

# **Extraction of the angle $\gamma$ from charmless 3-body $B$ decays**

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# CKM parameters and charmless B meson decays

## Measure CKM parameters:

- SM:  $V_{CKM}$  is unitary.
- SM + NP:  $V_{CKM}$  may not be unitary.
- Need to test unitarity and self-consistency.

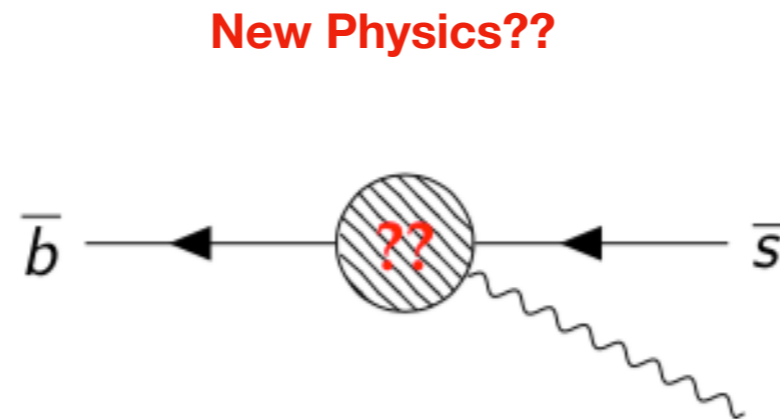
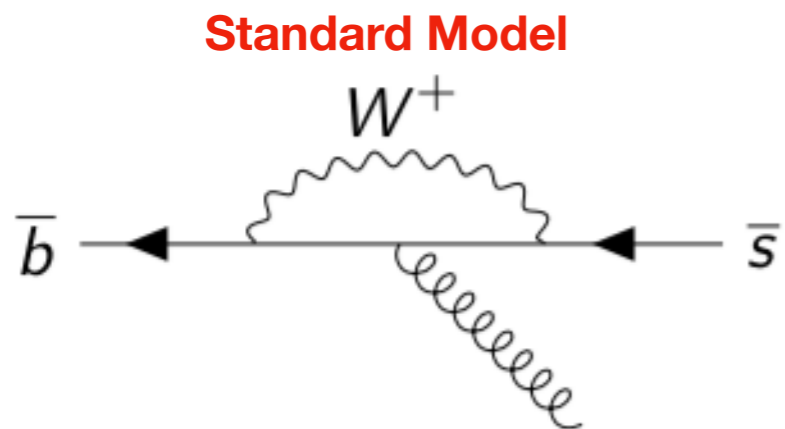
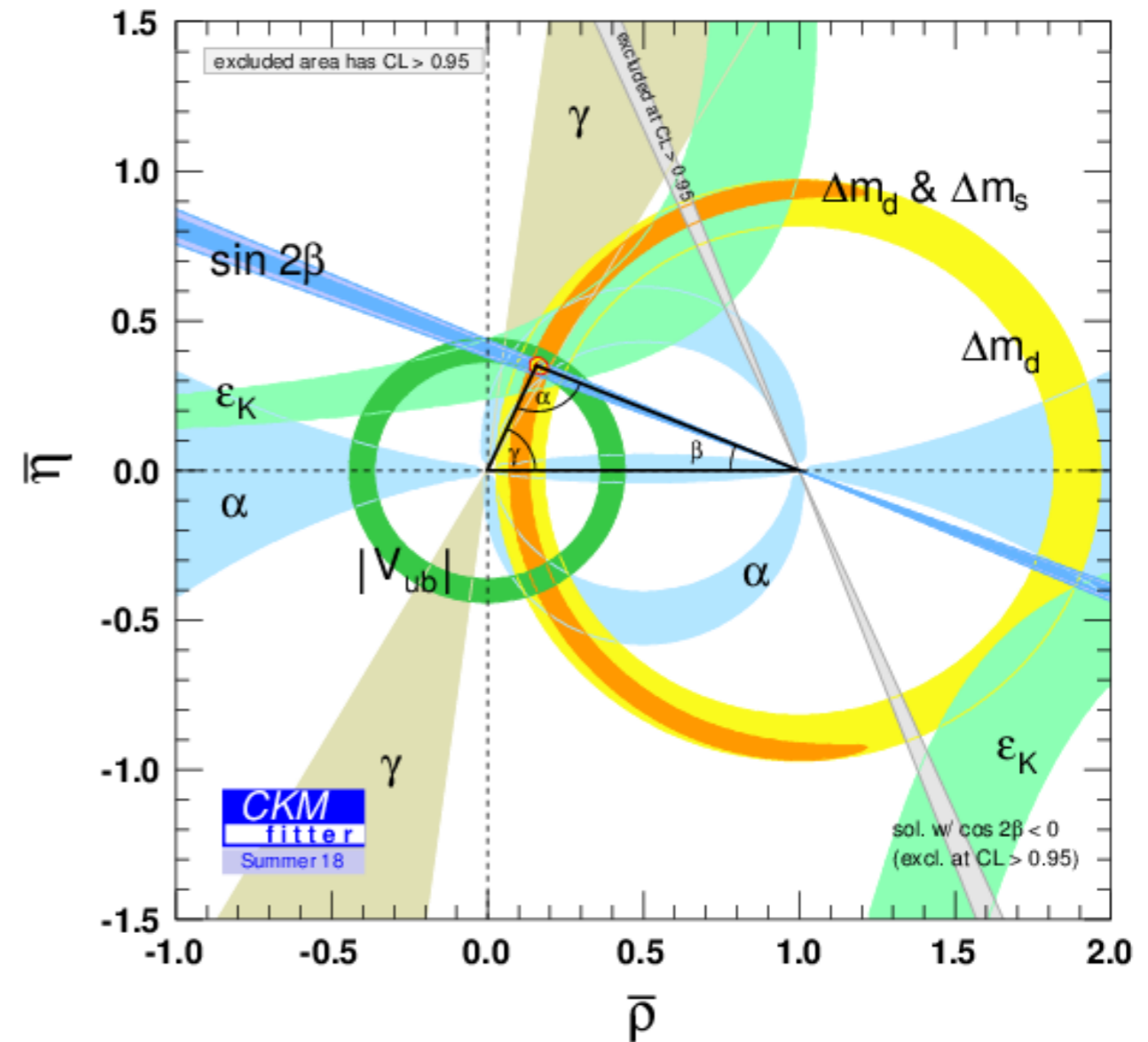
→ over-constrain the Unitarity Triangle.

## Measure $\gamma$ :

- from tree decays (eg.  $B \rightarrow DK$ ).
- from loop decays [charmless].

## Charmless B meson decays:

- Tree and penguin contributions can have similar size.
- CPV
- NP searches



$$\alpha = 86.4^{\circ+4.5}_{-4.3}$$

$$\beta = 22.14^{\circ+0.69}_{-0.67}$$

$$\gamma = 72.1^{\circ+5.4}_{-5.7}$$

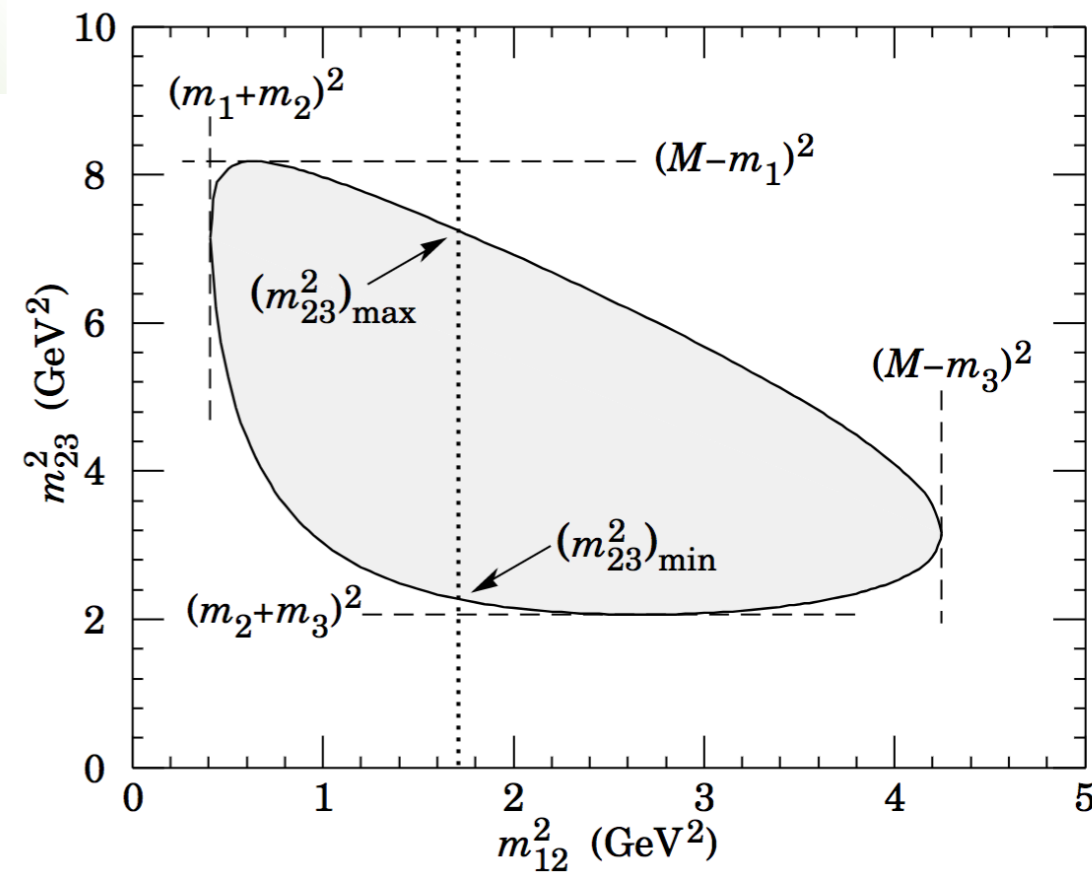
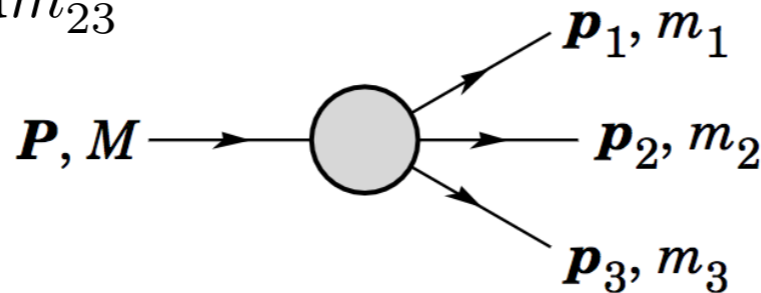
# 3-body decays

## Dalitz plot

- Information on the resonant structure.
- Direct access to phases
- Branching ratios, direct and indirect CP asymmetries.

- Partial width of the decay:

$$d\Gamma = \frac{1}{(2\pi^3)} \frac{1}{32M^2} |\bar{A}|^2 dm_{12}^2 dm_{23}^2$$



## Experimental parametrisation of the DP: Isobar Model

The total amplitude of a 3-body decay is described as a coherent sum of partial amplitudes:

$$A(m_{12}^2, m_{23}^2) = \sum_{j=1}^N c_j e^{i\phi_j} F_j(m_{12}^2, m_{23}^2)$$

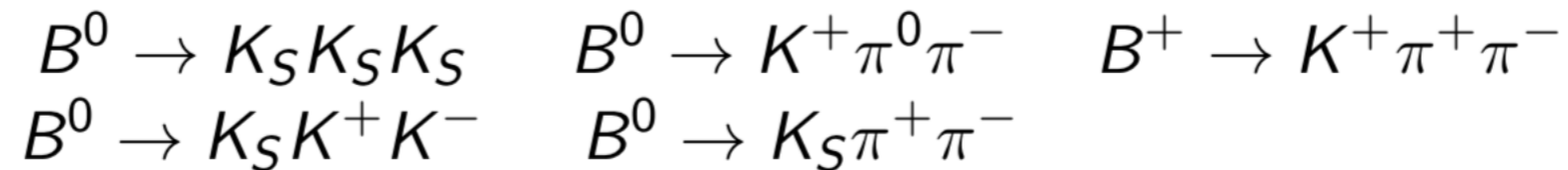
**Isobar parameters:**  
weak dynamics

**Lineshape:**  
strong dynamics

# Method overview

[Phys. Lett. B728 \(2014\) 206-209](#)

- Method to extract the CKM angle  $\gamma$  from charmless loop processes (NP sensitive) developed by Bhubanjyoti Bhattacharya, Maxime Imbeault and David London.
- Combine information from 5 charmless 3-body decays of  $B$  mesons

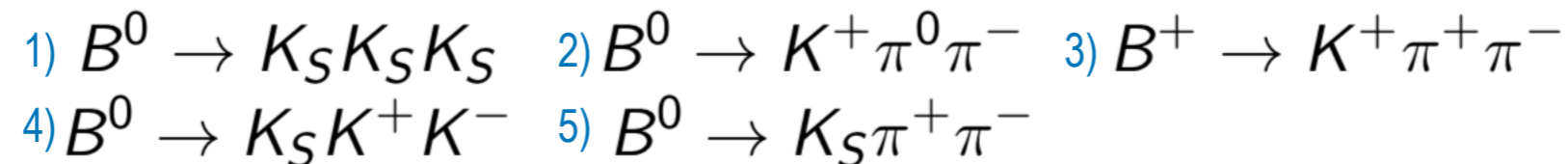


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- Combine information from 5 charmless 3-body decays of  $B$  mesons



- Use BABAR models (resonant content, lineshapes) and analysis results (isobar parameters, correlation matrices) to reconstruct the amplitudes of the different decay modes over the DP and extract  $\gamma$  with its uncertainty.

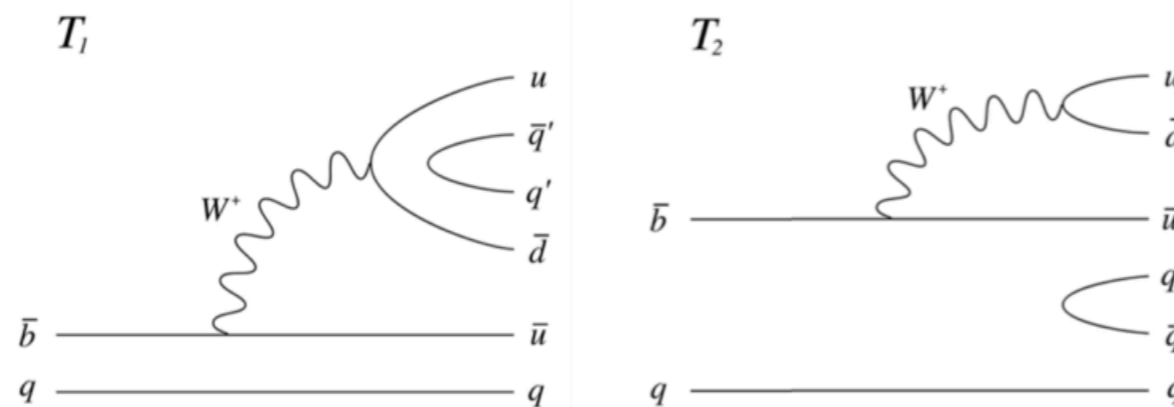
1) [Phys. Rev. D85 \(2012\) 054023](#)    2) [Phys. Rev. D83 \(2011\) 112010](#)    3) [Phys. Rev. D78 \(2009\) 112004](#)

4) [Phys. Rev. D78 \(2012\) 112010](#)    5) [Phys. Rev. D80 \(2009\) 112001](#)

# Flavour SU(3) diagrammatics

[Phys. Rev. D.84.034040](#)

- $\gamma$  from 2-body decays:  $N_{\text{obs}} > N_{\text{param}}$
  - $\gamma$  from 3-body decays:  $N_{\text{obs}} < N_{\text{param}}$
- Need to make assumptions to reduce the number of parameters**
- Different topologies for  $b \rightarrow s$  transitions:  $T'_1, T'_2, C'_1, C'_2, P'_1, P'_2 \dots$



**Flavour SU(3) limit:** tree and penguin diagrams are proportional

$$P_{EW}(C) = \kappa T(C) \quad \text{with} \quad \kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}| c_9 + c_{10}}{|\lambda_u^{(s)}| c_1 + c_2} \quad \left\{ \begin{array}{l} \lambda_p^{(s)} = V_{pb}^* V_{ps} \\ c_i : \text{Wilson coefficients} \end{array} \right.$$

This relation holds only for **fully symmetric amplitudes**:

$$A_{\text{fs}}(s_{12}, s_{13}) = \frac{1}{\sqrt{6}} (A(s_{12}, s_{13}) + A(s_{12}, s_{23}) + A(s_{13}, s_{23}) + A(s_{13}, s_{12}) + A(s_{23}, s_{12}) + A(s_{23}, s_{13}))$$

# Theoretical expressions for the amplitudes

**Theoretical amplitudes for each mode can be expressed in terms of:**

- 5 effective diagrams
- 1 weak phase
- 1 parameter related to flavour SU(3) breaking

$$2A_{\text{fs}}(B^0 \rightarrow K^+ \pi^0 \pi^-) = B e^{i\gamma} - \kappa C$$

$$\sqrt{2}A_{\text{fs}}(B^0 \rightarrow K^0 \pi^+ \pi^-) = -D e^{i\gamma} - \tilde{P}'_{\text{uc}} e^{i\gamma} - A + \kappa D$$

$$A_{\text{fs}}(B^0 \rightarrow K^0 K^0 \bar{K}^0) = \alpha_{\text{SU}(3)} (\tilde{P}'_{\text{uc}} e^{i\gamma} + A)$$

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**Parameter counting for 4 modes (5 modes)**  
**10 (11) theoretical parameters**

# Observables

From the extracted amplitudes of the 4 (5) modes, we construct **momentum dependent observables**

$$X(s_{13}, s_{23}) = |A_{fs}(s_{13}, s_{23})|^2 + |\bar{A}_{fs}(s_{13}, s_{23})|^2$$

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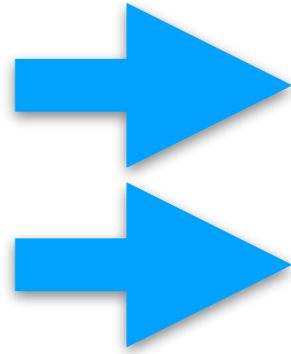
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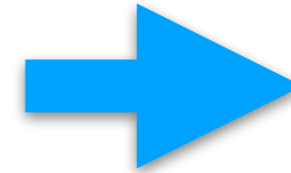
# Fit principle

10 (11) parameters

11 (13) observables



observables as  
functions of the  
parameters

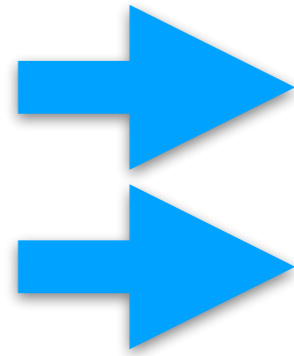


$\gamma$  extracted  
with a fit

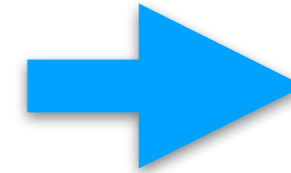
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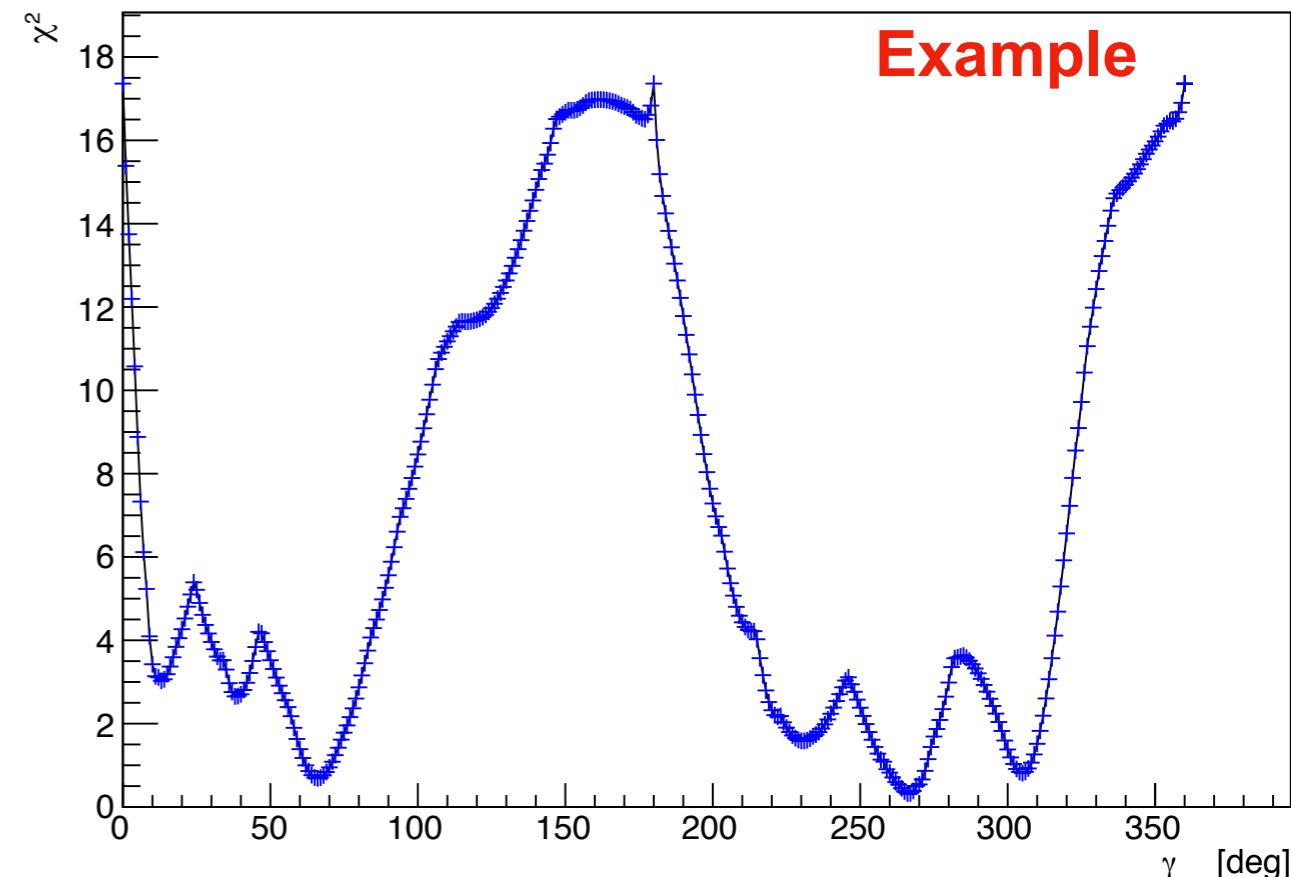


$\gamma$  extracted with a fit

## Extraction of $\gamma$ at one point ( $s_{13}$ , $s_{23}$ ) on the DP:

- Compute observables:  $X(s_{13}, s_{23})$ ,  $Y(s_{13}, s_{23})$ ,  $Z(s_{13}, s_{23})$ .
- Compute the **covariance matrix** including the correlations.
- Scan on  $\gamma$ : fix  $\gamma$  to consecutive values and evaluate the other parameters minimising a  $\chi^2$  function.

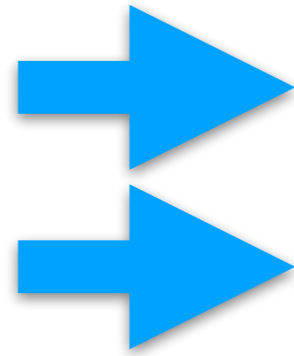
Cov matrix:  
11x11 (13x13)



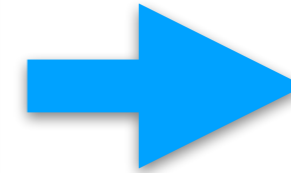
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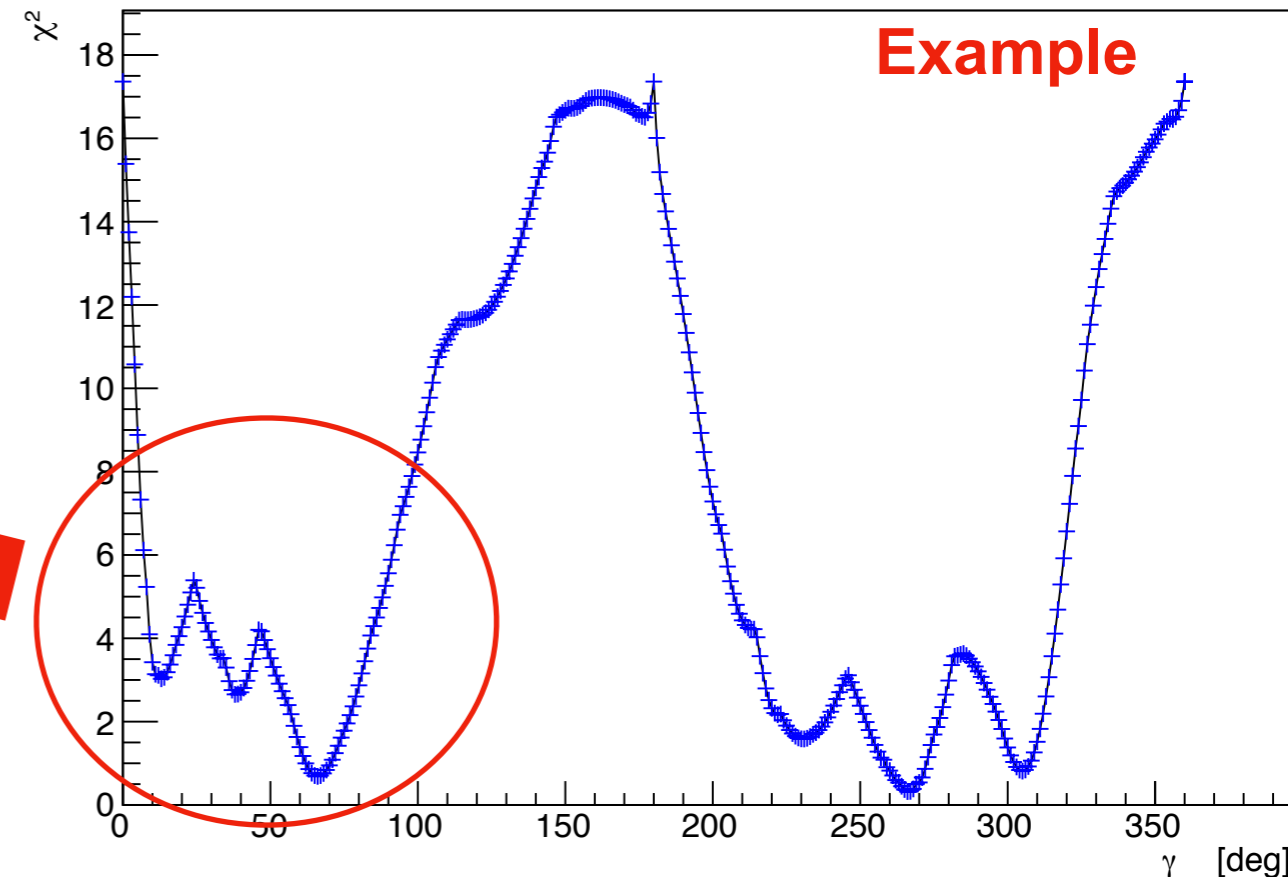
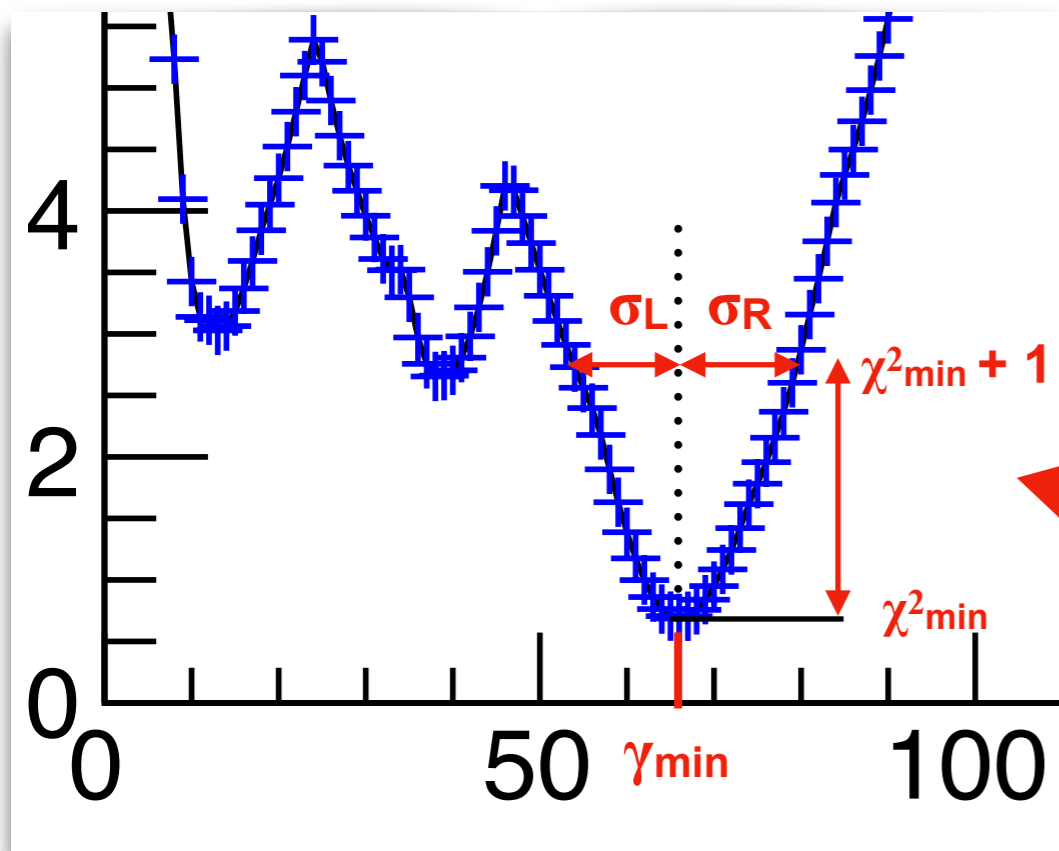


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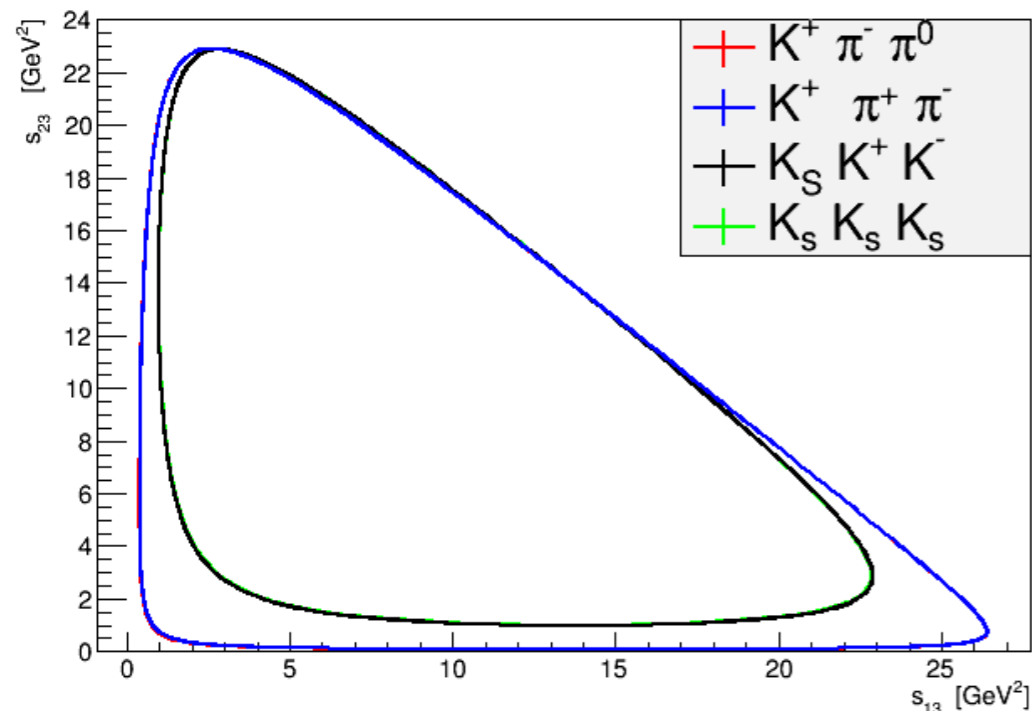
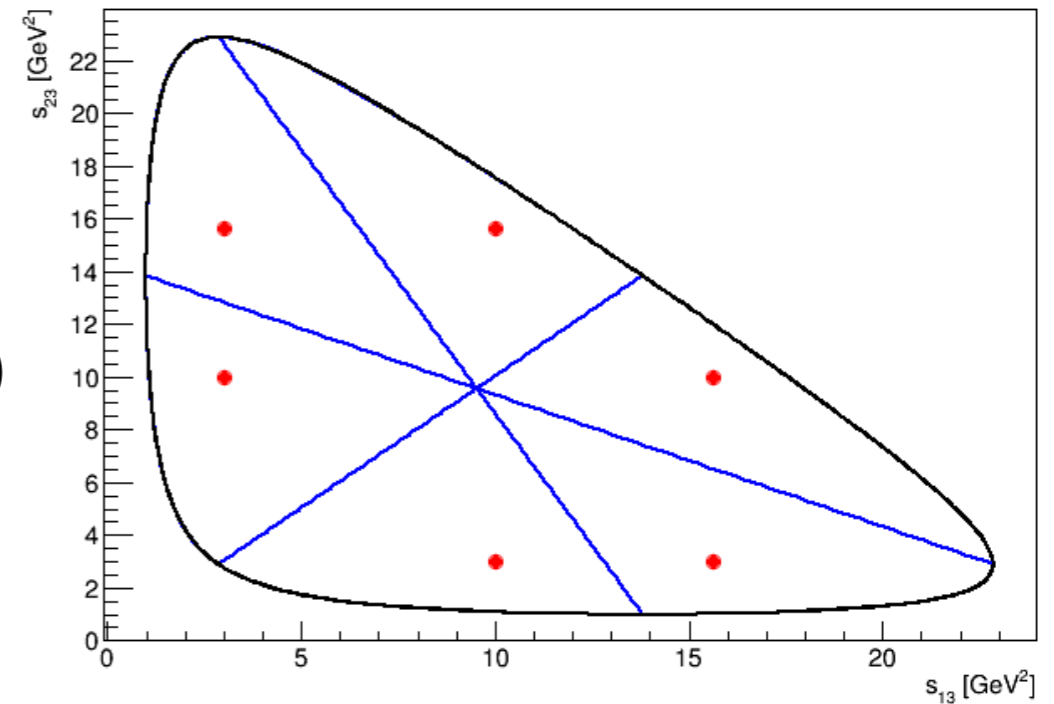


# Choice of points on the DP

## Fully symmetrised amplitudes

$$A_{fs}(s_{12}, s_{13}) = \frac{1}{\sqrt{6}} (A(s_{12}, s_{13}) + A(s_{12}, s_{23}) + A(s_{13}, s_{23}) \\ + A(s_{13}, s_{12}) + A(s_{23}, s_{12}) + A(s_{23}, s_{13}))$$

The fully symmetric DP is divided into 6 regions containing the same information.



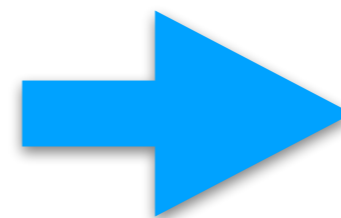
## Kinematic boundaries of the different modes

The information we can use is limited by the size of  $B^0 \rightarrow K_s K_s K_s$  DP (smallest one).

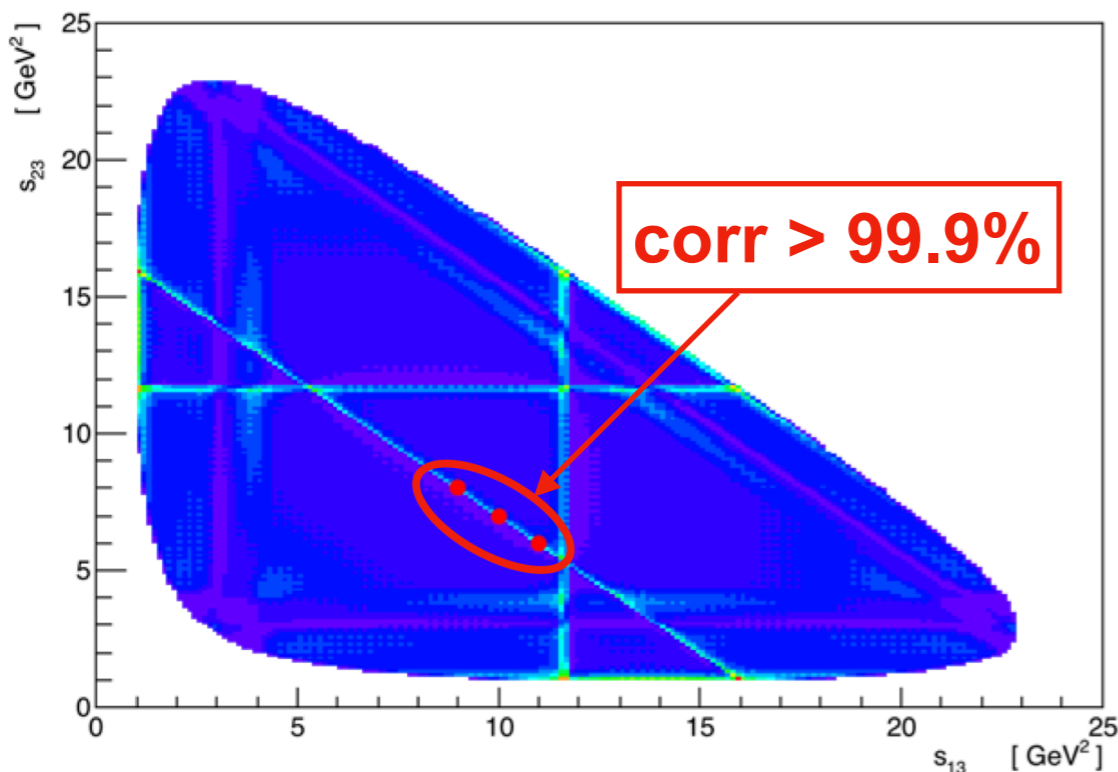
# Choice of points on the DP

## The use of several points allows:

- Improving the validity of flavour SU(3) hyp.
- Using the maximum amount of information.
- Improving the statistical uncertainties.



Extract  $\gamma$  using the **maximum possible** number of points on the DP.



In practice, due to very high correlations between certain points we are limited to the use of **3 simultaneous points**.

Cov matrix:  
33x33 (39x39)

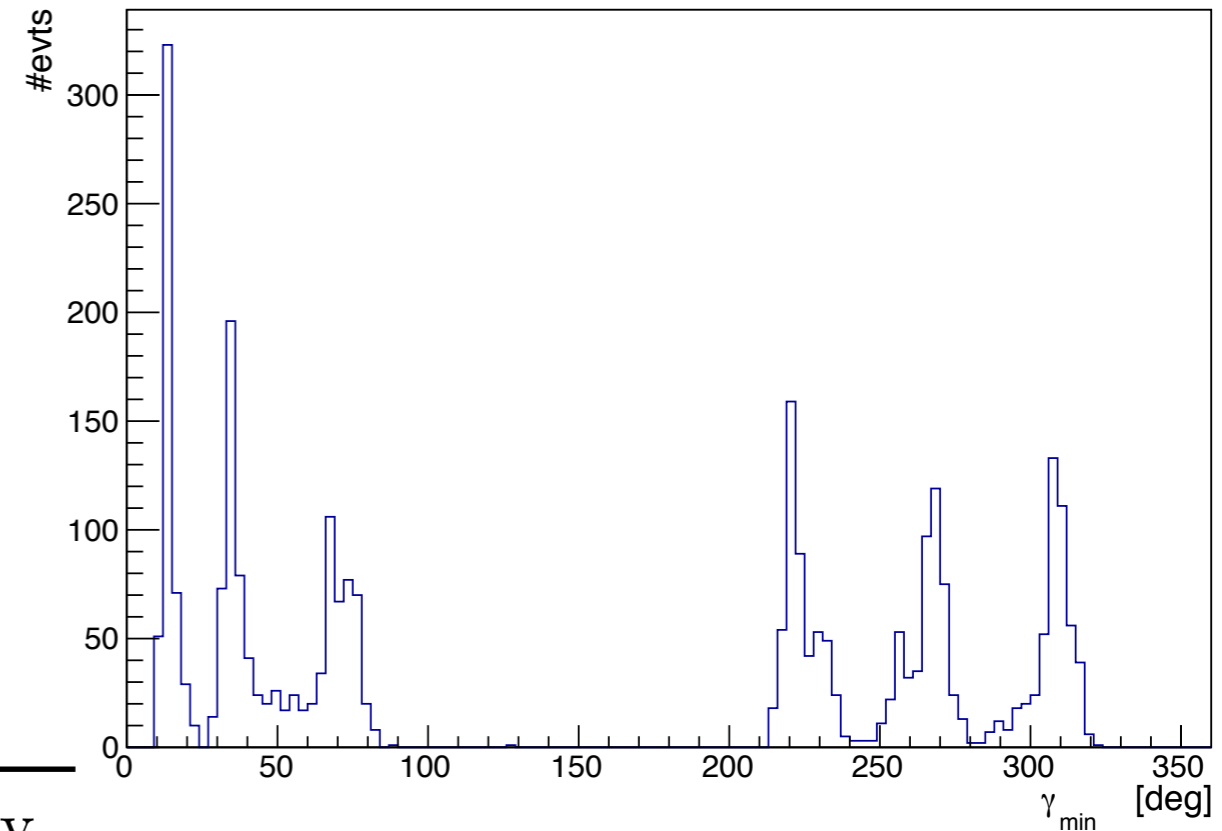
## Method for extracting the results

- Several hundred combinations of 3 points randomly scattered over the DP.
- For each set of points: scan on the value of  $\gamma$  (500 fits with random initial parameters).
- Extract minima and statistical uncertainties for each scan.
- Combine results of all scans.
- Estimate systematic uncertainties.

# Baseline results: extraction of $\gamma$ using 4 modes

- $\alpha_{\text{SU}(3)}$  fixed to 1 in the fit.
- 501 sets of random 3-points combinations (correlations  $< 70\%$ ).
- 500 fits randomising the initial values of the parameters per set.
- Fit convergence = 100%.

Histogram of the central values of the minima extracted from the 501 sets of points.



## Results

- 6 possible values for  $\gamma$ .
- 3rd minimum compatible with SM.
- Statistical error of the order of  $10^\circ$ .

Preferred values for  $\gamma$ : central values ( $\mu$ ) and statistical uncertainties ( $\sigma_L$ ,  $\sigma_R$ ).

	$\mu$	$\sigma_L$	$\sigma_R$	frequency
minimum 1	$12.9^\circ$	$4.3^\circ$	$8.4^\circ$	484
minimum 2	$36.6^\circ$	$6.1^\circ$	$6.6^\circ$	474
minimum 3	$68.9^\circ$	$8.6^\circ$	$8.6^\circ$	461
minimum 4	$223.2^\circ$	$7.5^\circ$	$10.9^\circ$	499
minimum 5	$266.4^\circ$	$10.8^\circ$	$9.2^\circ$	487
minimum 6	$307.5^\circ$	$8.1^\circ$	$6.9^\circ$	488

$$\gamma_{\text{SM}} = 72.1^\circ \begin{matrix} +5.4^\circ \\ -5.8^\circ \end{matrix}$$



# Systematic uncertainties

## Influence of "poorly resolved" minima

- To combine the results obtained from the different sets of 3 points we average on the central values of the minima.
- Some minima are not deep enough to extract statistical uncertainties. They are labelled as "**poorly resolved minima**" and are **not included** in the average for the baseline result.
- The central value including all the minima,  $\mu^{\text{all}}$ , is used to assign a systematic uncertainty

$$\text{Syst1} = |\mu - \mu^{\text{all}}|$$

## Influence flavour SU(3) breaking

- So far we do not take into account flavour SU(3) breaking.
- $\gamma$  is re-extracted with 5 modes, letting  $\alpha_{\text{SU}(3)}$  float in the fit. [next slide]
- Central values found with 5 modes are used to assign a systematic uncertainty

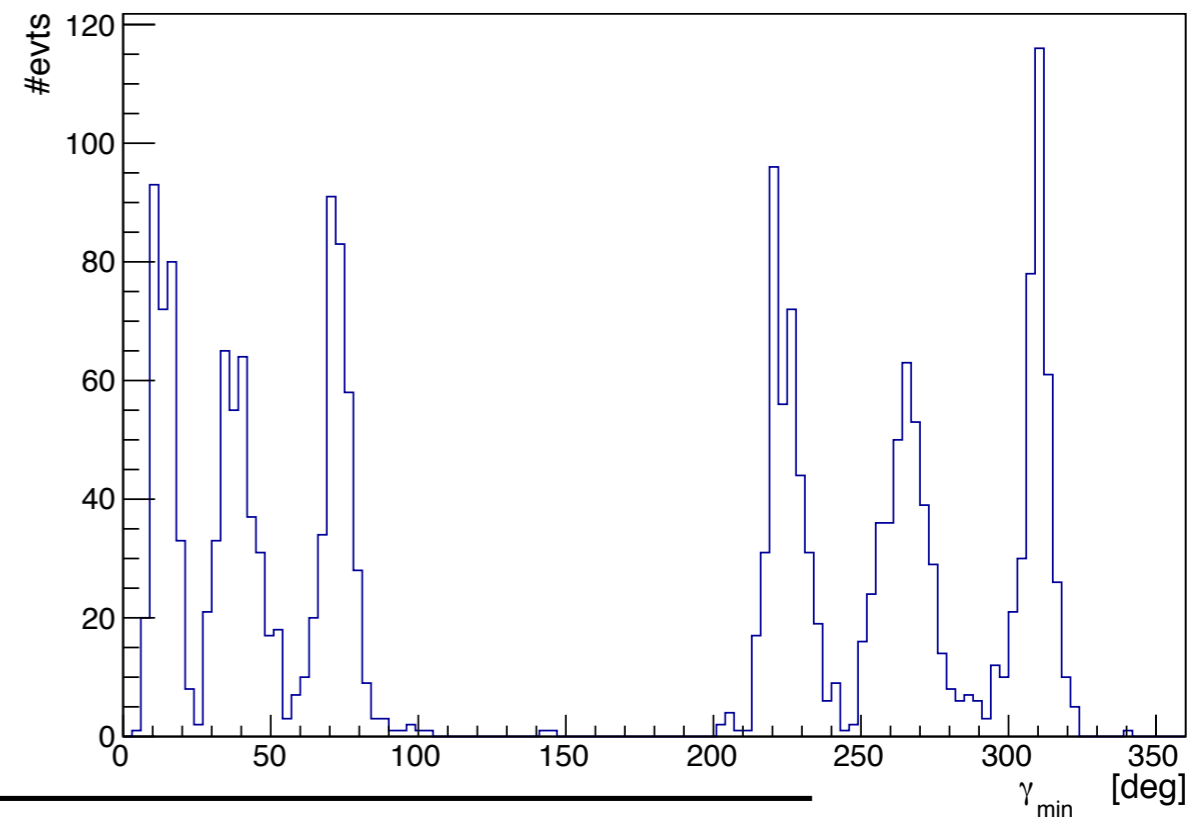
$$\text{Syst2} = |\mu^{4\text{modes}} - \mu^{5\text{modes}}|$$



# Extraction of $\gamma$ using 5 modes

- $\alpha_{SU(3)}$  free in the fit.
- 401 sets of random 3-points combinations (correlations < 80%).
- 500 fits randomising the initial values of the parameters per set.
- Fit convergence  $\gtrsim 80\%$ .

Histogram of the minima extracted from the 401 sets of points.



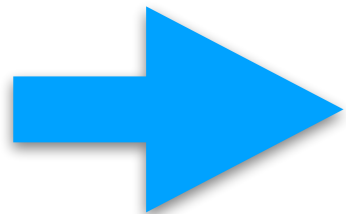
Preferred values for  $\gamma$ : central values ( $\mu$ ) and statistical uncertainties ( $\sigma_L$ ,  $\sigma_R$ ).

	$\mu$	$\sigma_L$	$\sigma_R$	$ \mu - \mu^{all} $	$ \mu^{4modes} - \mu^{5modes} $	frequency
minimum 1	11.9	5.8	9.1	1.3	1.0	306
minimum 2	39.2	6.3	6.7	1.2	2.6	329
minimum 3	71.3	9.5	9.3	0.4	2.4	372
minimum 4	223.9	7.4	9.5	0.1	0.7	383
minimum 5	265.0	11.0	10.0	1.2	1.3	378
minimum 6	308.4	8.8	7.0	0.6	0.9	391

**Central values and statistical uncertainties are compatible with those obtained extracting  $\gamma$  with 4 modes.**

# Summary of systematic uncertainties

	minimum 1	minimum 2	minimum 3	minimum 4	minimum 5	minimum 6
syst 1	0.8°	0.3°	0.2°	0.7°	1.4°	0.7°
syst 2	1.0°	2.6°	2.4°	0.7°	1.3°	0.9°



**Statistical uncertainties dominate ( $\approx 10^\circ$ )**

# Summary and results

- We studied a method for extracting  $\gamma$  from charmless 3-body decays relying on flavour SU(3) symmetry.
- Using BABAR results:
  - **6 values for  $\gamma$**  (1 consistent with SM).
  - Well separated, no overlap.
  - **Statistical error about  $10^\circ$**  (BABAR results only!).
  - Statistical error dominates over Systematics.

$$\gamma_1 = 12.9^\circ_{-4.3^\circ}^{+8.4^\circ} \quad (\text{stat}) \pm 1.3^\circ \quad (\text{syst}),$$

$$\gamma_2 = 36.6^\circ_{-6.1^\circ}^{+6.6^\circ} \quad (\text{stat}) \pm 2.6^\circ \quad (\text{syst}),$$

$$\gamma_3 = 68.9^\circ_{-8.6^\circ}^{+8.6^\circ} \quad (\text{stat}) \pm 2.4^\circ \quad (\text{syst}),$$

$$\gamma_4 = 223.2^\circ_{-7.5^\circ}^{+10.9^\circ} \quad (\text{stat}) \pm 1.0^\circ \quad (\text{syst}),$$

$$\gamma_5 = 266.4^\circ_{-10.8^\circ}^{+9.2^\circ} \quad (\text{stat}) \pm 1.9^\circ \quad (\text{syst}),$$

$$\gamma_6 = 307.5^\circ_{-8.1^\circ}^{+6.9^\circ} \quad (\text{stat}) \pm 1.1^\circ \quad (\text{syst}).$$

- Paper in preparation.

# Perspectives

The results of this study are very encouraging and we are following up in this direction.

- Take into account other symmetry states (under way):
  - totally anti-symmetric states
  - mixed states } may help to decrease the statistical uncertainties and reduce the number of solutions.
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**STAY TUNED !!!**

**BACKUP**

# Test no 1 of flavour SU(3) breaking

- From the theoretical expressions for the amplitudes:

$$A(B^0 \rightarrow K^+ K^0 K^-)_{\text{fs}} = \alpha_{SU(3)} A(B^+ \rightarrow K^+ \pi^+ \pi^-)_{\text{fs}}$$

- If flavour SU(3) symmetry is conserved,  $\alpha_{SU(3)} = 1$ , and thus these amplitudes are equal.
- We define the ratio  $R(s_{13}, s_{23})$

$$R(s_{13}, s_{23}) = \frac{A^{K^+ \pi^+ \pi^-}(s_{13}, s_{23}) + \bar{A}^{K^+ \pi^+ \pi^-}(s_{13}, s_{23})}{A^{K_S K^+ K^-}(s_{13}, s_{23}) + \bar{A}^{K_S K^+ K^-}(s_{13}, s_{23})}$$

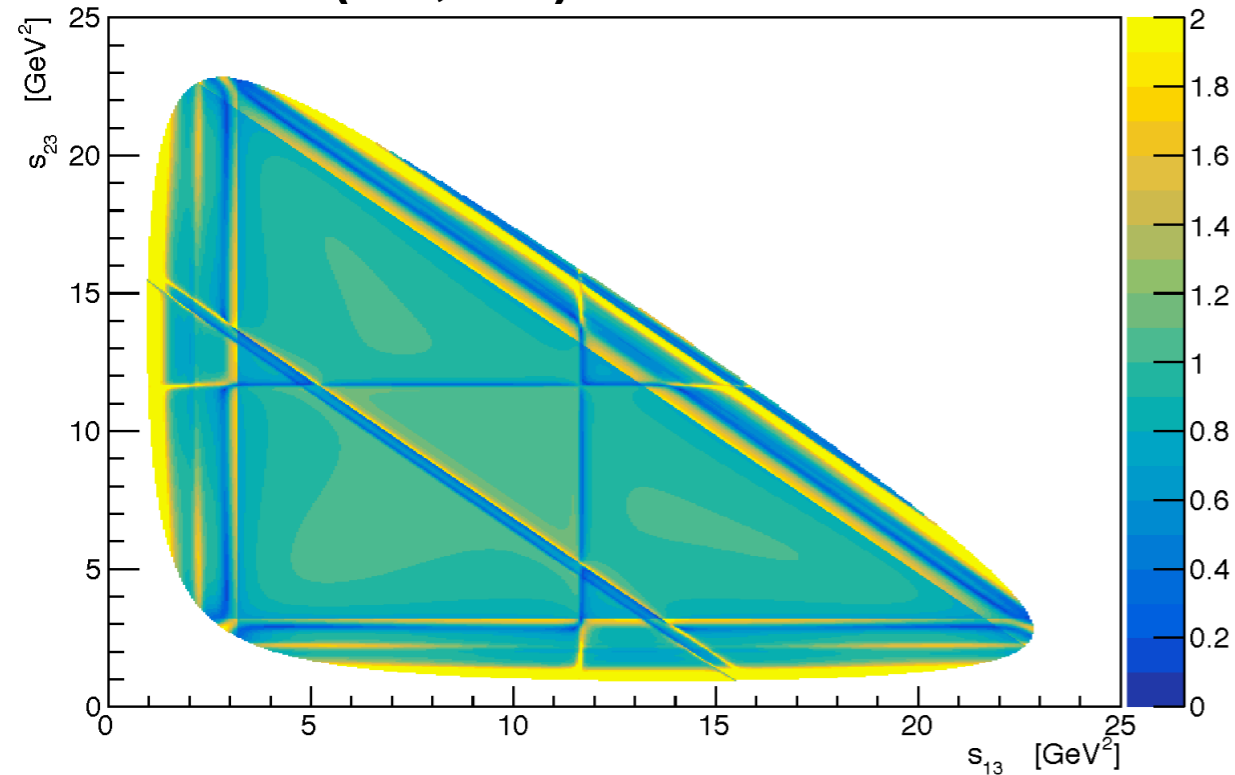
## Hypothesis:

- Flavour SU(3) symmetry is conserved when averaging over many points over the DP.

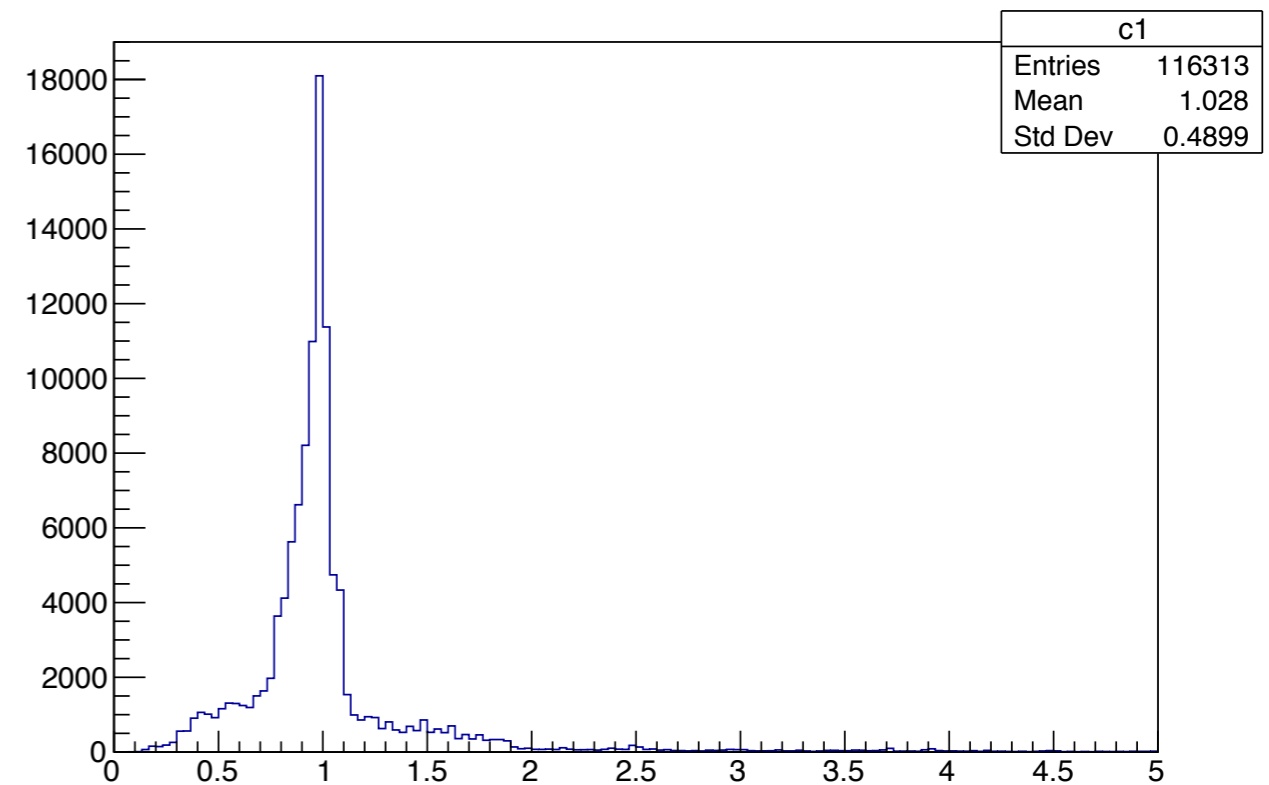


# Test no 1 of flavour SU(3) breaking

$R(s_{13}, s_{23})$  over the DP



Histogram of the values of  $R(s_{13}, s_{23})$

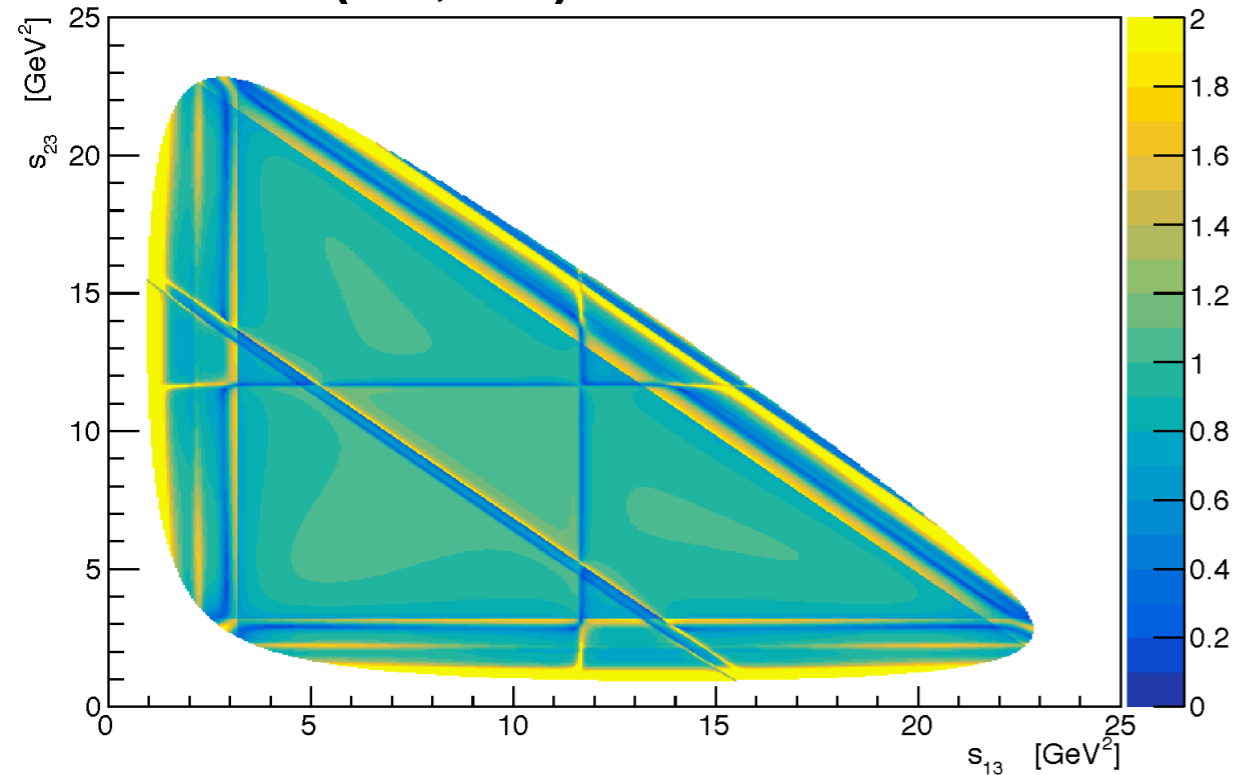


## Remarks:

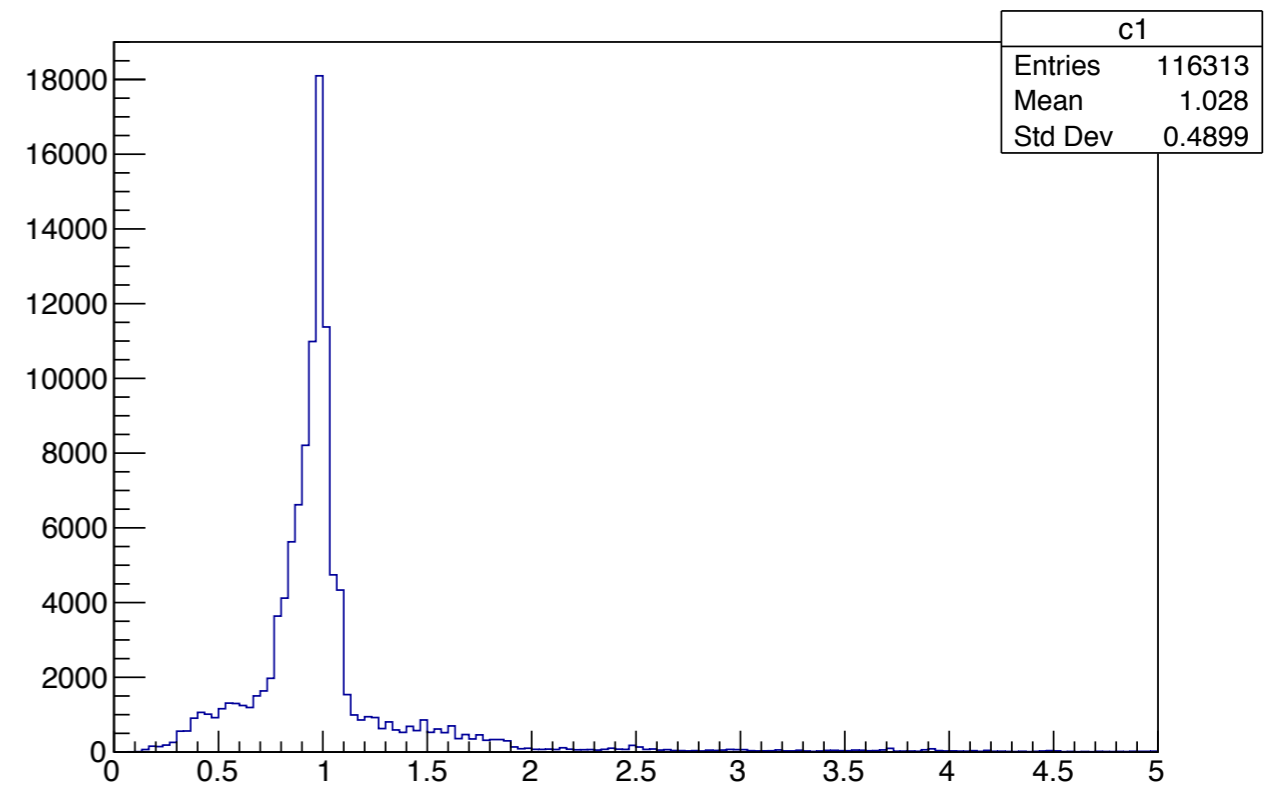
- $R(s_{13}, s_{23})$  varies over the DP, especially near resonances
- $\langle R(s_{13}, s_{23}) \rangle = 1.03 \approx 1$

# Test no 1 of flavour SU(3) breaking

$R(s_{13}, s_{23})$  over the DP



Histogram of the values of  $R(s_{13}, s_{23})$



## Remarks:

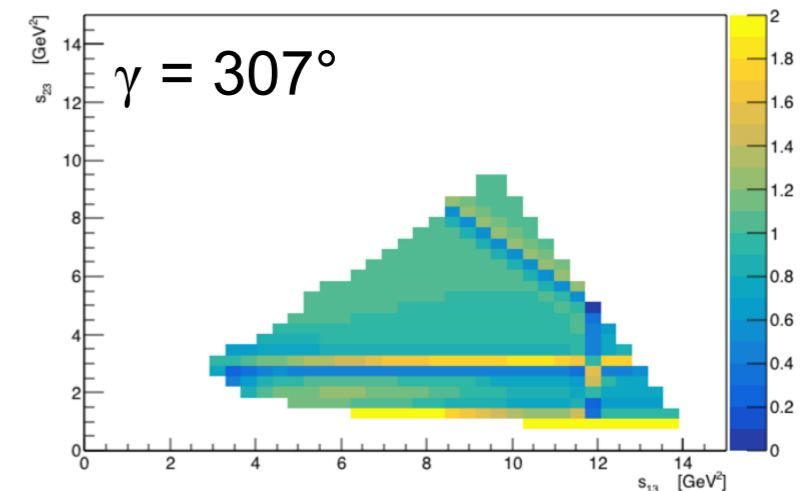
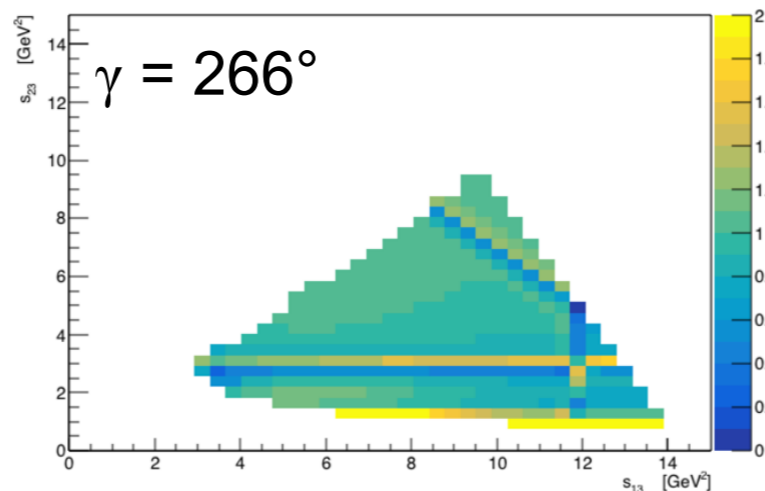
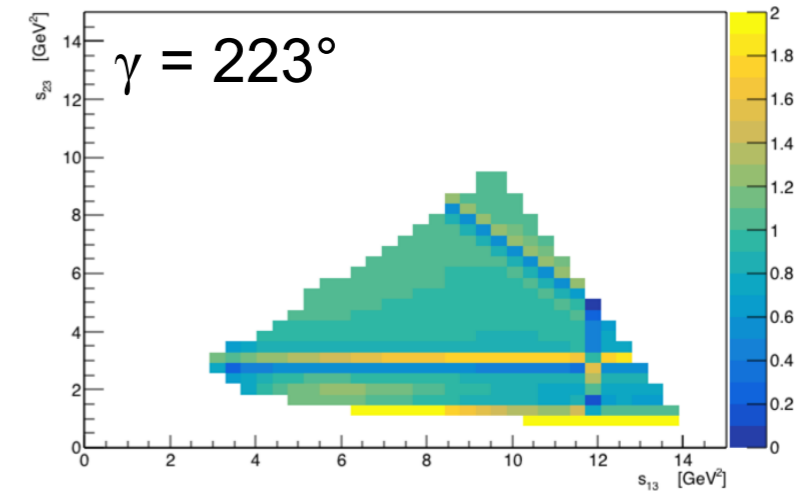
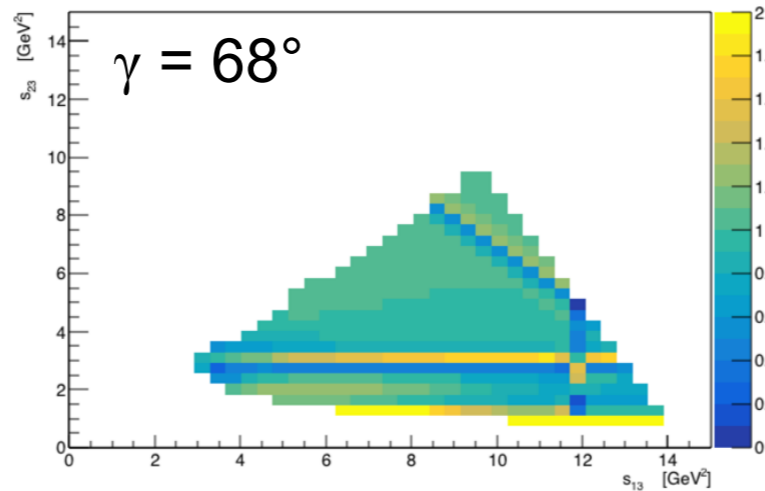
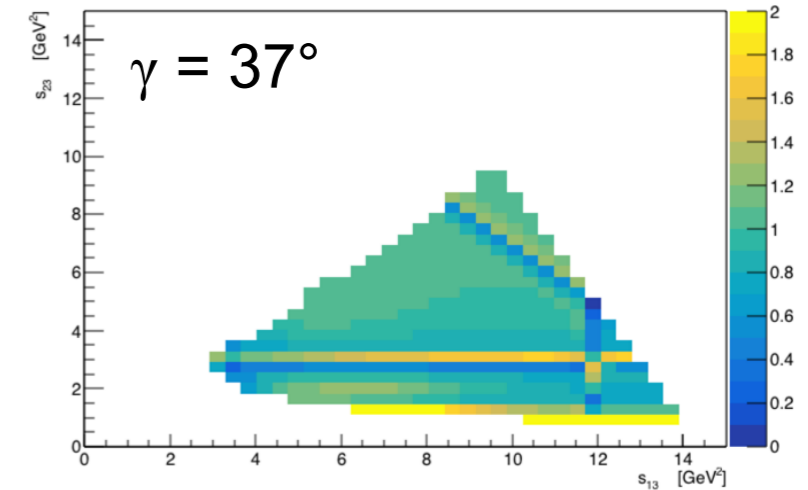
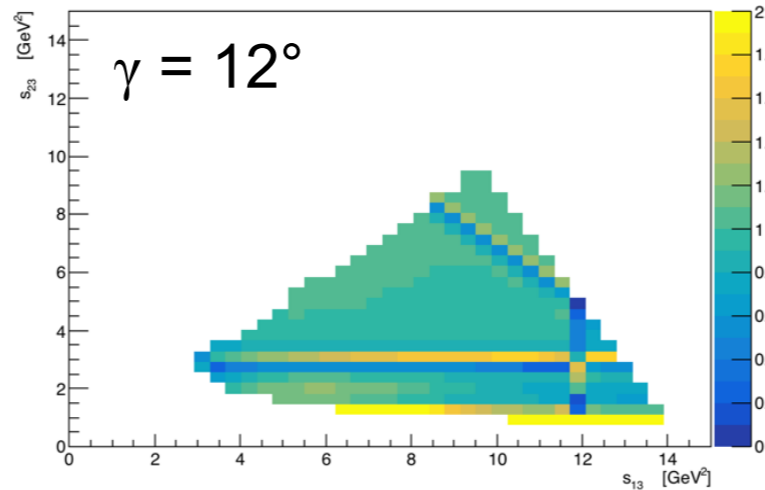
- $R(s_{13}, s_{23})$  varies over the DP, especially near resonances → **as expected.**
- $\langle R(s_{13}, s_{23}) \rangle = 1.03 \approx 1$  → **as expected.**

**The hypothesis of flavour SU(3) symmetry conserved "on average" holds.**

# Test no 2 of flavour SU(3) breaking

- Extract  $\alpha_{SU(3)}$  value by a fit at several single points ( $\approx 400$ ) over the DP fixing  $\gamma$  to the values of the 6 minima we found previously.

$\gamma_i$	$\langle \alpha_{SU(3)} \rangle_i$
$12^\circ$	1.06
$37^\circ$	1.06
$68^\circ$	1.05
$223^\circ$	1.06
$266^\circ$	1.05
$307^\circ$	1.05



**The hypothesis of flavour SU(3) symmetry conserved "on average" holds.**

# Observables

## Observables as functions of the theoretical parameters

$$A = ae^{i\phi_a}, B = be^{i\phi_b}, C = ce^{i\phi_c} \text{ and } D = de^{i\phi_d}$$

$$\phi_a = 0$$

$$X_{K^+\pi^+\pi^-}^{th}(s_1, s_2) = a^2 + (\kappa b)^2 + c^2 + 2ac \cos \phi_c \cos \gamma - 2\kappa ab \cos \phi_b - 2\kappa bc \cos(\phi_b - \phi_c) \cos \gamma$$

$$Y_{K^+\pi^+\pi^-}^{th}(s_1, s_2) = -2(ac \sin \phi_c + \kappa bc \sin(\phi_b - \phi_c)) \sin \gamma$$

$$X_{K_S K^+ K^-}^{th}(s_1, s_2) = \alpha_{SU(3)}^2 X_{K^+\pi^+\pi^-}^{th}$$

$$Y_{K_S K^+ K^-}^{th}(s_1, s_2) = \alpha_{SU(3)}^2 Y_{K^+\pi^+\pi^-}^{th}$$

$$Z_{K_S K^+ K^-}^{th}(s_1, s_2) = \alpha_{SU(3)}^2 (-c^2 \cos \gamma - ac \cos \phi_c + \kappa bc \cos(\phi_b - \phi_c)) \sin \gamma$$

$$X_{K_S \pi^+ \pi^-}^{th}(s_1, s_2) = a^2 + (\kappa d)^2 + d^2 + 2ad \cos \phi_d \cos \gamma - 2\kappa ad \cos \phi_d - 2\kappa d^2 \cos \gamma$$

$$Y_{K_S \pi^+ \pi^-}^{th}(s_1, s_2) = -2ad \sin \phi_d \sin \gamma$$

$$Z_{K_S \pi^+ \pi^-}^{th}(s_1, s_2) = (-d^2 \cos \gamma - ad \cos \phi_d + \kappa d^2) \sin \gamma$$

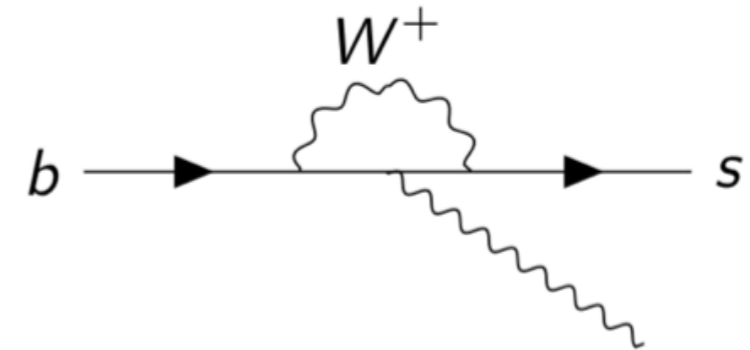
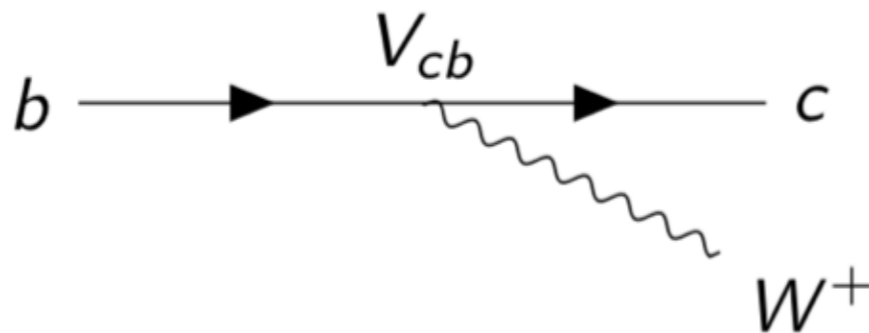
$$X_{K^+\pi^+\pi^0}^{th}(s_1, s_2) = \frac{1}{2} (b^2 + \kappa^2 c^2 - 2\kappa bc \cos \gamma \cos(\phi_b - \phi_c))$$

$$Y_{K^+\pi^+\pi^0}^{th}(s_1, s_2) = \kappa bc \sin \gamma \sin(\phi_b - \phi_c)$$

$$X_{K_S K_S K_S}^{th}(s_1, s_2) = 2\alpha_{SU(3)}^2 a^2$$

# CKM matrix

In the SM, transitions between different quark flavours can only happen in weak interaction through the exchange of a  $W^\pm$  boson.



## CKM matrix

- 3x3 complex unitary matrix.
- 3 angles + 1 phase (= unique source of CPV in SM).

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

## Wolfenstein parametrisation

- 3rd order development in  $\lambda = |V_{us}|$
- $\lambda \approx 0.23$ ,  $A \approx 0.8$ ,  $\rho \approx 0.14$ ,  $\eta \approx 0.35$

## Unexplained hierarchy between transitions

$$V_{CKM} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

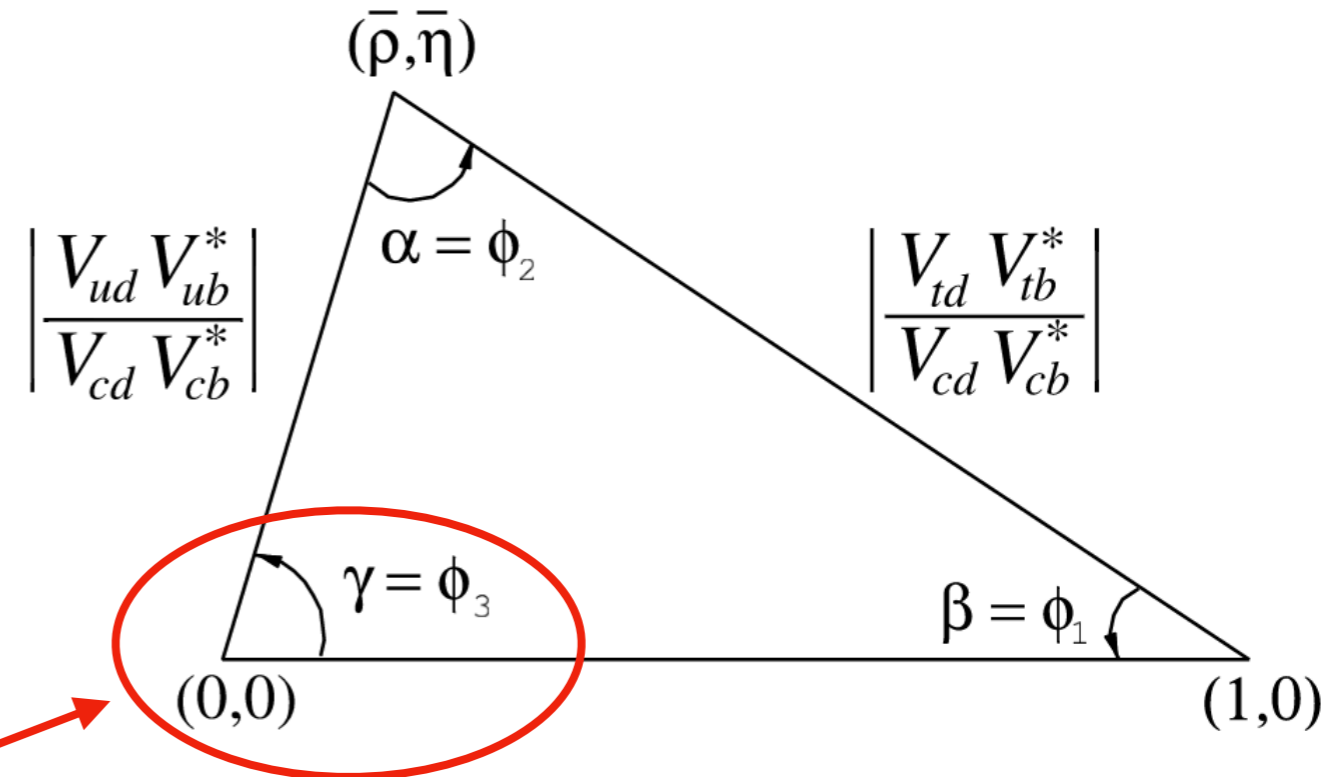
# Unitarity Triangle

CKM matrix is unitary  $\begin{cases} \sum_i V_{ij} V_{ik}^* = \delta_{jk} \\ \sum_j V_{ij} V_{kj}^* = \delta_{jk} \end{cases}$

## Unitarity Triangle defined by:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



$$\gamma = \arg \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]$$

**b → u highly suppressed:  
great precision on  $\gamma$  is hard to achieve.**