Another approach to track reconstruction: cluster analysis

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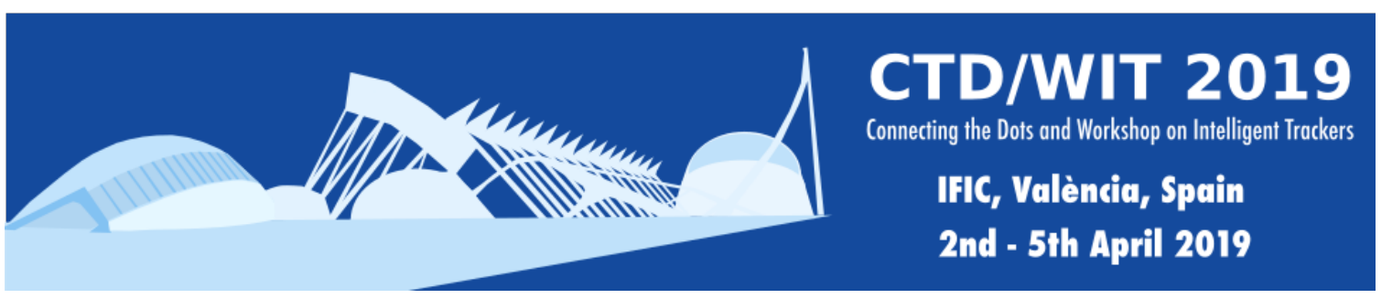
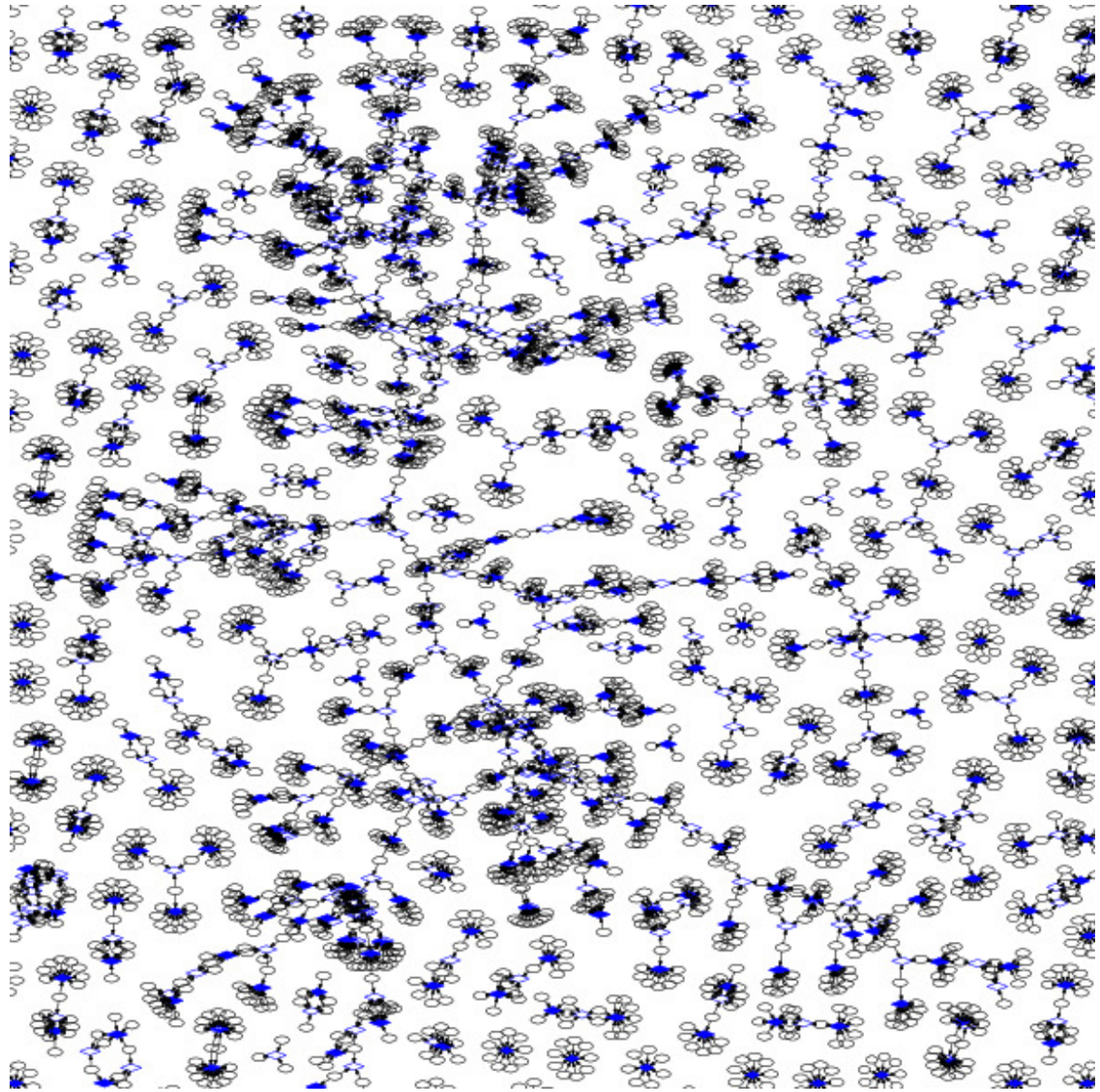


Valencia, 5 Apr 2019

Last time, CTD/WIT17 – disconnecting the graph



F Siklér, "Combination of analysis techniques for efficient track reconstruction in high multiplicity events", Eur Phys J A **54** (2018) 113

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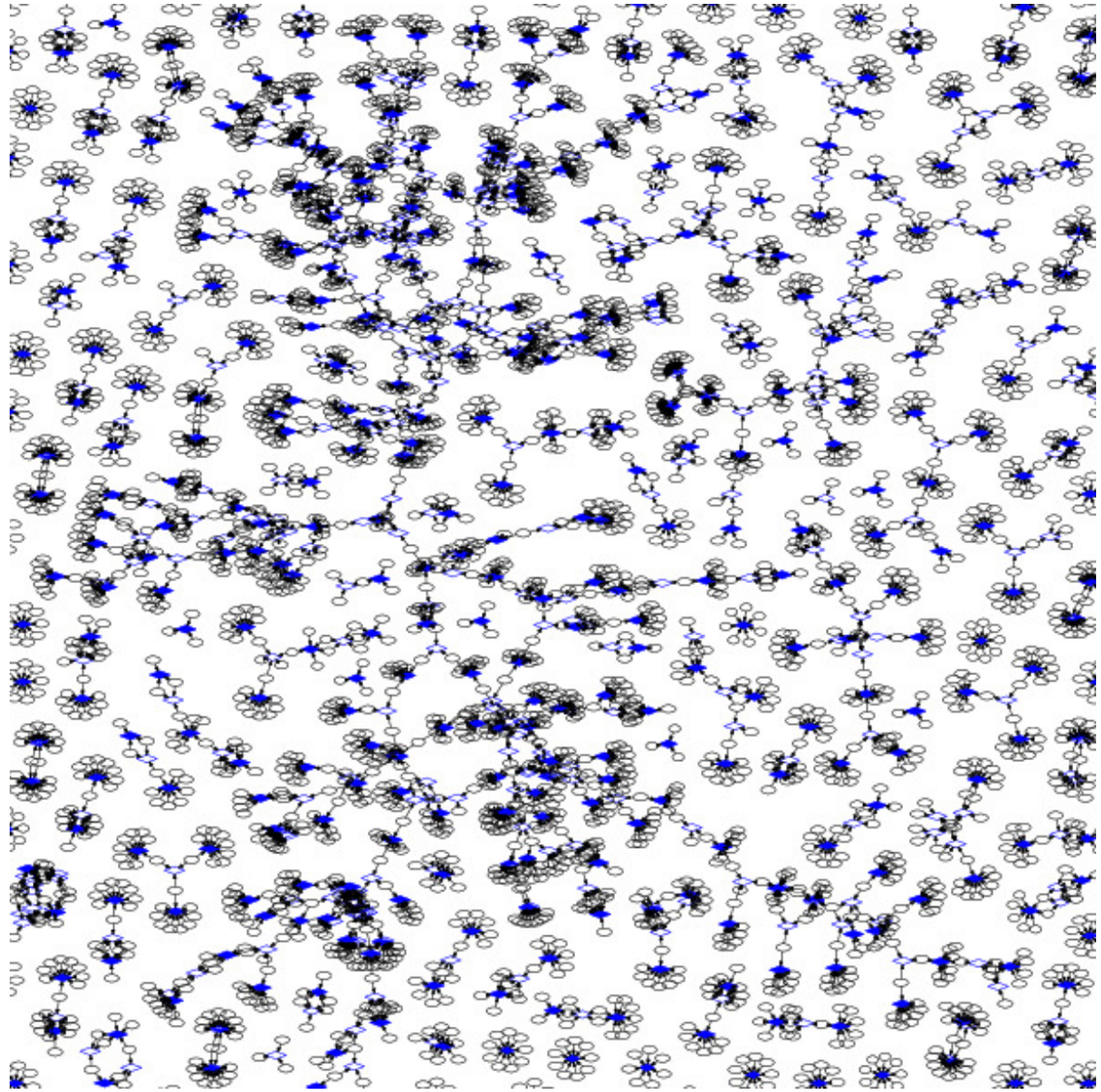

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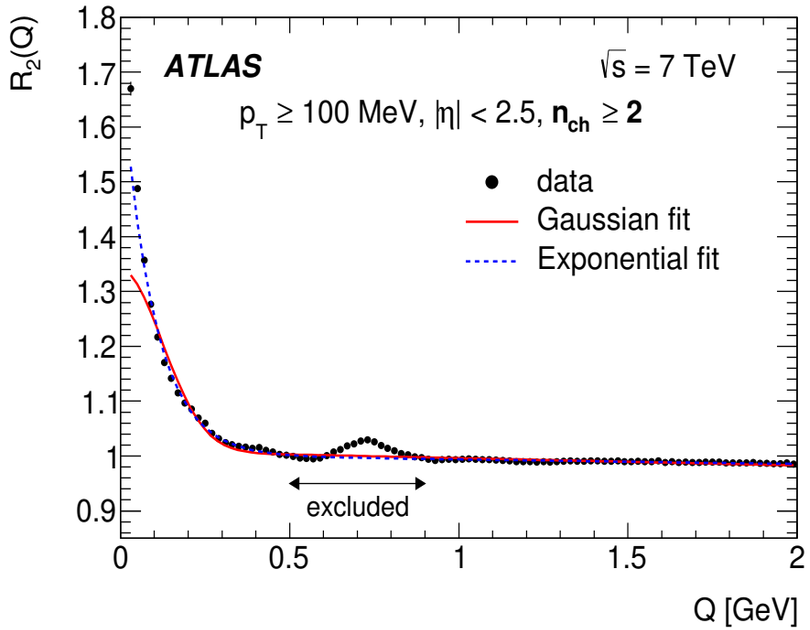
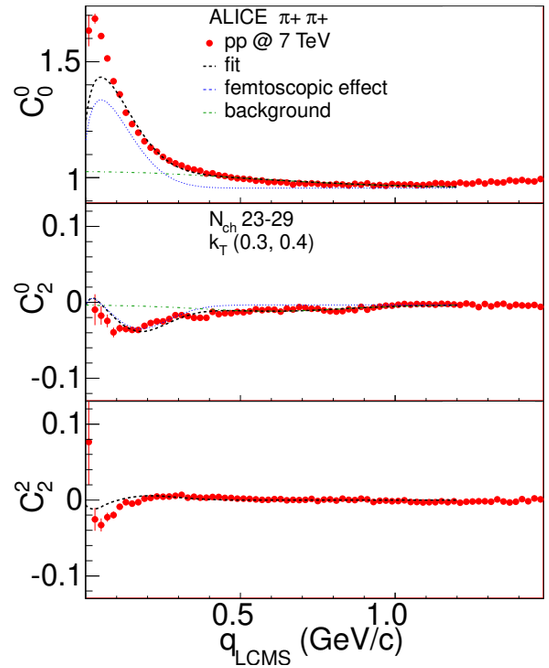
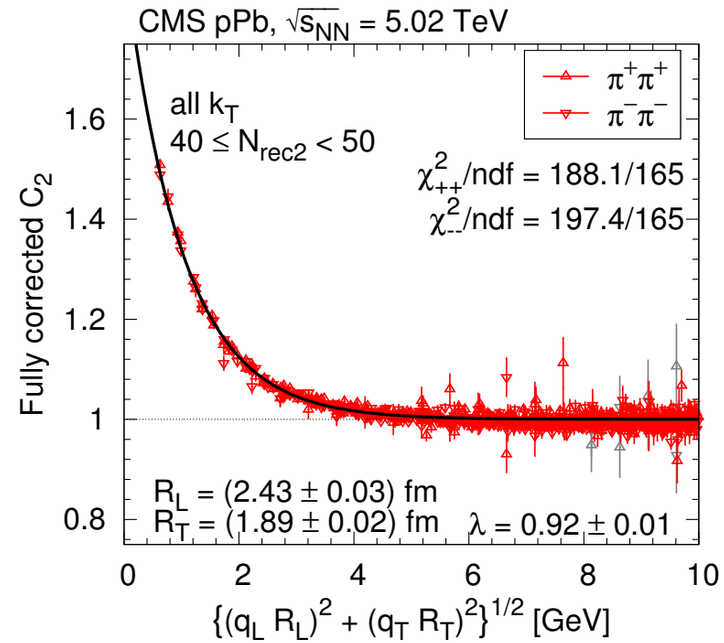
Motivation – quantum-correlations



ATLAS Coll, Eur Phys J C **75** (2015) 466



ALICE Coll, Phys Rev D **84** (2011) 112004

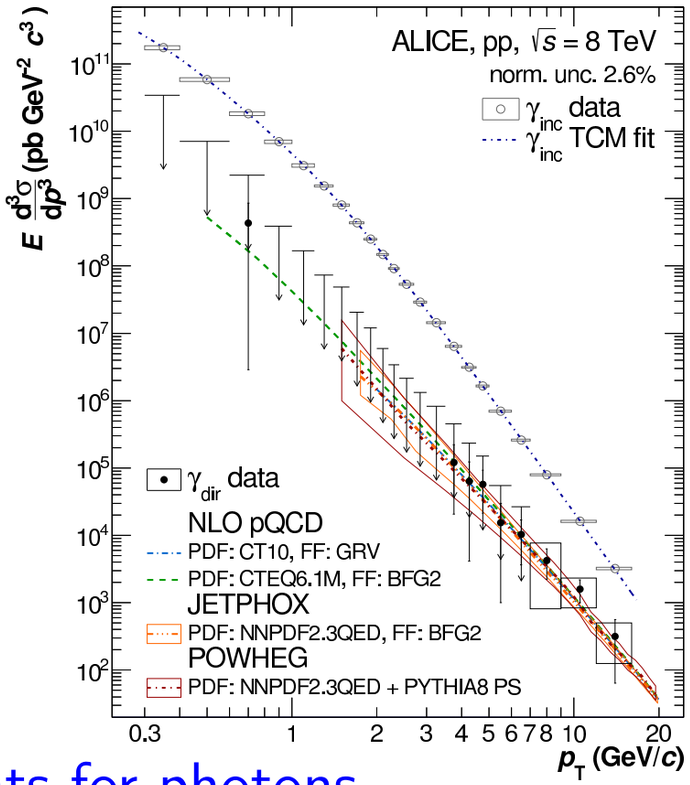


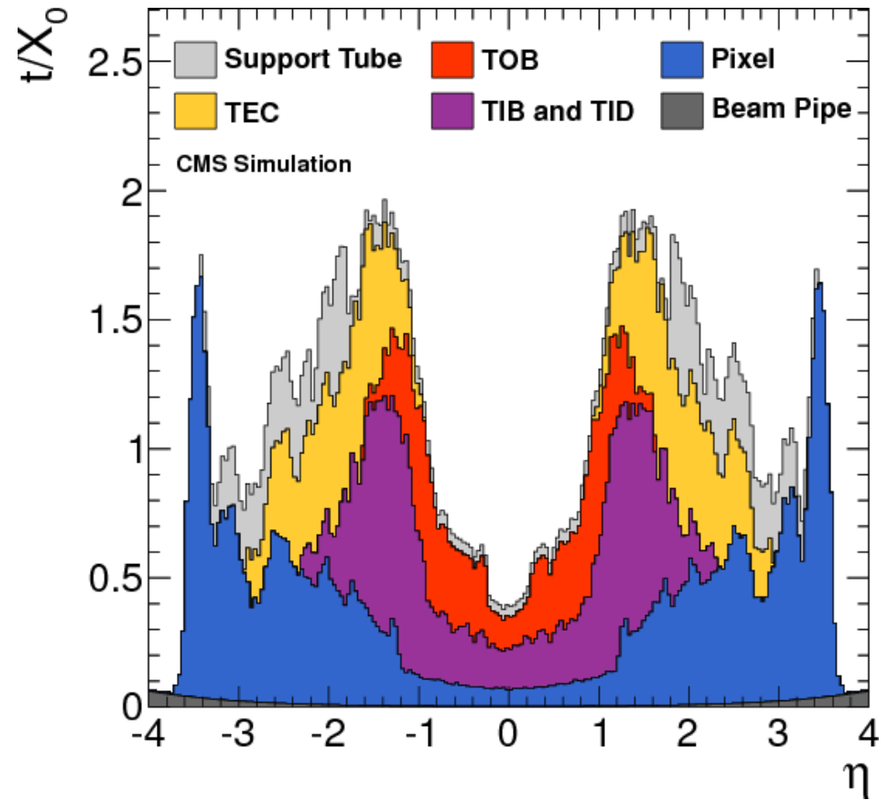
CMS Coll, Phys Rev C **97** (2018) 064912

- Bose-Einstein correlations
  - interesting quantum physical background; sensitive to source size and shape
  - usually explored by electrically charged and strongly interacting bosons ($\pi^{\pm}$, $\mathrm{K}^{\pm}$)
  - drawbacks: have to correct for FSI (Coulomb and strong); also jet-related background

Let's try $\gamma\gamma$ **correlations**! No charge, no strong, reach down to $\mathbf{q} = 0$

Motivation – photon reconstruction using silicon trackers

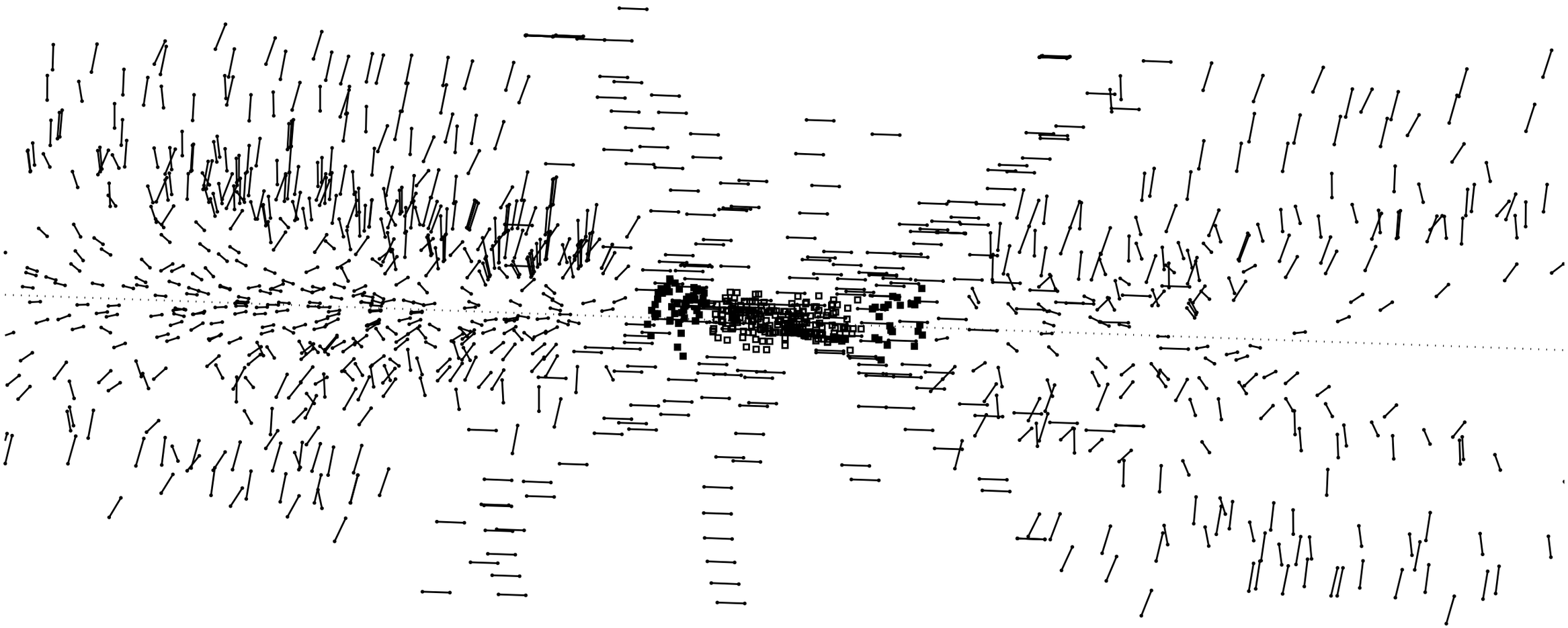




- Requirements for photons
  - it is a correlation measurement, we want **pairs of photons**: high efficiency!
  - photons are **at low momentum**: calorimetry alone not enough
  - high chance of conversion! for $e^{\pm}$ calo-based seeding won't work; **highly curved** tracks

Task: reconstruction of primaries and **detached** photon conversions, offline

Track reconstruction with silicon detectors – pixels and strips

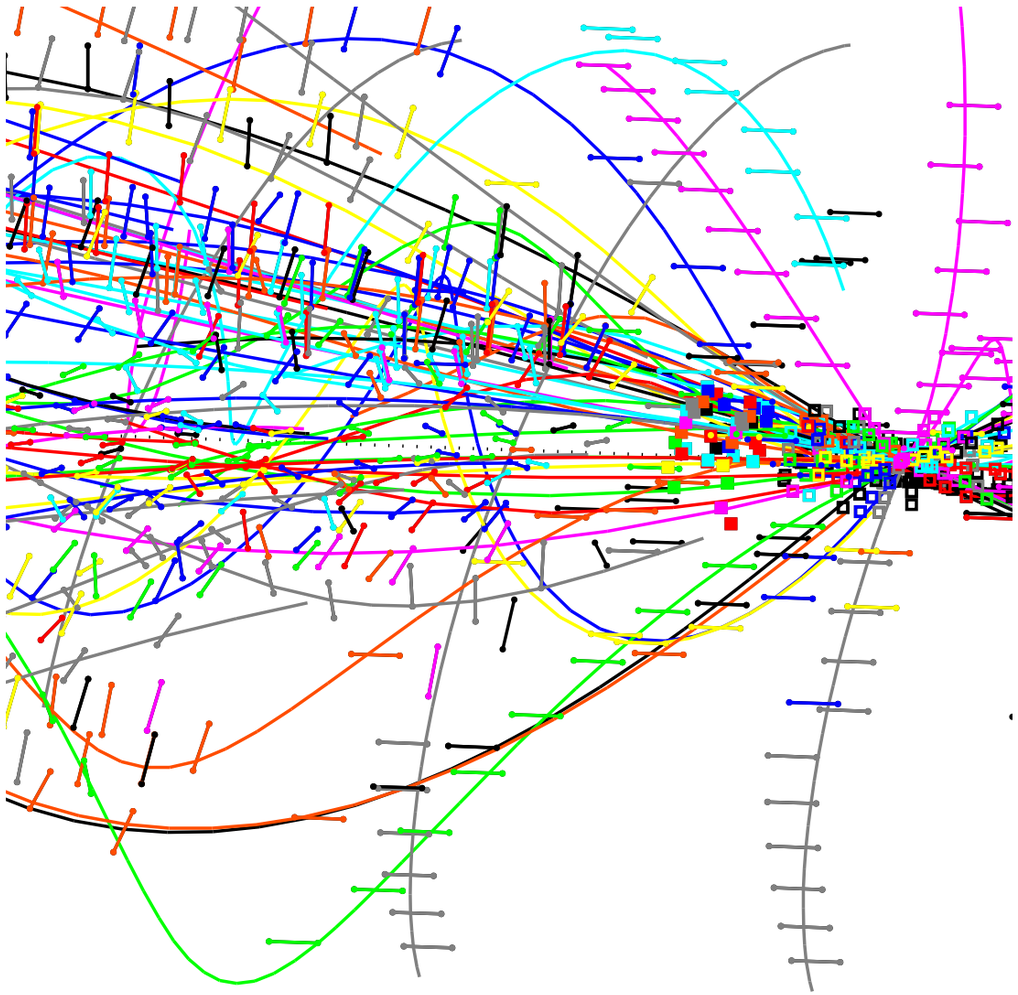


Name of the game: put (most of) the hits on a limited number of trajectories

How to do it? Default solution: combinatorial track finder via Kalman filter

R E Kalman, Journal of Basic Engineering **82** (1960) 35, R Frühwirth, Nucl Instrum Meth A **262** (1987) 444

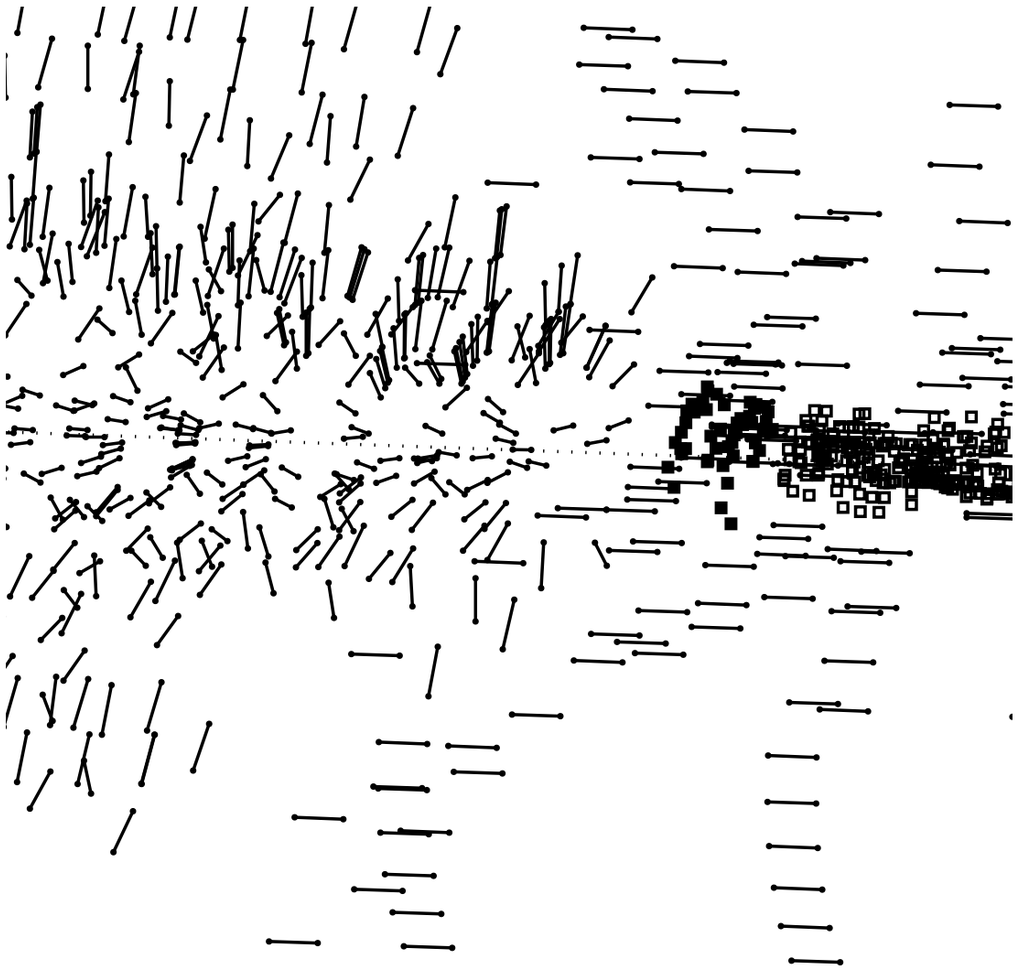
Motivation – personal



- Looking for new combinations
  - no classical trajectory building
  - no image-transformation-type approach
  - must be simple
- Clustering methods
  - EM algorithms vs mixture models?
  - k-means or k-medians (for robustness)
  - hits as objects, tracks as clusters
  - mobile hit-to-track assoc, minimize sum-χ^2
- Evolution
  - allow the solution to evolve
  - from Metropolis-Hastings MCMC
  - adding and removing track candidates

Efficient use of both local and global information

Details: detector model



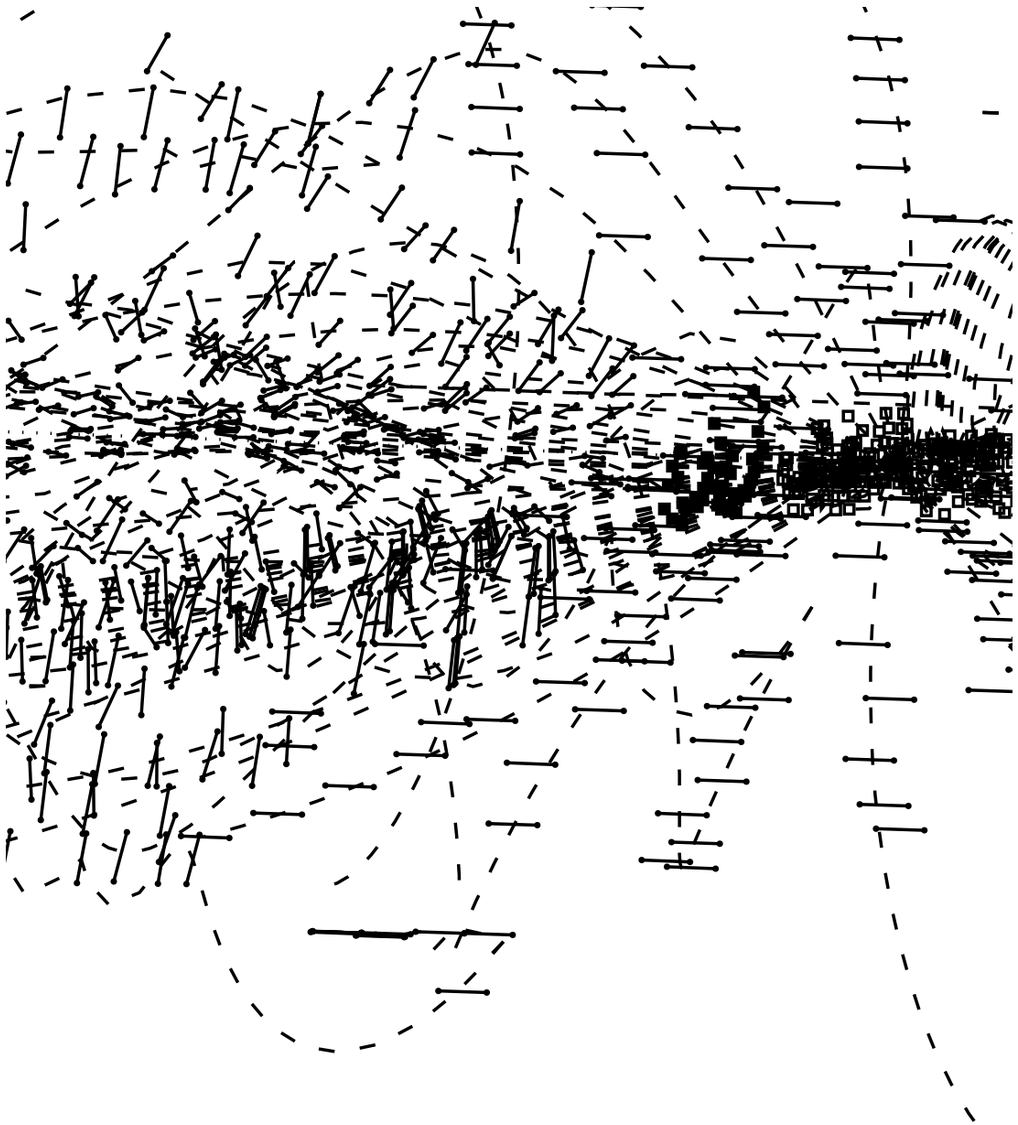
- Silicon layers

Barrel	r [cm]	z_{max} [cm]
pixels	4, 7, 10	25
strips	20, 30, 40, 50	55
strips	60, 70, 80, 90, 100, 110	55
Endcap	z [cm]	r_{min}–r_{max} [cm]
pixels	35, 45	5–15
strips	75, 90, 105	20–50
strips	125, 140, 155	20–110
strips	170, 185, 200	30–110
strips	220, 245	40–110
strips	270	50–110

- Other data
  - axial magn field B = 3 T; hit effic $\varepsilon_{\text{eff}} = 0.98$
  - material $x/X_0 = 2\%$ (pixels), 5% (strips)
  - strip length 10 cm, pos resolution 50 μm

Barrel and endcap

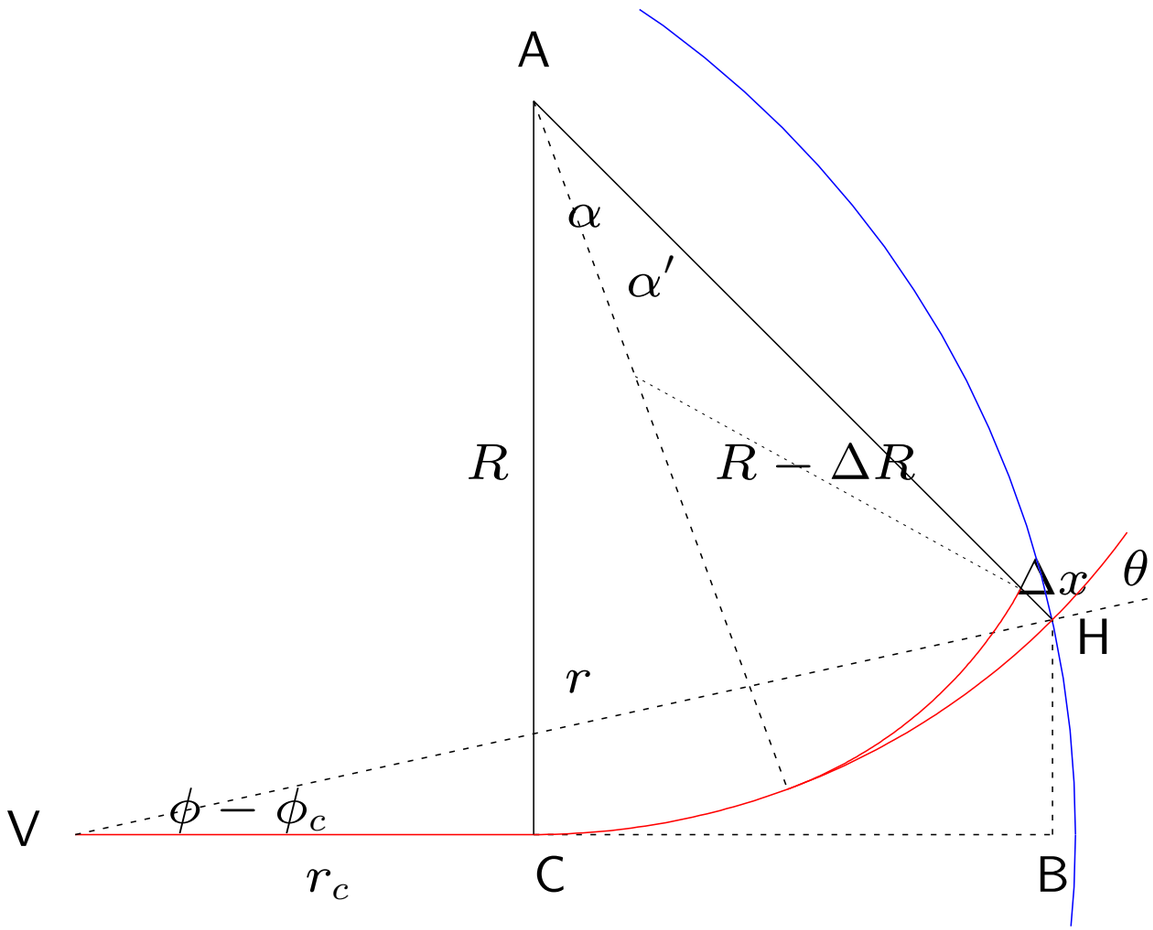
Details: known physics effects



- Effects modelled
  - Multiple scattering (charged)
    $\theta_0 = \frac{13.6\ \text{MeV}}{\beta c p} q \sqrt{x/X_0}\left[1 + 0.038 \ln(x/X_0)\right]$
  - Energy loss (charged)
    $\Delta_p = \xi \left[\ln \frac{2mc^2\beta^2\gamma^2\xi}{I^2} + 0.2000 - \beta^2 - \delta\right]$
    $\Gamma_\Delta = 4.018\xi$, where $\xi = \frac{K}{2} z^2 \frac{Z}{A} \rho \frac{x}{\beta^2}$
  - Bremsstrahlung ($\mathrm{e}^-/\mathrm{e}^+$)
    $\frac{d\sigma}{dk} = \frac{A}{X_0 N_A k}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$, with $y = \frac{k}{E}$
    (or as $\frac{dE}{dx} = -\frac{E}{X_0}$ in reconstruction)
  - Photon conversion (γ)
    conversion length $9/7 X_0$
    $\frac{d\sigma}{dx} = \frac{A}{X_0 N_A}\left[1 - \frac{4}{3}x(1-x)\right]$, where $x = \frac{E}{k}$

Compact standalone code, c++

Details: track model and variances



- Basics
  - track parameters
    $$(\eta, q/p_{\mathrm{T}}, \phi_0, z_0, r_c)$$
  - homogeneous axial magnetic field
- Variances of known physics effects
  - multiple scattering
    $$\sigma_x^2 \propto \frac{\alpha^3 p_{\mathrm{T}} \cosh \eta}{\beta^2}$$
  - energy loss
    $$\Delta x \propto \frac{\alpha^3 p_{\mathrm{T}}}{\beta^3}$$
  - bremsstrahlung

Produce candidate hits for each candidate track, then match to measured hits

Variances calculable, some nice maths behind

Details: global covariance?

Hit positions are highly correlated between neighboring layers

Covariance between hit positions decays roughly as $\sim \rho^{-|i-j|}$, $\rho \approx 0.8 - 0.9$

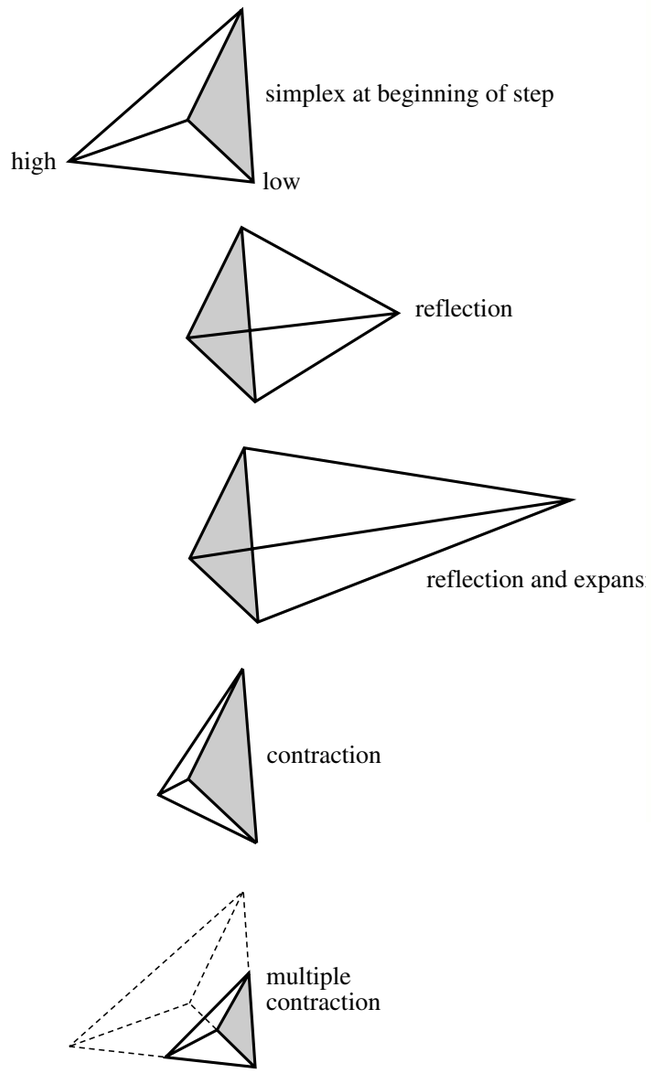
$$V^{-1} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 & \rho^2\sigma_1\sigma_4 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho\sigma_2\sigma_4 \\ \rho^2\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ \rho^3\sigma_1\sigma_4 & \rho^2\sigma_2\sigma_4 & \rho\sigma_3\sigma_4 & \sigma_4^2 \end{pmatrix}^{-1} =$$

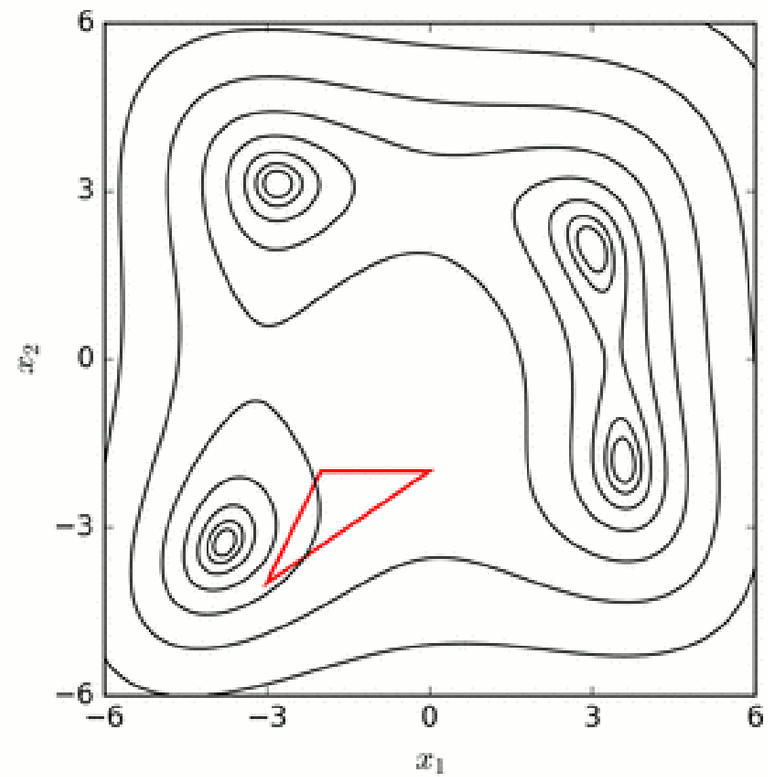
$$= \frac{1}{1-\rho^2} \begin{pmatrix} 1/\sigma_1^2 & -\rho/(\sigma_1\sigma_2) & 0 & 0 \\ -\rho/(\sigma_1\sigma_2) & (1+\rho^2)/\sigma_2^2 & -\rho/(\sigma_2\sigma_3) & 0 \\ 0 & -\rho/(\sigma_2\sigma_3) & (1+\rho^2)/\sigma_3^2 & -\rho/(\sigma_3\sigma_4) \\ 0 & 0 & -\rho/(\sigma_3\sigma_4) & 1/\sigma_4^2 \end{pmatrix} \quad (1)$$

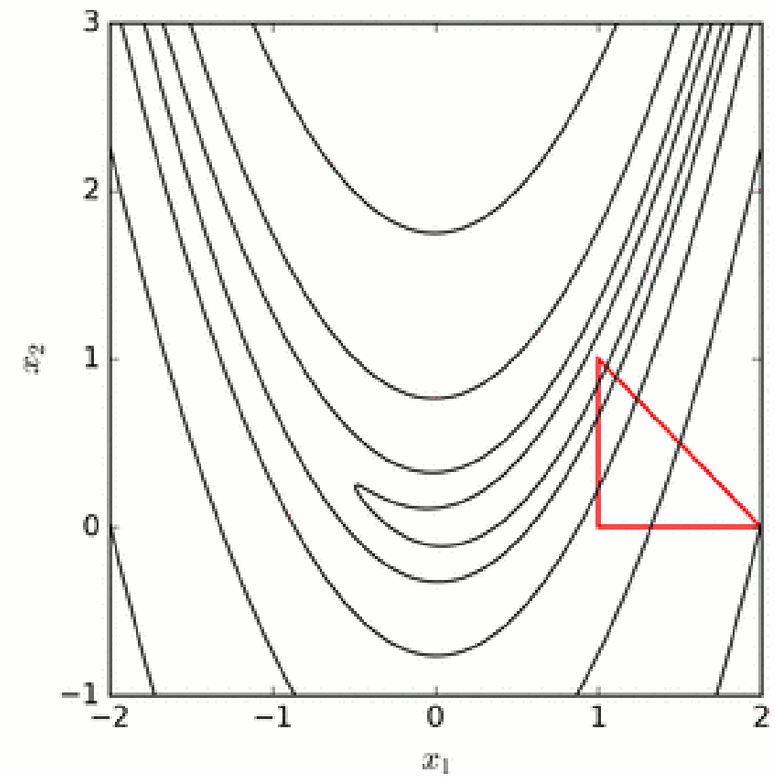
Thus, the inverse of the covariance matrix is close to tridiagonal

$\sum_{\text{all hits}} x^T V^{-1} x$

Details: how to fit tracks?







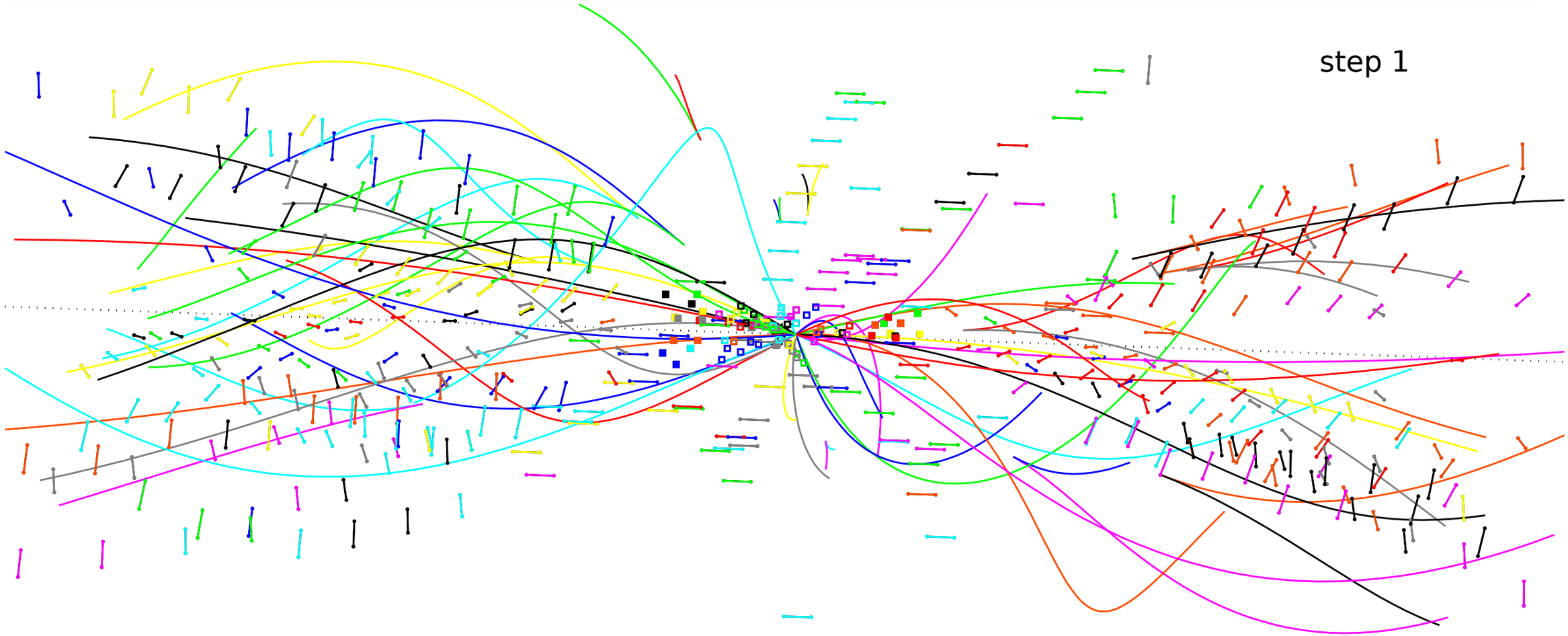
Downhill simplex method

JA Nelder and R Mead, Computer Journal **7** (1965) 308

Robust: derivatives are not used, only evaluations at vertices

Merit function is flexible, promoting increase of r_c for conversion

k-me(di)ans clustering – random initial condition

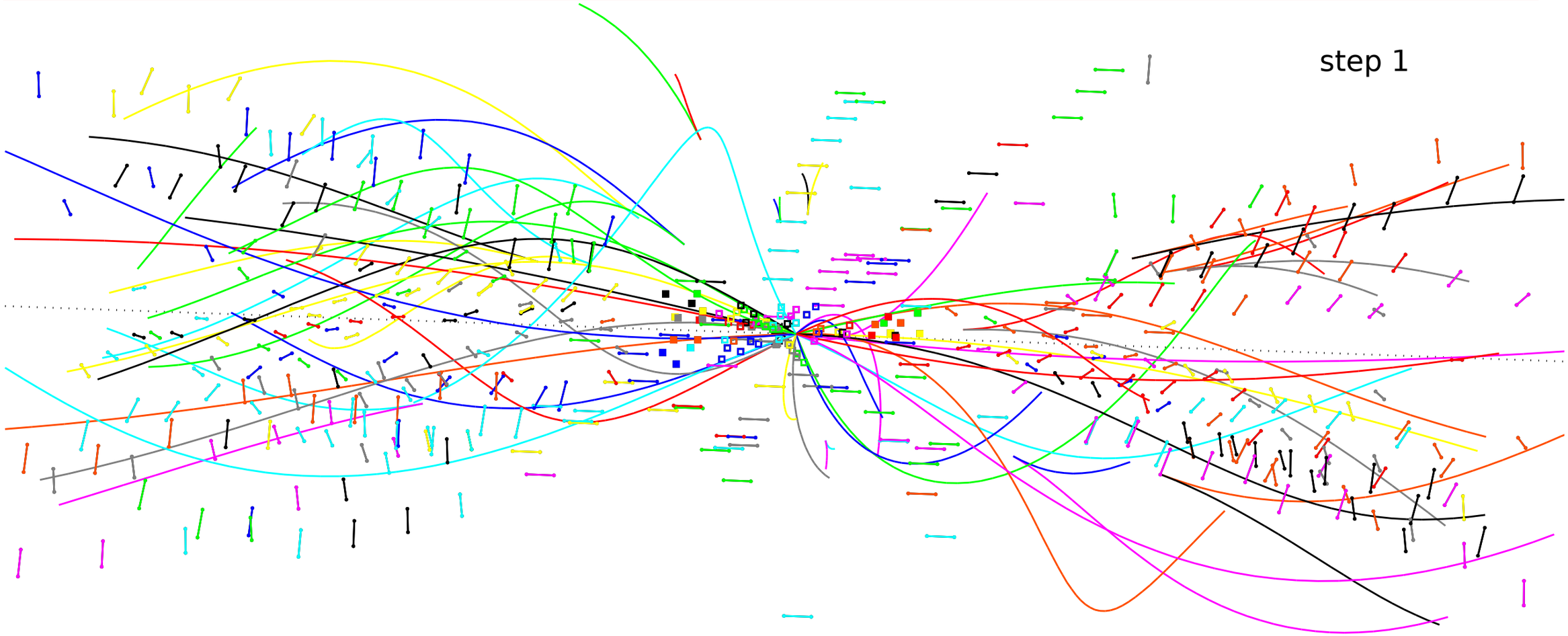


Step 1: assign (and relocate) each **hit to the closest track** (lowest χ^2, highest p-value)

Step 2: **update track parameters** by refitting to its associated hits (via global covariance)

Alternate between these steps, until convergence

k-me(di)ans clustering – steps

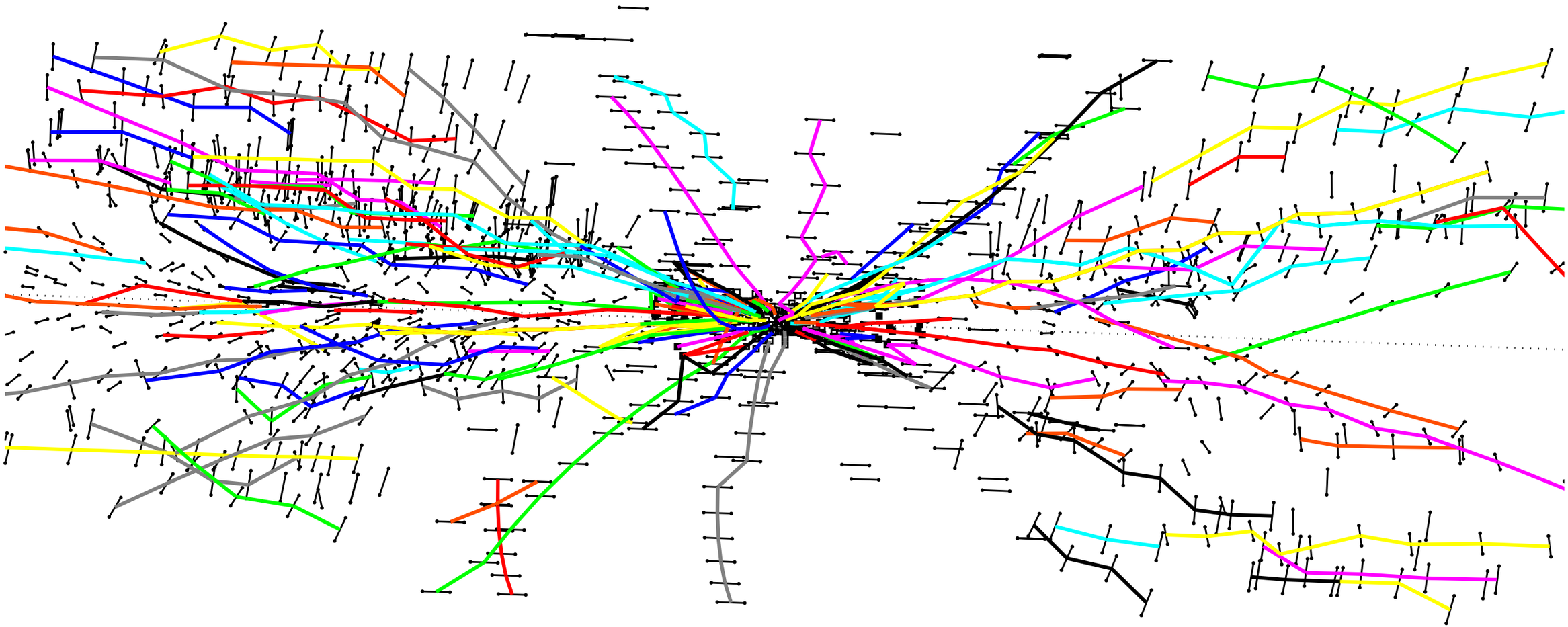


Goodness-of-fit? Problem with outlier hits, also several local minima

Use hit-to-track distances ("medians")? **Soften with Cauchy-motivated** $\tan^{-1}(\beta x)/\beta$

Punish non-matched candidate hits by $-2\log(1-\varepsilon_{\text{eff}})$, and measured hits by $-2\log p_{\text{noise}}$

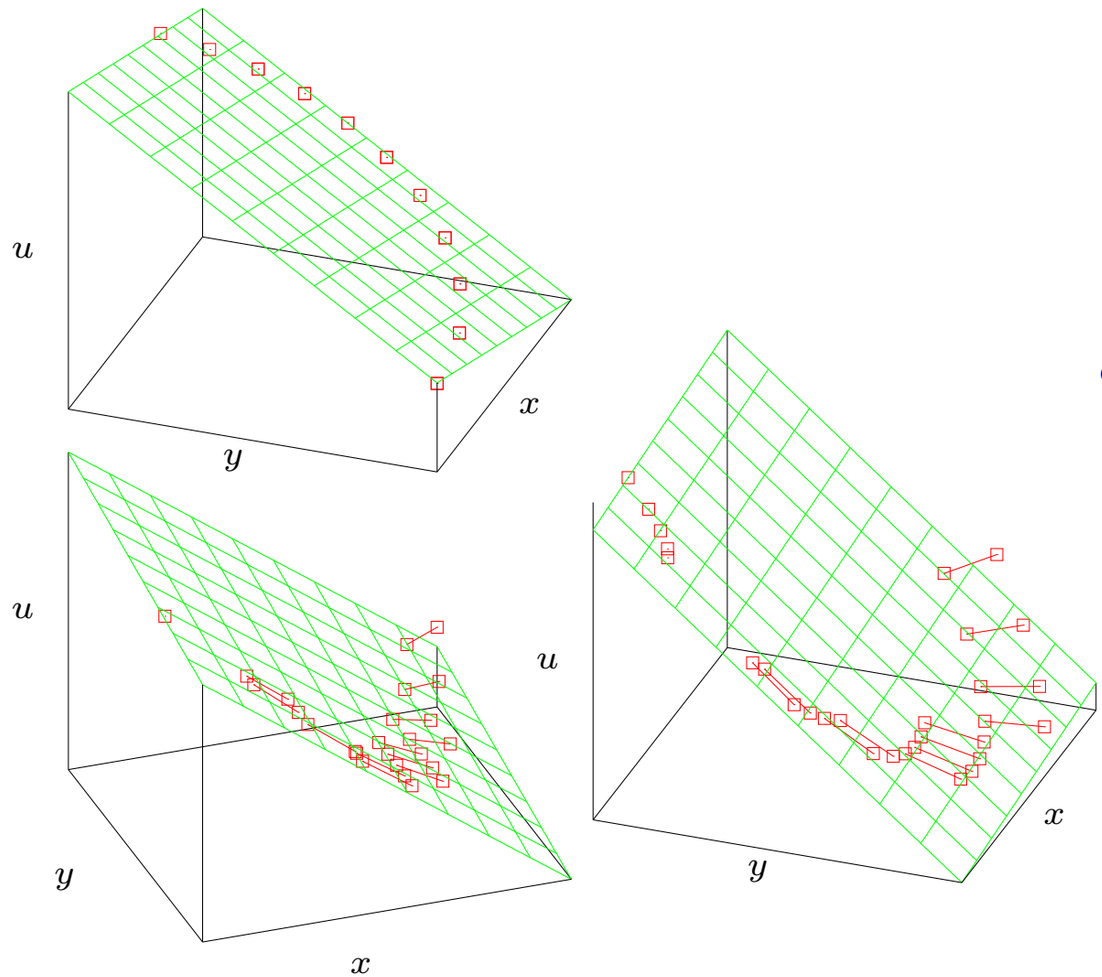
k-me(di)ans: better guess for initial clusters? how many are there?



Mutual nearest neighbor search in angular distance wrt (0,0,0)

Take chains of connected hits as initial clusters

Details: initial tracks, robust helix fit

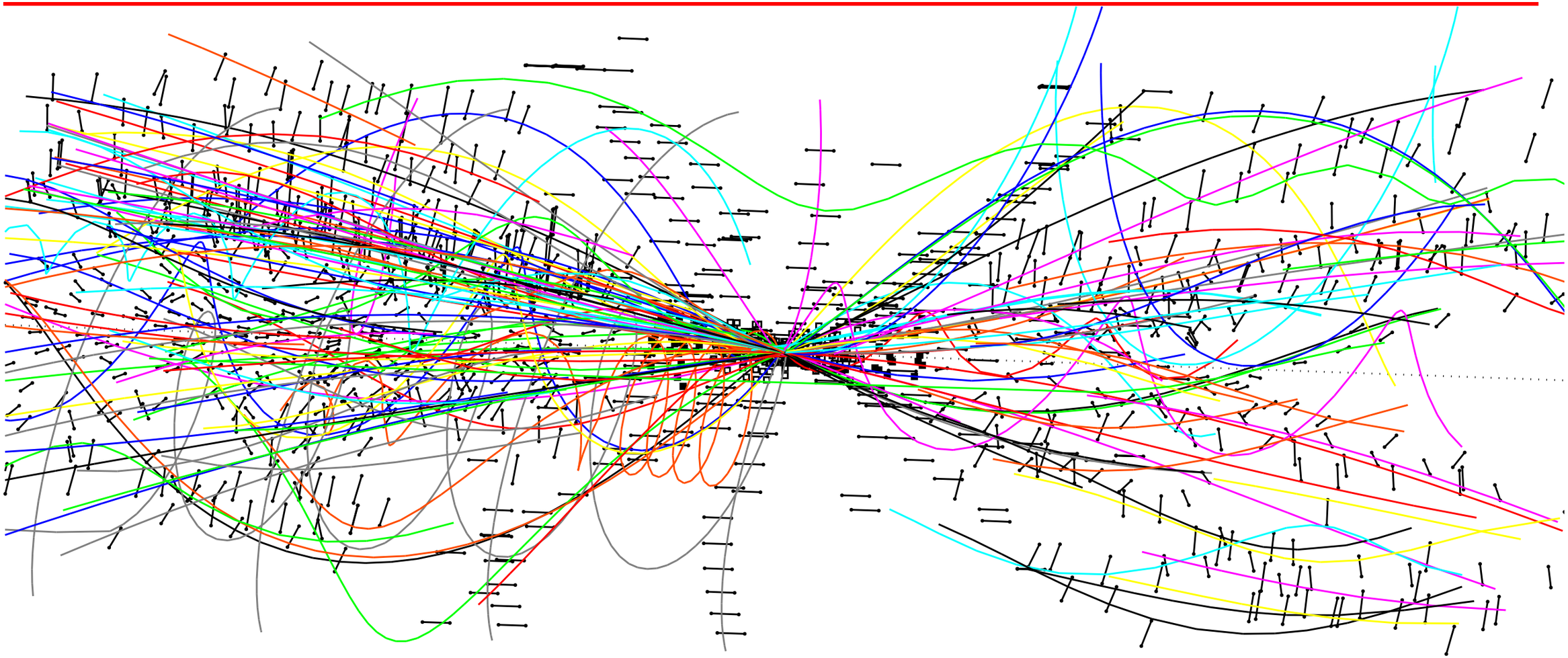


- Find circles in the bending plane
  - center, scale; project to the Riemann sphere
  - planes are fitted to triplet permutations of the projected points

R Frühwirth and A Strandlie, J Phys Conf Ser **1085** (2018) 042004

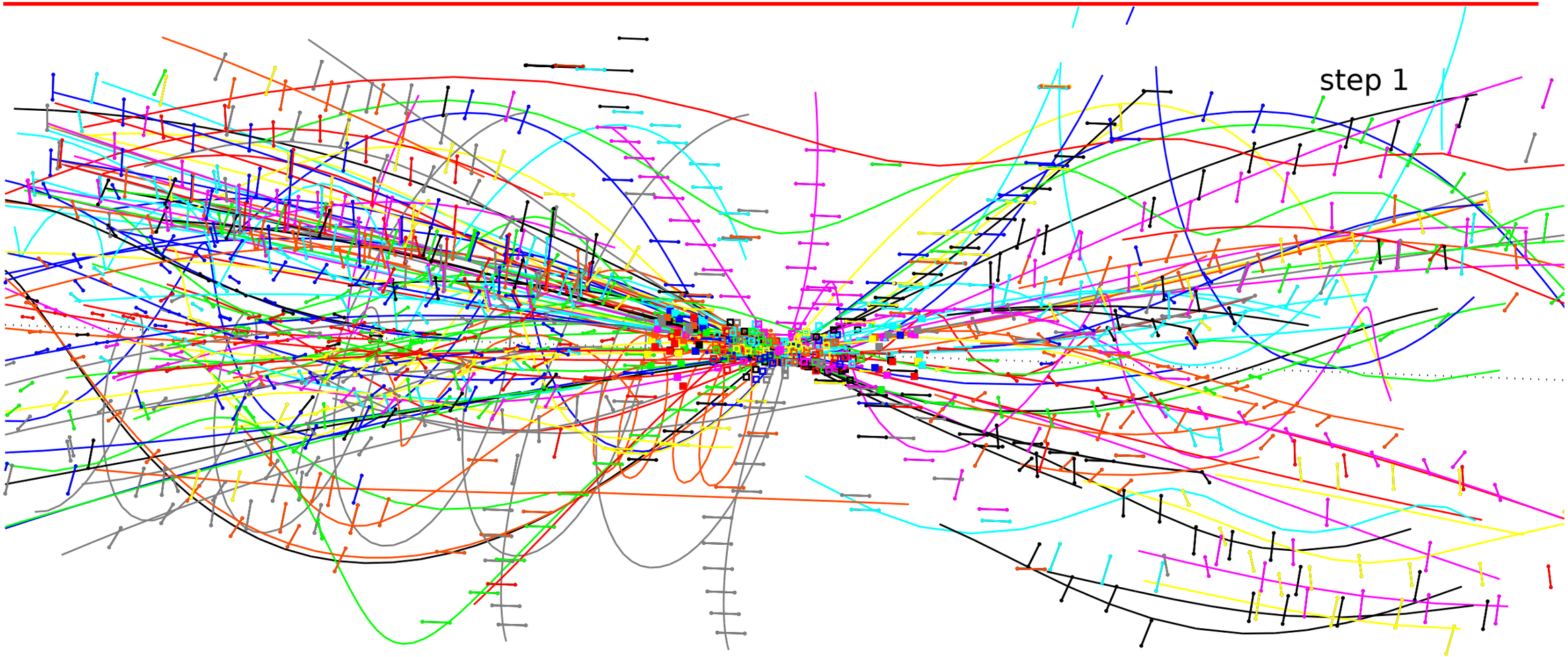
- Additional complications
  - mix of pixels and strips: **points vs segments**
  - use a few (2-4) **internal segment points**
  - take triplets: **ask for** nearly equal number of (end)points on both sides of the plane; and minimize sum of hit-to-plane distances
  - **prefer** circles which do not contain (0,0)
  - **prefer** circles where no hit is closer than the tangent point from (0,0) [for secondaries]
  - then, robust $r\alpha$ vs z linear fit for the helix

k-me(di)ans: initial track candidates



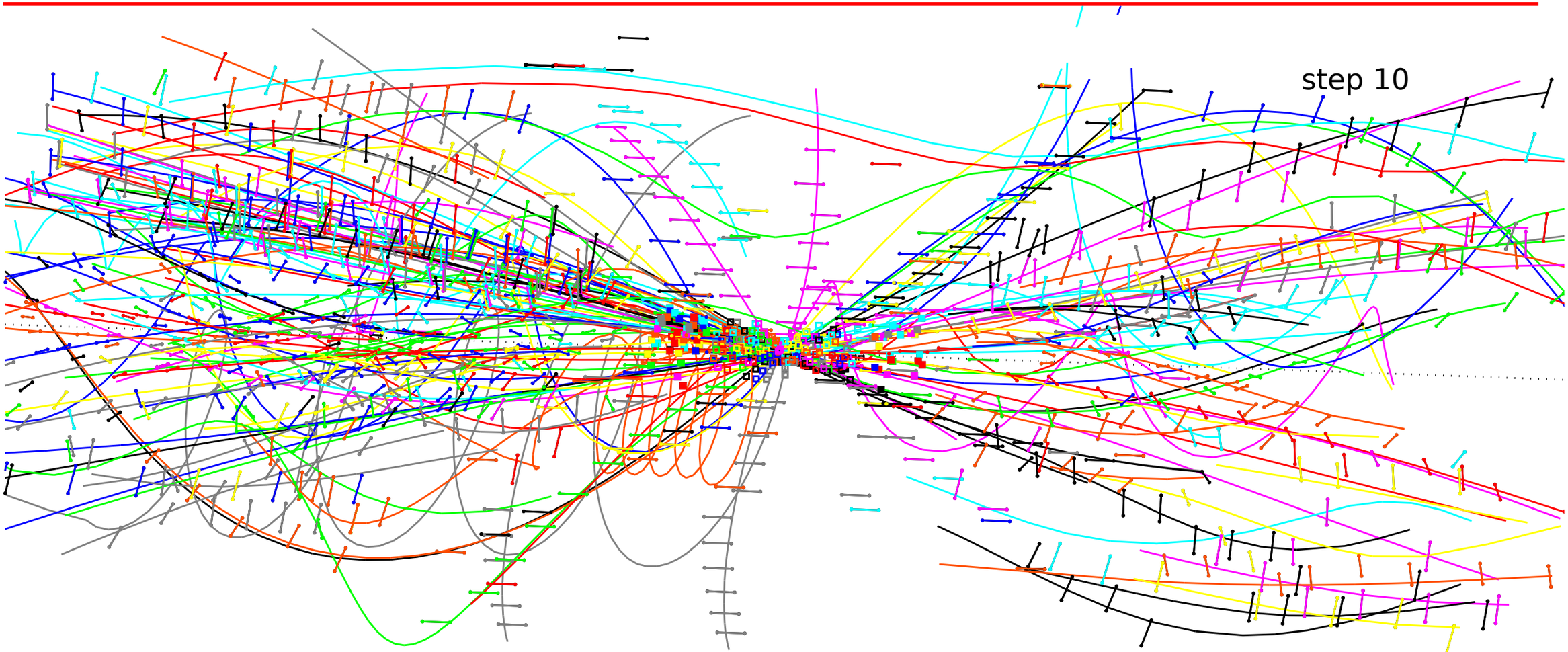
Reasonable starting guesses

k-me(di)ans: steps



Looks good, but can we do better?

k-me(di)ans: steps



[Related to deformable templates, elastic nets or arms?]

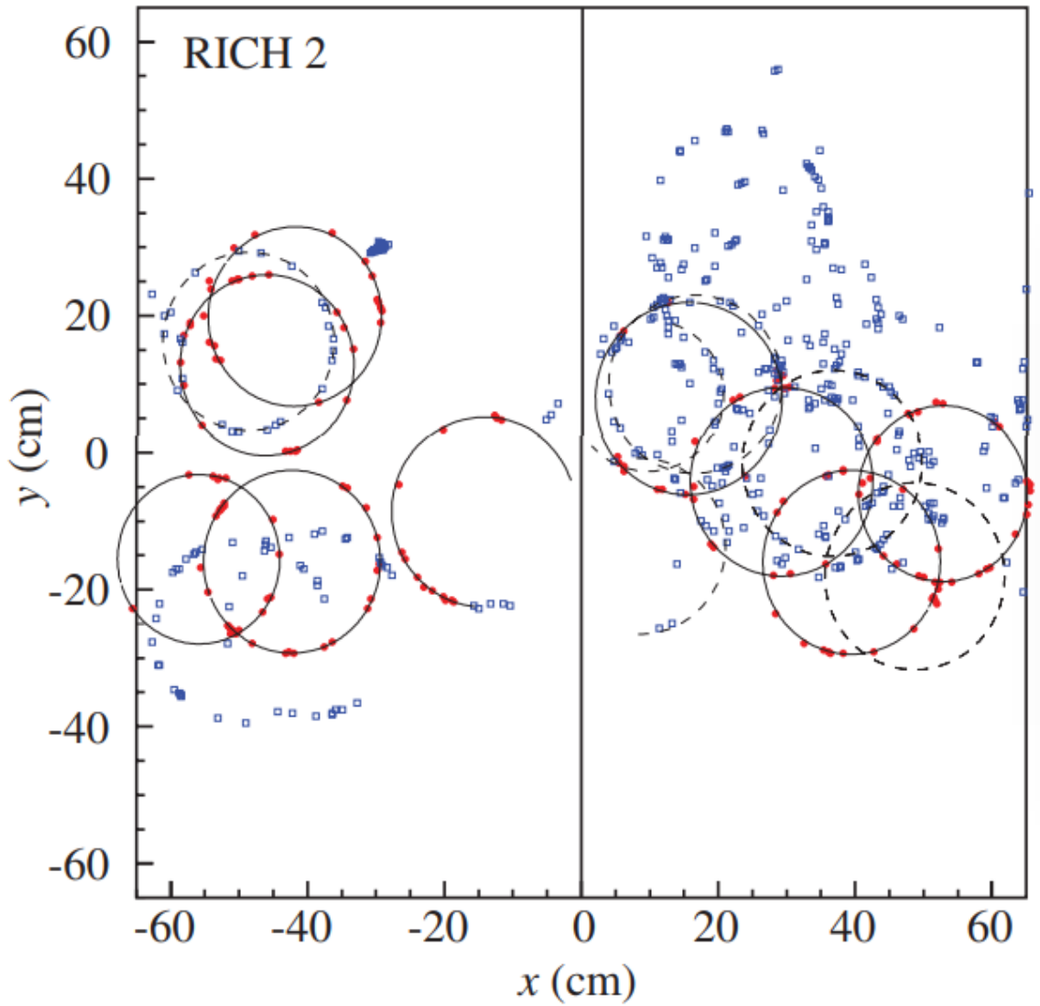
[Hough trans estimates the number of tracks; w/ det annealing]

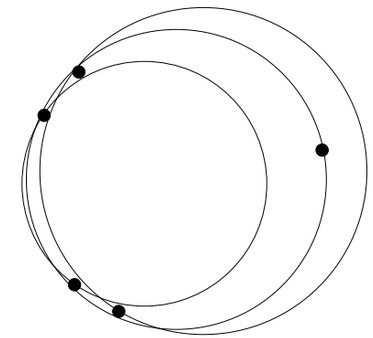
M Gyulassy and H Harlander, Comput Phys Commun **66** (1991) 31

M, Ohlsson, C Peterson, AL Yuille, Comput Phys Commun **71** (1992) 778

Motivation – evolution

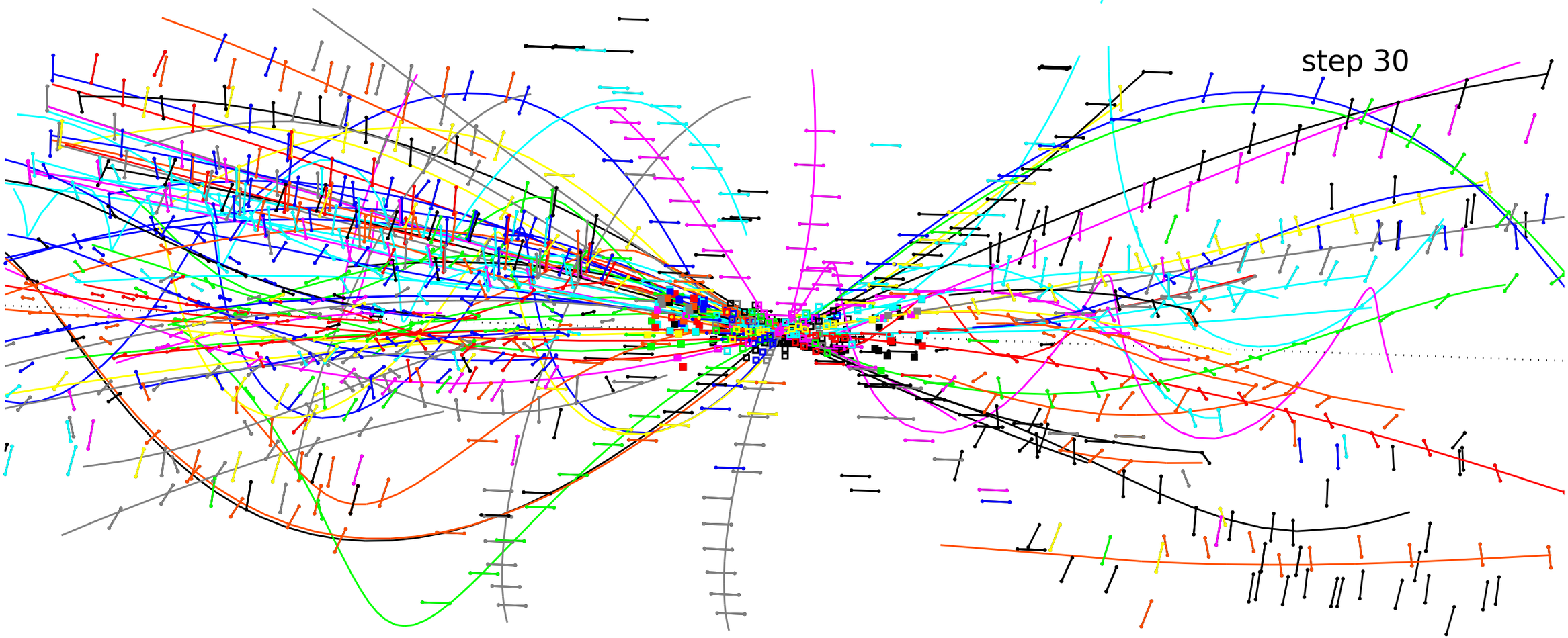
CG Lester, Nucl Instrum Meth A **560** (2006) 621





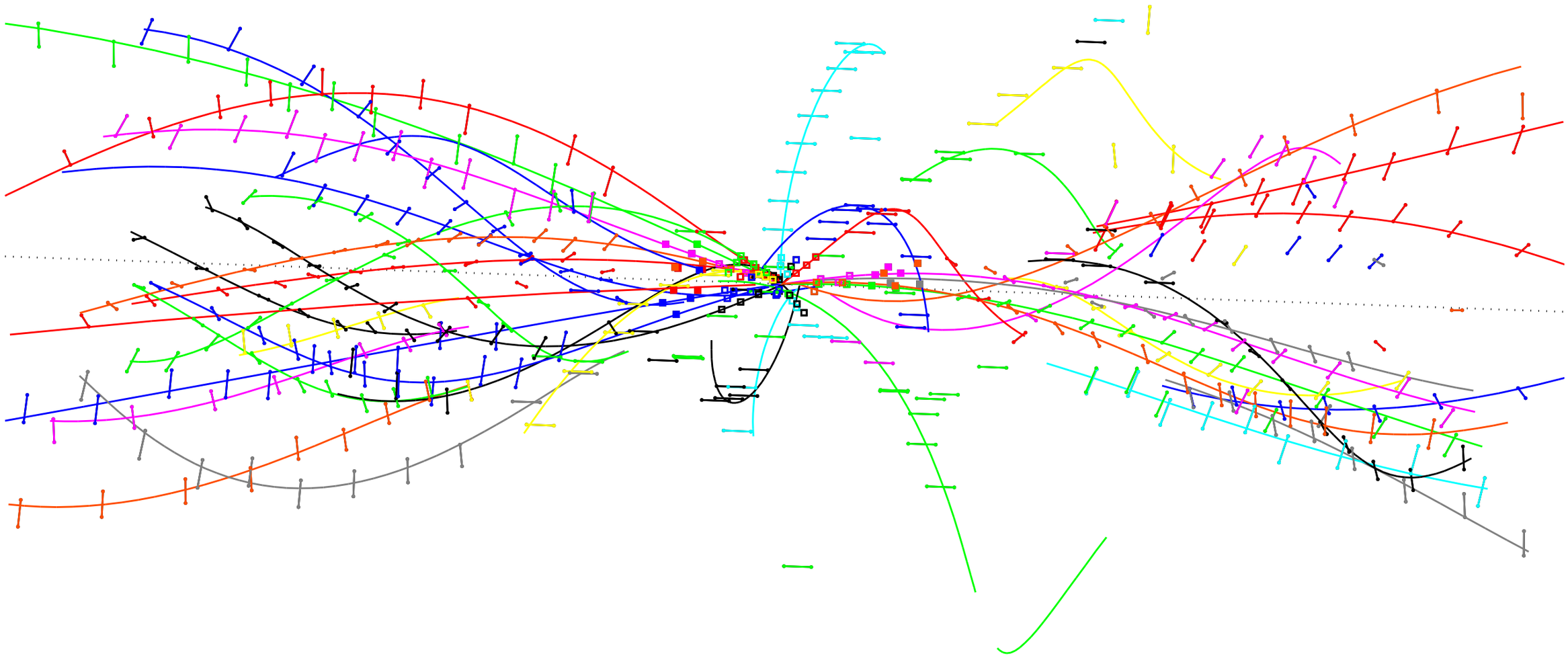
- Cherenkov-ring identification
  - Metropolis-Hastings sampler with a novel proposal distribution
  - efficiently visits the phase space
  - addition, removal, or alteration
- Can we learn something from hits?
  - allow the solution to evolve
  - our parameter space is 5-dimensional, need to be careful

k-me(di)ans: result with evolution



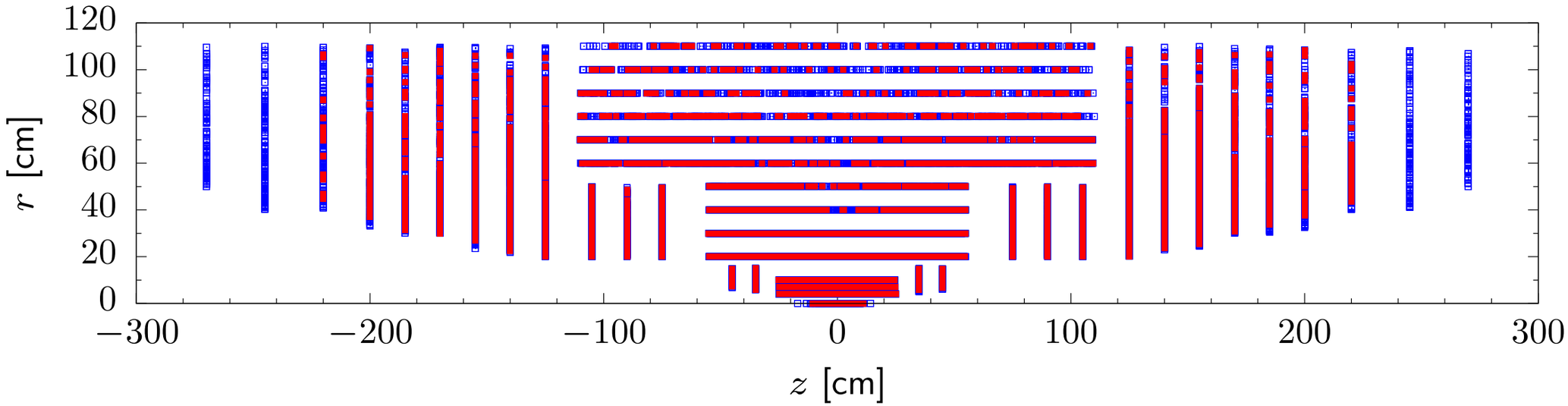
Inspect after every 5th step: **remove candidates** with too few matched hits;
add new candidates by redoing hit-chain search and helix finding on unused hits

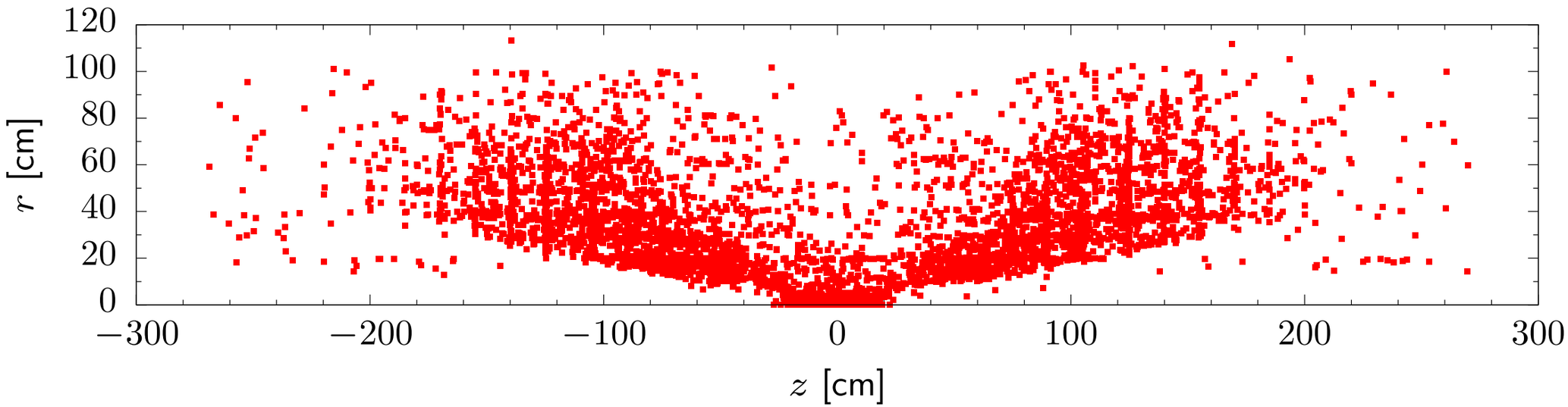
Results – a photon-rich event



How to identify conversions?

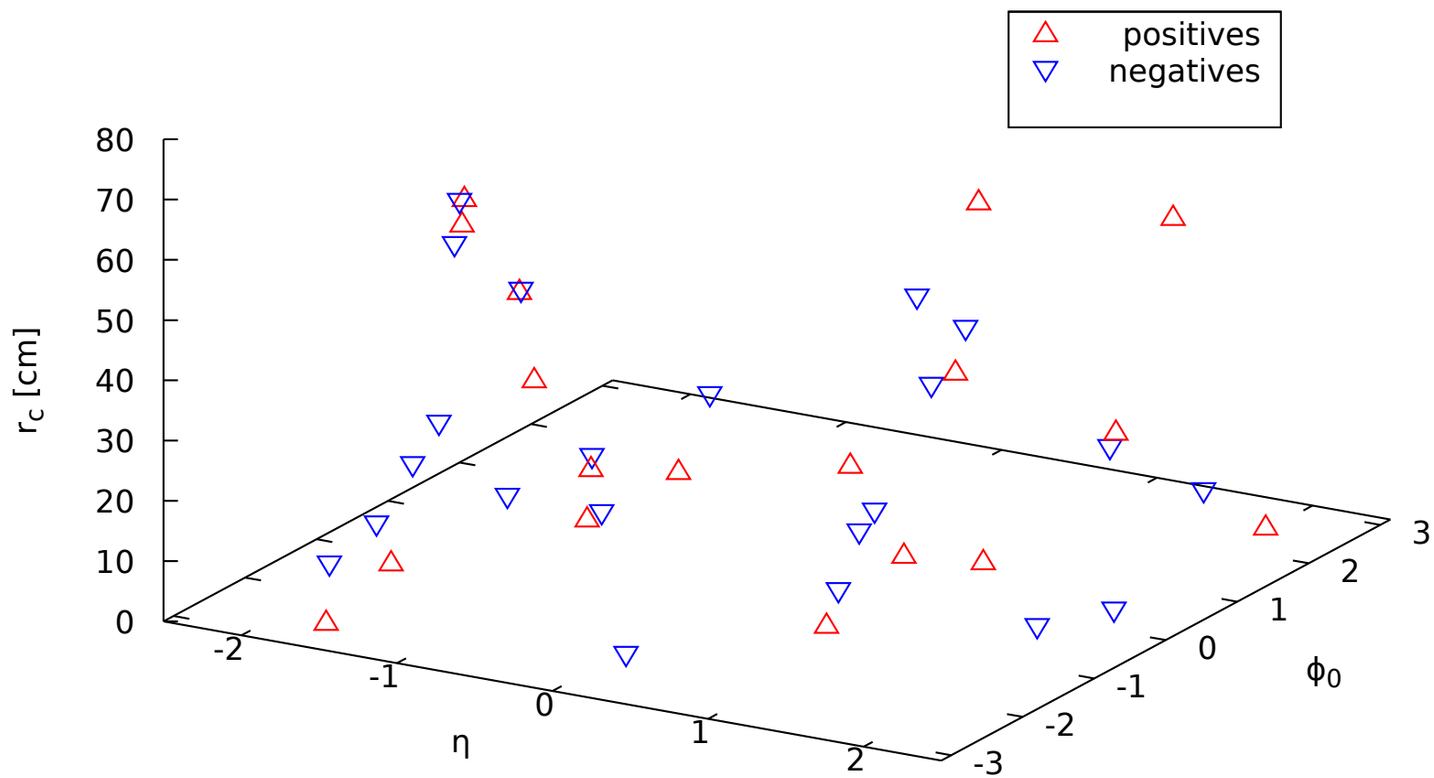
Result – e^+/e^- origins





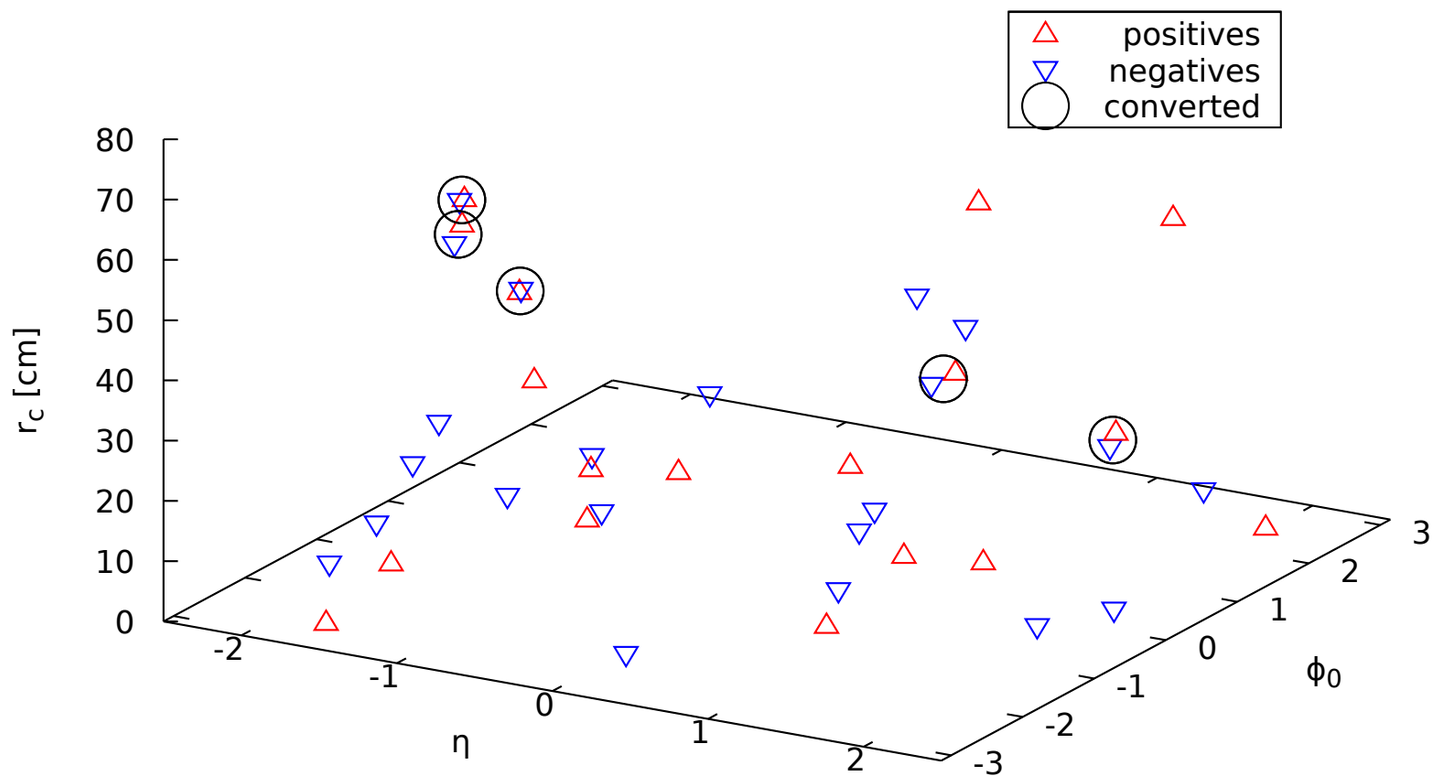
Summary for all events processed

Results – search for photons



Found tracks, plotted in (η, ϕ_0, r_c) space

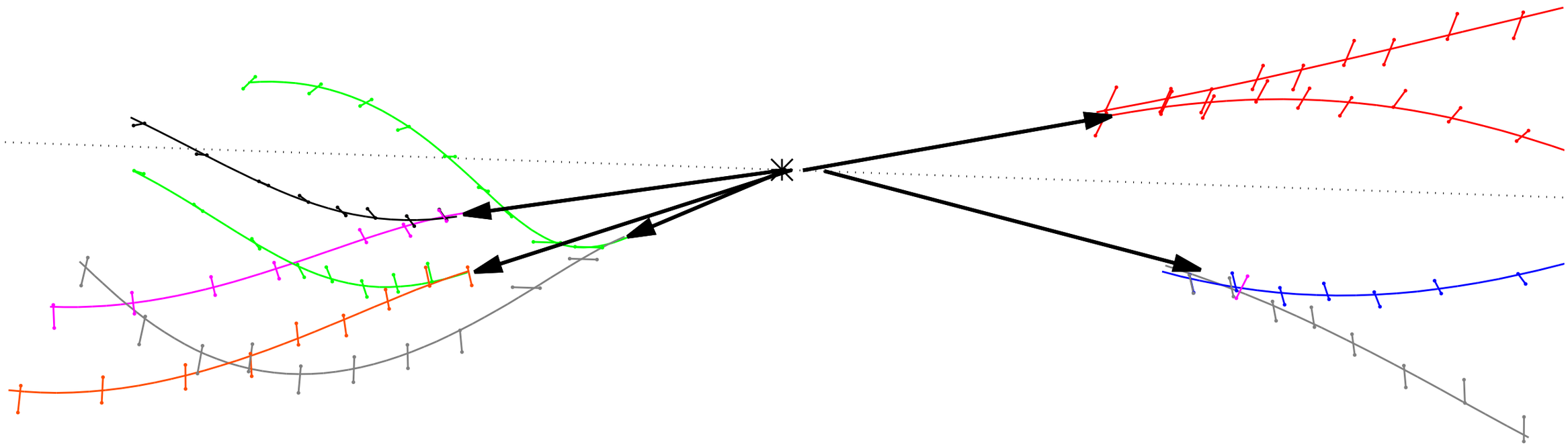
Results – search for photons



Found tracks, plotted in (η, ϕ_0, r_c) space

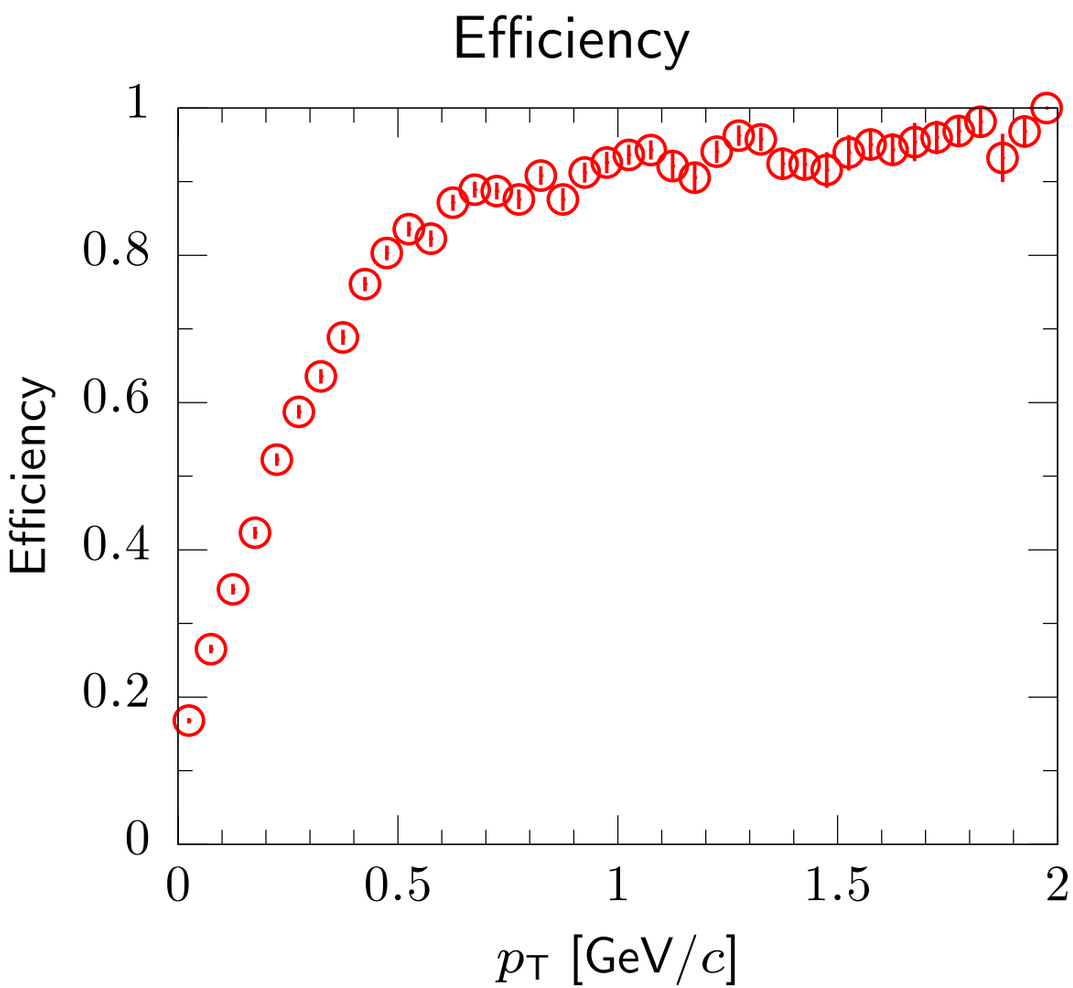
Close positives and negatives; if the conversion is far $\rightarrow$ converted photon candidate

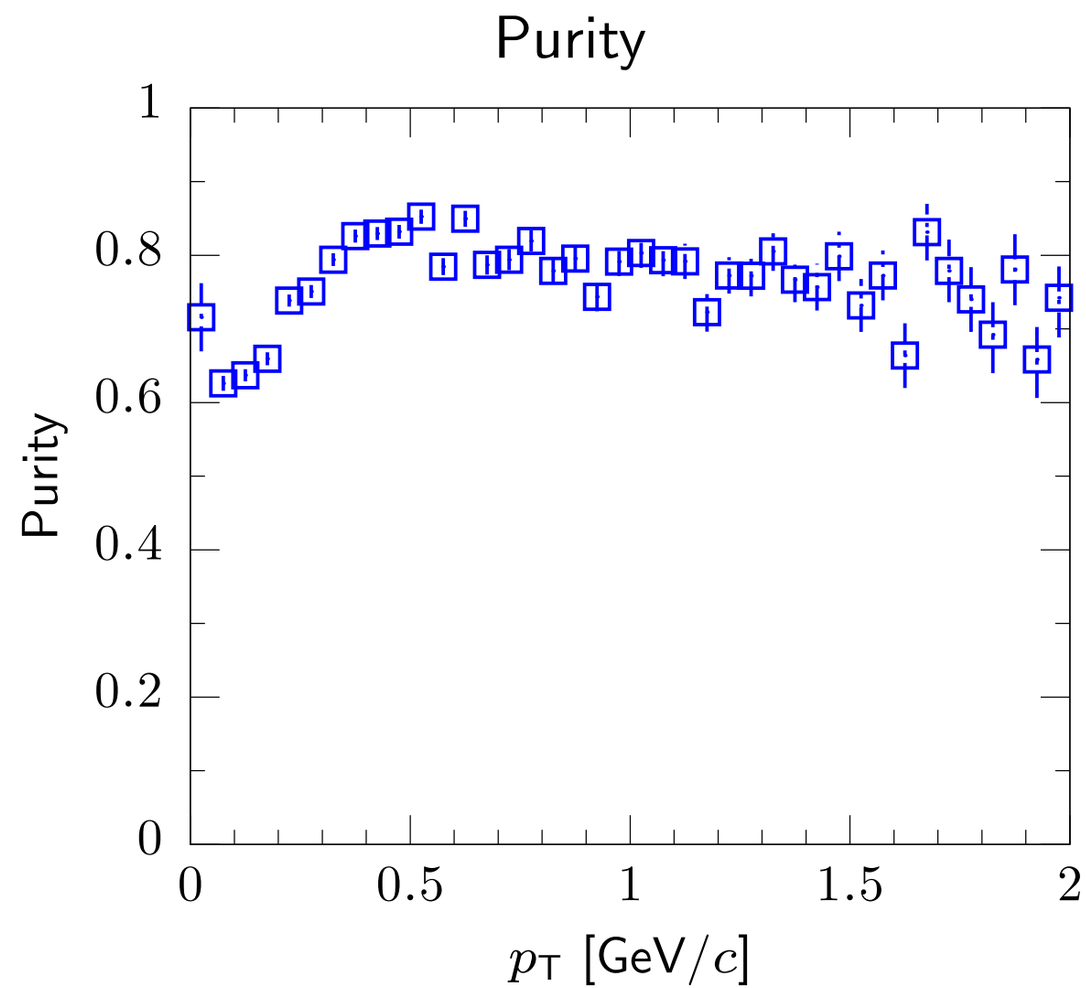
Results – photon candidates



Only candidates of e^+e^- pairs are shown

Performance – charged particles

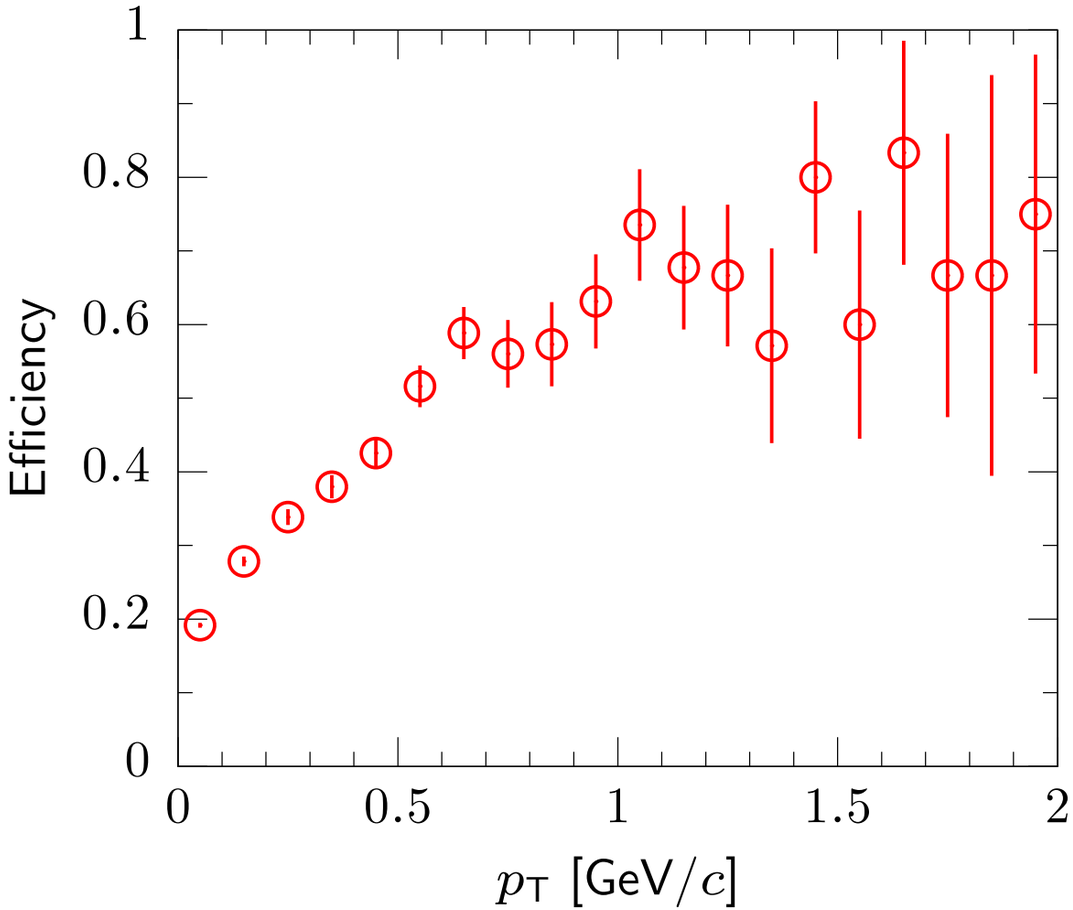




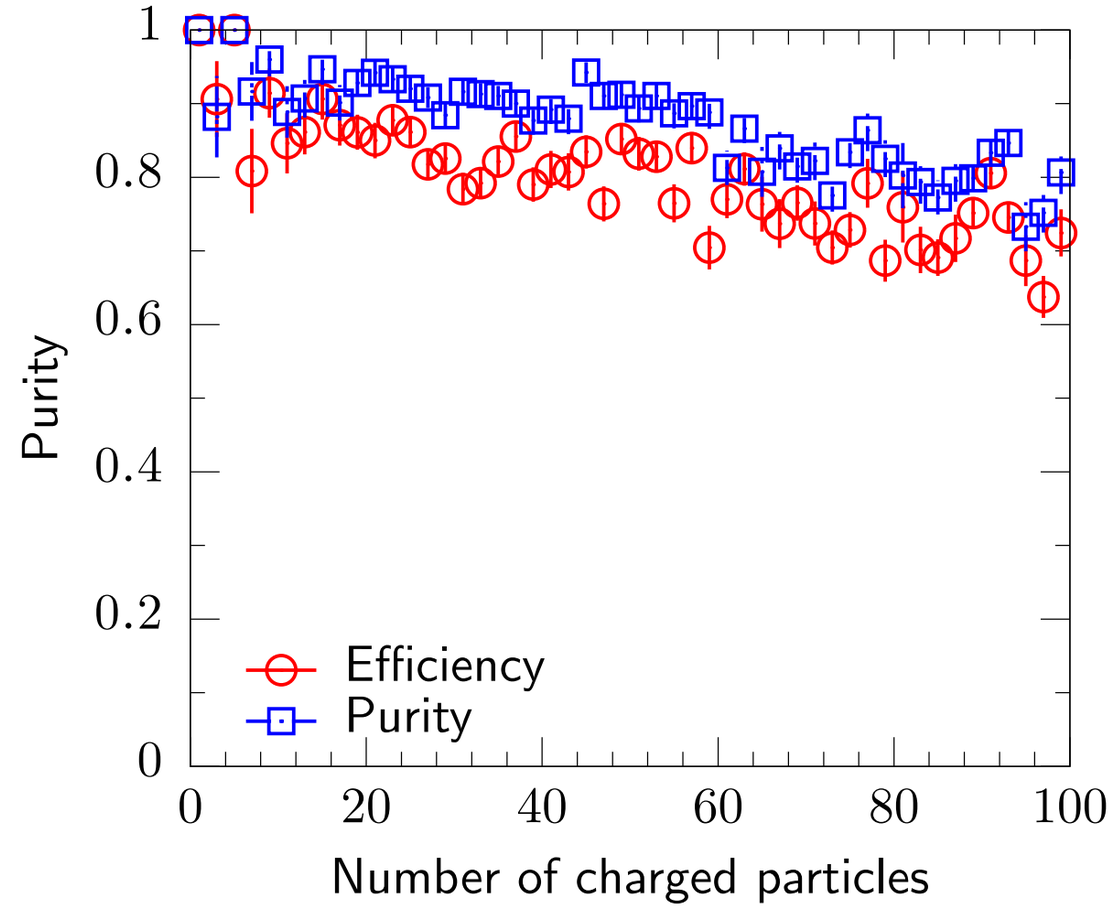
Tests with thousand minimum bias pp events (Pythia8) at $\sqrt{s} = 13$ TeV

Performance

For conversion electrons



Multiplicity dependence



Initial look, a starting point for optimization

Summary and outlook

- Novel combination of data analysis techniques
  - for primary charged particles + daughters of displaced vertices
    (mostly photon conversions, but also decays and interactions)
  - hits of adjacent tracking layers are clustered,
    mutual nearest neighbor search in angular distance
  - k-me(di)ans clustering,
    alternating between the hit-to-track assignment and the track-fit update steps
  - through the global covariance of the measured hits,
    with the possibility of adding new track hypotheses or removing unnecessary ones

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    alternating between the hit-to-track assignment and the track-fit update steps
  - through the global covariance of the measured hits,
    with the possibility of adding new track hypotheses or removing unnecessary ones
- Plans
  - now a proof of concept $\rightarrow$ optimize performance
  - apply to real collision data, do physics with photons

Thanks for your attention!