Another approach to track reconstruction: cluster analysis

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Valencia, 5 Apr 2019
Last time, CTD/WIT17 – disconnecting the graph

F Siklér, “Combination of analysis techniques for efficient track reconstruction in high multiplicity events”, Eur Phys J A 54 (2018) 113
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## Motivation – quantum-correlations

### ATLAS

\[ \sqrt{s} = 7 \text{ TeV} \]

\[ p_T \geq 100 \text{ MeV}, |\eta| < 2.5, n_{\text{ch}} \geq 2 \]

- data
- Gaussian fit
- Exponential fit

\[ R_2(Q) \]

\[ Q \text{ [GeV]} \]

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \]

\[ (Q)^2 R \]

\[ 0.9 \quad 1 \quad 1.1 \quad 1.2 \quad 1.3 \quad 1.4 \quad 1.5 \quad 1.6 \quad 1.7 \quad 1.8 \]

### ATLAS Coll, Eur Phys J C 75 (2015) 466

### ALICE Coll, Phys Rev D 84 (2011) 112004

### CMS pPb, \[ \sqrt{s_{NN}} = 5.02 \text{ TeV} \]

\[ \lambda = 0.92 \pm 0.01 \]

\[ R_L = (2.43 \pm 0.03) \text{ fm} \]

\[ R_T = (1.89 \pm 0.02) \text{ fm} \]

\[ \chi^2_{+}/\text{ndf} = 188.1/165 \]

\[ \chi^2_{--}/\text{ndf} = 197.4/165 \]

### Bose-Einstein correlations

- interesting quantum physical background; sensitive to source size and shape
- usually explored by electrically charged and strongly interacting bosons (\( \pi^{\pm}, K^{\pm} \))
- drawbacks: have to correct for FSI (Coulomb and strong); also jet-related background

**Let’s try \( \gamma \gamma \) correlations!** No charge, no strong, reach down to \( q = 0 \)
Motivation – photon reconstruction using silicon trackers

- Requirements for photons
  - it is a correlation measurement, we want pairs of photons: high efficiency!
  - photons are at low momentum: calorimetry alone not enough
  - high chance of conversion! for $e^\pm$ calo-based seeding won’t work; highly curved tracks

Task: reconstruction of primaries and detached photon conversions, offline
Name of the game: put (most of) the hits on a limited number of trajectories

How to do it? Default solution: combinatorial track finder via Kalman filter

Motivation – personal

- Looking for new combinations
  - no classical trajectory building
  - no image-transformation-type approach
  - must be simple

- Clustering methods
  - EM algorithms vs mixture models?
  - $k$-means or $k$-medians (for robustness)
  - hits as objects, tracks as clusters
  - mobile hit-to-track assoc, minimize sum-$\chi^2$

- Evolution
  - allow the solution to evolve
  - from Metropolis-Hastings MCMC
  - adding and removing track candidates

Efficient use of both local and global information
Details: detector model

- Silicon layers

<table>
<thead>
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<th>Barrel</th>
<th>$r$ [cm]</th>
<th>$z_{\text{max}}$ [cm]</th>
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<tr>
<td>strips</td>
<td>20, 30, 40, 50</td>
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<tr>
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<td>60, 70, 80, 90, 100, 110</td>
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<table>
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<th>Endcap</th>
<th>$z$ [cm]</th>
<th>$r_{\text{min}}$–$r_{\text{max}}$ [cm]</th>
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<td>5–15</td>
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<tr>
<td>strips</td>
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<tr>
<td>strips</td>
<td>125, 140, 155</td>
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<tr>
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<tr>
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<td>220, 245</td>
<td>40–110</td>
</tr>
<tr>
<td>strips</td>
<td>270</td>
<td>50–110</td>
</tr>
</tbody>
</table>

- Other data
  - axial magn field $B = 3$ T; hit effic $\varepsilon_{\text{eff}} = 0.98$
  - material $x/X_0 = 2\%$ (pixels), 5\% (strips)
  - strip length 10 cm, pos resolution 50 $\mu$m

Barrel and endcap
Details: known physics effects

- Effects modelled
  - Multiple scattering (charged)
    \[ \theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} q \sqrt{x/X_0} [1 + 0.038 \ln(x/X_0)] \]
  - Energy loss (charged)
    \[ \Delta \rho = \xi \left[ \ln \frac{2mc^2 \beta^2 \gamma^2 \xi}{I^2} + 0.2000 - \beta^2 - \delta \right] \]
    \[ \Gamma_\Delta = 4.018\xi, \quad \text{where} \quad \xi = \frac{K}{2} \frac{Z^2 Z}{A} \rho \frac{x}{\beta^2} \]
  - Bremsstrahlung (e⁻/e⁺)
    \[ \frac{d\sigma}{dk} = \frac{A}{X_0 N_A k} \left( \frac{4}{3} - \frac{4}{3}y + y^2 \right), \quad \text{with} \quad y = \frac{k}{E} \]
    \( \text{(or as} \quad \frac{dE}{dx} = -\frac{E}{X_0} \text{ in reconstruction)} \)
  - Photon conversion (γ)
    conversion length \( 9/7X_0 \)
    \[ \frac{d\sigma}{dx} = \frac{A}{X_0 N_A} \left[ 1 - \frac{4}{3}x(1 - x) \right], \quad \text{where} \quad x = \frac{E}{k} \]

Compact standalone code, c++
Details: track model and variances

- Basics
  - track parameters
    \((\eta, q/p_T, \phi_0, z_0, r_c)\)
  - homogeneous axial magnetic field

- Variances of known physics effects
  - multiple scattering
    \[ \sigma_x^2 \propto \frac{\alpha^3 p_T \cosh \eta}{\beta^2} \]
  - energy loss
    \[ \Delta x \propto \frac{\alpha^3 p_T}{\beta^3} \]
  - bremsstrahlung

Produce candidate hits for each candidate track, then match to measured hits
Variance calculable, some nice maths behind
Hit positions are highly correlated between neighboring layers. Covariance between hit positions decays roughly as $\sim \rho^{-|i-j|}$, $\rho \approx 0.8 - 0.9$.

$$V^{-1} = \begin{pmatrix}
\sigma_1^2 & \rho \sigma_1 \sigma_2 & \rho^2 \sigma_1 \sigma_3 & \rho^2 \sigma_1 \sigma_4 \\
\rho \sigma_1 \sigma_2 & \sigma_2^2 & \rho \sigma_2 \sigma_3 & \rho \sigma_2 \sigma_4 \\
\rho^2 \sigma_1 \sigma_3 & \rho \sigma_2 \sigma_3 & \sigma_3^2 & \rho \sigma_3 \sigma_4 \\
\rho^3 \sigma_1 \sigma_4 & \rho^2 \sigma_2 \sigma_4 & \rho \sigma_3 \sigma_4 & \sigma_4^2
\end{pmatrix}^{-1} = \\
\frac{1}{1 - \rho^2} \begin{pmatrix}
\frac{1}{\sigma_1^2} & -\frac{\rho}{(\sigma_1 \sigma_2)} & 0 & 0 \\
-\frac{\rho}{(\sigma_1 \sigma_2)} & \frac{(1 + \rho^2)}{\sigma_2^2} & -\frac{\rho}{(\sigma_2 \sigma_3)} & 0 \\
0 & -\frac{\rho}{(\sigma_2 \sigma_3)} & \frac{(1 + \rho^2)}{\sigma_3^2} & -\frac{\rho}{(\sigma_3 \sigma_4)} \\
0 & 0 & -\frac{\rho}{(\sigma_3 \sigma_4)} & \frac{1}{\sigma_4^2}
\end{pmatrix} \quad (1)$$

Thus, the inverse of the covariance matrix is close to tridiagonal $\sum_{\text{all hits}} x^T V^{-1} x$. 

Ferenc Siklér: Track reconstruction through cluster analysis
Details: how to fit tracks?

Downhill simplex method
JA Nelder and R Mead, Computer Journal 7 (1965) 308

Robust: derivatives are not used, only evaluations at vertices
Merit function is flexible, promoting increase of $r_c$ for conversion
Step 1: assign (and relocate) each hit to the closest track (lowest $\chi^2$, highest $p$-value)

Step 2: update track parameters by refitting to its associated hits (via global covariance)

Alternate between these steps, until convergence
$k$-me(di)ans clustering – steps

Goodness-of-fit? Problem with outlier hits, also several local minima
Use hit-to-track distances ("medians")? **Soften with Cauchy-motivated** $\tan^{-1}(\beta x)/\beta$
Punish non-matched candidate hits by $-2 \log(1 - \varepsilon_{\text{eff}})$, and measured hits by $-2 \log p_{\text{noise}}$
$k$-me(di)ans: better guess for initial clusters? how many are there?

Mutual nearest neighbor search in angular distance wrt $(0,0,0)$
Take chains of connected hits as initial clusters
Details: initial tracks, robust helix fit

- Find circles in the bending plane
  - center, scale; project to the Riemann sphere
  - planes are fitted to triplet permutations of the projected points

R Frühwirth and A Strandlie, J Phys Conf Ser 1085 (2018) 042004

- Additional complications
  - mix of pixels and strips: points vs segments
  - use a few (2-4) internal segment points
  - take triplets: ask for nearly equal number of (end)points on both sides of the plane; and minimize sum of hit-to-plane distances
  - prefer circles which do not contain (0,0)
  - prefer circles where no hit is closer than the tangent point from (0,0) [for secondaries]
  - then, robust $r\alpha$ vs $z$ linear fit for the helix
$k$-me(di)ans: initial track candidates

Reasonable starting guesses
Looks good, but can we do better?
Related to deformable templates, elastic nets or arms?

[Hough trans estimates the number of tracks; w/ det annealing]

M, Ohlsson, C Peterson, AL Yuille, Comput Phys Commun 71 (1992) 778
Motivation – evolution

CG Lester, Nucl Instrum Meth A 560 (2006) 621

- Cherenkov-ring identification
  - Metropolis-Hastings sampler with a novel proposal distribution
  - efficiently visits the phase space
  - addition, removal, or alteration

- Can we learn something from hits?
  - allow the solution to evolve
  - our parameter space is 5-dimensional, need to be careful
Inspect after every 5th step: **remove candidates** with too few matched hits; **add new candidates** by redoing hit-chain search and helix finding on unused hits.
Results – a photon-rich event

How to identify conversions?
Result – $e^+ / e^-$ origins

Summary for all events processed
Results – search for photons

Found tracks, plotted in $(\eta, \phi_0, r_c)$ space
Results – search for photons

Found tracks, plotted in $(\eta, \phi_0, r_c)$ space

Close positives and negatives; if the conversion is far $\rightarrow$ converted photon candidate
Results – photon candidates

Only candidates of $e^+e^-$ pairs are shown
Performance – charged particles

Tests with thousand minimum bias pp events (Pythia8) at $\sqrt{s} = 13$ TeV
Initial look, a starting point for optimization
Summary and outlook

- Novel combination of data analysis techniques
  - for primary charged particles + daughters of displaced vertices
    (mostly photon conversions, but also decays and interactions)
  - hits of adjacent tracking layers are clustered,
    mutual nearest neighbor search in angular distance
  - $k$-me(di)ans clustering,
    alternating between the hit-to-track assignment and the track-fit update steps
  - through the global covariance of the measured hits,
    with the possibility of adding new track hypotheses or removing unnecessary ones
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- Plans
  - now a proof of concept $\rightarrow$ optimize performance
  - apply to real collision data, do physics with photons

Thanks for your attention!