

# EXOTIC HADRONS - SEMIEMPIRICAL APPROACH

J. Rosner (Univ. of Chicago) – LHCb Workshop - CERN - Oct. 18, 2018

1964: M. Gell-Mann and G. Zweig proposed that the known mesons were  $q\bar{q}$  and baryons  $qqq$ , with quarks known at the time  $u$  (“up”),  $d$  (“down”), and  $s$  (“strange”) having charges  $(2/3, -1/3, -1/3)$ . Mesons and baryons would then have integral charges.

Mesons such as  $qqq\bar{q}$  and baryons such as  $qqqq\bar{q}$  would also have integral charges. Why weren't they seen?

They *have* now been seen, as “molecules” of heavy-quark hadrons or as deeply bound states involving heavy quarks (charm and bottom).

Charm-anticharm and bottom-antibottom molecules; “pentaquark” as a charmed meson – baryon molecule;  $\Xi_{cc}^{++} = ccu$  as the first doubly charmed baryon;  $ccs$  mass; stable  $bb\bar{u}\bar{d}$  tetraquark; quark fusion.

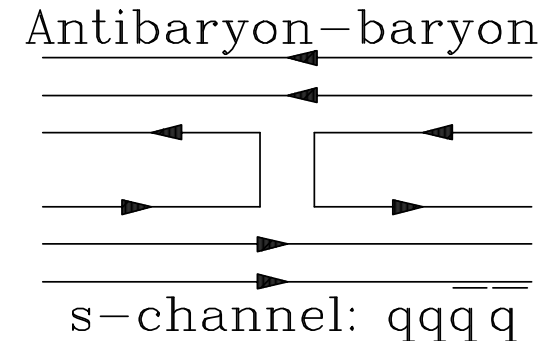
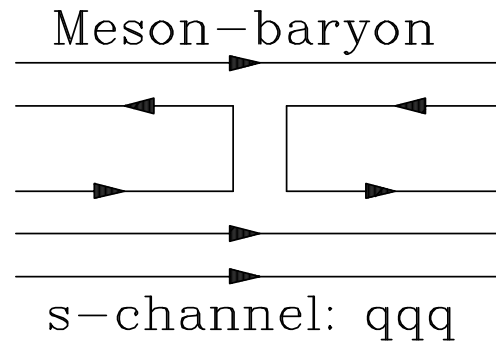
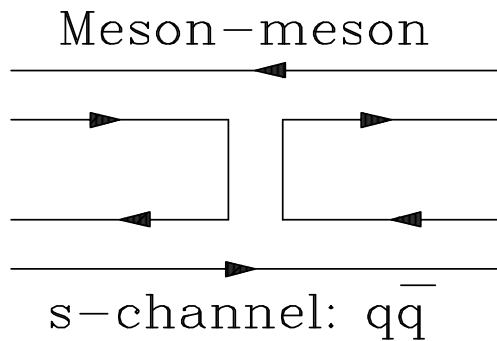
Thanks to Marek Karliner, Michael Gronau, and Shmuel Nussinov for enjoyable collaborations. References at end; review by M. Karliner, T. Skwarnicki, JLR, arXiv:1711.10626.

Today:  $\Xi_{cc}^{++} = ccu$ ,  $\Omega_c = ccs$  mass; stable  $bb\bar{u}\bar{d}$  tetraquark.

# EARLY HINT OF EXOTICS

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Processes “dual” (JLR, 1968) to  $t$ -channel  $q\bar{q}$  exchange:

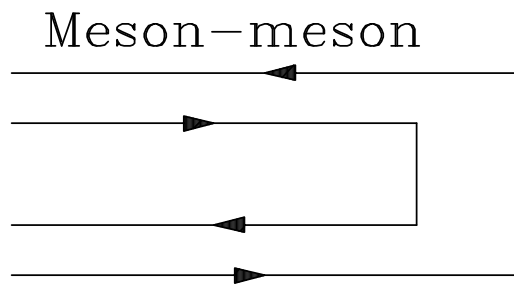


$s$ -channel resonances  $\Leftrightarrow t$ -channel Regge trajectories

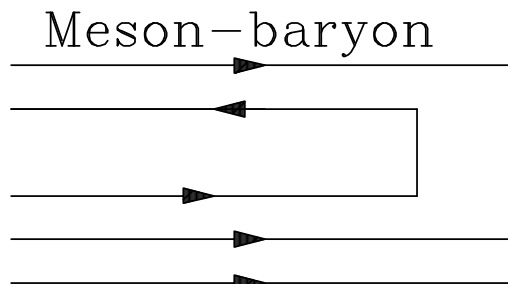
In antiproton-proton scattering,  $q\bar{q}$  dual to  $qqq\bar{q}$

Predicts “exotic”  $qqq\bar{q}$  mesons, but where?

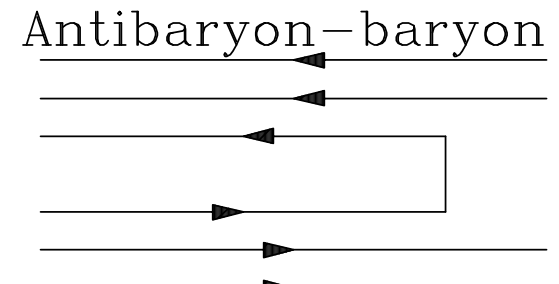
Do resonances form via  $q\bar{q}$  annihilation? (JLR, 1972):



$p^* \leq 350 \text{ MeV}/c$



$p^* \leq 250 \text{ MeV}/c$



$p^* \leq 200 \text{ MeV}/c?$

# HEAVY QUARKS STABILIZE EXOTICS

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Compton wavelengths are shorter; deeper QCD binding

Molecular states (predicted in 1976) make their appearance

Heavier quarks: less kinetic energy; favorable to binding

Two or more heavy quarks have an affinity for one another

First heavy exotic:  $X(3872)$ ,  $\rightarrow J/\psi\pi\pi$ ,  $J/\psi3\pi$

Appears to be an S-wave bound state of  $D^0\bar{D}^{*0} + c.c.$  mixed with a charmonium  $\chi_{c1}(2P)$  state

Mass within 0.2 MeV of  $D^0\bar{D}^{*0}$  threshold; bound?

Similar behavior with  $b$  quarks:  $Z_b(10610)$  near  $B\bar{B}^*$  threshold,  $Z_b(10650)$  near  $B^*\bar{B}^*$  threshold

$P_c = c\bar{c}uud$  candidates at (4380,4450) MeV:  $\Sigma_c\bar{D}^*$  molecules or tightly bound pentaquarks?

# BARYONS WITH $> 1$ HEAVY QUARK

4/19

So far:  $Q\bar{Q}q\bar{q}'$  or  $Q\bar{Q}qqq'$  ( $Q = \text{heavy}, q, q' = \text{light}$ ). Can we predict masses of (simpler)  $QQ'q$  systems?

SELEX at Fermilab (2002-5) claimed  $\Xi_{cc}^{++}(3520) = ccu$  and  $\Xi_{cc}^+(3460) = ccd$ ; not confirmed by others

M. Karliner + JLR (PR D **90**): Constituent-quark masses, hyperfine splittings, estimates of  $QQ'$  binding ( $q = u, d$ ):

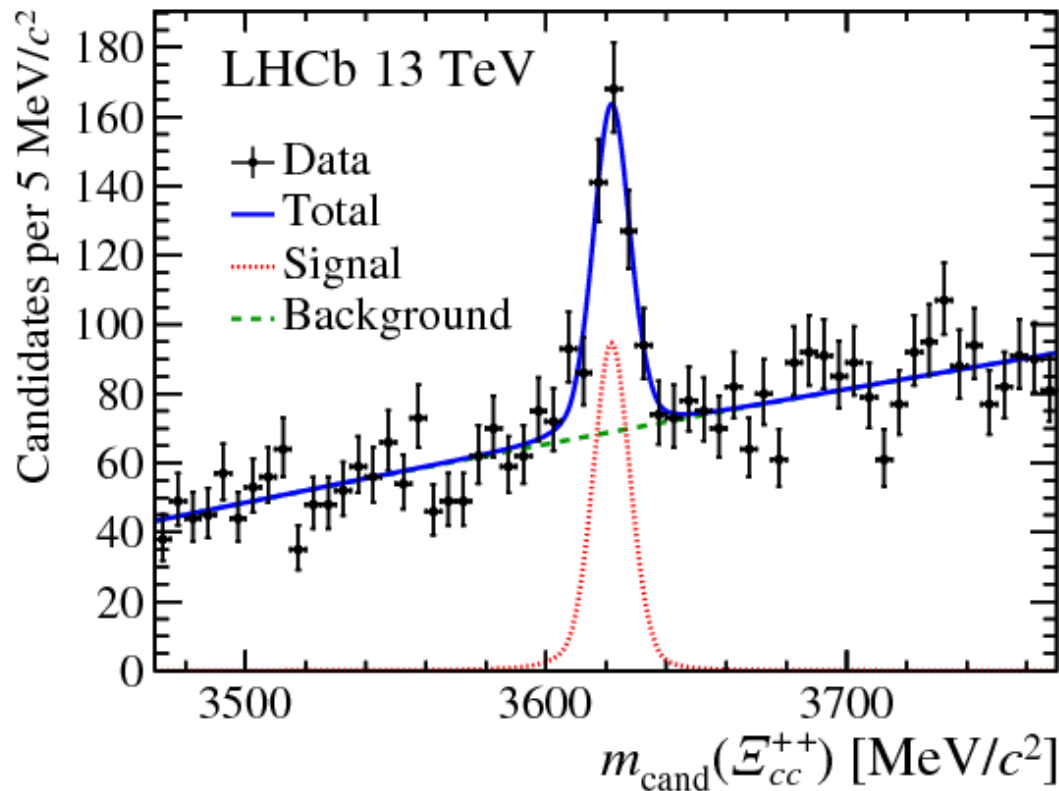
State	Quark content	$M(J = 1/2)$	$M(J = 3/2)$
$\Xi_{cc}^{(*)}$	$ccq$	$3627 \pm 12$	$3690 \pm 12$
$\Xi_{bc}^{(*)}$	$b[cq]$	$6914 \pm 13$	$6969 \pm 14$
$\Xi'_{bc}$	$b(cq)$	$6933 \pm 12$	–
$\Xi_{bb}^{(*)}$	$bbq$	$10162 \pm 12$	$10184 \pm 12$

LHCb PRL **119**, 112001:  $M(\Xi_{cc}^{++}) = 3621.40 \pm 0.78$  MeV

Other estimates ( $> 30$ ): spread of at least 100 MeV

# $\Lambda_c K^- \pi^+ \pi^+$ SPECTRUM

5/19



Similar peak seen in 8 TeV data; no  $\Lambda_c K^- \pi^+$  peak

We predicted  $\tau(\Xi_{cc}^{++,+}) = (185,53)$  fs;  $\Lambda_c K^- \pi^+$  peak disfavored by LHCb lifetime cut  $\tau > 150$  fs

LHCb (PRL **121**, 052001):  $\tau(\Xi_{cc}^{++}) = 256_{-22}^{+24} \pm 14$  fs

# INPUTS

6/19

Describe ground-state baryons containing  $u, d, s$  taking  $m_u^b = m_d^b \equiv m_q^b = 363$  MeV,  $m_s^b = 538$  MeV, and hyperfine interaction term  $a/(m_q^b)^2 = 50$  MeV

State (mass in MeV)	Spin	Expression for mass	Predicted mass (MeV)
$N(939)$	1/2	$3m_q^b - 3a/(m_q^b)^2$	939
$\Delta(1232)$	3/2	$3m_q^b + 3a/(m_q^b)^2$	1239
$\Lambda(1116)$	1/2	$2m_q^b + m_s^b - 3a/(m_q^b)^2$	1114
$\Sigma(1193)$	1/2	$2m_q^b + m_s^b + a/(m_q^b)^2 - 4a/m_q^b m_s^b$	1179
$\Sigma(1385)$	3/2	$2m_q^b + m_s^b + a/(m_q^b)^2 + 2a/m_q^b m_s^b$	1381
$\Xi(1318)$	1/2	$2m_s^b + m_q^b + a/(m_s^b)^2 - 4a/m_q^b m_s^b$	1327
$\Xi(1530)$	3/2	$2m_s^b + m_q^b + a/(m_s^b)^2 + 2a/m_q^b m_s^b$	1529
$\Omega(1672)$	3/2	$3m_s^b + 3a/(m_s^b)^2$	1682

Describe mesons with quark masses  $m_{u,d,s}^m \sim 55$  MeV less

$$M(\Lambda_{c,b}) - M(\Lambda) \Rightarrow m_{c,b}^b = (1710.5, 5043.5) \text{ MeV}$$

# CHARMED & BOTTOM BARYONS <sup>7/19</sup>

Above choices of mass sufficient to describe nonstrange baryons with one  $c$  or  $b$  quark

When taking account of deeper  $cs$  or  $bs$  binding in baryons with one or two strange quarks and one charm or bottom fit all baryons with one  $c$  or  $b$

Charmed baryons			Bottom baryons		
State ( $M$ in MeV)	Spin	Predicted $M$ (MeV)	State ( $M$ in MeV)	Spin	Predicted $M$ (MeV)
$\Lambda_c(2286.5)$	1/2	Input	$\Lambda_b(5619.5)$	1/2	Input
$\Sigma_c(2453.4)$	1/2	2444.0	$\Sigma_b(5814.3)$	1/2	5805.1
$\Sigma_c^*(2518.1)$	3/2	2507.7	$\Sigma_b^*(5833.8)$	3/2	5826.7
$\Xi_c(2469.3)$	1/2	2475.3	$\Xi_b(5792.7)$	1/2	5801.5
$\Xi_c'(2575.8)$	1/2	2565.4	$\Xi_b'(-)$	1/2	5921.3
$\Xi_c^*(2645.9)$	3/2	2628.6	$\Xi_b^*(5949.7)$	3/2	5944.1
$\Omega_c(2695.2)$	1/2	2692.1	$\Omega_b(6046.4)$	1/2	6042.8
$\Omega_c^*(2765.9)$	3/2	2762.8	$\Omega_b^*(-)$	3/2	6066.7

# HEAVY QUARK PAIR BINDING<sup>8/19</sup>

Quark pair more deeply bound when neither is  $u$  or  $d$

$$B(c\bar{s}) = [3M(D_s^*) + M(D_s)]/4 - m_s^m - m_c^m = -69.9 \text{ MeV}$$

Assume  $B(cs)/B(c\bar{s}) = 1/2$  as for single-gluon exchange

Then  $B(cs) = -35 \text{ MeV}$ ; also find  $B(bs) = -41.8 \text{ MeV}$

Rescale hyperfine interactions when neither quark is  $u$  or  $d$ ; take a cue from  $M(D_s^*) - M(D_s) \simeq M(D^*) - M(D)$

Now we are ready to deal with  $cc$ ,  $cb$ ,  $bb$

Charm-anticharm binding:  $B(c\bar{c}) = [3M(J/\psi) + M(\eta_c)]/4 - 2m_c^m = -258 \text{ MeV}$ , so  $B(cc) = -129 \text{ MeV}$

Similar calculations give  $B(bb) = -281.4 \text{ MeV}$  and  $B(bc) = -167.8 \pm 3.0 \text{ MeV}$  (uncertainty in  $B_c^*$  mass)



# CONTRIBUTIONS

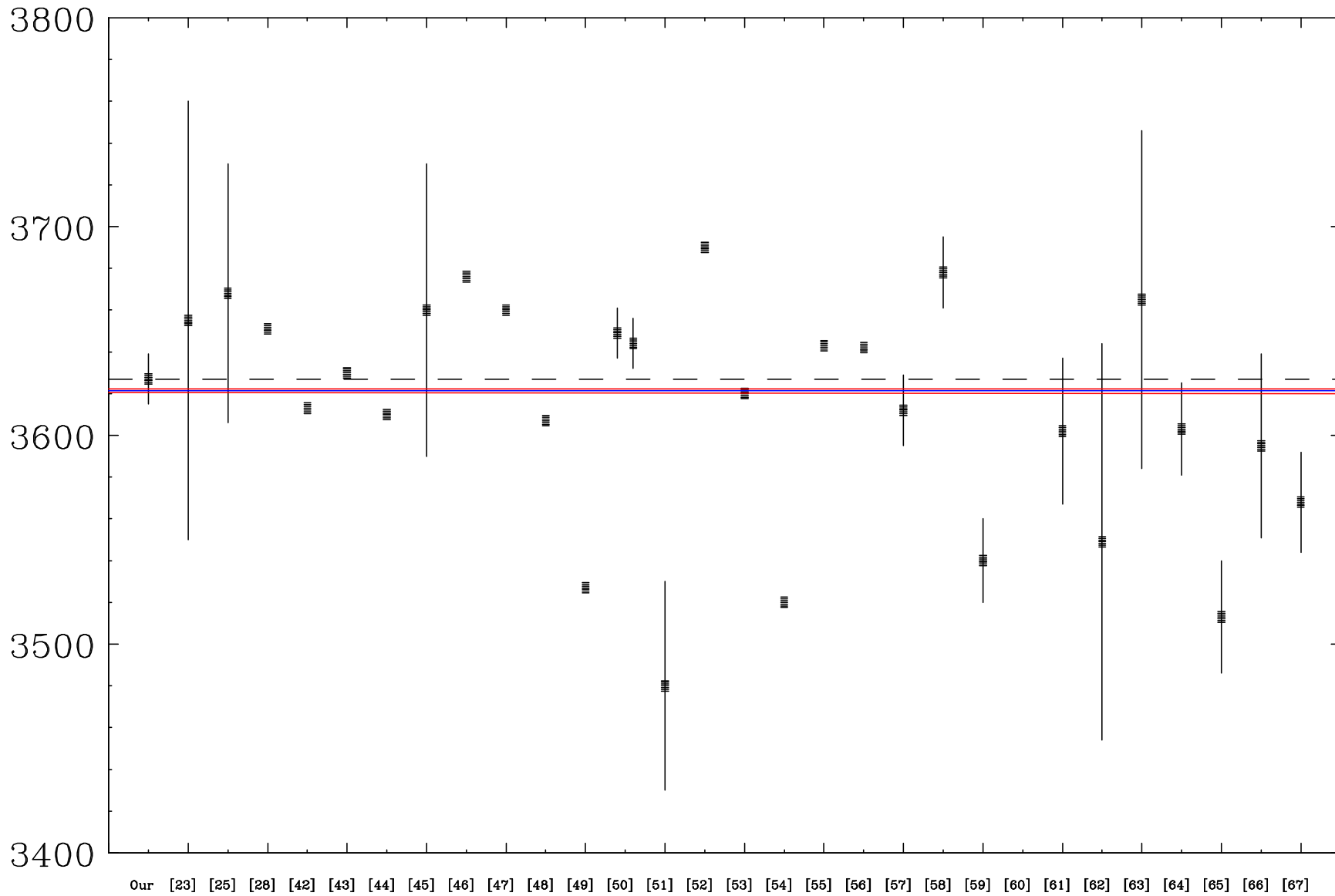
9/19

Contribution	Value (MeV)	Contribution	Value (MeV)
$2m_c^b + m_q^b$	3783.9	$2m_b^b + m_q^b$	10450.0
$cc$ binding	-129.0	$bb$ binding	-281.4
$a_{cc}/(m_c^b)^2$	14.2	$a_{bb}/(m_b^b)^2$	7.8
$-4a/m_q^b m_c^b$	-42.4	$-4a/m_q^b m_b^b$	-14.4
$M(\Xi_{cc})$	$3627 \pm 12$	$M(\Xi_{bb})$	$10162 \pm 12$

Contribution	Value (MeV)	Contribution	Value (MeV)
$m_b^b + m_c^b + m_q^b$	7117.0	$m_b^b + m_c^b + m_q^b$	7117.0
$bc$ binding	-170.8	$bc$ binding	-170.8
$-3a/(m_c^b m_q^b)$	-31.8	$a/(m_c^b m_q^b)$	10.6
-	-	$-2a/(m_b^b m_q^b)$	-24.2
-	-	$-2a_{bc}/(m_b^b m_c^b)$	
$M(\Xi_{bc})$	$6914 \pm 13$	$M(\Xi'_{bc})$	$6933 \pm 12$

# PREDICTIONS FOR $\Xi_{cc}$

10/19



Our prediction is on the left; others: PR D **90**, 094007

# PREDICTIONS FOR $\Omega_{cc} = cc s$ 11/19

Strange quark is about 175 MeV heavier than nonstrange but more deeply bound to  $cc$  diquark than nonstrange

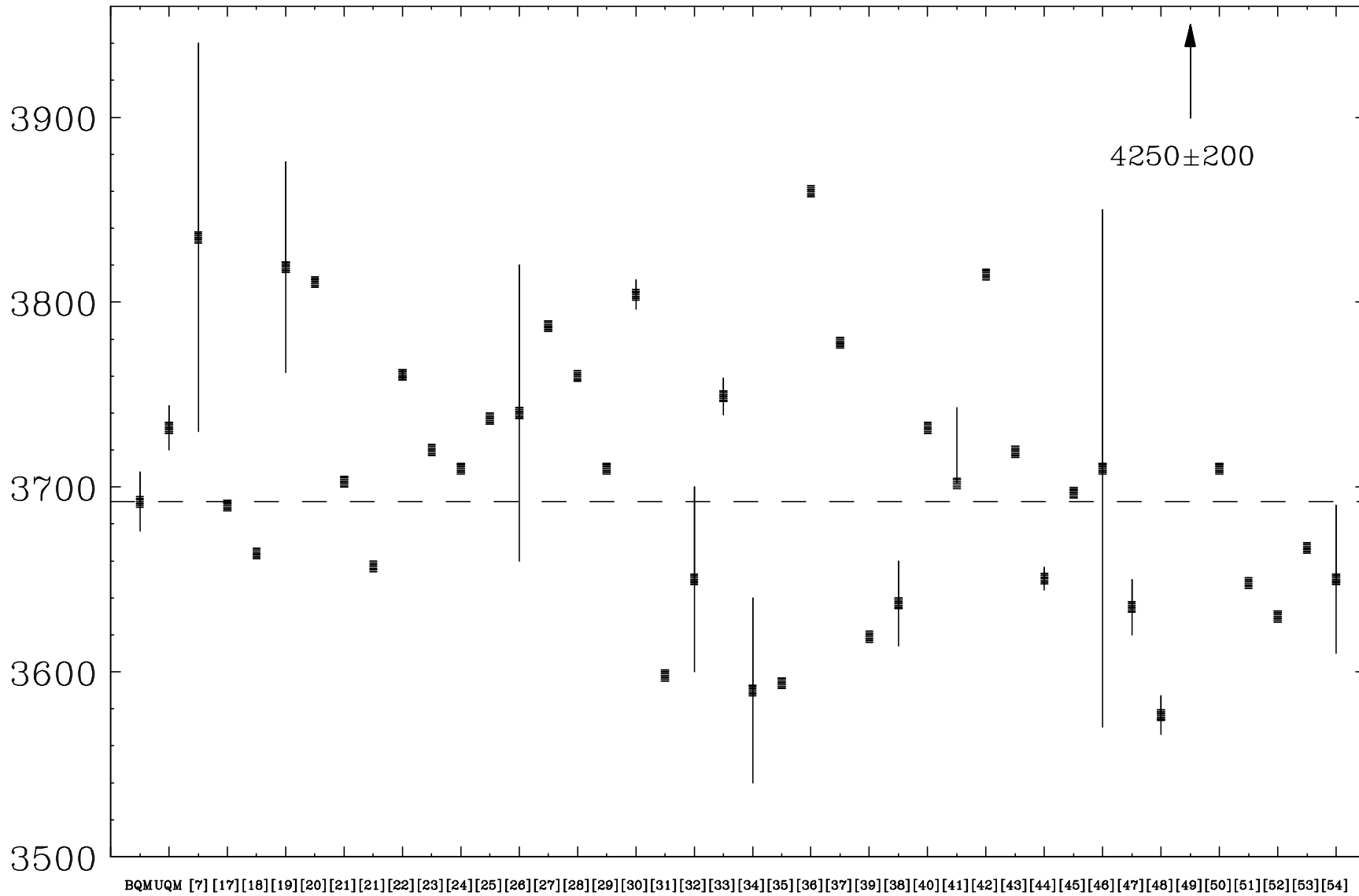
$\Xi_{cc} = ccq$		$\Omega_{cc} = cc s$	
Contribution	Value (MeV)	Contribution	Value (MeV)
$2m_c^b + m_q^b$	3789.0	$2m_c^b + m_s^b$	3959.0
$cc$ binding	-129.0	$cc$ binding	-129.0
$a_{cc}/(m_c^b)^2$	14.2	$a_{cc}/(m_c^b)^2$	14.2
$-4a/m_q^b m_c^b$	-42.4	$-4a'/m_s^b m_c^b$	-42.4
Total	3626.8 $\pm$ 12	Subtotal	3801.8 $\pm$ 12

Additional binding of  $s$  to  $cc$ :  $-109.4 \pm 10.5$  MeV, giving  $M(\Omega_{cc}) = 3692 \pm 16$  MeV,  $M(\Omega_{cc}^*) = 3756 \pm 16$  MeV

Superscripts on quark masses: value in a baryon

Universal quark masses and 165 MeV “string junction” term for baryons: predict  $M(\Omega_{cc}) \sim 40$  MeV higher

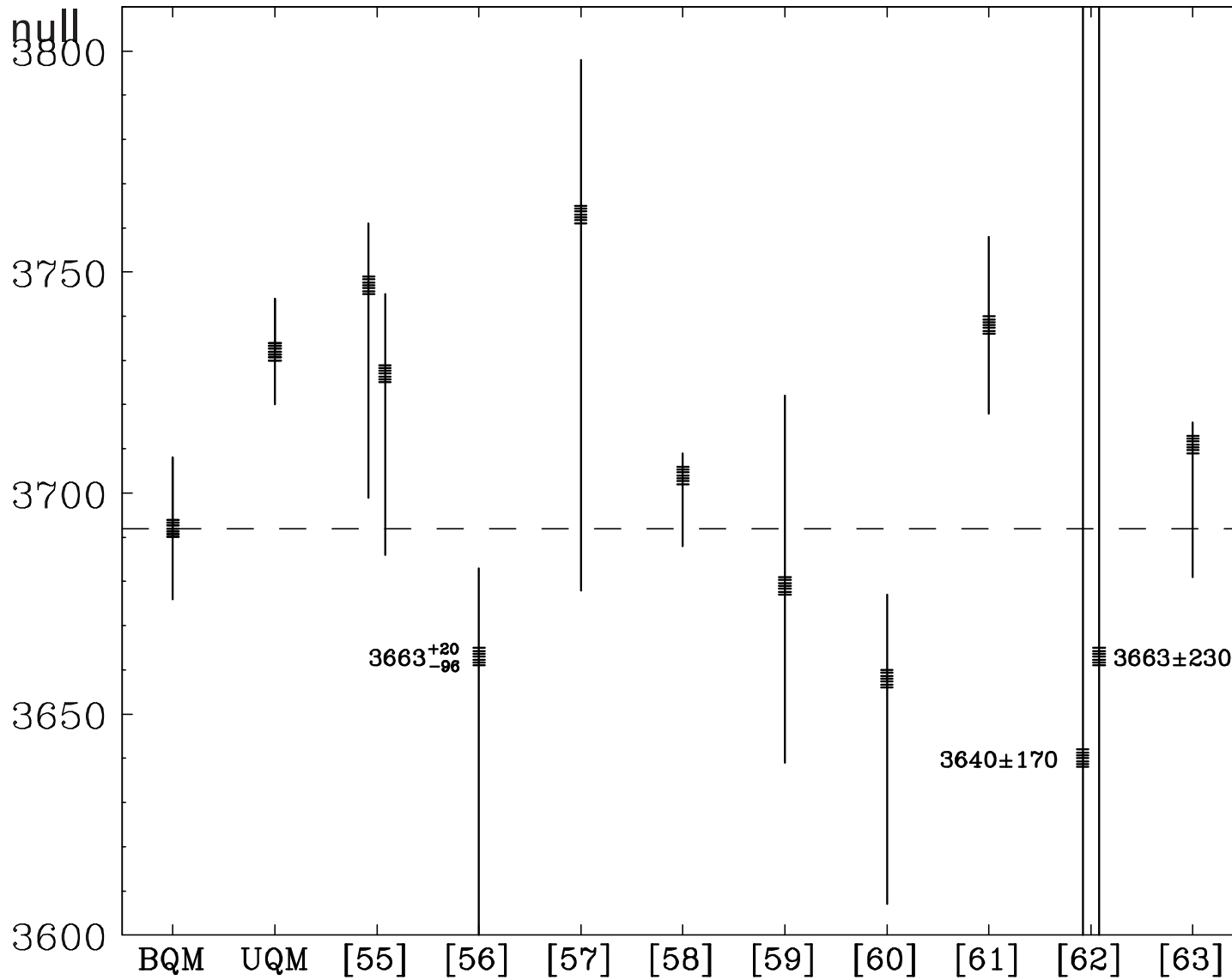
# COMPARISONS (NON-LATTICE) <sup>12/19</sup>



Our prediction is on the left; others: PR D **97**, 094006

# COMPARISONS (LATTICE)

13/19



Our prediction is on the left; others: PR D **97**, 094006

# DECAYS AND LIFETIMES

14/19

$\Xi_{cc}^{++} = ccu$ : A  $c$  quark decays to  $W^{*+} s \rightarrow (\pi^+, \rho^+, a_1^+) s$

Remnant  $csu$  can decay to  $W^{*+} s$  and  $ssu$  remnant which then can be  $\Xi^0$  (hard to detect) or decay to (e.g.)  $\Xi^- \pi^+$

Or remnant  $csu$  fragments  $\rightarrow (\Lambda_c K^- \pi^+, \Xi_c^0 \pi^+, \dots)$

$\Xi_{cc}^+ = ccd$ : Subprocess  $c(cd) \rightarrow c(su)$  shortens lifetime

$\Xi_{bc}^+ = bcu$ :  $\Gamma(W^{*-}(\Xi_{cc}, \Xi_b)) = (6.9, 20.1) \times 10^{-13}$  GeV

Some predicted lifetimes, in fs:

Baryon	This work	Anikeev	Kiselev	bj2	Moinester	Expt.
$\Xi_{cc}^{++} = ccu$	185	430±100	460±50	500	~ 200	256± 27
$\Xi_{cc}^+ = ccd$	53	120±100	160±50	150	~ 100	—
$\Xi_{bc}^+ = bcu$	244	330±80	300±30	200	—	—
$\Xi_{bc}^0 = bcd$	93	280±70	270±30	150	—	—
$\Xi_{bb}^0 = bbu$	370	—	790±20	—	—	—
$\Xi_{bb}^- = bbd$	370	—	800±20	—	—	—

# PRODUCTION

15/19

Production of two heavy quarks: need also two antiquarks

Production of  $\Xi_{cc} = ccu$  related to that of  $B_c = \bar{b}c$

Somewhat smaller because have to pick up a light quark

PR D **90**, 2014:  $\sigma(\Xi_{cc}, \Xi_{bc})/\sigma(B_c + B_c^*) \simeq 10 - 50\%$

Result: estimate  $\Sigma(pp \rightarrow \Xi_{cc} + X) \leq 40$  nb at 7 TeV

A. V. Berezhnoy +, arXiv:1809.10058:

$\sigma(\Xi_{bc})/\sigma(B_c) \leq 1/3$ ;  $\sigma(\Xi_{bc}) \sim 10 - 25$  nb at 13 TeV

A useful ratio to be measured in  $e^+e^-$ :

$$\sigma(e^+e^- \rightarrow \Xi_{cc} + X) \sim \sigma(e^+e^- \rightarrow \Xi_c + X) \cdot \frac{\sigma(e^+e^- \rightarrow \Xi_c + X)}{\sigma(e^+e^- \rightarrow \Xi + X)}$$

# STABLE $bb\bar{u}\bar{d}$ TETRAQUARK 16/19

We looked at  $QQ'\bar{u}\bar{d}$  systems ( $Q = c$  or  $b$ ) (PRL **119**)

We found  $cc\bar{u}\bar{d}$  unbound; it could decay to  $DD^*$  or  $DD\gamma$

Lowest-lying  $bc\bar{u}\bar{d}$  state was near  $BD\gamma$  threshold and we could not tell for sure whether it was bound or unbound

Predicted  $M(bb\bar{u}\bar{d}) = 10,389 \pm 12$  MeV, 215 MeV below  $B^-B^{*0}$  threshold and 170 MeV below  $B^-B^0\gamma$  threshold

Regard  $bb$  as a color- $3^*$  diquark (transforming under QCD as an antiquark); fermi statistics require its spin to be 1

Lightest  $\bar{q}\bar{q}'$  state ( $q, q' = u, d$ ) is a color-3  $\bar{u}\bar{d}$  state with isospin zero; fermi statistics require its spin to be zero

Mass prediction then relies on accounting for constituent-quark masses, hyperfine interactions, and binding effects



# TETRAQUARKS $QQ'\bar{u}\bar{d}$

17/18

Contributions (MeV) to mass of lightest tetraquark:

$cc\bar{u}\bar{d}, J^P = 1^+$		$bc\bar{u}\bar{d}, J^P = 0^+$		$bb\bar{u}\bar{d}, J^P = 1^+$	
Contribution	Value	Contribution	Value	Contribution	Value
$2m_c^b$	3421.0	$m_b + m_c$	6754.0	$2m_b^b$	10087.0
$2m_q^b$	726.0	$2m_q^b$	726.0	$2m_q^b$	726.0
$cc$ hyperfine	14.2	$bc$ hyperfine	-25.5	$bb$ hyperfine	7.8
$-3a/(m_q^b)^2$	-150.0	$-3a/(m_q^b)^2$	-150.0	$-3a/(m_q^b)^2$	-150.0
$cc$ binding	-129.0	$bc$ binding	-170.8	$bb$ binding	-281.4
Total	$3882 \pm 12$	Total	$7134 \pm 13$	Total	$10389 \pm 12$

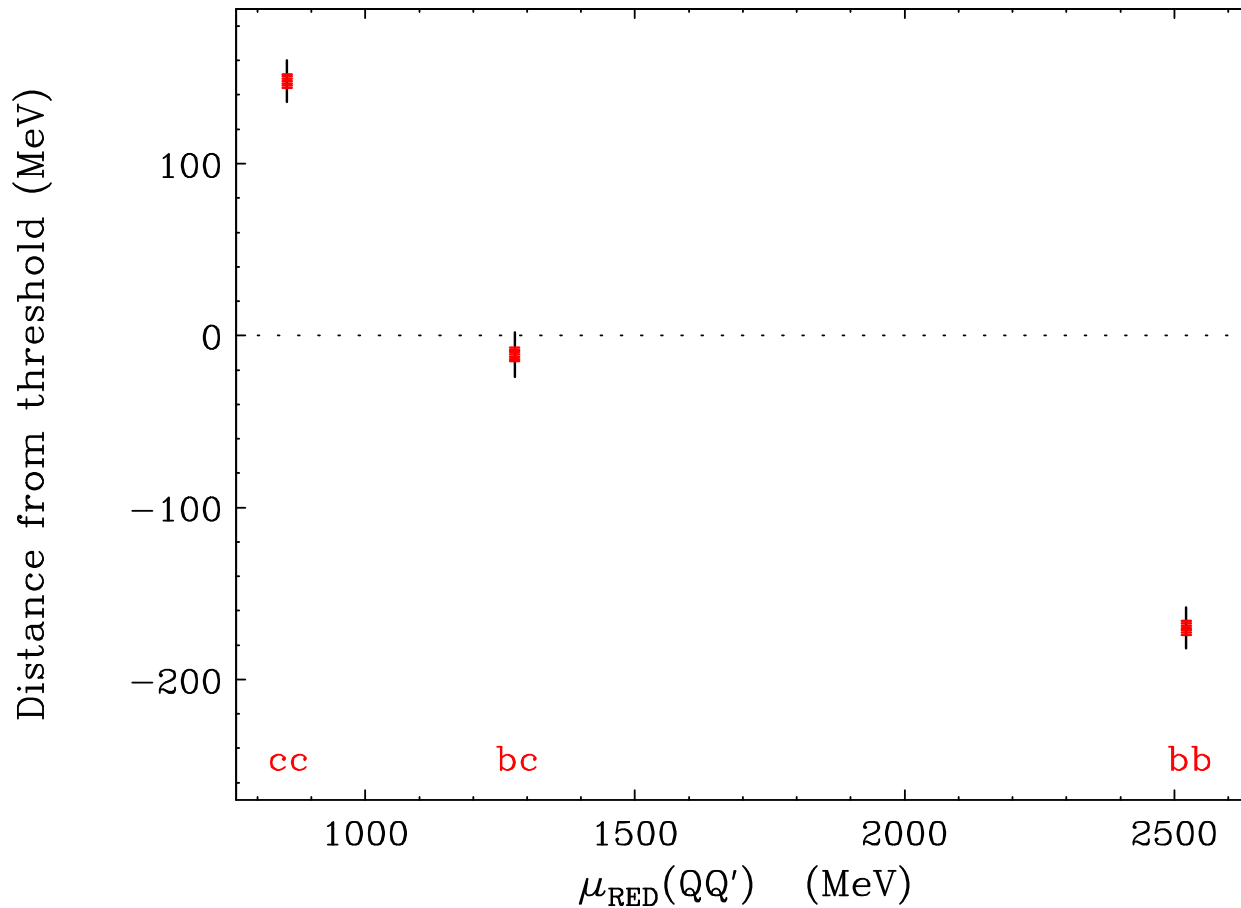
Spin zero allowed for the  $bc\bar{u}\bar{d}$  state, taking advantage of the attractive  $bc$  hyperfine interaction

Since  $M(cc\bar{u}\bar{d}) > M(D^0) + M(D^+) = 3734$  MeV, it can decay to  $D^0 D^+ \gamma$  (decay to  $D^0 D^+$  is forbidden)

$M(bc\bar{u}\bar{d}) < M(D^0) + \bar{M}(B^0) = 7144$  MeV?

Estimated lifetime of  $bb\bar{u}\bar{d}$  state: 367 fs

# COMPARISON OF TQ MASSES 18/19



Distance in MeV of the  $cc\bar{u}\bar{d}$ ,  $bc\bar{u}\bar{d}$  and  $bb\bar{u}\bar{d}$  tetraquark masses from corresponding thresholds  $D^0 D^+ \gamma$ ,  $\bar{B}^0 D^0$ , and  $\bar{B}^0 B^- \gamma$ , plotted against reduced masses of the doubly-heavy diquarks  $\mu(QQ')$ ,  $Q, Q' = c, b$ .

# PROSPECTS

19/19

Exotic mesons and baryons (beyond  $q\bar{q}$  and  $qqq$ ) *do* exist; molecular configurations are at least part of the story

Heavy quarks have a lower kinetic energy and help to stabilize exotic configurations containing them

Techniques for mass estimation (constituent-quark masses, hyperfine interactions, binding effects) relatively straightforward and starting to be tested for  $QQ'q$  baryons

Frontier:  $Q_1Q_2\bar{Q}_3\bar{Q}_4$ ; any  $cc\bar{c}\bar{c}$  lighter than  $2M(\eta_c)$ ? Any  $bb\bar{b}\bar{b}$  lighter than  $2M(\eta_b)$ ?

Can quark-level analogue of nuclear fusion be put to use?

Still to be known: What does it cost to produce one or more extra heavy quarks via strong interactions? When do two heavy quarks end up in the same hadron?

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29/19

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M. Gronau, JLR, and C. G. Wohl, arXiv:1808.03720 (depiction of  $\Lambda_c$  branching fractions)

M. Karliner and JLR, arXiv:1808.07869 (P-wave excitation energies)

# P-WAVE EXCITATIONS

22/19

What is the splitting between S and P wave levels?

M. Karliner and JLR, arXiv:1808.07869, to be published in PR D

Shortcut avoids solving relativistic wave equation

Quarks have effective masses and pair binding energies

Effective quark masses, MeV    Binding terms  $B(q_1q_2)$ , MeV

Quark	In a meson	In a baryon
$u$	$m_u^m = 310$	$m_u^b = 363$
$d$	$m_d^m = 310$	$m_d^b = 363$
$s$	$m_s^m = 483$	$m_s^b = 538$
$c$	$m_c^m = 1663.3$	$m_c^b = 1710.5$
$b$	$m_b^m = 5003.8$	$m_b^b = 5043.5$

Pair $q_1q_2$	$B(q_1q_2)$	$B(q_1\bar{q}_2)$
$cs$	35.0	70.0
$bs$	41.8	83.6
$cc$	129	258
$bc$	170.8	341.5
$bb$	281.4	562.8

Know S–P splitting of  $\Lambda$ ,  $\Lambda_c$ ,  $\Lambda_b$ ,  $\Xi_c$ , and  $\Omega_c$  baryons (treat as 2-body systems);  $D_s$ ,  $c\bar{c}$ , and  $b\bar{b}$  mesons

Plot differences  $\Delta E_R \equiv \Delta E_{P-S} - \sum B$  vs. reduced mass  $\mu_{12}$

# RESIDUAL ENERGY DIFFERENCES $\Delta E_R$ 23/19

System	$q_1$	$q_2$	$m_1$	$m_2$	$\mu_{12}$	$\Delta E_{P-S}$	$\sum B$	$\Delta E_R$
$\Lambda$	$[ud]$	$s$	576.0	538	278.2	365.7	0	365.7
$\Lambda_c$	$[ud]$	$c$	576.0	1710.5	430.9	329.7	0	329.7
$\Lambda_b$	$[ud]$	$b$	576.0	5043.5	517.0	297.8	0	297.8
$\Xi_c$	$[qs]$	$c$	799.8	1710.5	545.0	340.3	35.0	305.3
$\Omega_c$	$(ss)$	$c$	1098.8	1710.5	669.0	337.6	70.0	267.6
$D_s$	$c$	$s$	1663.3	483	374.3	436.0	70.0	366.0
$c\bar{c}$	$c$	$c$	1663.3	1663.3	831.6	456.8	258.0	198.8
$b\bar{b}$	$b$	$b$	5003.8	5003.8	2501.9	454.8	563	-108.2

$[ud]$  is a spin-zero isospin-zero color- $3^*$  diquark

$[qs]$  is a spin-zero isospin-1/2 color- $3^*$  diquark ( $q = u, d$ )

$(ss)$  is a spin-1 isospin-zero color  $3^*$  diquark

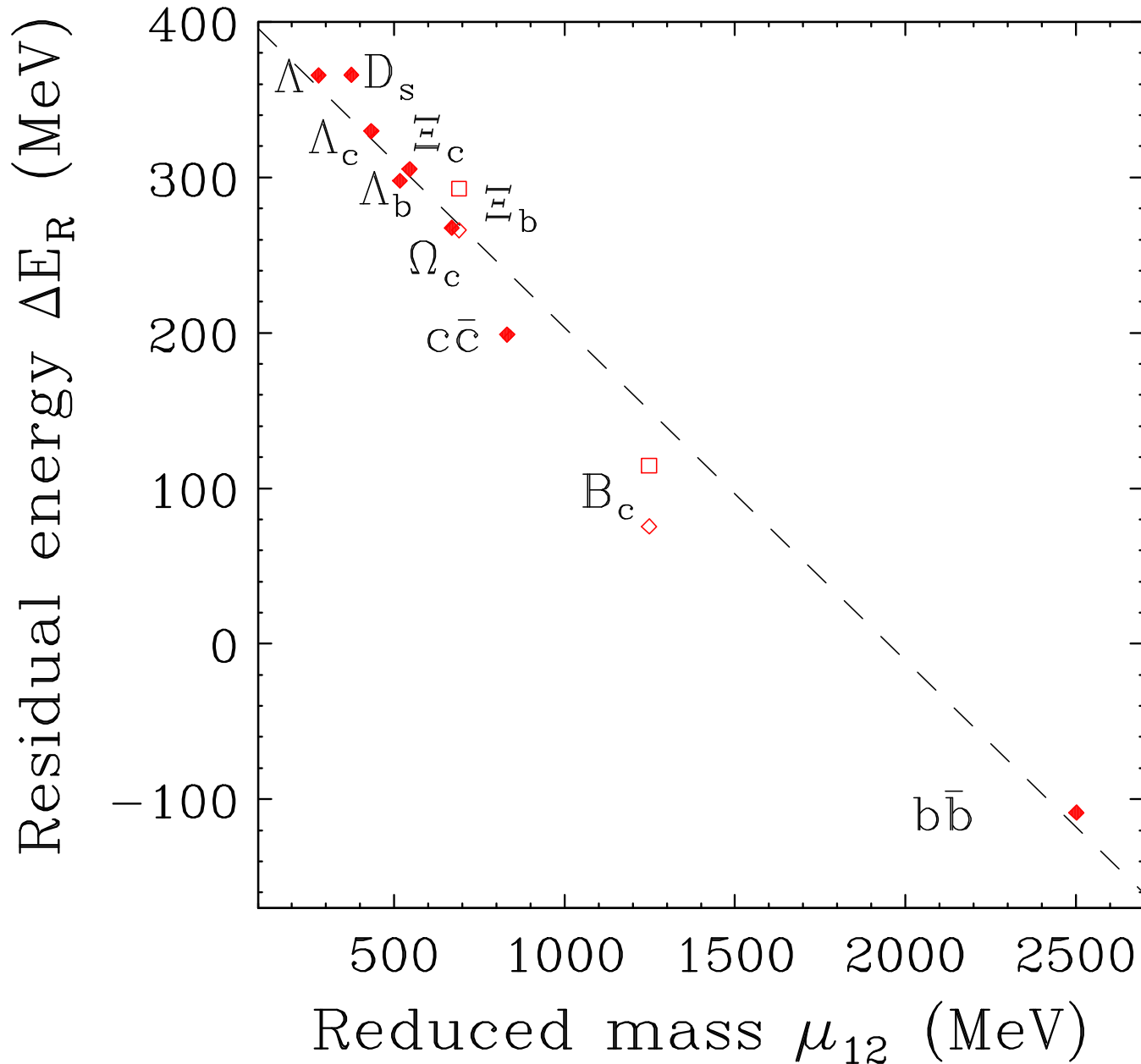
Effective masses and hyperfine interactions  $\Rightarrow m_1$  values

Approximate relation  $\Delta E_R \simeq (417.37 - 0.2141 \mu_{12}) \text{ MeV}$

# $\Delta E_R$ VS. REDUCED MASS

24/19

P-wave excitations



Dashes: lin. fit

$D, B$ : Not all

P waves known

Filled diamonds:  
data used in fit

Hollow symbols:  
Theor. preds.

arXiv:1808.07869:

consequences for

$\Sigma_{c,b}, \Omega_c, \Xi_b$  baryons,

$D, D_s, B, B_c$  mesons