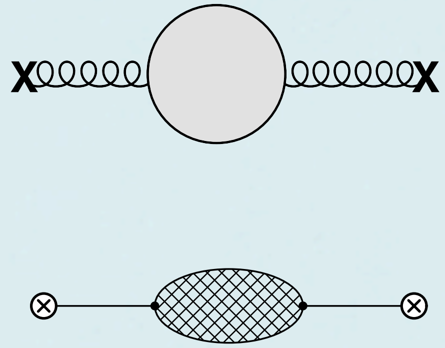
  - (there are different partonic channels, with different H, J, S and f's)

Hard, jet and soft functions

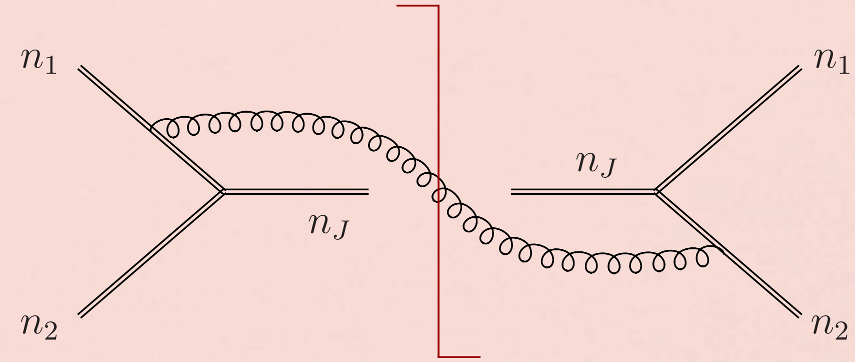
Hard function



Jet function



Soft function

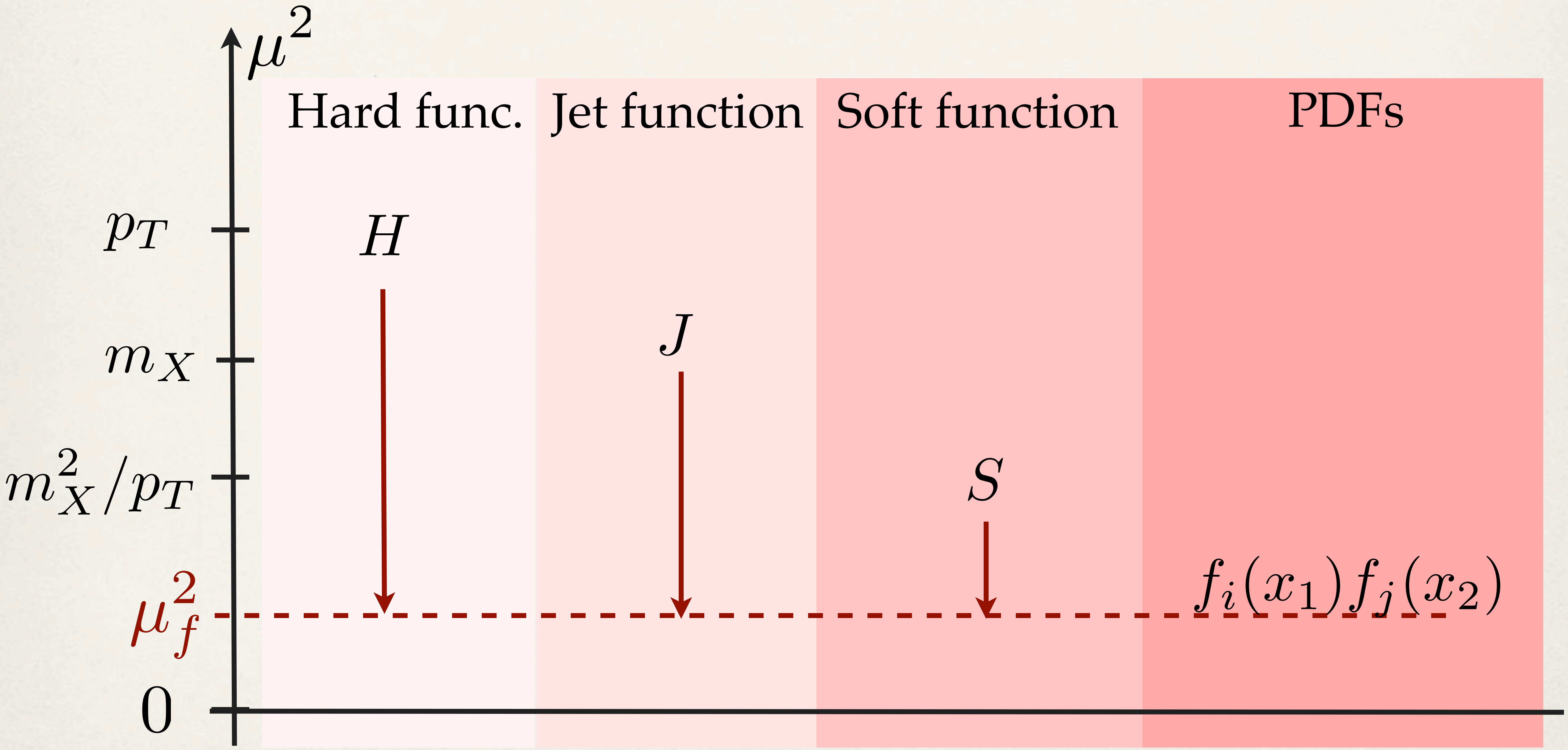


- Hard function is square of on-shell qg→qγ amplitude. Ellis et al. ’83, Arnold and Reno ’89
- Jet functions are quark and gluon propagators in light-cone gauge.
- Soft function is matrix element of Wilson lines Y_i from 0 ... ∞ along the beam and jet directions.

$$S_{\bar{q}q}(k_+) = \frac{1}{N_c} \sum_{X_s} \left| \left\langle X_s \left| \boldsymbol{T} \left[Y_1^\dagger(0)\, Y_J(0) t^a\, Y_J^\dagger(0) Y_2(0) \right] \right| 0 \right\rangle \right|^2 (2\pi)\delta(n_J \cdot p_{X_s} - k_+)$$

Resummation by RG evolution

- Evaluate each part at its characteristic scale, evolve to common scale:



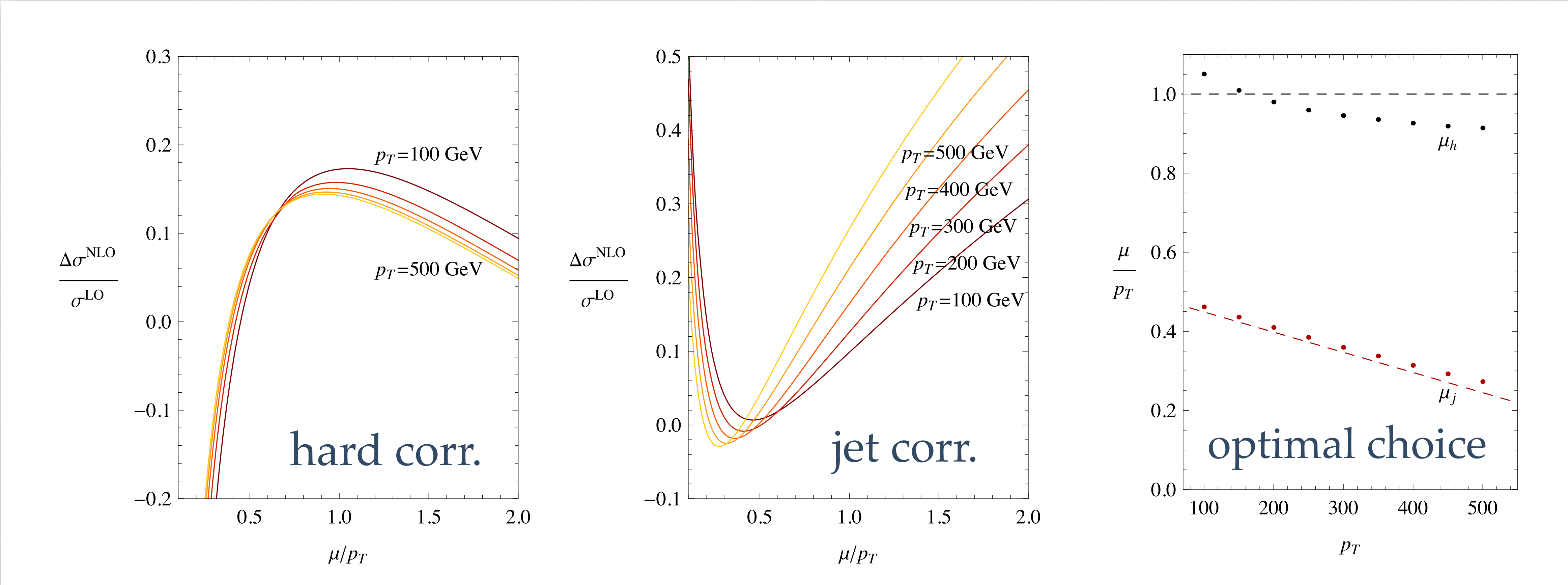
Anomalous dimensions

- Have analytic solution for the RGs of H, J and S. TB and Neubert '06
- Using RG invariance and known results, we are able to extract all anomalous dimensions to *three loops*
  - Hard anomalous dimension Γ_H from general result TB and Neubert '09 (see also Gardi and Magnea '09 + Dixon '09)
    - For n=3, the constraints determine Γ_H uniquely.
  - Quark-jet function anomalous dimension Γ_{J_q} known
  - Soft anom. dim. for qg channel is $\Gamma_{S_{qg}} = \Gamma_{H_{qg}} - \Gamma_{J_q}$
  - Soft anom dim. for qq channel is $\Gamma_{S_{\bar{q}q}} = \frac{2C_F - C_A}{C_A} \Gamma_{S_{qg}}$
  - Gluon-jet function anomalous dimension is $\Gamma_{J_g} = \Gamma_{H_{\bar{q}q}} - \Gamma_{S_{\bar{q}q}}$

Scale choice

- Natural choice for scale in hard function is $\mu_h \sim p_T$
- Choice of jet scale μ_j is more difficult, since *partonic* invariant mass varies $m_X = 0 \ldots M_X$ where the *hadronic* $M_X^2 = E_{\rm CM}^2(1 - p_T/p_T^{\rm max})$
  - For small M_X, *i.e. very* large p_T, $\mu_j \sim M_X$ is appropriate
  - Choice $\mu_j = m_X$ leads to Landau pole ambiguities; is implicit in trad. resummation method.
- Convolution with PDF dynamically enhances threshold region of low m_X .
  - Would like to set μ_j to the average value of m_X, but convolution with PDFs can only be done numerically.
  - Determine μ_j by looking at jet-function corrections as a function of μ_j. Reasonable scale choice gives moderate corrections.

Scale choice



- As a default, we choose

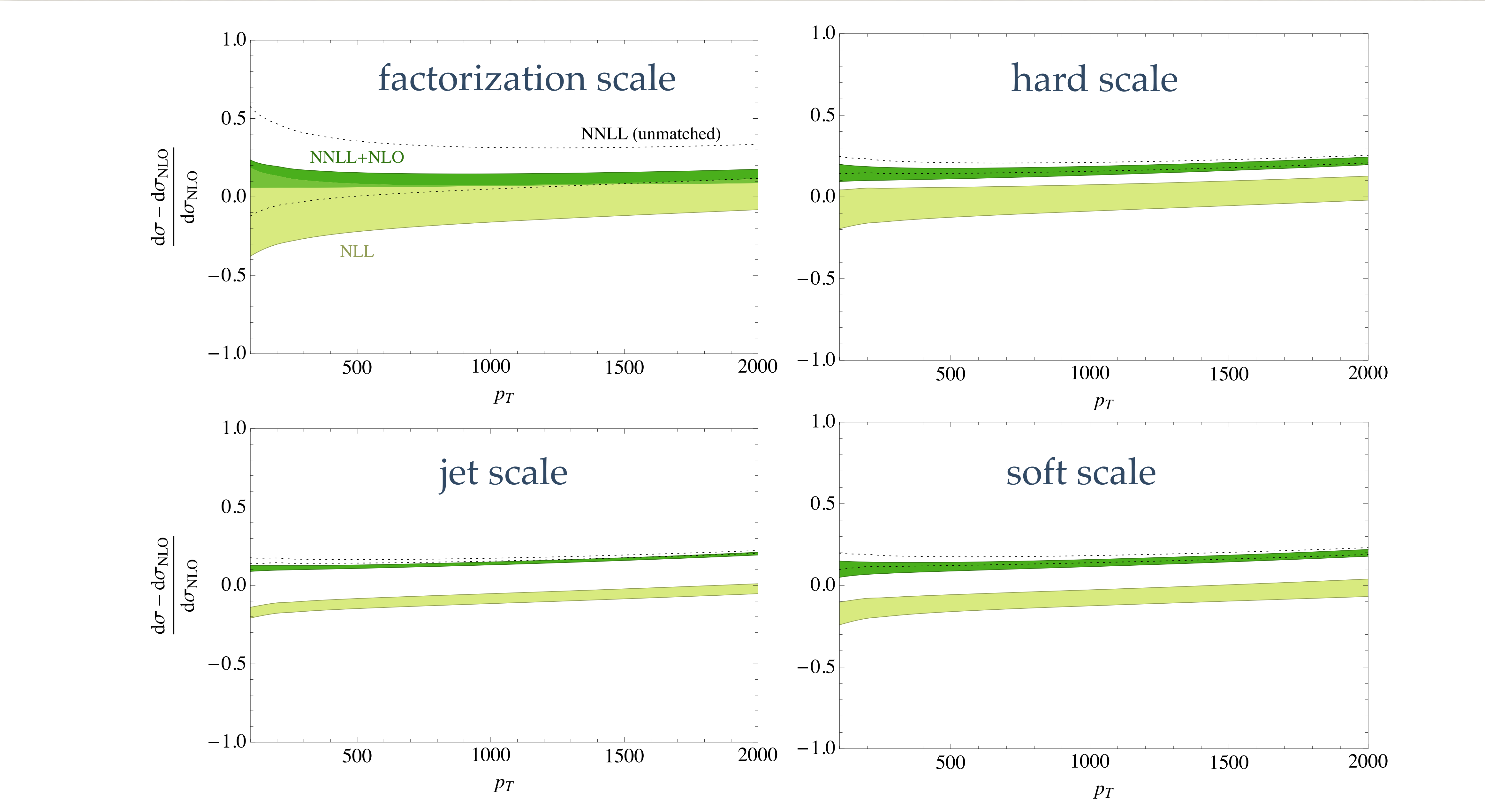
$$\mu_h = p_T\,,$$

$$\mu_j = \frac{p_T}{2}\left(1 - 2\frac{p_T}{E_{\rm CM}}\right),$$

$$\mu_s = \mu_j^2/\mu_h$$

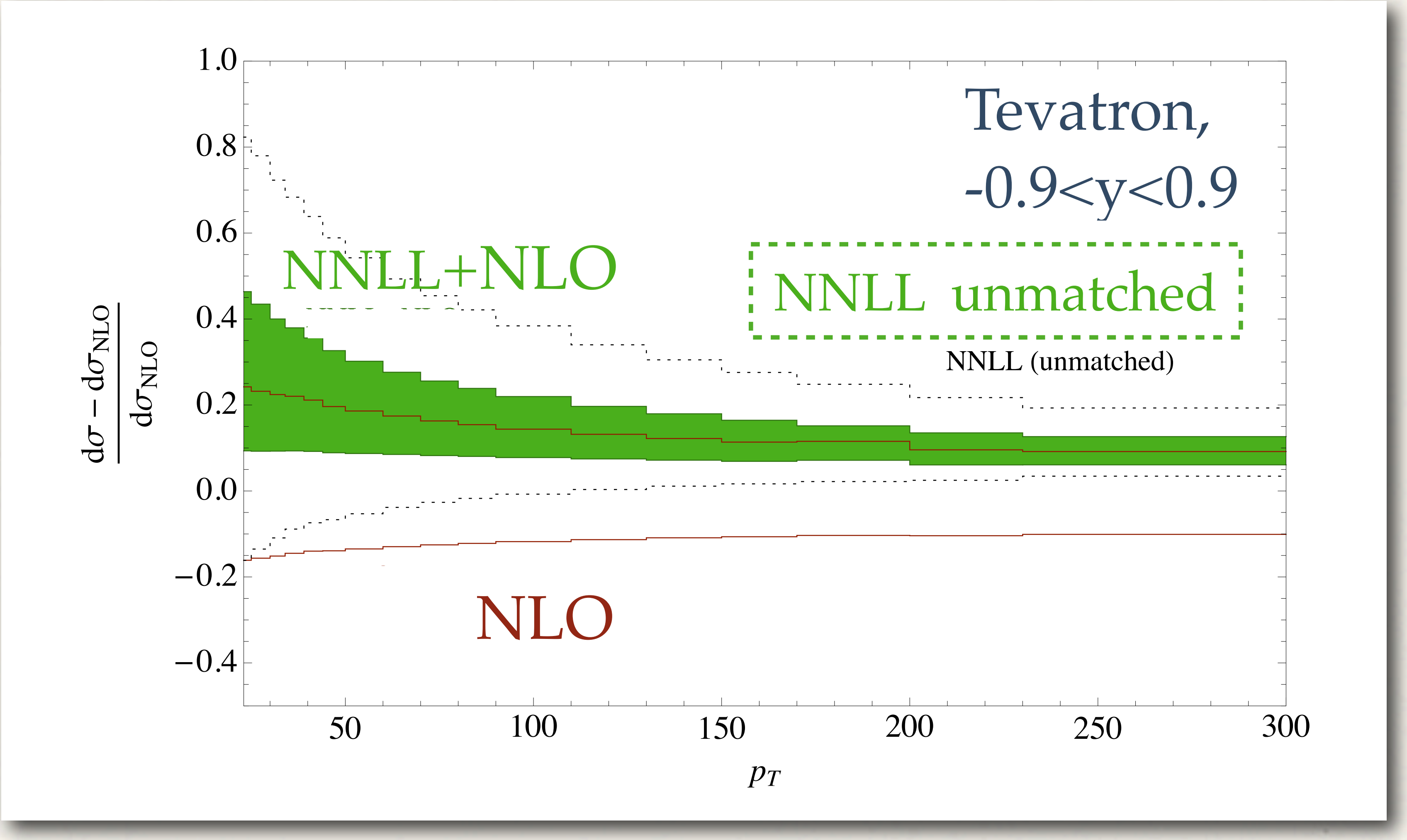
and vary by a factor two.

Scale variations



- Matching scales variations are small, factorizations scale uncertainty dominates. Matching to NLO reduces factorization scale dep.

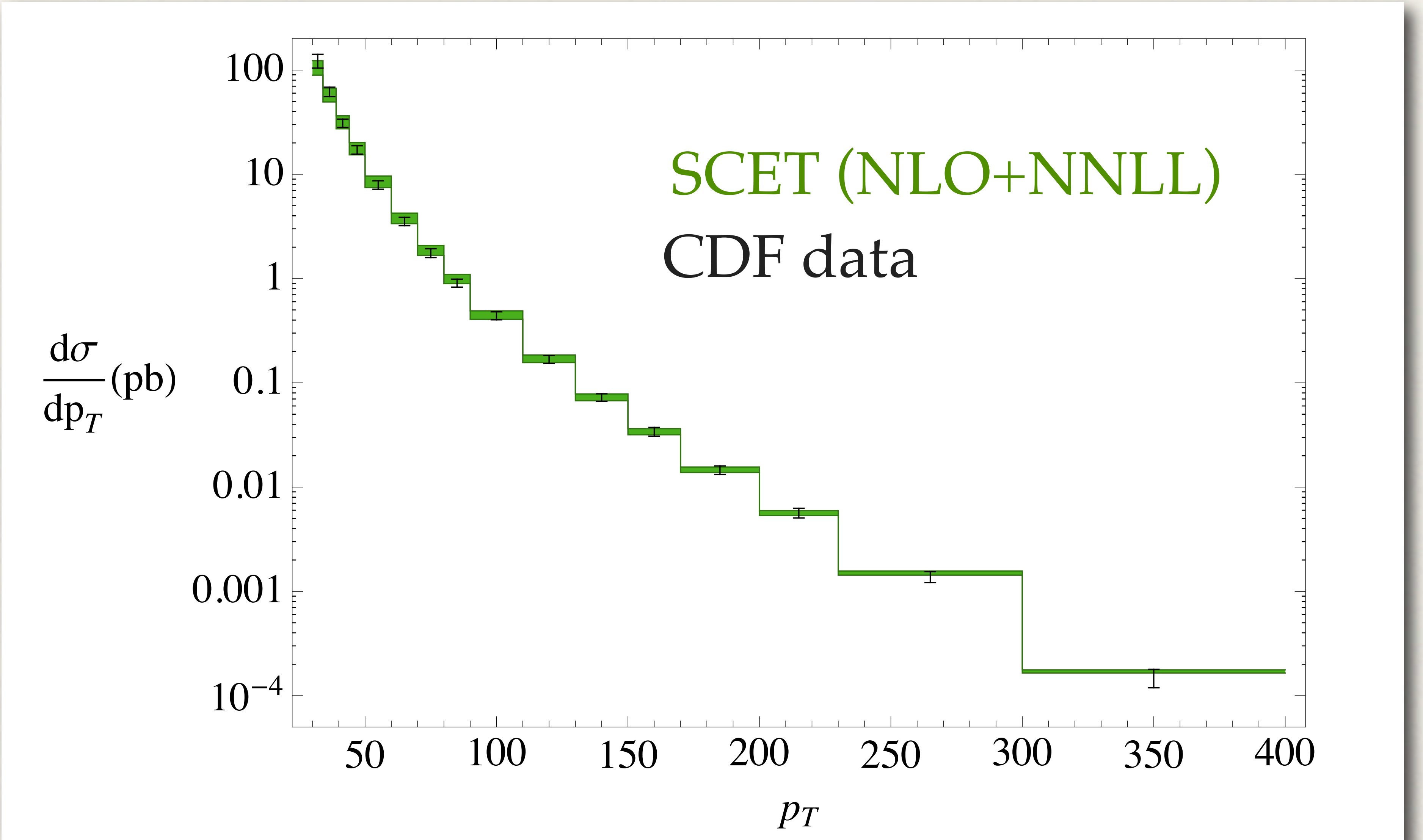
Matching to fixed order



- We match the NLO fixed order result in JETPHOX. This allows us to account for isolation cuts and fragmentation contributions.

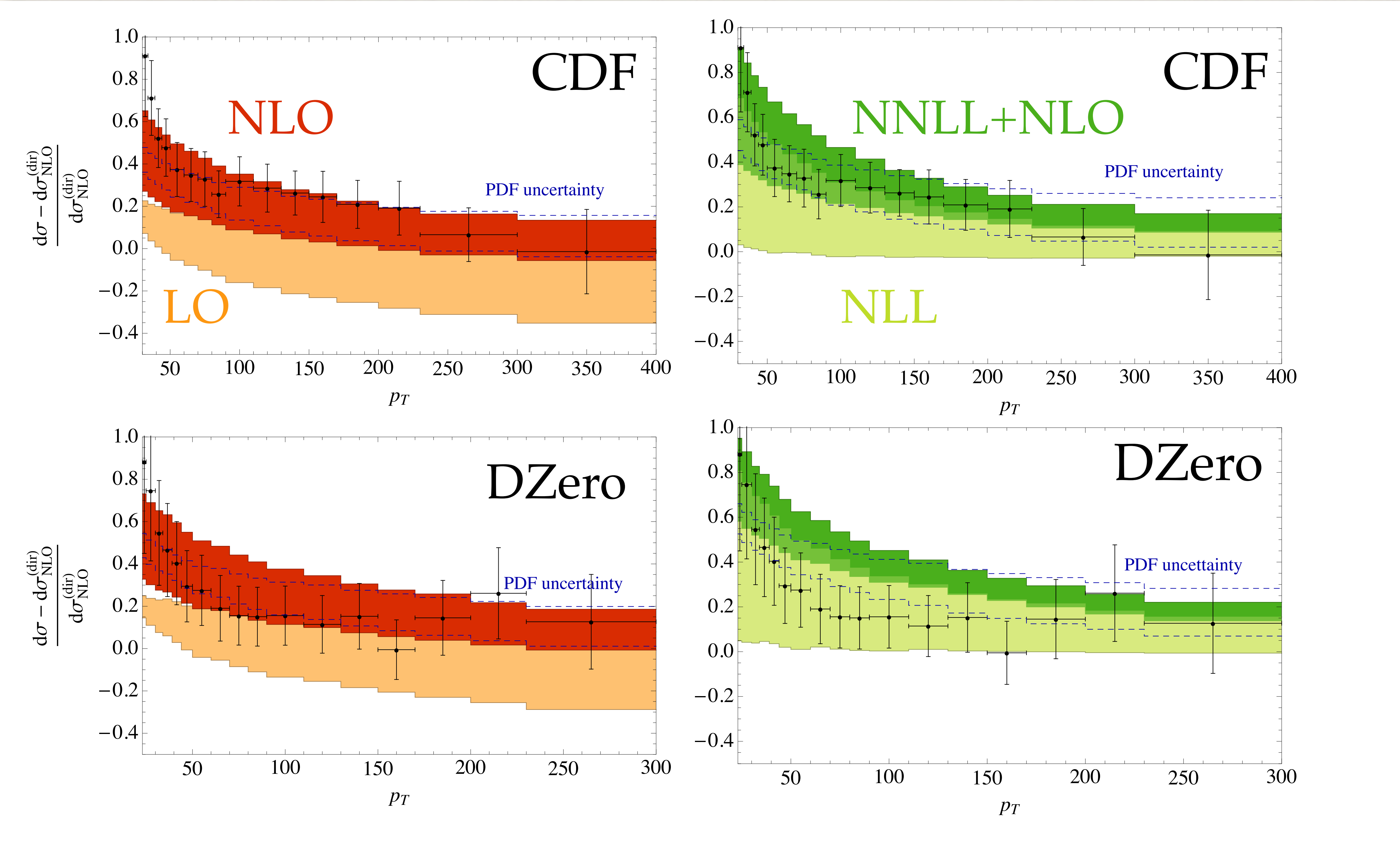
$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}v\mathrm{d}w}\right)^{\text{matched}} = \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}v\mathrm{d}w}\right)^{\text{NNLL}} - \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}v\mathrm{d}w}\right)^{\text{NNLL}}_{\mu_h=\mu_j=\mu_s=\mu_f} + \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}v\mathrm{d}w}\right)^{\text{NLO}}_{\mu_f} .$$

Cross section at Tevatron



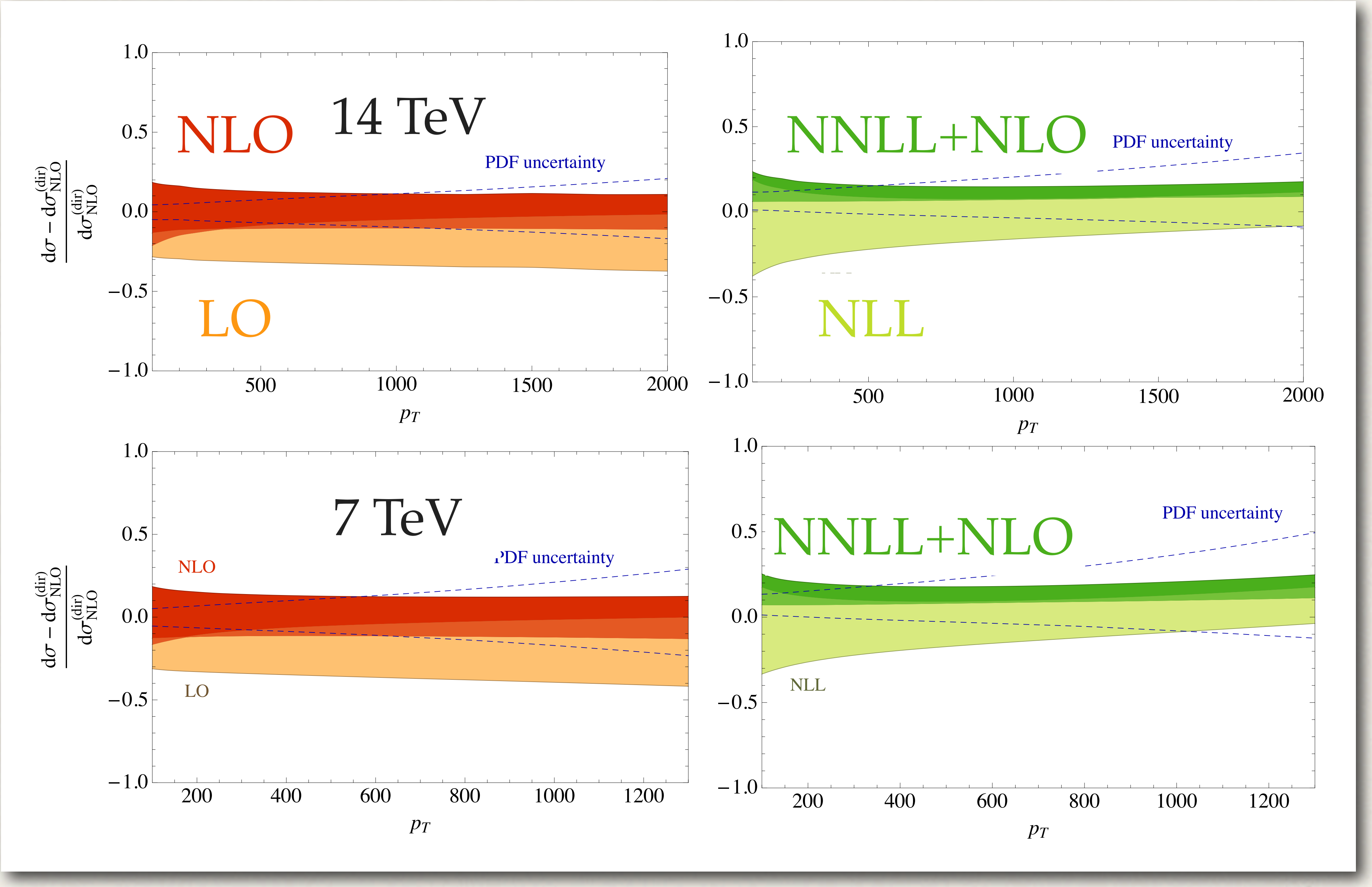
- Rapidly falling, so in the next slides I will plot $\dfrac{d\sigma - d\sigma_{\mathrm{NLO}}^{(\mathrm{dir})}}{d\sigma_{\mathrm{NLO}}^{(\mathrm{dir})}}$
  - $d\sigma_{\mathrm{NLO}}^{(\mathrm{dir})}$ is the *direct* photon production w/o isolation cuts.

Tevatron results



Fragmentation and isolation from JETPHOX. Additional hadronisation correction (a factor 0.91) as determined in CDF paper from MC studies.

LHC results



- Direct contribution only: no fragmentation or isolation cuts.

Conclusions

- A lot of progress during the past year towards the analysis of more complex collider observables in SCET
  - n-jet anomalous dimension
    - completely known to NNLL
    - fulfills stringent all-order constraints
  - implementation of jet algorithms
  - treatment of the beam
- First application involving three directions of large momentum flow
  - Photon production at large p_T to NNLL
    - hadronically still inclusive
- However, ...

... many multi-scale problems at colliders still await effective theory treatment, e.g. scattering in Regge kinematics:

On Regge kinematics in SCET

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We discuss the kinematics of the particles that make up a Reggeon in field theory, using the terminology of the Soft Collinear Effective Theory (SCET). Reggeization sums a series of strongly-ordered collinear emissions resulting in an overall Reggeon exchange that falls in the Glauber or Coulomb kinematic region. This is an extremely multi-scale problem and appears to fall outside of the usual organizing scheme of SCET.