Light Dark World International Forum 2018

Flavourful Axion Phenomenology

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Introduction

- PQ symmetry, a QCD-anomalous global symmetry, solves the strong CP problem.
- It may be identified with a flavour symmetry.

Wilczek, '82; … ; Ema, et.al.; Calibbi, et.al., '16

- It can be an approximate symmetry respected by operators up to D=10 to guarantee $\delta\theta < 10^{-10}$.

Holman, et.al.; Kamionkowski, et.al.; Barr-Seckel, '92

- An approximate PQ symmetry could arise accidentally from a discrete gauge symmetry.

EJC, Lukas, '92

Introduction

- (Discrete) Flavour symmetry as the origin of quark and lepton masses and mixing may enforce an accidental PQ symmetry

F. Bjoerkeroth, EJC, S. King, 1711.05741

- A general study of phenomenology of flavourful axion

F. Bjoerkeroth, EJC, S. King, 1806.00660

Flavourful axion phenomenology

- PQ charges may be generation-dependent: QCD axion = phase field of flavons

$$\phi_i = \frac{v_i}{\sqrt{2}} e^{i\frac{x_i a}{v_{PQ}}} \qquad a = \sum_i x_i \frac{v_i a_i}{v_{PQ}};\ v_{PQ}^2 = \sum_i x_i^2 v_i^2$$

- Fermion Yukawa couplings take a general form:

Feng, et.al., '98

$$-\mathcal{L}_{\text{Yuk}} = e^{i\frac{a}{v_{PQ}}\left(x_{f_{Li}} - x_{f_{Rj}}\right)} M_{ij}^f \overline{f_{Li}} f_{Rj}$$

- After the transformation: $f_{L/Ri} \to e^{i\frac{a}{v_{PQ}} x_{f_{L/Ri}}} f_{L/Ri}$

$$-\mathcal{L} = \frac{\partial_\mu a}{v_{PQ}} \left(x_{f_{Li}} \overline{f_{Li}} \gamma^\mu f_{Li} + x_{f_{Ri}} \overline{f_{Ri}} \gamma^\mu f_{Ri} \right) - \mathcal{L}_{\text{anomaly}} + M_{ij}^f \overline{f_{Li}} f_{Rj}$$

General flavourful axion couplings

- In the mass basis: $f_{L/R} \to U_{fL/R}\, f_{L/R},\ M^f \to U_{fL}^\dagger M^f U_{fR} = m^f$

$$-\mathcal{L} = \frac{\partial_\mu a}{v_{PQ}} \bar{f}_i \gamma^\mu \left(V_{ij}^f - A_{ij}^f \gamma_5\right) f_j + \frac{a}{f_a}\left(\frac{\alpha_s}{8\pi} G\tilde{G} + c_{a\gamma}\frac{\alpha}{8\pi} F\tilde{F}\right) + m_i^f \overline{f_{Li}} f_{Rj}$$

$$V^f = \frac{1}{2}\left(U_{fL}^\dagger x_{f_L} U_{fL} + U_{fR}^\dagger x_{f_R} U_{fR}\right)$$

$$A^f = \frac{1}{2}\left(U_{fL}^\dagger x_{f_L} U_{fL} - U_{fR}^\dagger x_{f_R} U_{fR}\right)$$

$f_a = v_{PQ}/N_{DW}$ (N_{DW} = QCD anomaly)

Lepton decays to axion

- LFV decays $l_i \to l_j a$ with the couplings (V^e_{ij}, A^e_{ij}):

$$B(l_i \to l_j a) \equiv \tilde{c}_{l_i \to l_j} \left|C^e_{ij}\right|^2 \left(\frac{10^{12} GeV}{v_{PQ}}\right)^2$$

$$\tilde{c}_{l_i \to l_j} \approx \frac{1}{16\pi\Gamma(l_i)} \frac{m^3_{l_i}}{(10^{12} GeV)^2} \qquad \left|C^e_{ij}\right|^2 = \left|V^e_{ij}\right|^2 + \left|A^e_{ij}\right|^2$$

- Angular distribution:

$$\frac{d\Gamma}{d\cos\theta} = \frac{\left|C^e_{ij}\right|^2}{32\pi} \frac{m^3_{l_i}}{v^2_{PQ}} \left(1 - A\, P_{l_i} \cos\theta\right) \qquad A = \frac{2\Re\left(A^e_{ij} V^{e\,*}_{ij}\right)}{\left|C^e_{ij}\right|^2} = \begin{Bmatrix} A = 0 \ \ (\text{isotropy}) \\ A = -1 \ \ (V - A\text{: SM}) \\ A = +1 \ \ (V + A\text{: our model}) \end{Bmatrix}$$

Decay	Branching ratio	Experiment	$\tilde{c}_{\ell_1 \to \ell_2}$	v_{PQ}/GeV
$\mu^+ \to e^+ a$	$< 2.6 \times 10^{-6}$	$(A=0)$ Jodidio *et al* [86]	7.82×10^{-11}	$> 5.5 \times 10^9 \vert V^e_{21}\vert$
	$< 2.1 \times 10^{-5}$	$(A=0)$ TWIST [87]		$> 1.9 \times 10^9 \vert C^e_{21}\vert$
	$< 1.0 \times 10^{-5}$	$(A=1)$ TWIST [87]		$> 2.8 \times 10^9 \vert C^e_{21}\vert$
	$< 5.8 \times 10^{-5}$	$(A=-1)$ TWIST [87]		$> 1.2 \times 10^9 \vert C^e_{21}\vert$
	$\lesssim 5 \times 10^{-9*}$	Mu3e (future) [88]		$\gtrsim 1 \times 10^{11} \vert C^e_{21}\vert$
$\tau^+ \to e^+ a$	$< 1.5 \times 10^{-2}$	ARGUS [89]	4.92×10^{-14}	$> 1.8 \times 10^6 \vert C^e_{31}\vert$
$\tau^+ \to \mu^+ a$	$< 2.6 \times 10^{-2}$	ARGUS [89]	4.87×10^{-14}	$> 1.4 \times 10^6 \vert C^e_{32}\vert$

Radiative LFV decay: $l_1 \to l_2 a\gamma$

- Cristal Box: $Br(\mu \to ea\gamma) < 1.1 \times 10^{-9} \Rightarrow v_{PQ} > 9.4 \times 10^8 |C^e_{21}|\ GeV$
  $[Br(\mu \to e\gamma) < 4.9 \times 10^{-11}]$

$$\frac{d^2\Gamma}{dx\,dc_\theta} = \frac{\alpha \left|C^e_{\ell_1\ell_2}\right|^2 m^3_{\ell_1}}{32\pi^2 v^2_{PQ}} f(x, c_\theta),\ \ f(x, c_\theta) = \frac{1 - x(1 - c_\theta) + x^2}{(1 - x)(1 - c_\theta)}. \qquad x = 2E_{\ell_2}/m_{\ell_1}, \quad c_\theta \equiv \cos\theta_{2\gamma}$$

- Future search?

Decay	Branching ratio	Experiment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	MEG [92]
	$\lesssim 6 \times 10^{-14}*$	MEG-II (future) ?? [93]
$\tau^- \to e^-\gamma$	$< 3.3 \times 10^{-8}$	BaBar [95]
$\tau^- \to \mu^-\gamma$	$< 4.4 \times 10^{-8}$	BaBar [95]

Axion-mediated LFV processes

- Mu to 3e: $Br(\mu \to eee) < 10^{-12}$ (SINDRUM); 10^{-16}(Mu3e)

$$\mathrm{Br}(\mu^+ \to e^+ e^- e^+) \approx \frac{m_e^2 m_\mu^3}{16\pi^3 \Gamma(\mu)} \frac{|A_{11}^e|^2 |C_{21}^e|^2}{v_{PQ}^4} \left(\ln \frac{m_\mu^2}{m_e^2} - \frac{15}{4} \right),$$

$$\approx 1.43 \times 10^{-41} |A_{11}^e|^2 |C_{21}^e|^2 \left(\frac{10^{12}\ \mathrm{GeV}}{v_{PQ}} \right)^4 .$$

- Mu-to-e conversion: $R_{\mu e} < 7 \times 10^{-13}$ (SINDRUM II); 6×10^{-17}(Mu2e)

$$R_{\mu e}^{(A,Z)} \equiv \frac{\Gamma(\mu^- \to e^-(A,Z))}{\Gamma_{\mu^- \mathrm{cap}}^{(A,Z)}} \sim \frac{m_\mu^5}{(q^2 - m_a^2)^2} \frac{(\alpha Z)^3}{\pi^2\, \Gamma_{\mu^- \mathrm{cap}}^{(A,Z)}} \frac{m_\mu^2 m_N^2}{v_{PQ}^4} |C_{21}^e|^2 |S_N^{(A,Z)} C_{aN}|^2$$

↑ Nucleon spin-dependent

Cirigliano, Davidson, Kuno, ‘17

- Both put bounds around $v_{PQ} \gtrsim 10^6$ GeV

Meson decay to axion

- Meson decays $P \to P'a$ from flavourful vector couplings:

$$B\left(P(q_i) \to P'\left(q_j\right) a\right) = \frac{1}{16\pi\Gamma(P)} \left|V_{ij}^q\right|^2 |f_+(0)|^2 \frac{m_P^3}{v_{PQ}^2} \left(1 - \frac{m_{P'}^2}{m_P^2}\right)^3$$

$$\equiv \tilde{c}_{P\to P'} \left|V_{ij}^q\right|^2 \left(\frac{10^{12} GeV}{v_{PQ}}\right)^2$$

Decay	Branching ratio	Experiment	$\tilde{c}_{P\to P'}$	v_{PQ}/GeV
$K^+ \to \pi^+ a$	$< \mathbf{0.73 \times 10^{-10}}$	E949 + E787 [59]	3.51×10^{-11}	$> 6.9 \times 10^{11} \lvert V_{21}^d \rvert$
	$< 0.01 \times 10^{-10}*$	NA62 (future) [62]		$> 5.9 \times 10^{12} \lvert V_{21}^d \rvert$
	$< 1.2 \times 10^{-10}$	E949 + E787 [58]		
	$< 0.59 \times 10^{-10}$	E787 [73]		
$K_L^0 \to \pi^0 a$	$< \mathbf{5 \times 10^{-8}}$	KOTO [68]	3.67×10^{-11}	$> 2.7 \times 10^{10} \lvert V_{21}^d \rvert$
$(K_L^0 \to \pi^0 \nu\bar{\nu})$	$(< 2.6 \times 10^{-8})$	E391a [66]		
$B^\pm \to \pi^\pm a$	$< \mathbf{4.9 \times 10^{-5}}$	CLEO [71]	5.30×10^{-13}	$> 1.0 \times 10^{8} \lvert V_{31}^d \rvert$
$(B^\pm \to \pi^\pm \nu\bar{\nu})$	$(< 1.0 \times 10^{-4})$	BaBar [74]		
	$(< 1.4 \times 10^{-4})$	Belle [75]		
$B^\pm \to K^\pm a$	$< \mathbf{4.9 \times 10^{-5}}$	CLEO [71]	7.26×10^{-13}	$> 1.2 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^\pm \to K^\pm \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	BaBar [76]		
	$(< 1.9 \times 10^{-5})$	Belle [75]		
	$(< 1.5 \times 10^{-6})*$	Belle-II (future) [77]		
$B^0 \to \pi^0 a$			4.92×10^{-13}	
$(B^0 \to \pi^0 \nu\bar{\nu})$	$(< 0.9 \times 10^{-5})$	Belle [75]		$\gtrsim 2.3 \times 10^{8} \lvert V_{31}^d \rvert$
$B^0 \to K^0_{(S)} a$	$< \mathbf{5.3 \times 10^{-5}}$	CLEO [71]	6.74×10^{-13}	$> 1.1 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^0 \to K^0 \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	Belle [75]		
$D^\pm \to \pi^\pm a$	< 1		1.11×10^{-13}	$> 3.3 \times 10^{5} \lvert V_{21}^u \rvert$
$D^0 \to \pi^0 a$	< 1		4.33×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$D_s^\pm \to K^\pm a$	< 1		4.38×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$B_s^0 \to \overline{K}^0 a$	< 1		3.64×10^{-13}	$> 6.0 \times 10^{5} \lvert V_{31}^d \rvert$

Axion-meson mixing

- After QCD condensation,

$$\frac{c_{ij}}{v_{PQ}} \partial_\mu a \, \bar{f}_i \gamma^\mu \gamma_5 f_j \; \Rightarrow c_P \frac{f_P}{f_a} \partial_\mu a \partial^\mu P$$

- Kinetic mixing diagonalization & mass re-diagonalizaton:

$$a \to a + c_P \frac{f_P}{f_a} \frac{m_P^2}{m_P^2 - m_a^2} P \,, \qquad P \to P - c_P \frac{f_P}{f_a} \frac{m_a^2}{m_P^2 - m_a^2} a$$

mass-dependent mixing

Axion-pion mixing

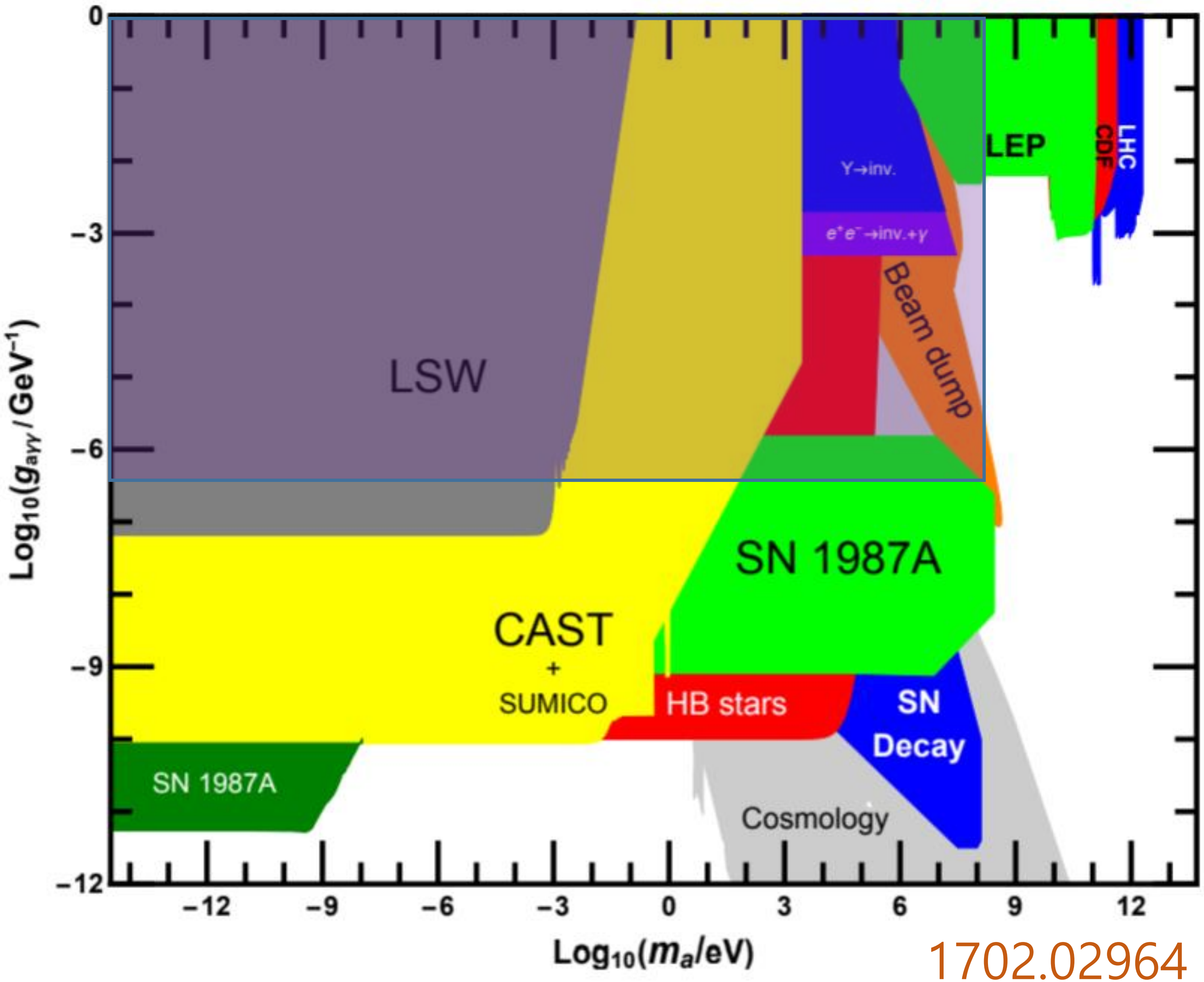
- For QCD axion, the mixing is negligible ($f_P \ll f_a, m_a \ll m_\pi$).
- For ALP, it can lead to sizable contribution to $K^+ \to \pi^+ a$ & $a \to \gamma\gamma$ induced from $K^+ \to \pi^+\pi^0$ & $\pi^0 \to \gamma\gamma$:

$$\Gamma(K^+ \to \pi^+ a) \approx \left(c_\pi \frac{f_\pi}{f_a} \frac{m_a^2}{m_\pi^2 - m_a^2}\right)^2 \Gamma(K^+ \to \pi^+\pi^0)$$

$$\Gamma(a \to \gamma\gamma) \approx \left(c_\pi \frac{f_\pi}{f_a} \frac{m_a^2}{m_\pi^2 - m_a^2}\right)^2 \left(\frac{m_a}{m_\pi}\right)^3 \Gamma(\pi^0 \to \gamma\gamma) \Rightarrow \left(g_{a\gamma}\right)_{mix} = \frac{\alpha}{\pi} \frac{c_\pi m_a^2}{m_\pi^2 - m_a^2} \frac{1}{f_a}$$

- $B(K^+ \to \pi^+ a) < 10^{-10}$ puts a limit:

$$f_a > 4 \left(\frac{c_\pi m_a^2}{m_\pi^2 - m_a^2}\right) TeV \Rightarrow \left(g_{a\gamma}\right)_{mix} < 5.8 \times 10^{-7}\, GeV^{-1} \text{ for } m_a < 110 MeV$$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Impact on neutral meson mass splitting

- Flavourful axion couplings ($A^{u,d}_{12,23}$) induces mixing with heavy mesons (K, D, B) contributing to their mass splitting:

$$\Delta m_P \approx |\eta_P|^2 m_P = |c_P|^2 \frac{f_P^2}{v_{PQ}^2} m_P$$

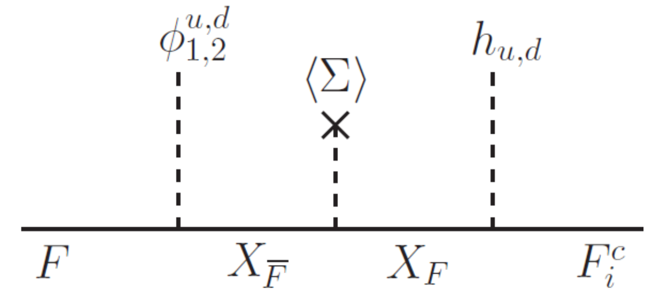
System	$(\Delta m_P)_{\rm exp}/\mathrm{MeV}$	v_{PQ}/GeV
$K^0 - \overline{K}^0$	$(3.484 \pm 0.006) \times 10^{-12}$	$\gtrsim 2 \times 10^6 \lvert c_{K^0}\rvert$
$D^0 - \overline{D}^0$	$(6.25\,^{+2.70}_{-2.90}) \times 10^{-12}$	$\gtrsim 4 \times 10^6 \lvert c_{D^0}\rvert$
$B^0 - \overline{B}^0$	$(3.333 \pm 0.013) \times 10^{-10}$	$\gtrsim 8 \times 10^5 \lvert c_{B^0}\rvert$
$B^0_s - \overline{B}^0_s$	$(1.1688 \pm 0.0014) \times 10^{-8}$	$\gtrsim 1 \times 10^5 \lvert c_{B^0_s}\rvert$

A-to-Z flavour Pati-Salam Model

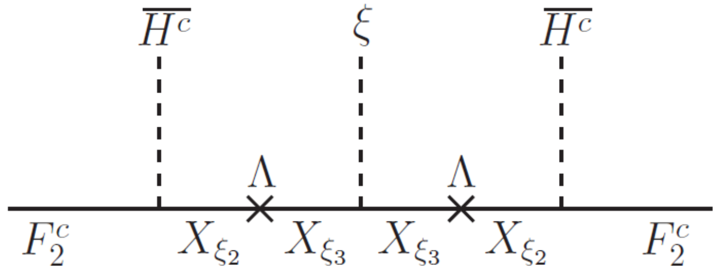
King, '14

- A Pati-Salam unification model where PQ symmetry, protected up to D=10 operators, arises accidentally from discrete flavour symmetry $A_4 \times Z_5 \times Z_3 \times Z_5'$:

$$W_Y^{\rm eff} = \lambda_3 (F \cdot h_3) F_3^c + \lambda_{1u} \frac{(F \cdot \phi_1^u) h_u F_1^c}{\langle \Sigma_u \rangle} + \lambda_{2u} \frac{(F \cdot \phi_2^u) h_u F_2^c}{\langle \Sigma_u \rangle} + \lambda_{1d} \frac{(F \cdot \phi_1^d) h_d F_1^c}{\langle \Sigma_{15}^d \rangle} + \lambda_{2d} \frac{(F \cdot \phi_2^d) h_{15}^d F_2^c}{\langle \Sigma_d \rangle} + \lambda_{ud} \frac{(F \cdot \phi_1^u) h_d F_1^c}{\langle \Sigma_d \rangle}$$



$$W_{\rm Maj}^{\rm eff} = \frac{\overline{H^c}\,\overline{H^c}}{\Lambda} \left(\frac{\xi^2}{\Lambda^2} F_1^c F_1^c + \frac{\xi}{\Lambda} F_2^c F_2^c + F_3^c F_3^c + \frac{\xi}{\Lambda} F_1^c F_3^c \right)$$



Protection up to D=10

	Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
Fermions	F	$(4,2,1)$	3	1	1	1	1	0
	$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	$\alpha,\alpha^3,1$	$\beta,\beta^2,1$	$\gamma^3,\gamma^4,1$	1	$-2,-1,0$
	$\overline{H^c}$	$(4,1,2)$	1	1	1	1	0	0
	H^c	$(\bar{4},1,2)$	1	1	1	1	0	0
Flavons	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	$2,1$
	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	$2,1$
Higgses	h_3	$(1,2,2)$	3	1	1	1	0	0
	h_u	$(1,2,2)$	$1''$	α	1	1	0	0
	h^u_{15}	$(15,2,2)$	1	α	1	1	0	0
	h_d	$(1,2,2)$	$1'$	α^3	1	1	0	0
	h^d_{15}	$(15,2,2)$	$1'$	α^4	1	1	0	0
	Σ_u	$(1,1,1)$	$1''$	α	1	1	0	0
	Σ_d	$(1,1,1)$	$1'$	α^3	1	1	0	0
	Σ^d_{15}	$(15,1,1)$	$1'$	α^2	1	1	0	0
	ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2
Messengers	$X_{F''_{1,3}}$	$(4,2,1)$	$1''$	α,α^3	β^2,β	γ^2,γ	1	$2,1$
	$X_{F'_{1,3}}$	$(4,2,1)$	$1'$	α,α^3	β,β^2	γ,γ^2	1	$1,2$
	$X_{\overline{F}_i}$	$(\bar{4},2,1)$	1	α^i	$\beta,\beta,\beta^2,\beta^2$	$\gamma^3,\gamma^3,\gamma^4,\gamma^4$	1	$-2,-2,-1,-1$
	X_{ξ_i}	$(1,1,1)$	1	α^i	$\beta,\beta,\beta^2,\beta^2,1$	$\gamma^3,\gamma,\gamma^4,\gamma^2,1$	1	$-2,1,-1,2,0$
	$\bar{\phi}^u_{1,2}$	$(1,1,1)$	3	α,α^3	β,β^2	γ^3,γ^4	0	$-2,-1$
	$\bar{\phi}^d_{1,2}$	$(1,1,1)$	3	α^2,α^4	β,β^2	γ^3,γ^4	0	$-2,-1$
	$\bar{\xi}$	$(1,1,1)$	1	α	β	γ^3	0	-2

$x_{f_L} = (0,0,0)$

$x_{f_R} = (2,1,0)$

$N_{DW} = 6$

"Family-dependent DFSZ"

Axion-dependent Yukawa

- Flavon fields $\phi_{1,2}^{u,d}$ and right-handed fermions $F_{1,2,3}^{c}$ are charged under the PQ symmetry:

$$
\begin{aligned}
-\mathcal{L}_Y = {} & \lambda_3(\bar{f}\cdot\langle h_3\rangle^*)f_{R3} + \frac{\lambda_{1u}v_u}{\sqrt{2}v_{\Sigma_u}}(\bar{u}_L\cdot\langle\varphi_1^u\rangle^*)u_{R1}\exp\left[\frac{-ix_{\varphi_1^u}a}{v_{PQ}}\right] \\
& + \frac{\lambda_{2u}v_u}{\sqrt{2}v_{\Sigma_u}}(\bar{u}_L\cdot\langle\varphi_2^u\rangle^*)u_{R2}\exp\left[\frac{-ix_{\varphi_2^u}a}{v_{PQ}}\right] + \frac{\lambda_{1d}v_d}{\sqrt{2}v_{\Sigma_d}}(\bar{d}_L\cdot\langle\varphi_1^d\rangle^*)d_{R1}\exp\left[\frac{-ix_{\varphi_1^d}a}{v_{PQ}}\right] \\
& + \frac{\lambda_{2d}v_d}{\sqrt{2}v_{\Sigma_d}}(\bar{d}_L\cdot\langle\varphi_2^d\rangle^*)d_{R2}\exp\left[\frac{-ix_{\varphi_2^d}a}{v_{PQ}}\right] + \frac{\lambda_{ud}v_d}{\sqrt{2}v_{\Sigma_d}}(\bar{d}_L\cdot\langle\varphi_1^u\rangle^*)d_{R1}\exp\left[\frac{-ix_{\varphi_1^u}a}{v_{PQ}}\right] \\
& + \left\{d_L\to e_L, d_R\to e_R, \lambda_{1d}\to\tilde{\lambda}_{1d}, \lambda_{2d}\to\tilde{\lambda}_{2d}, \lambda_{ud}\to\tilde{\lambda}_{ud}\right\} + \text{h.c.}.
\end{aligned}
$$

$x_{\varphi_1^{u,d}} = 2$

$x_{\varphi_2^{u,d}} = 1$

Fitting fermion masses and mixing

- Fermion mass matrices from 15 input parameters:

$$M^u = v_u \begin{pmatrix} 0 & b & \epsilon_{13} c \\ a & 4b & \epsilon_{23} c \\ a & 2b & c \end{pmatrix}, \quad M^d = v_d \begin{pmatrix} y_d^0 & 0 & 0 \\ B y_d^0 & y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}, \quad M^e = v_d \begin{pmatrix} -(y_d^0/3) & 0 & 0 \\ B y_d^0 & x y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}$$

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{i\xi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

→ Specific flavour prediction for axion

- Best-fit:

Parameter	Value
$a\,/10^{-5}$	$1.246\, e^{4.047i}$
$b\,/10^{-3}$	$3.438\, e^{2.080i}$
c	-0.545
$y_d^0\,/10^{-5}$	$3.053\, e^{4.816i}$
$y_s^0\,/10^{-4}$	$3.560\, e^{2.097i}$
$y_b^0\,/10^{-2}$	3.607

Parameter	Value
$\epsilon_{13}\,/10^{-3}$	$6.215\, e^{2.434i}$
$\epsilon_{23}\,/10^{-2}$	$2.888\, e^{3.867i}$
B	$10.20\, e^{2.777i}$
x	5.880

Parameter	Value
m_a /meV	3.646
m_b /meV	1.935
m_c /meV	1.151
η	2.592
ξ	2.039

Our model Predictions

- Best-fit values for axion-fermion couplings:

$$V^u = -A^u \simeq \begin{pmatrix} 1.0 & 4.3\times10^{-3}e^{-0.05i} & -1.7\times10^{-5}e^{-0.015i} \\ 4.3\times10^{-3}e^{0.05i} & -0.5 & -6.0\times10^{-4} \\ -1.7\times10^{-5}e^{0.015i} & -6.0\times10^{-4} & 7.3\times10^{-7} \end{pmatrix}$$

$$V^d = -A^d \simeq \begin{pmatrix} 0.78 & 0.25 & -0.0065 \\ 0.25 & 0.72 & -0.0057 \\ -0.0065 & -0.0057 & 7.5\times10^{-5} \end{pmatrix}, \qquad V^e = -A^e \simeq \begin{pmatrix} 0.99 & 0.073 & -0.0085 \\ 0.073 & 0.51 & -0.0013 \\ -0.0085 & -0.0013 & 7.5\times10^{-5} \end{pmatrix}$$

- Rare decays to axion:

Process	Branching ratio ($v_{PQ} = 10^{12}$ GeV)	Experimental sensitivity
$K^+ \to \pi^+ a$	2.19×10^{-12}	$\lesssim 1\times10^{-12}$ (NA62 future)
$K_L^0 \to \pi^0 a$	2.29×10^{-12}	$< 5\times10^{-8}$ (KOTO)
$\mu^+ \to e^+ a$	8.3×10^{-13}	$\lesssim 5\times10^{-9}$ (Mu3e future)

- Correlated probes: $R_{\mu/K} \equiv \dfrac{\mathrm{Br}(\mu^+ \to e^+ a)}{\mathrm{Br}(K^+ \to \pi^+ a)} \simeq 4.45\,\dfrac{|V^e_{21}|^2}{|V^d_{21}|^2} \approx 31\,e^{-1.8\sqrt{x}}$ $\approx 0.38.$

Conclusion

- Approximate PQ symmetric may arise accidentally from discrete family symmetry.
- A specific Pati-Salam unified model is worked out.
- It leads to interesting flavourful axion phenomenology.
- Rare Kaon and muon decays may provide a sensitive probe for QCD axion.
- There appear numerous flavour observables interesting for ALP.