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Another approach to track reconstruction: cluster analysis

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ABSTRACT

A novel combination of data analysis techniques is proposed for the reconstruction of all tracks of primary charged particles, as well as of daughters of displaced vertices (decays, photon conversions, nuclear interactions), created in high energy collisions. Instead of performing a classical trajectory building or an image transformation, an efficient use of both local and global information is undertaken while keeping competing choices open. The measured hits of adjacent tracking layers are clustered first with the help of a mutual nearest neighbor search in the angular distance. The resulted chains of connected hits are used as initial clusters and as input for cluster analysis algorithms, such as the robust k-medians clustering. This latter proceeds by alternating between the hit-to-track assignment and the track-fit update steps, until convergence. The calculation of the hit-to-track distance and that of the track-fit χ^2 is performed through the global covariance of the measured hits. The clustering is complemented with elements from a more sophisticated Metropolis–Hastings MCMC algorithm, with the possibility of adding new track hypotheses or removing unnecessary ones. Simplified but realistic models of today's silicon trackers, including the relevant physics processes, are employed to test and study the performance (efficiency, purity) of the proposed method as a function of the particle multiplicity in the collision event.

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1 Introduction

The reconstruction of charged particles, of their trajectories, is an active area of research in high energy particle and nuclear physics. The task is usually computationally difficult (NP-hard). Detectors at today's particle colliders mostly employ large surface silicon-based tracking devices which sample the trajectory of the emitted charged particles at several locations. When a charged particle crosses the semiconducting material, it deposits energy and creates a hit by exciting electrons to the valence band producing electron-hole pairs. The electrons or holes, or both, are transported with an applied electric field, and their charge is read out, amplified, and digitized.

The silicon-based trackers are highly segmented, they consist of several millions of tiny pixels (dimensions of $\sim$100 μm) and of narrow but long strips ($\sim$10 cm in length). In a high energy collision event, several thousands of pixel and strip hits are created. Our task is to solve a mathematical puzzle: the goal is to identify particle trajectories by associating most of these hits to a limited number of true trajectories. The default solution for this problem is the combinatorial track finding and fitting [1] via the Kalman filter [2]. On the one hand, classical trajectory building utilizes mostly local information by extending the trajectory and picking up compatible hits. On the other hand, image transformation methods (e.g. variants of the Hough transform [3]) collect global information on the parameters of potential track candidates [4]. In the following, elements of an alternative track reconstruction method are outlined, with the aim of efficiently using both local and global information at the same time.

One of the goals of this study is to develop a reasonably efficient reconstruction method for (converted) photons, this way paving the way for a potential two-photon Bose–Einstein correlation measurement at LHC energies. Such study needs at least two reconstructed photons in a collision event, hence good efficiency is required. Most of the emitted photons are at low momentum, thus their identification through calorimetry is hopeless. Fortunately, in today's silicon trackers the chance of photon conversion is on average in the range 60-80%. Since a calorimetry-based track seeding for the conversion products (e^+, e^-) would not work for the above reasons, the task is to develop a track reconstruction method that works well for detached photon conversions.

2 Methods

The k-medians clustering is a robust classification method [5, 6]. It aims to partition the observations into k clusters where each observation belongs to the cluster with the nearest center. In our case the observations are the pixel or strip hits, and the centers are the track candidates with parameters $(\eta, q/p_\mathrm{T}, \phi_0, z_0, r_c)$, where η is the pseudorapidity, q is the electric charge, p_T is the momentum in the transverse plane, ϕ_0 is the

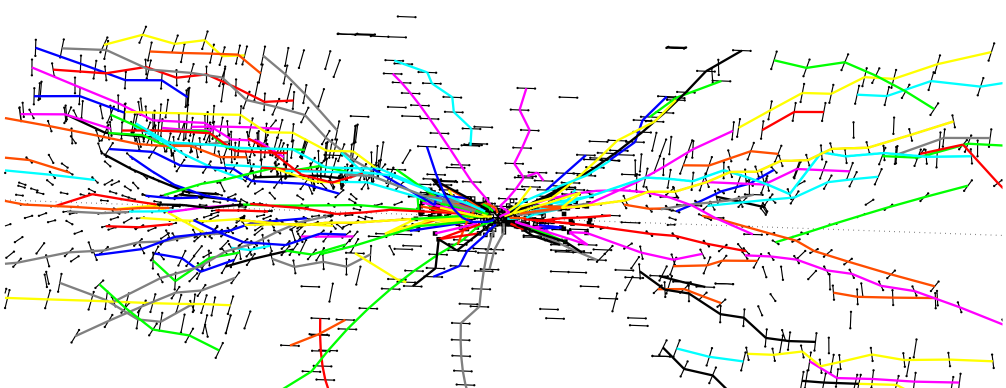

Figure 1: Chains of connected hits, taken as initial clusters in the k-medians clustering method.

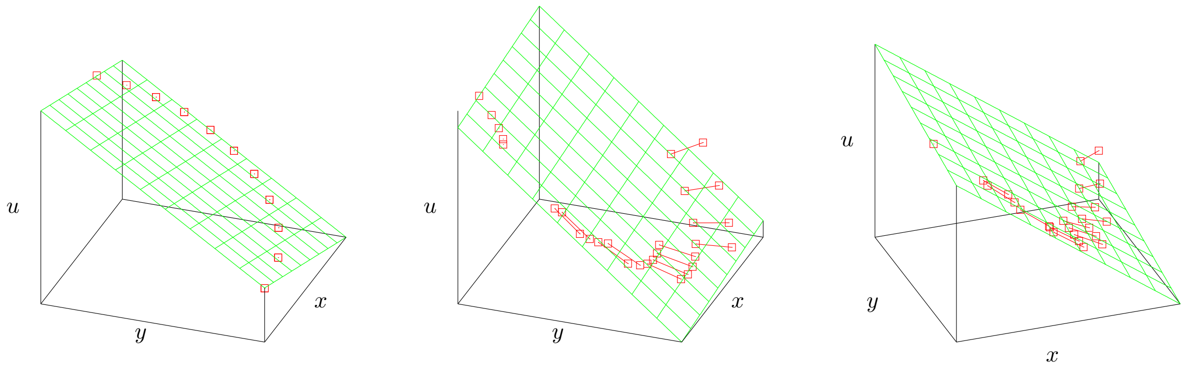

Figure 2: Transformed hits (red squares and segments) and the fitted plane (green) corresponding to the Riemann-type fit.

initial azimuth angle, and z_0 is the longitudinal, while r_c is the radial coordinate of the emission point. The method consists of two alternating steps. First, each hit is assigned to the closest track candidate, and then the parameters of the track candidates are updated by refitting their associated hits to an analytic model. The process is stopped if there are no hits changing their association (convergence) or if the number of steps exceeds a given limit.

It is important to choose a suitable measure of proximity. Because of outlier hits, the use of the sum of normalized hit-to-track distances (instead of the ordinary χ^2) provides a more robust method. In our implementation, the normalized distances are calculated through the global covariance of the measured hits, this way, no classical trajectory building through the Kalman filter is needed. This approach requires an analytic but precise description of the main physical processes, such as multiple scattering, continuous energy loss, and bremsstrahlung, with conversion to electron-positron pairs for photons [7]. In addition, non-matched candidate hits are punished by a $-2\ln(1-\varepsilon_{\text{eff}})$ terms, where ε_{eff} is the hit reconstruction efficiency. Non-matched measured hits come with a $-2\ln p_{\text{noise}}$ contribution to the global χ^2, where p_{noise} the frequency of reconstructed noise hits.

The locations of trajectory hits are obviously highly correlated. The covariance between hits in layers i and j decays roughly proportionally to $\rho^{-|i-j|}$, where $\rho \approx 0.8 - 0.9$. With that approximation, the inverse of the covariance matrix (in the example below with four hits) is

$$V^{-1} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \rho^2\sigma_1\sigma_3 & \rho^2\sigma_1\sigma_4 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_2\sigma_3 & \rho\sigma_2\sigma_4 \\ \rho^2\sigma_1\sigma_3 & \rho\sigma_2\sigma_3 & \sigma_3^2 & \rho\sigma_3\sigma_4 \\ \rho^3\sigma_1\sigma_4 & \rho^2\sigma_2\sigma_4 & \rho\sigma_3\sigma_4 & \sigma_4^2 \end{pmatrix}^{-1} =$$

$$= \frac{1}{1-\rho^2} \begin{pmatrix} 1/\sigma_1^2 & -\rho/(\sigma_1\sigma_2) & 0 & 0 \\ -\rho/(\sigma_1\sigma_2) & (1+\rho^2)/\sigma_2^2 & -\rho/(\sigma_2\sigma_3) & 0 \\ 0 & -\rho/(\sigma_2\sigma_3) & (1+\rho^2)/\sigma_3^2 & -\rho/(\sigma_3\sigma_4) \\ 0 & 0 & -\rho/(\sigma_3\sigma_4) & 1/\sigma_4^2 \end{pmatrix}.$$

As can be seen, the inverse is tridiagonal and in the calculation of the goodness-of-fit measure ($\sum x^T V^{-1} x$) only the differences between hits on neighboring layers have to be taken into account. Track fit to the associated hits is best accomplished by the downhill simplex method of Nelder and Mead [8]. It employs no function derivatives but only function evaluations at the vertices of a simplex, in our case a 5-simplex.

The choice for initial clusters (tracks) is an important one. The initial tracks could be chosen randomly, but much better performance can be achieved. We first find all mutual nearest hit neighbors in the angular distance, with respect to the nominal interaction point (center of the detector). Then, we take the chains of connected hits as initial clusters (Fig. 1).

The initial estimate of track parameters is obtained through a robust helix fit. First, circles are found in the bending plane after centering and scaling the hits in a given cluster. It is followed by a projection to

Table 1: Main characteristics of tracking detector (silicon layers) used in the simulation. For the barrel layers, the layer type is shown along with the radii (r) of the concentric cylinders, and their longitudinal extent ($-z_{\max}$ to $z_{\max}$) in the beam direction. For the endcap layers, the layer type is shown along with their $|z|$ positions, and with the inner ($r_{\min}$) and outer radii ($r_{\max}$) of their disks.

Barrel	r [cm]	$z_{\max}$ [cm]
pixels	4, 7, 10	25
strips	20, 30, 40, 50	55
strips	60, 70, 80, 90, 100, 110	55
Endcap	$\lvert z\rvert$ [cm]	$r_{\min}$–$r_{\max}$ [cm]
pixels	35, 45	5–15
strips	75, 90, 105	20–50
strips	125, 140, 155	20–110
strips	170, 185, 200	30–110
strips	220, 245	40–110
strips	270	50–110

the Riemann sphere [9], and finally planes are fitted to triplet permutations of the projected hits. In order to allow for pixel-less tracks, strip hits are also employed by using a few (4) representative internal points of their segments (Fig. 2). During the process, we prefer a nearly equal number of (end)points on both sides of the fitted plane, while minimizing the sum of hit-to-plane distances.

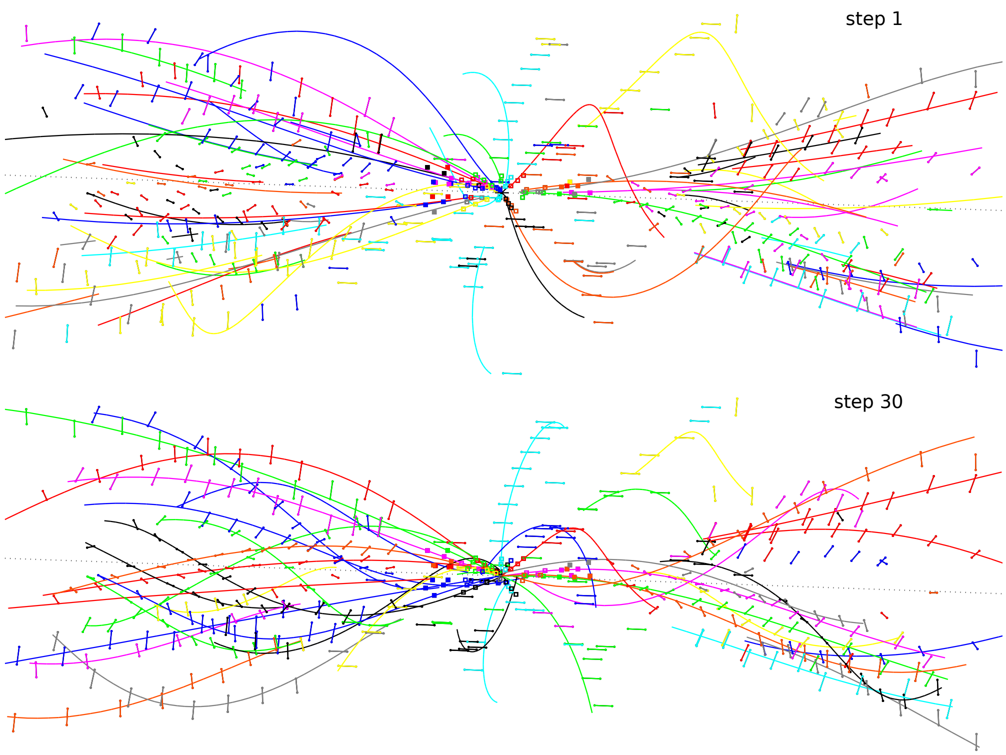

Figure 3: Hits and track candidates, their trajectories (colored curves), after the first iteration (top), and after the 30th iteration (bottom). The event is identical to the one displayed in Fig. 1.

3 Simulation results

The above ideas are demonstrated on a simplified detector model, with cylindrical and disk-type layers of pixel and strip silicon sensors, in a barrel-and-endcap layout (Table 1). The thickness of the pixel layers is 2% in radiation length units (X_0), while for strip layers it is set to 5%. The tracker detector is immersed in a homogeneous magnetic field of $B_z = 3$ T, where z is in the beam direction. Altogether thousand collision events with 24, 48, or 96 primary charged particles, and half as many converted photons, are generated. The primary interaction points are chosen on the z-axis, according to a normal distribution with a standard deviation of $\sigma_z = 5$ cm.

The generated charged particles have a uniform distribution in pseudorapidity in the range $-2.5 < \eta < 2.5$ and in azimuthal angle ϕ. Their p_{T} distribution is proportional to $p_{\mathrm{T}}^2 \exp(-p_{\mathrm{T}}/p_0)$, where p_0 is chosen to be 0.2 GeV/c. Photons are generated with similar η, ϕ, and p_{T} distributions but with $p_0 = 0.1$ GeV/c. The momentum distribution of their conversion products (electrons and positrons) are chosen according to the simplified Tsai's formula [7].

The layer-to-layer tracking of charged particles in the homogeneous magnetic field is performed by piecewise helices. All relevant physical processes are properly modelled, such as multiple scattering and specific energy loss (for all charged particles), bremsstrahlung (for electrons and positrons), and photon conversion [7]. The uncertainty of the local position measurement is modeled according to a normal distribution with a standard deviation of 50 μm. The efficiency of hit ($\varepsilon_{\mathrm{eff}}$) reconstruction is taken to be 98%.

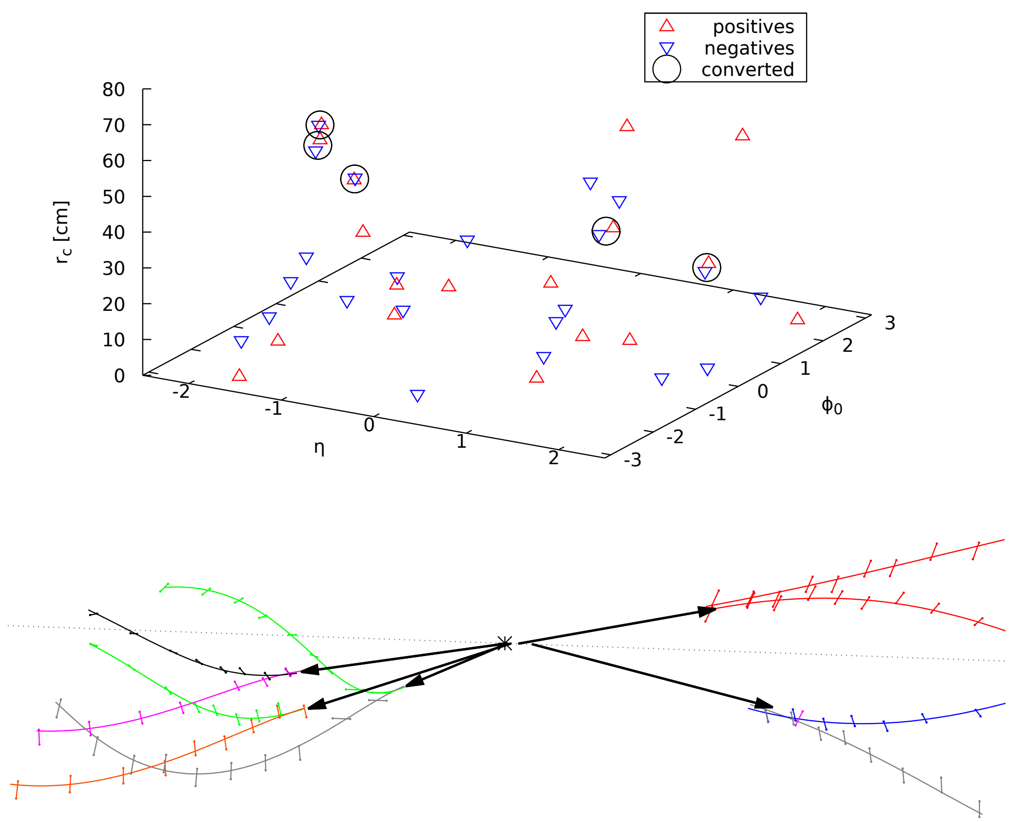

Figure 4: Top: identification of photon conversions in the (η, ϕ_0, rc) space of track candidates. Bottom: hits and track candidates, their trajectories, corresponding to the electron and positron tracks (colored curves) coming from photon conversions (thick black arrows). The event is identical to the one displayed in Fig. 3.

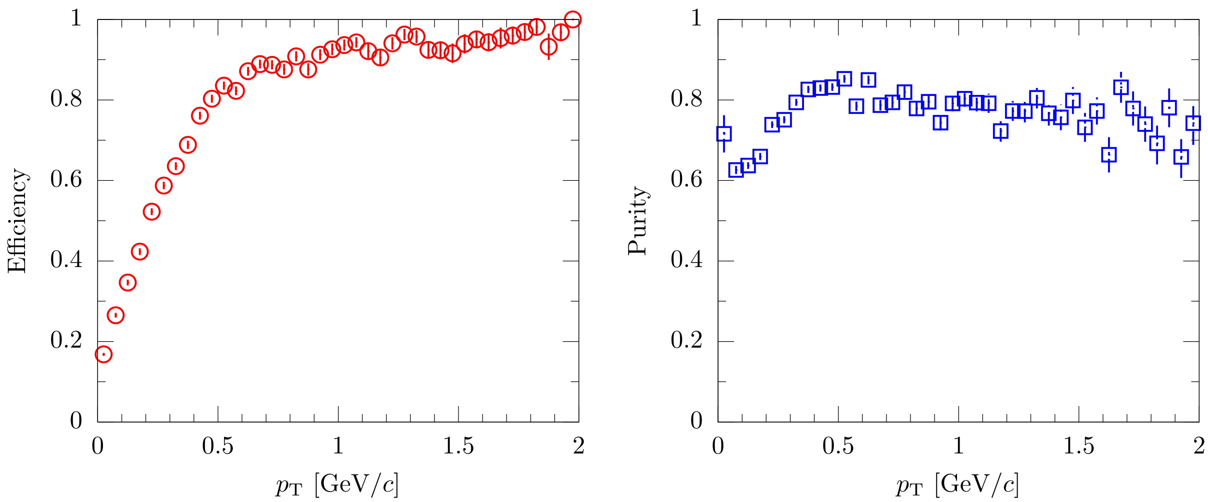

Figure 5: Efficiency (left) and purity (right) of the proposed reconstruction method for primary charged particles.

As results of the track finding steps outlined in Section 2, hits and track candidates, their trajectories, after the first and the 30th (final) k-medians iterations are shown in Fig. 3. For primary particles the tracking efficiency in the range $p_T > 0.5$ GeV/c is observed to be around of 90–95% (Fig. 5). It decreases towards very low transverse momenta and reaches 50% near 0.2 GeV/c. The purity is around 80%, independent of p_T. Photon conversions are found by searching for close positively- and negatively-charged track candidates in the (η, ϕ_0, rc) space (Fig. 4-top). The corresponding electron and positron tracks are plotted in Fig. 4-bottom. For conversion electrons (and positrons), the tracking efficiency in the range $p_T > 0.6$ GeV/c is around 70%, with a slight decrease towards lower transverse momenta, and it reaches 30% near 0.2 GeV/c (Fig. 6-left).

According to these simple tests, the measures mentioned above only slightly depend on the number of primary charged particles in the studied multiplicity range (Fig. 6-right). The performance is further increased by using elements from a more sophisticated Metropolis–Hastings MCMC algorithm [10], namely by sometimes adding new track hypotheses and removing unnecessary ones during the iteration process.

4 Conclusions

A novel combination of data analysis techniques was proposed for the reconstruction of all tracks of primary charged particles, as well as of daughters of displaced vertices, created in high energy collisions. Instead of performing a classical trajectory building or an image transformation, an efficient use of both local and global information is undertaken while keeping competing choices open.

The measured hits of adjacent tracking layers are clustered first with the help of a mutual nearest neighbor search in the angular distance. The resulting chains of connected hits are used as initial clusters and as input for a cluster analysis algorithm, the robust k-medians clustering. This latter proceeds by alternating between the hit-to-track assignment and the track-fit update steps, until convergence. The calculation of the hit-to-track distance and that of the track-fit χ^2 is performed through the global covariance of the measured hits. The clustering is complemented with elements from a more sophisticated Metropolis–Hastings MCMC algorithm, with the possibility of adding new track hypotheses or removing unnecessary ones.

Preliminary studies show that the proposed method provides reasonable efficiency and purity for the reconstruction of converted photons, this way, it opens the way towards an efficient identification of low momentum converted photons.

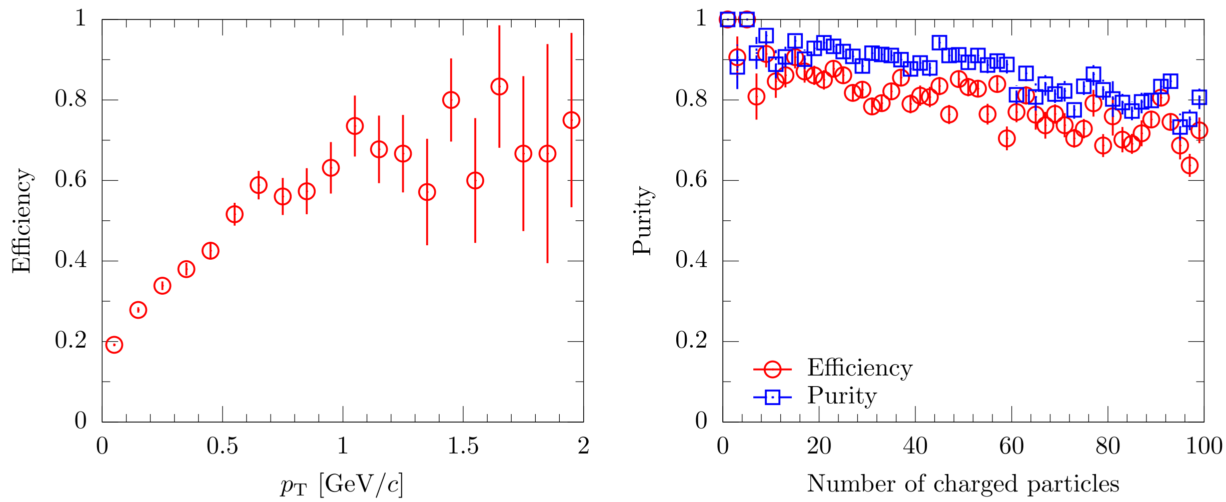

Figure 6: Left: efficiency of the proposed reconstruction method for conversion electrons. Right: multiplicity dependence of efficiency and purity for primary charged particles.

References

[1] R. Frühwirth, "Application of Kalman filtering to track and vertex fitting," Nucl. Instrum. Meth. A **262** (1987) 444–450.

[2] R. E. Kalman, "A new approach to linear filtering and prediction problems," Journal of Basic Engineering **82** (1960) no. 1, 35–45.

[3] P. V. C. Hough, "Method and means for recognizing complex patterns," tech. rep., 1962. US Patent 3069654.

[4] F. Siklér, "A combination of analysis techniques for efficient track reconstruction of high multiplicity events in silicon detectors," European Physical Journal A **54** (2018) no. 6, 113, `arXiv:1712.06146 [physics.data-an]`.

[5] H. Steinhaus, "Sur la division des corps matériels en parties.," Bull. Acad. Pol. Sci., Cl. III **4** (1957) 801–804.

[6] J. MacQueen, "Some methods for classification and analysis of multivariate observations.." Proc. 5th Berkeley Symp. Math. Stat. Probab., Univ. Calif. 1965/66, 1, 281-297 (1967)., 1967.

[7] Particle Data Group Collaboration, "Review of Particle Physics," Phys. Rev. D **98** (2018) no. 3, 030001.

[8] J. A. Nelder and R. Mead, "A simplex method for function minimization," The computer journal **7** (1965) no. 4, 308–313.

[9] R. Frühwirth and A. Strandlie, "Robust circle reconstruction with the Riemann fit," Journal of Physics: Conf. Ser. **1085** (2018) 042004.

[10] W. K. Hastings, "Monte Carlo Sampling Methods Using Markov Chains and Their Applications," Biometrika **57** (1970) no. 1, 97–109.