Lattice QCD overview: form-factors, V_{cb}, V_{ub}, R-ratios, ...

Implications of LHCb measurements and future prospects
CERN, October 2018

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UNIVERSITY OF Southampton

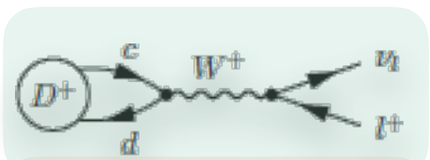
Lattice Flavour Physics and CKM

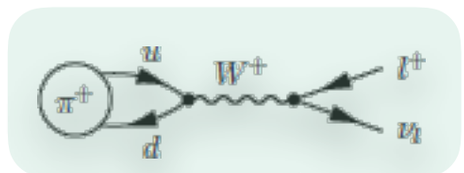
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

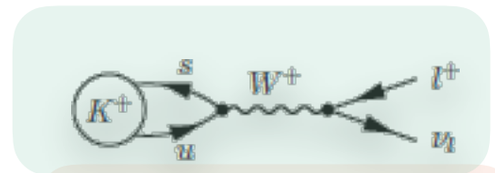
Lattice Flavour Physics and CKM

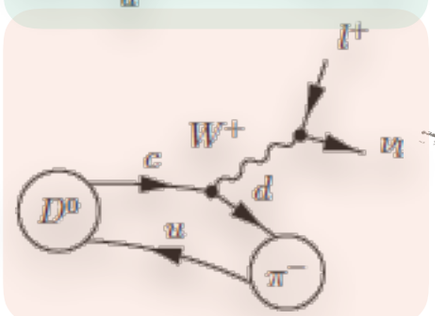
illustrations from
L. Lellouch's Les Houches
Lecture arXiv:1104.5484

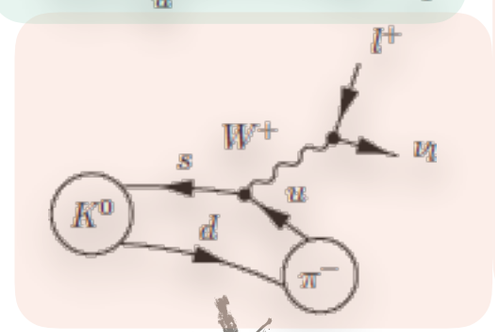
leptonic decays

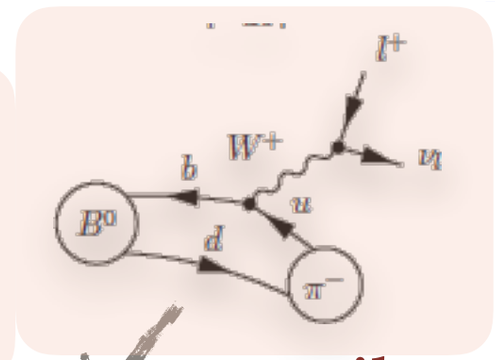




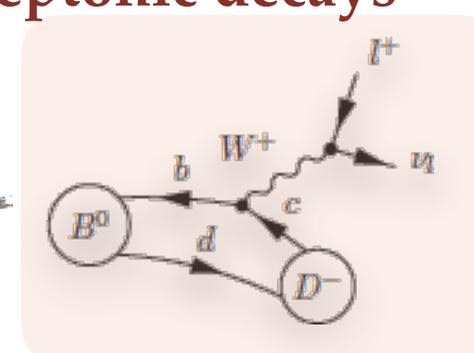




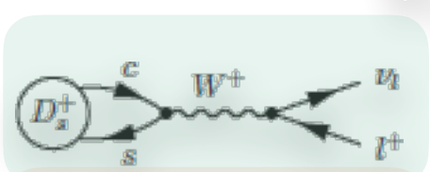


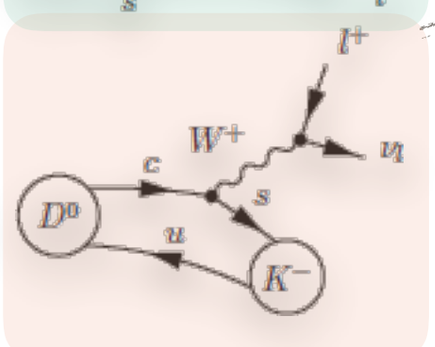


semileptonic decays

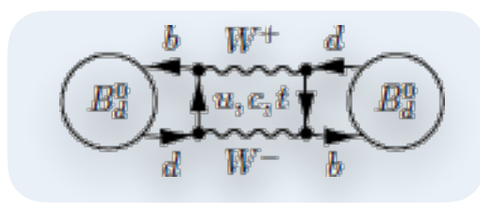


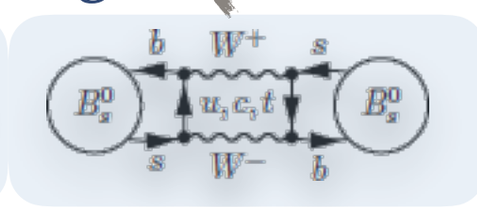
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





mixing





Assumptions

We assume factorisation of SM:

$$\Gamma_{\text{exp.}} \overset{???}{=} V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$$

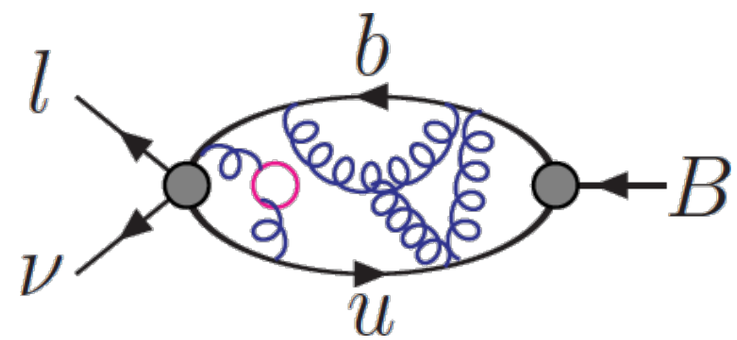
Strong contribution given in terms of hadronic form factors (lattice)

Weak & EM & strong treated separately — although in real world all three SM sectors talk to each other

In particular EM:
Note that $O(\alpha_{EM}) \approx 1\%$ — so OK as long as we keep it in mind

Quark Flavour Physics

e.g tree level leptonic *B* decay:



Assumed factorisation: $\Gamma_{\text{exp.}} \overset{???}{=} V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$

$\langle 0|A_\mu|B\rangle_{\text{QCD}}$

$$\underbrace{\Gamma(B \to l\nu_l)}_{\text{experiment}} = \underbrace{|V_{ub}|^2}_{\text{output}} \underbrace{\frac{m_B}{8\pi} G_F^2 m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2}_{\text{theory prediction}}$$

Lattice QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

Free parameters:

- gauge coupling $g \rightarrow \alpha_s = g^2/4\pi$
- quark masses $m_f = u,d,s,c,b,t$

- Lagrangian of massless gluons and *almost massless quarks*
- What experiment sees are bound states, e.g. $m_\pi, m_P \gg m_{u,d}$
- Underlying physics non-perturbative

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Path integral quantisation:

$$\begin{aligned} \langle 0|O|0\rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-i S_{\mathrm{lat}}[U,\psi,\bar{\psi}]} \\ \langle 0|O|0\rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O \boxed{e^{-\ S_{\mathrm{lat}}[U,\psi,\bar{\psi}]}} \end{aligned}$$

Euclidean space-time Boltzmann factor

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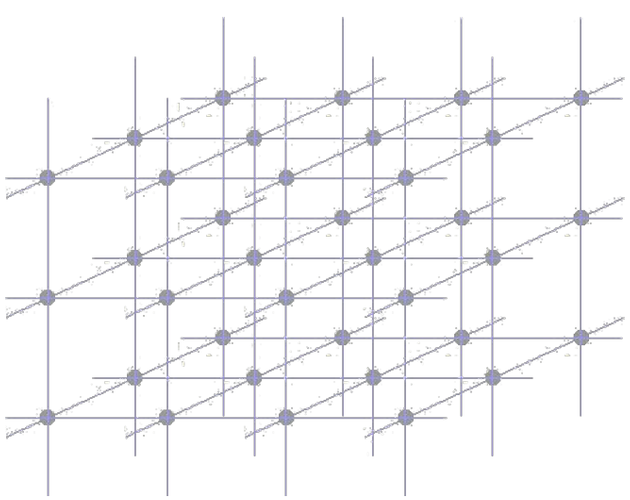
Euclidean space-time Boltzmann factor



finite volume, space-time grid (IR and UV regulators)

$\propto L^{-1}$ $\propto a^{-1}$

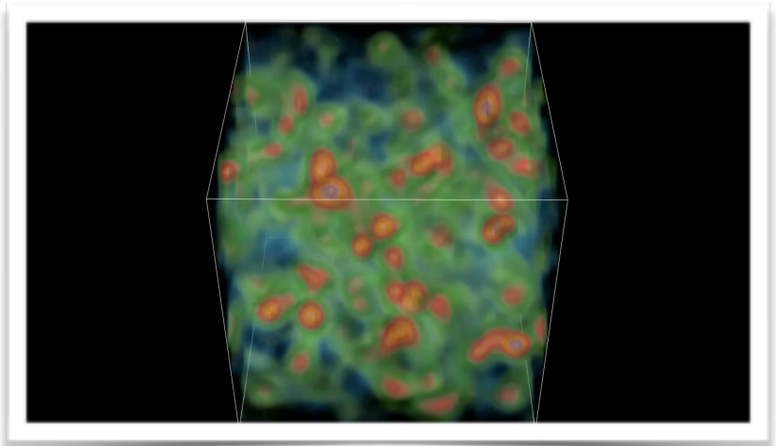
→ Well defined, finite dimensional Euclidean path integral

→ From first principles, solve via MCMC

State-of-the-art lattice QCD

What we can do

- simulations of QCD with dynamical (sea) u,d,s,c quarks with masses as found in nature
- bottom as valence quark
- cut-off $a^{-1} \leq 4\mathrm{GeV}$
- volume $L \leq 6fm$
- m_u=m_d, α_{EM}=*0* — this is changing!



action density of RBC/UKQCD physical point DWF ensemble

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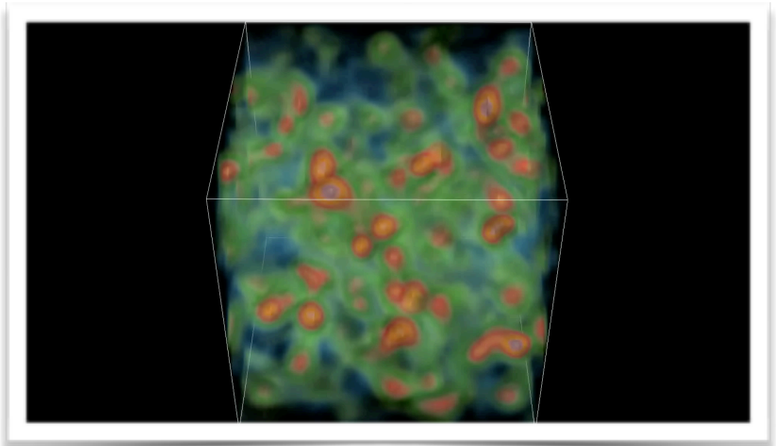

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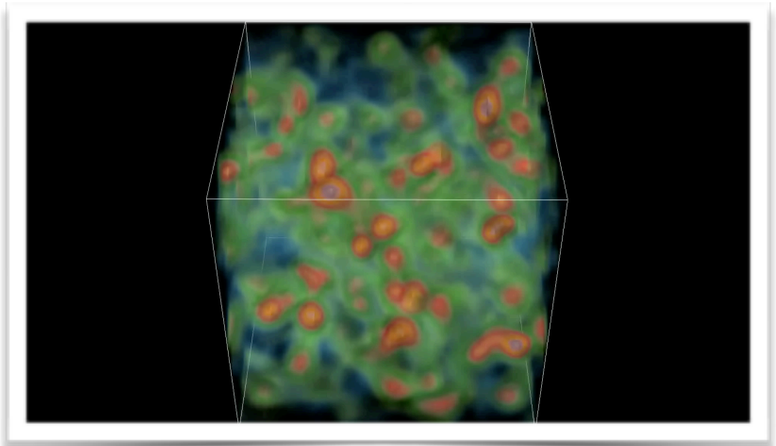

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Parameter tuning

need to fix quark masses and coupling by sacrificing a few hadronic inputs (typically meson/baryon masses and/or simple matrix elements)

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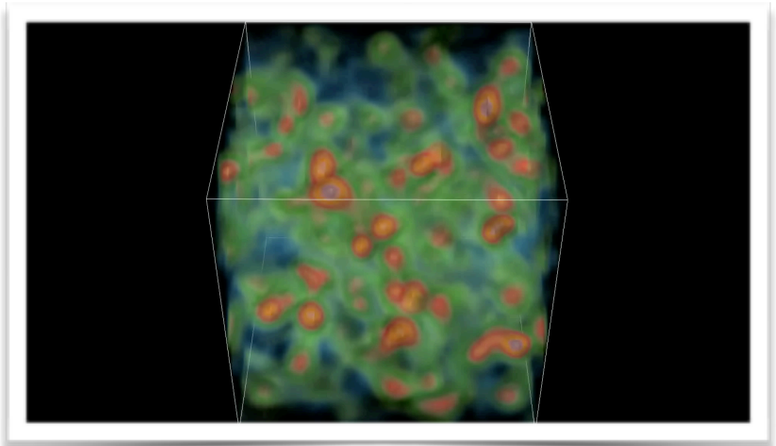

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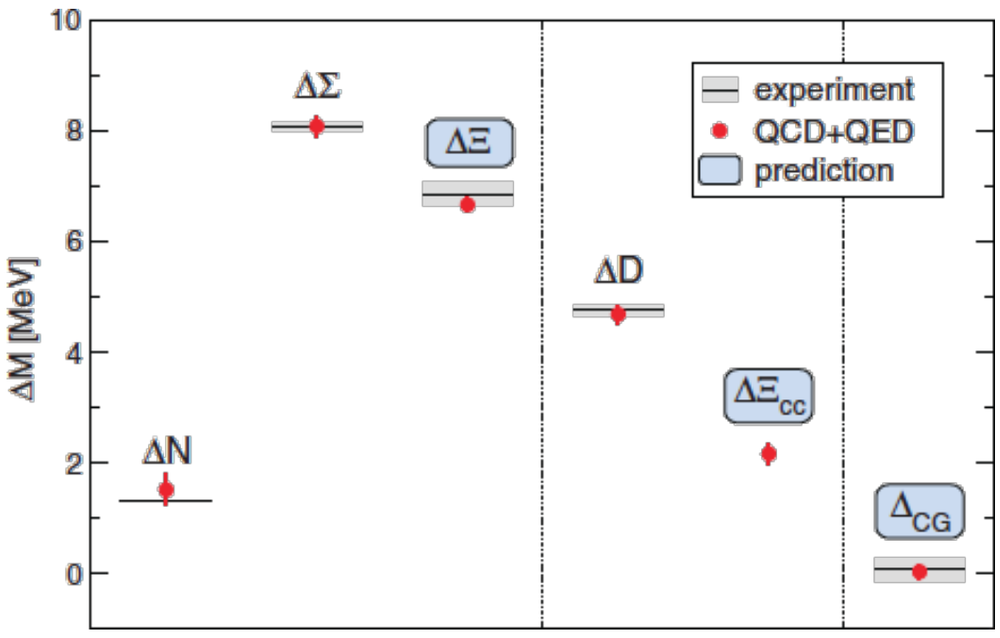


BMW Collaboration
Science 347 (2015)
1452-1455 arXiv:1406.4088

Advertisement:

Flavour Lattice Averaging Group

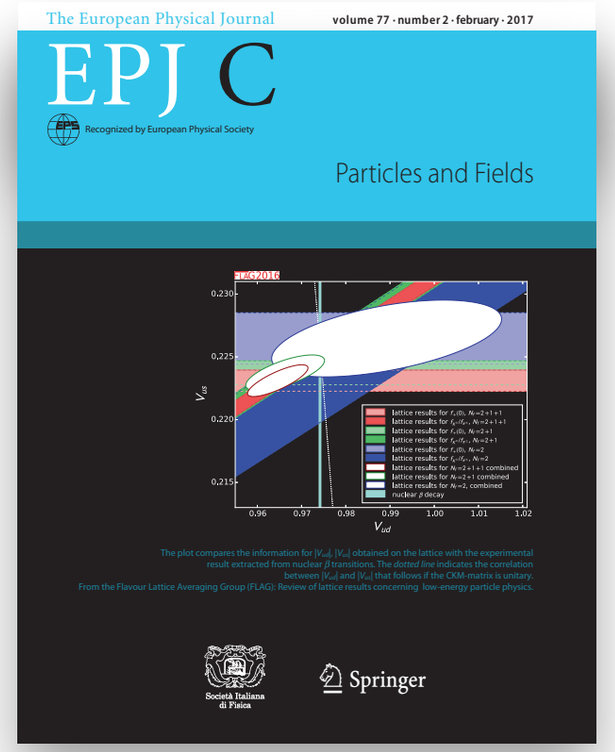
“What's currently the best lattice value for a particular quantity?”

FLAG-1 (Eur. Phys. J. C71 (2011) 1695)

FLAG-2 (Eur. Phys.J. C74 (2014) 2890)

FLAG-3 (Eur.Phys.J. C77 (2017) no.2, 112, http://itpwiki.unibe.ch/flag/)

WEB UPDATES: http://flag.unibe.ch



- quantities:

$$m_{u,d,s,c,b}$$

$$f_K/f_\pi,\ f_+^{K\pi}(0),\ B_K,\ SU(2) \text{ and } SU(3) \text{ LECs}$$

$$f_{D_{(s)}},\ f_{B_{(s)}},\ B_{B_{(s)}},\ B_{(s)}\text{– and } D_{(s)}\text{–semileptonics}$$

$$\alpha_s$$

- summary of results
  - evaluation according to FLAG quality criteria (colour coding)
  - averages of best values where possible
  - detailed summary of properties of individual simulations

FLAG-4 aiming for early 2019

Advertisement: Flavour Lattice Averaging Group

From the 2017 web update (http://flag.unibe.ch)

When using FLAG averages please cite the underlying original work!!

Challenges for heavy quark on the lattice

A multi-scale problem

a^{-1} << physics of interest << L^{-1}

finite cutoff — finite box size

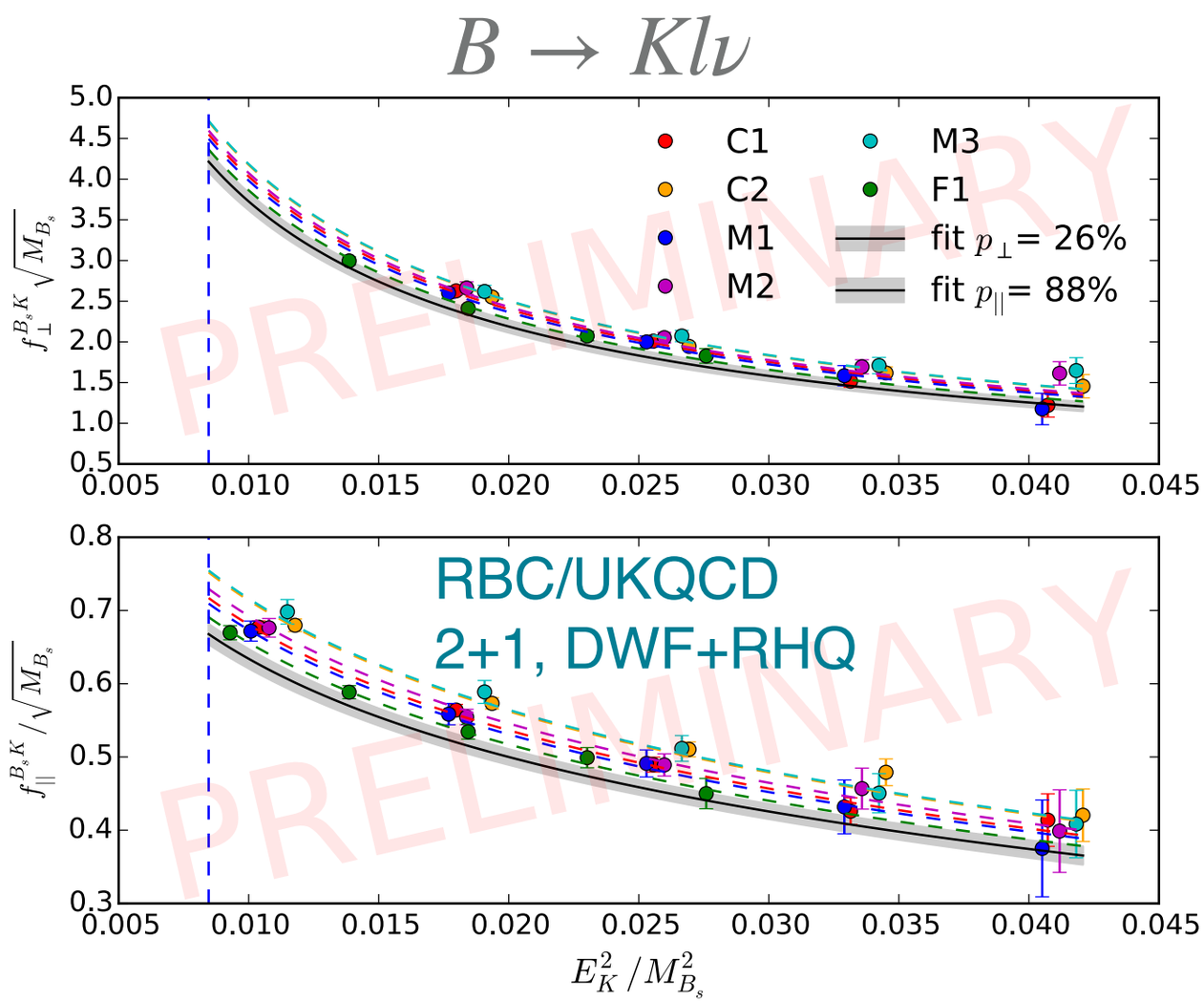
finite lattice spacing

- hard to discretise *b*-quarks (slowly getting there but need to play and control tricks like effective theory, improvement, extrapolation in m_h,…, which are not needed for light quarks)

finite lattice volume

- physical pion mass ‘expensive’ to reconcile with above bounds — we often dial heavier (cheaper) m_π and then extrapolate (model or EFT)

Extrapolation of lattice data



lattice data points with ‘heavy’ pion

model/EFT/z-parametrisation

extrapolate into into kinematic region inaccessible by lattice data

Challenges for heavy quark on the lattice

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}\,\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_{B_{(s)}}^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_{B_{(s)}}^2(E_P^2-m_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_{B_{(s)}}^2-m_P^2)^2|f_0(q^2)|^2\right]$$

$$\frac{d\Gamma_{B^-\to D^0\ell^-\bar\nu}}{dw} = \frac{G_{\rm F}^2 m_D^3}{48\pi^3}(m_B+m_D)^2(w^2-1)^{3/2}|\eta_{\rm EW}|^2|V_{cb}|^2|\mathcal{G}(w)|^2,$$

$$\frac{d\Gamma_{B^-\to D^{0*}\ell^-\bar\nu}}{dw} = \frac{G_{\rm F}^2 m_{D^*}^3}{4\pi^3}(m_B-m_{D^*})^2(w^2-1)^{1/2}|\eta_{\rm EW}|^2|V_{cb}|^2\chi(w)|\mathcal{F}(w)|^2$$

Challenges for heavy quark on the lattice

a^{-1} << physics of interest << L^{-1}

finite cutoff — finite box size

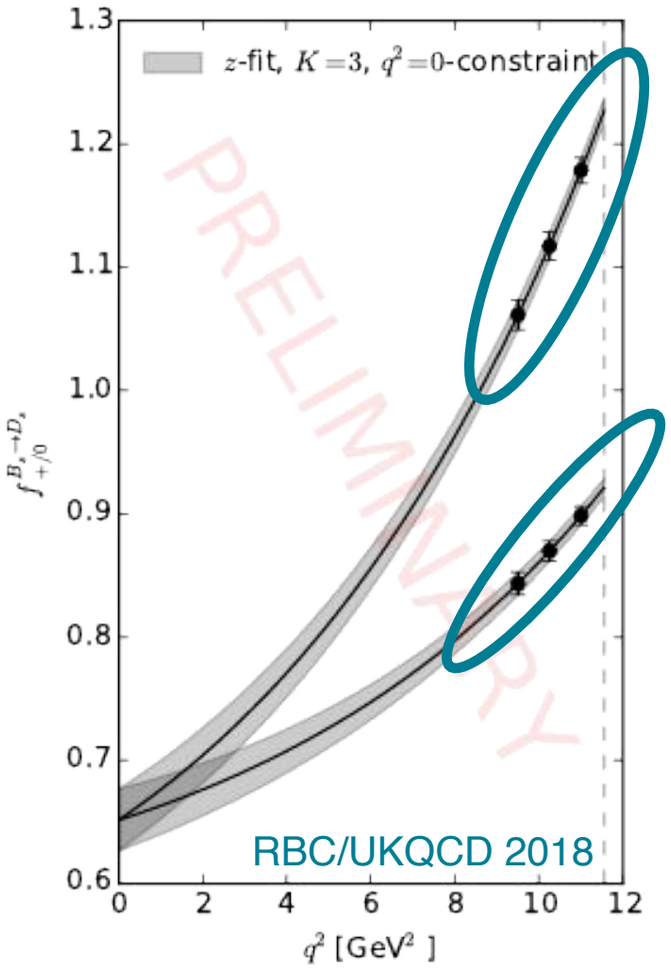
kinematics

$$q^2 = \left(E_B - E_{\text{light}}\right)^2 - \left(\vec{p}_B - \vec{p}_{\text{final}}\right)^2$$

lattice does best with mesons at rest (statistical error and cutoff effects smaller)

this is at tension with the suppression of the decay rate at large momenta

Kinematical reach limited in lattice QCD → extract value of V_{CKM} from simultaneous analysis of exp. and lattice data



And now some results…

The following is an incomplete list of mostly recent results (for comprehensive list see e.g. FLAG 2017 web update)

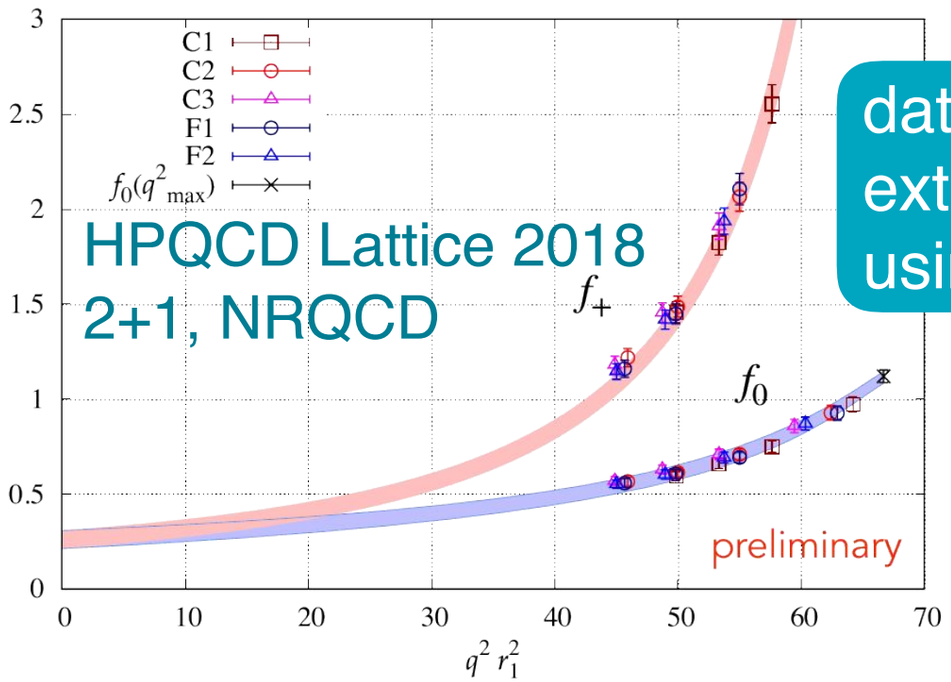
Activity has picked up lately and more collaborations are now contributing on $B_{(s)}$ decays

That's great since different lattice techniques are being used, each with slightly different systematic effects

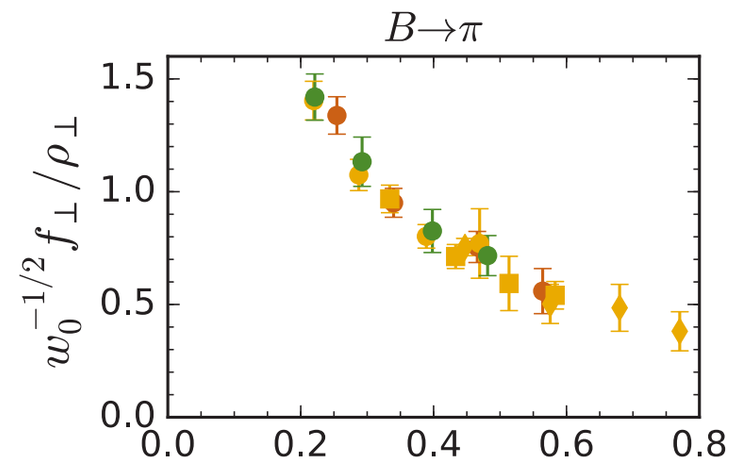
In the following as per request:
concentrate on $b \to u$ and $b \to c$ form factors, CKM MEs and R-ratios

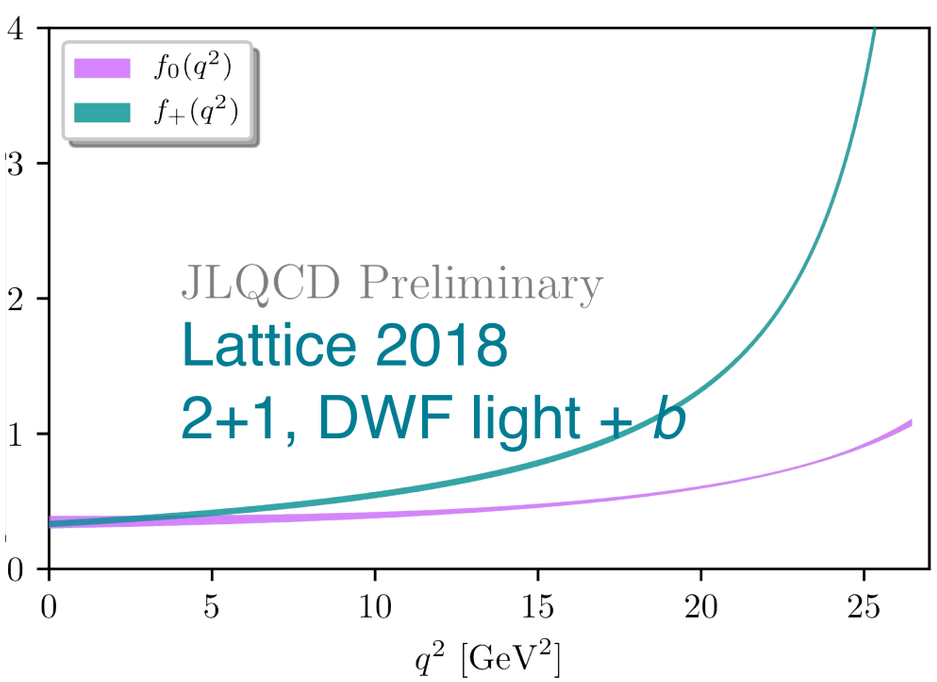
$$b \longrightarrow u$$

$B \to \pi$



data near q^2_{max} extrapolated using z-expansion



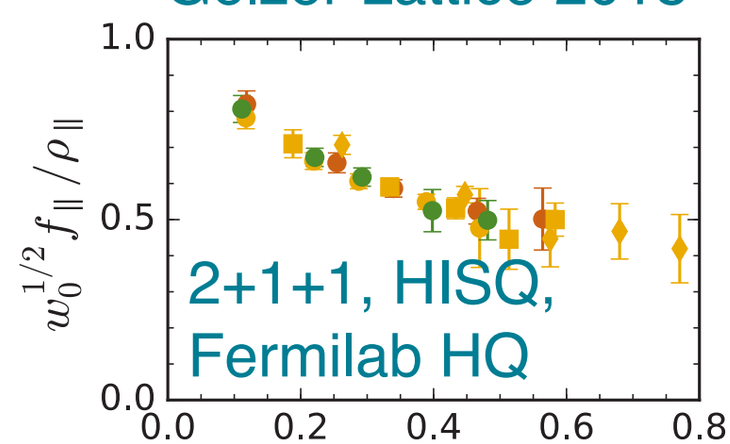


Current activity on

$$B \to \pi l \nu$$

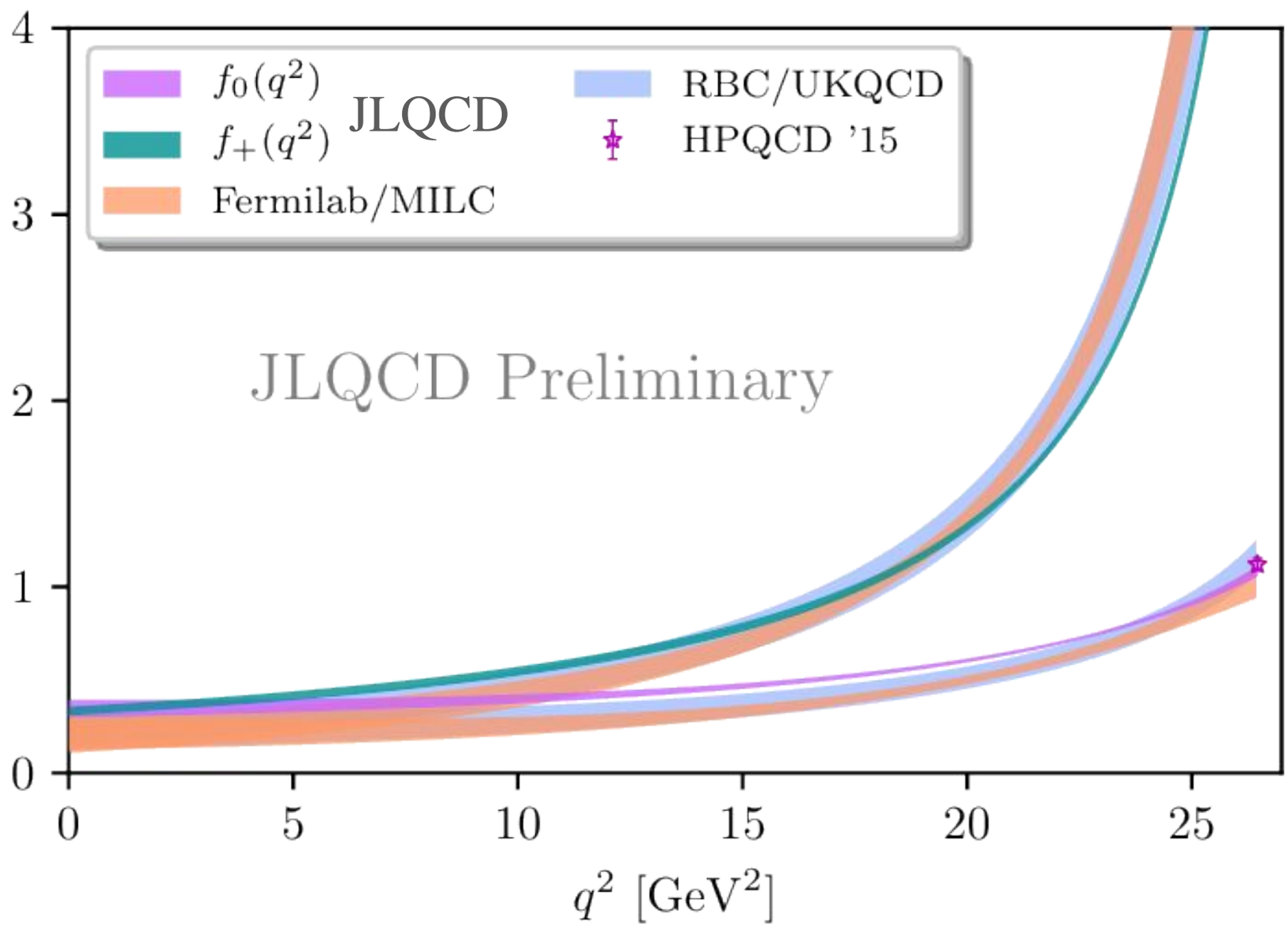
RBC/UKQCD is working on update

Gelzer Lattice 2018



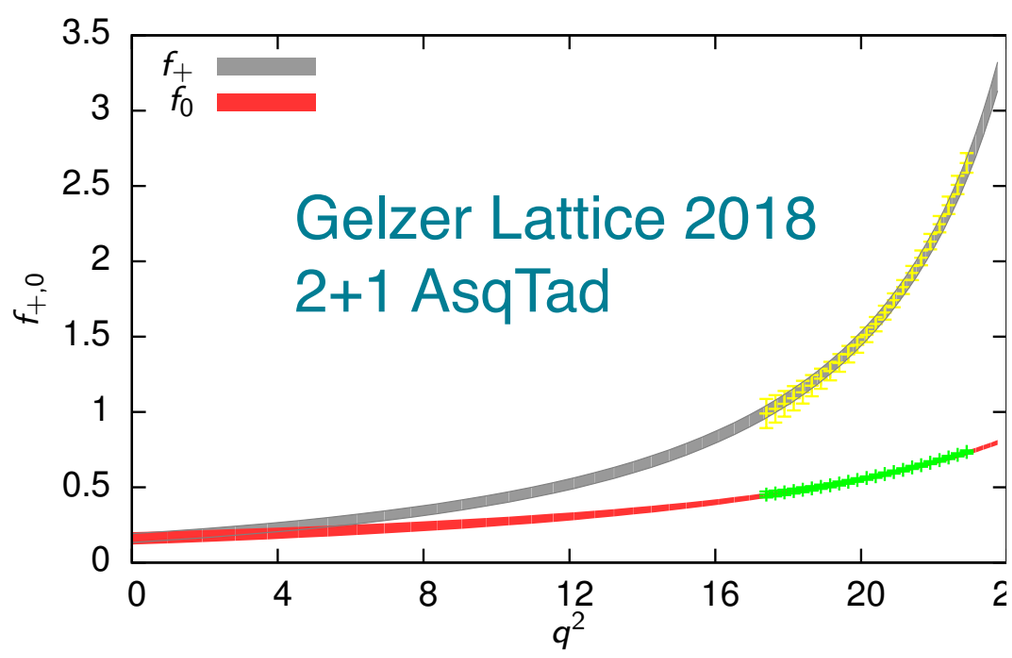
$B \to \pi$ comparison

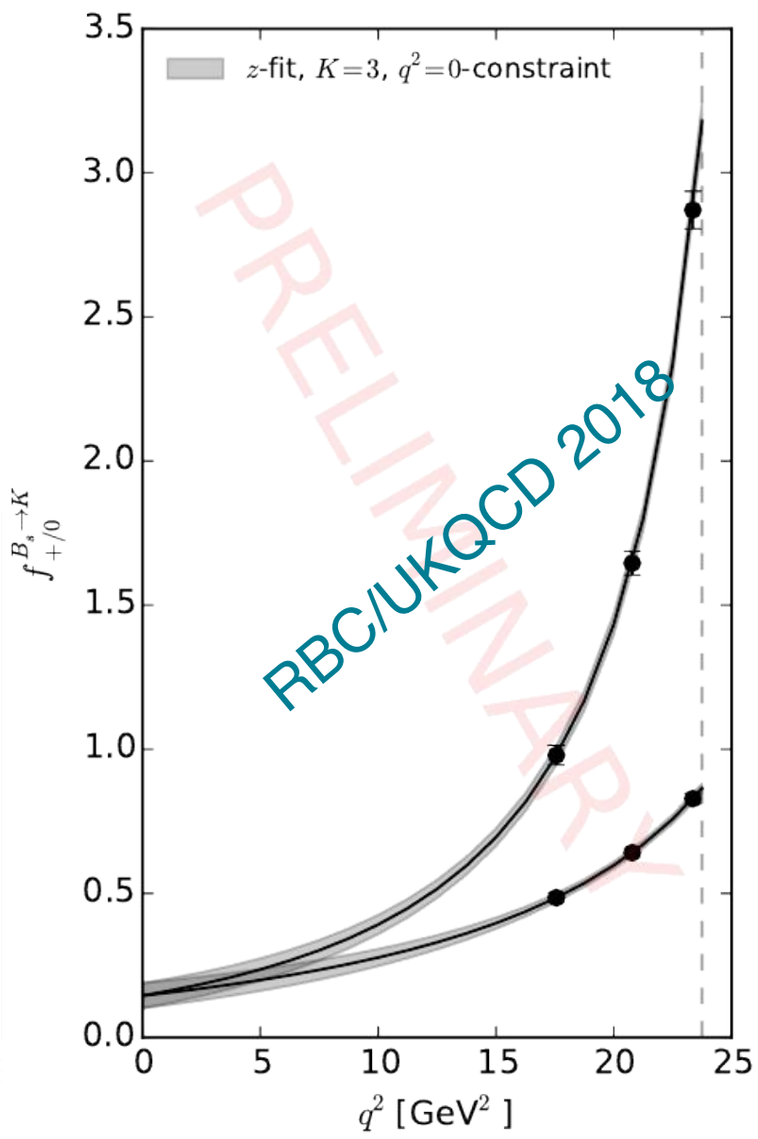
Overlay plot by Colquhoun showing (preliminary) agreement

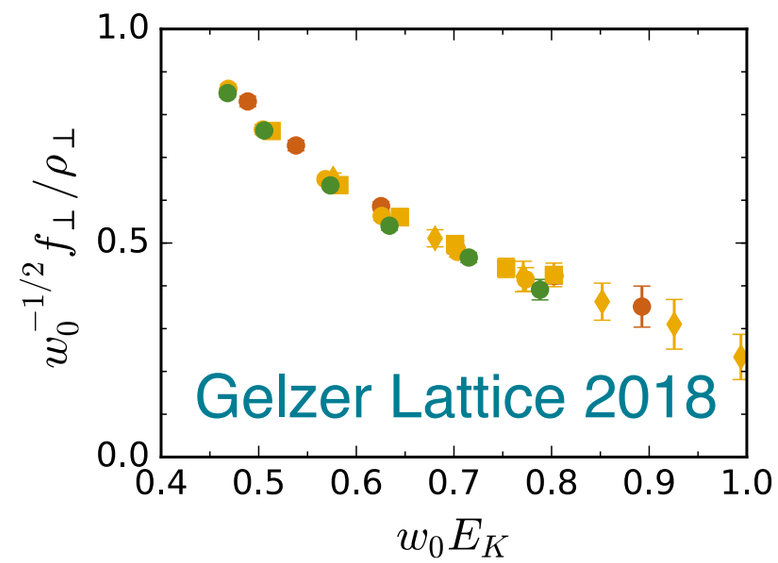


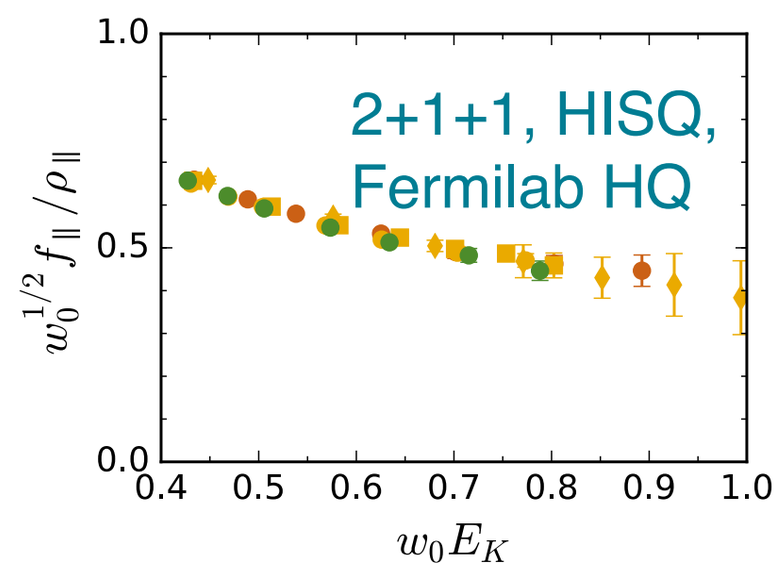
$B_s \to K$

Current activity on
$B_s \to Kl\nu$



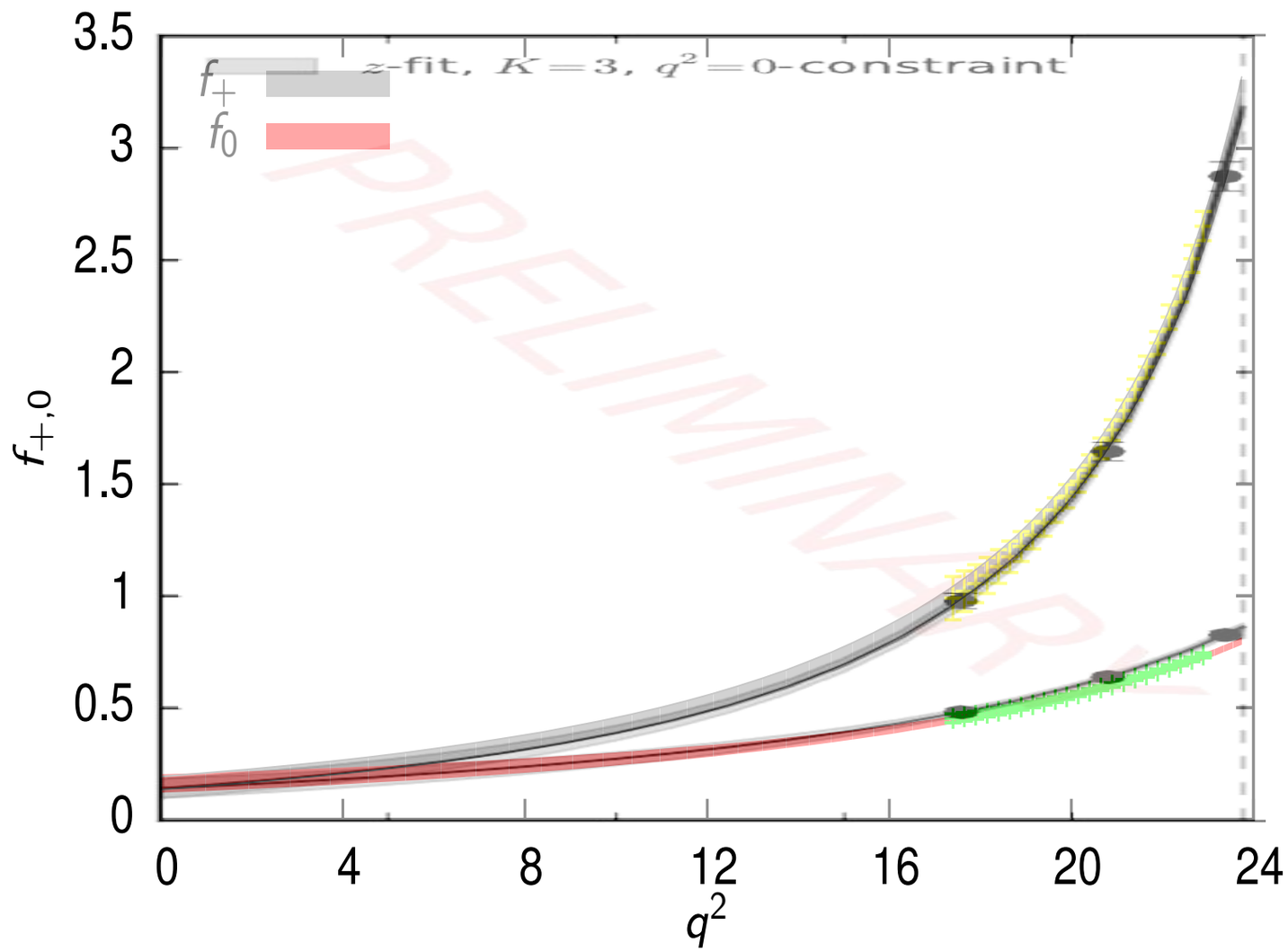






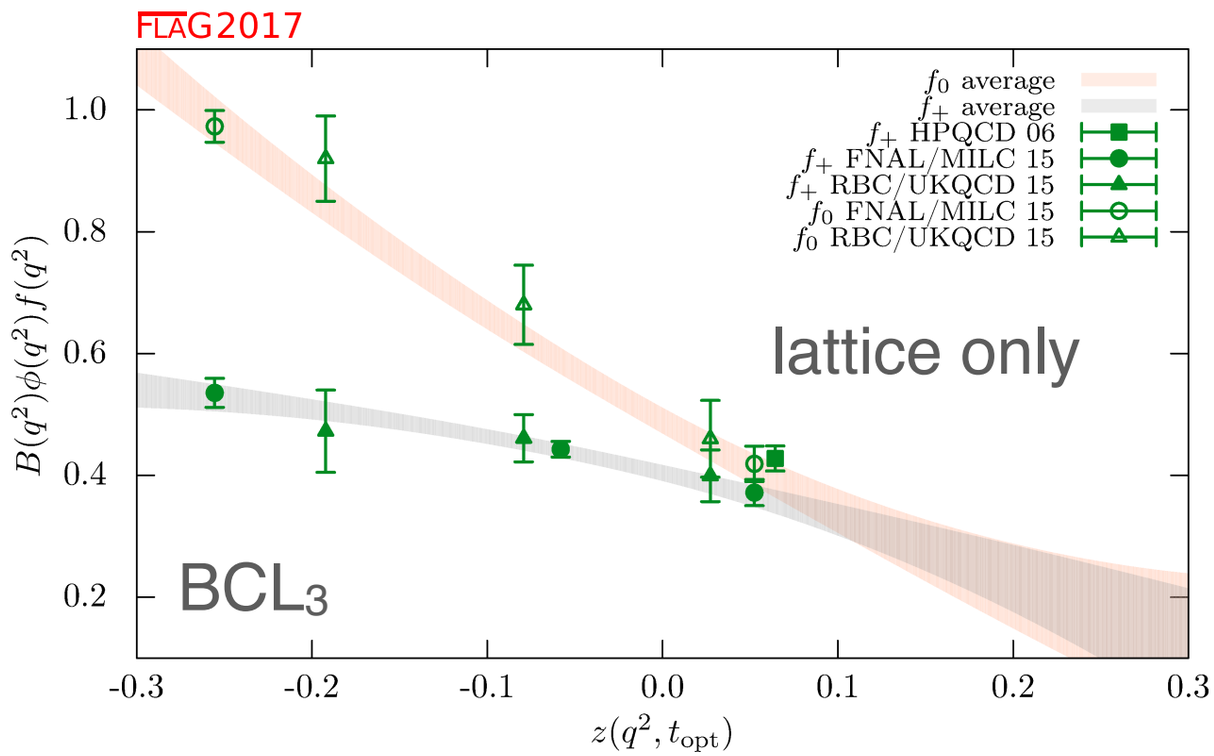
$B_s \to K$ comparison

Overlay plot of RBC/UKQCD and Gelzer (Fermilab MILC)
a bit of tension maybe — need to wait for final error budget

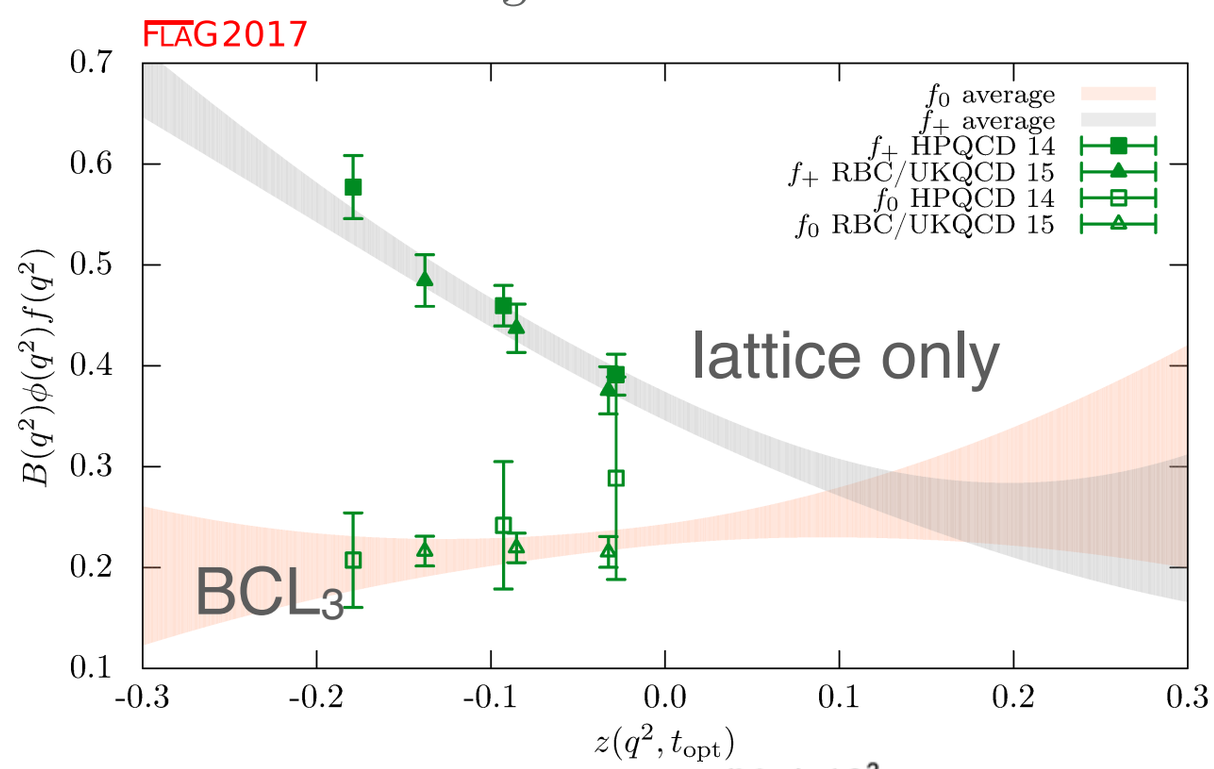


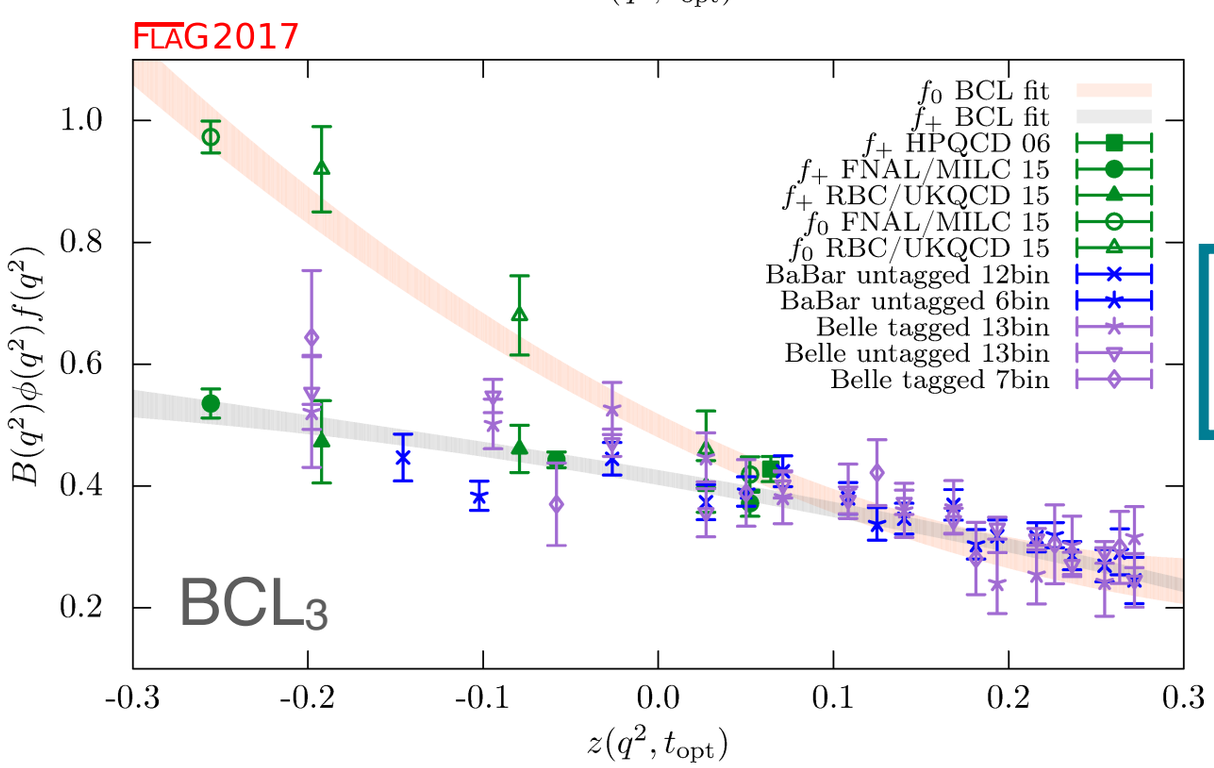
V_{ub}

$B \to \pi l \nu$

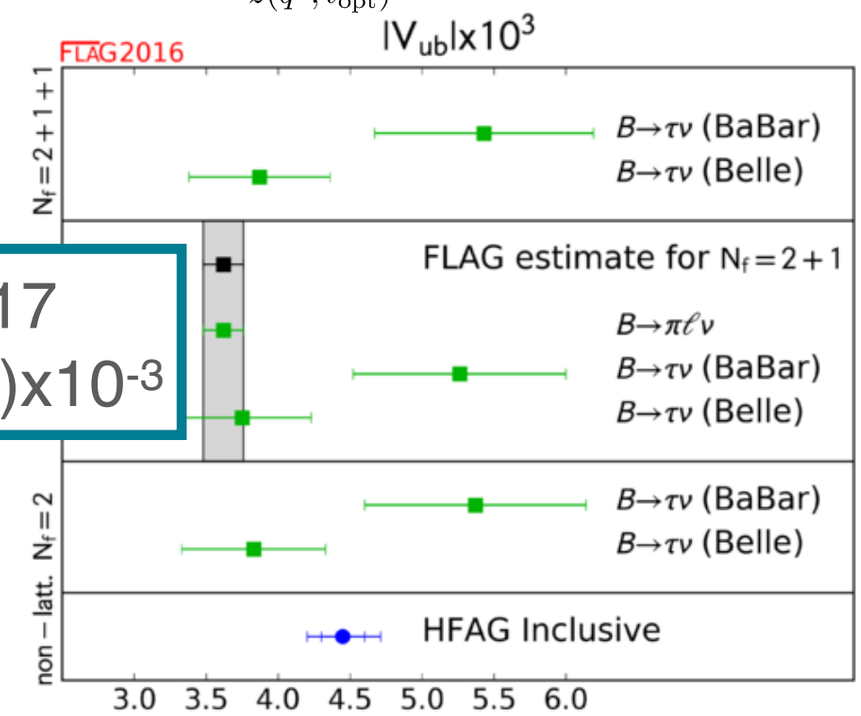


$B_s \to K l \nu$



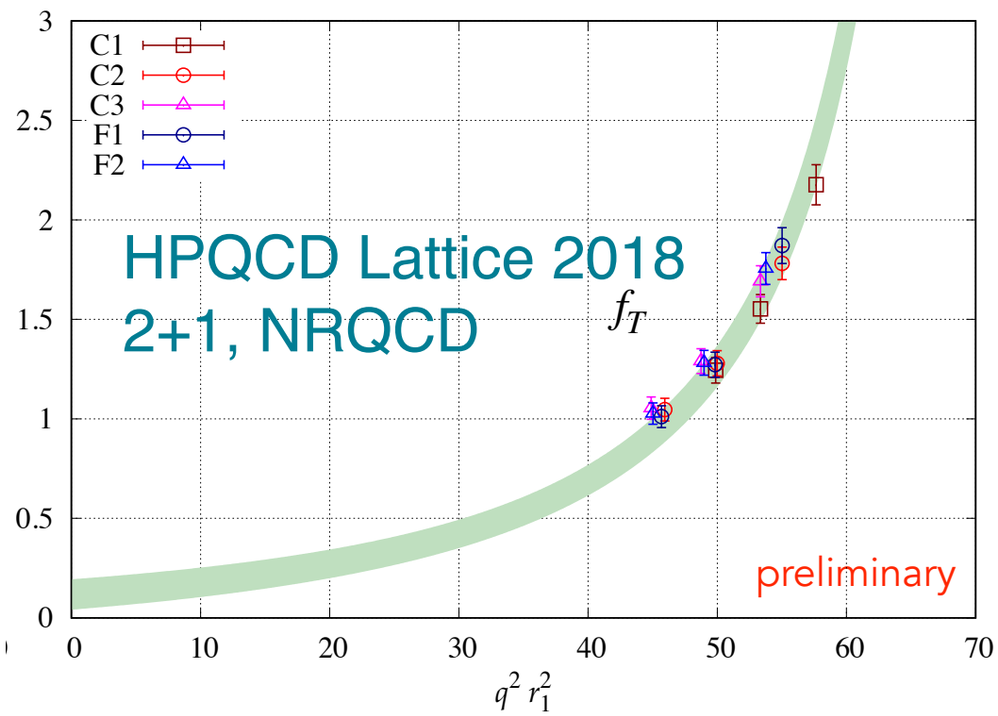


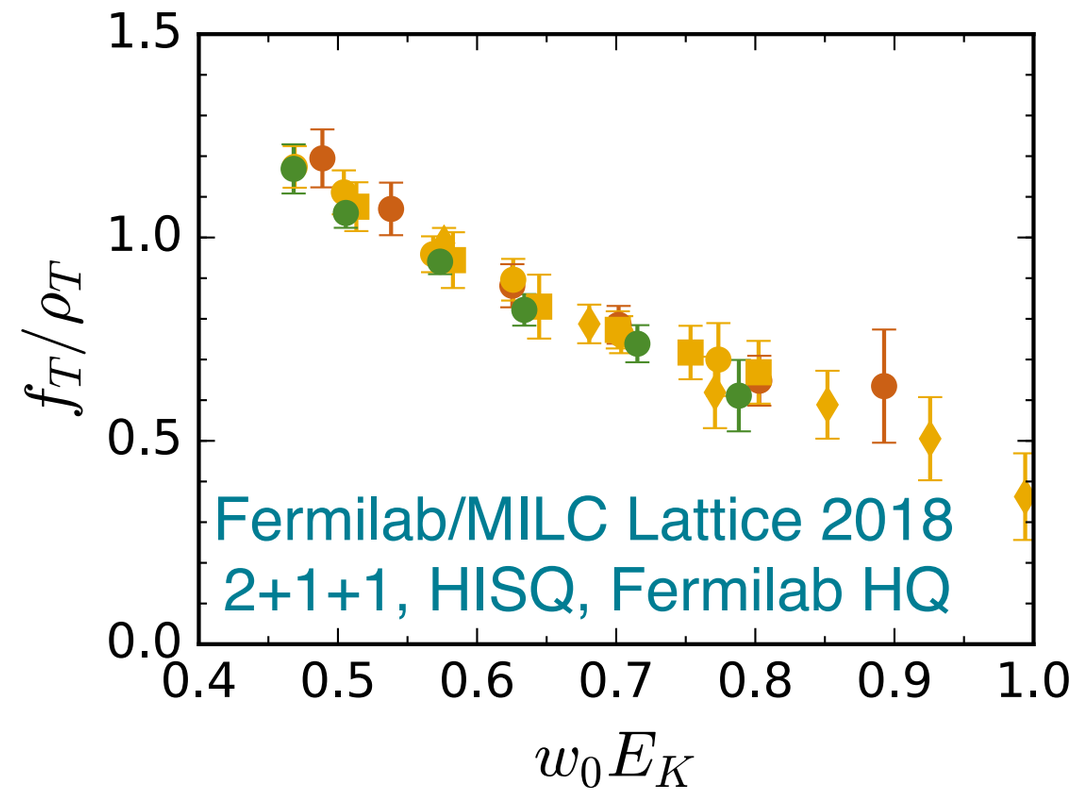
FLAG 2017
$|V_{ub}|=3.73(14)\times10^{-3}$



$B \to K$

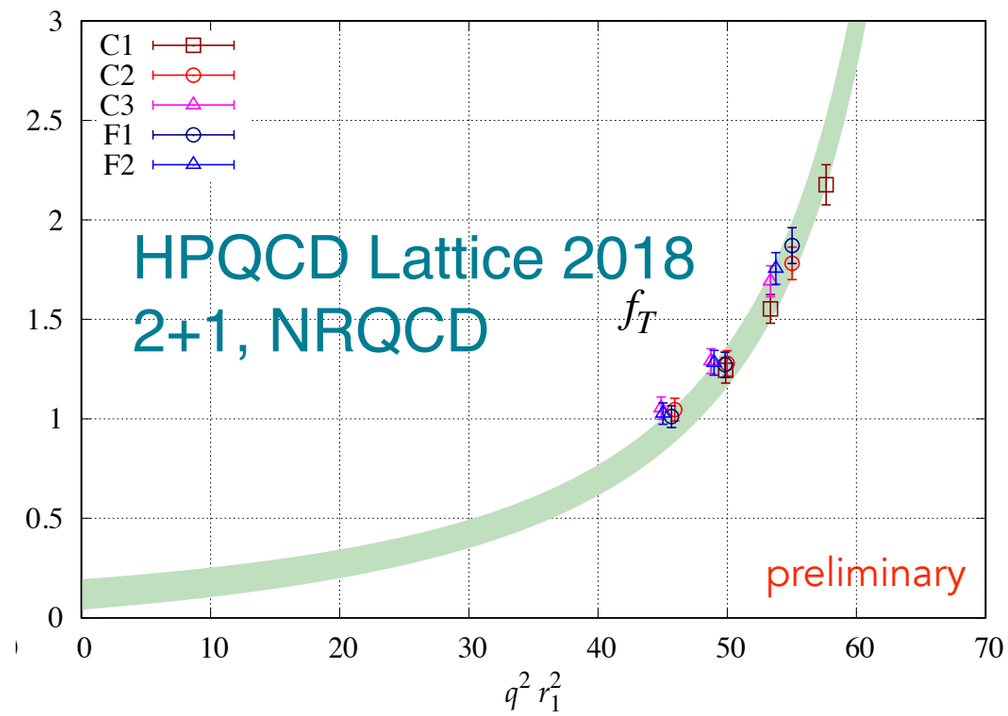
Current activity on $B \to Kll$

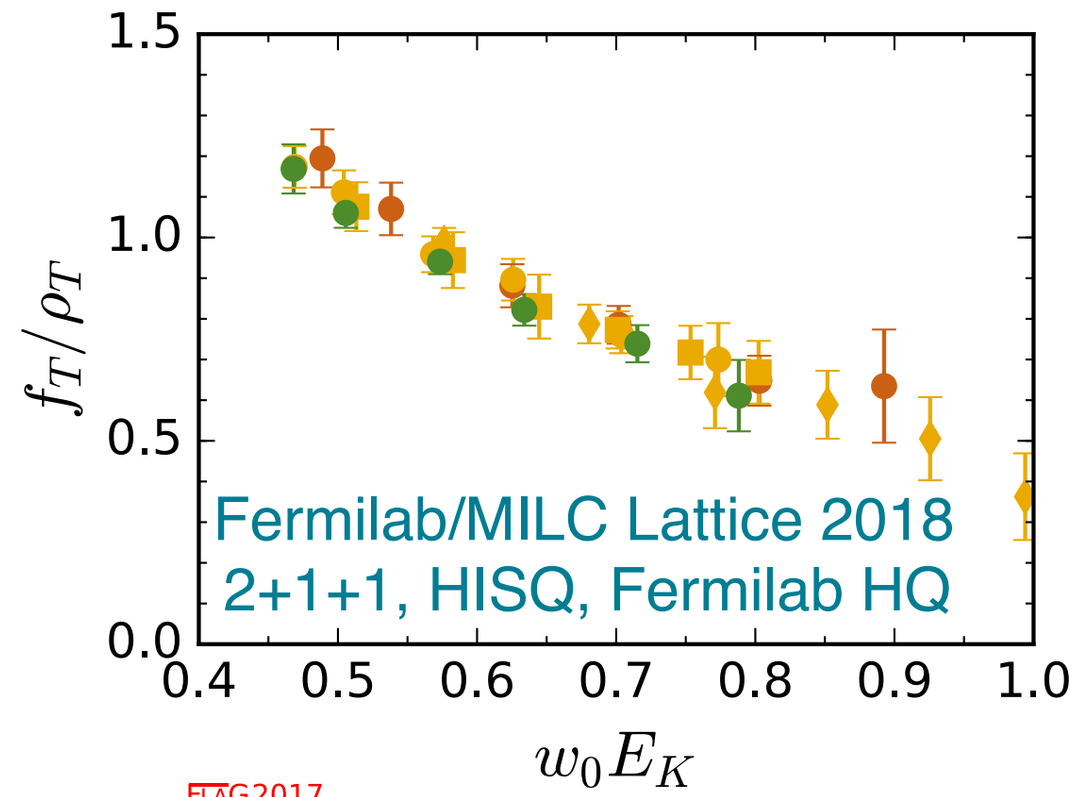


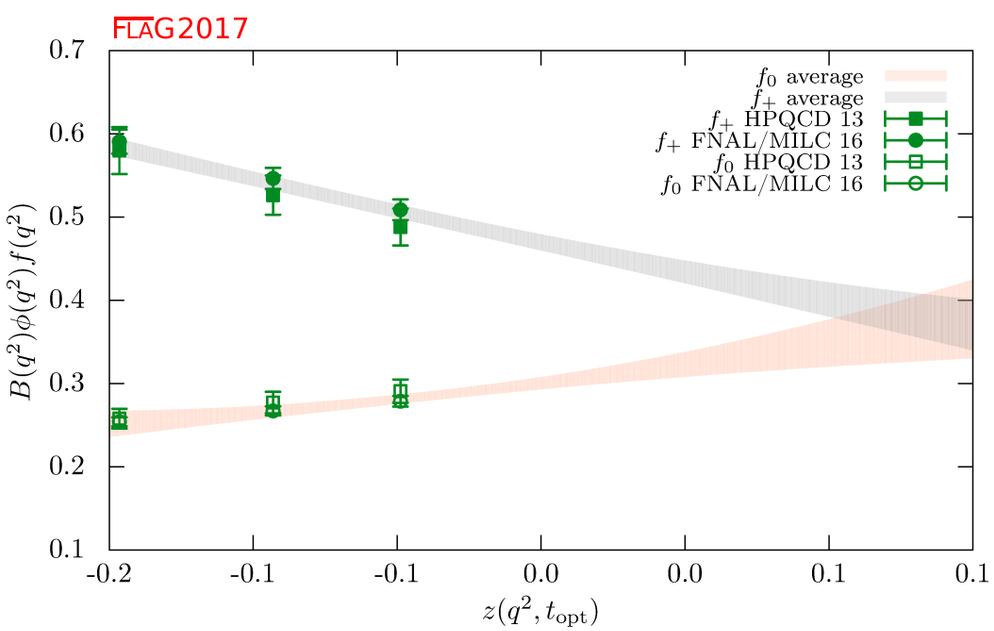


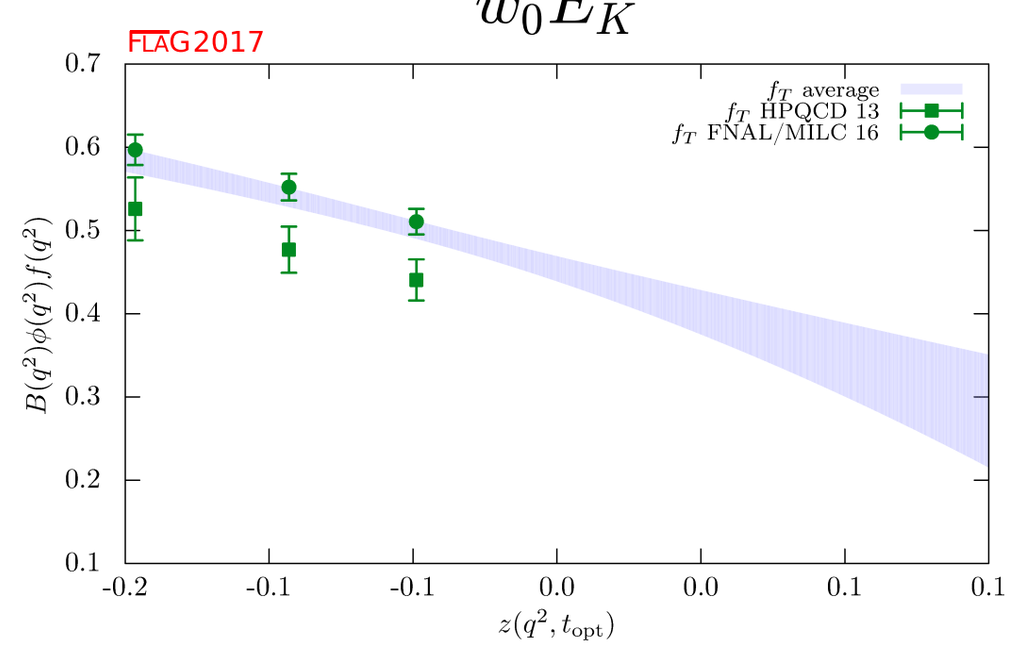
$B \to K$

Current activity on $B \to Kll$



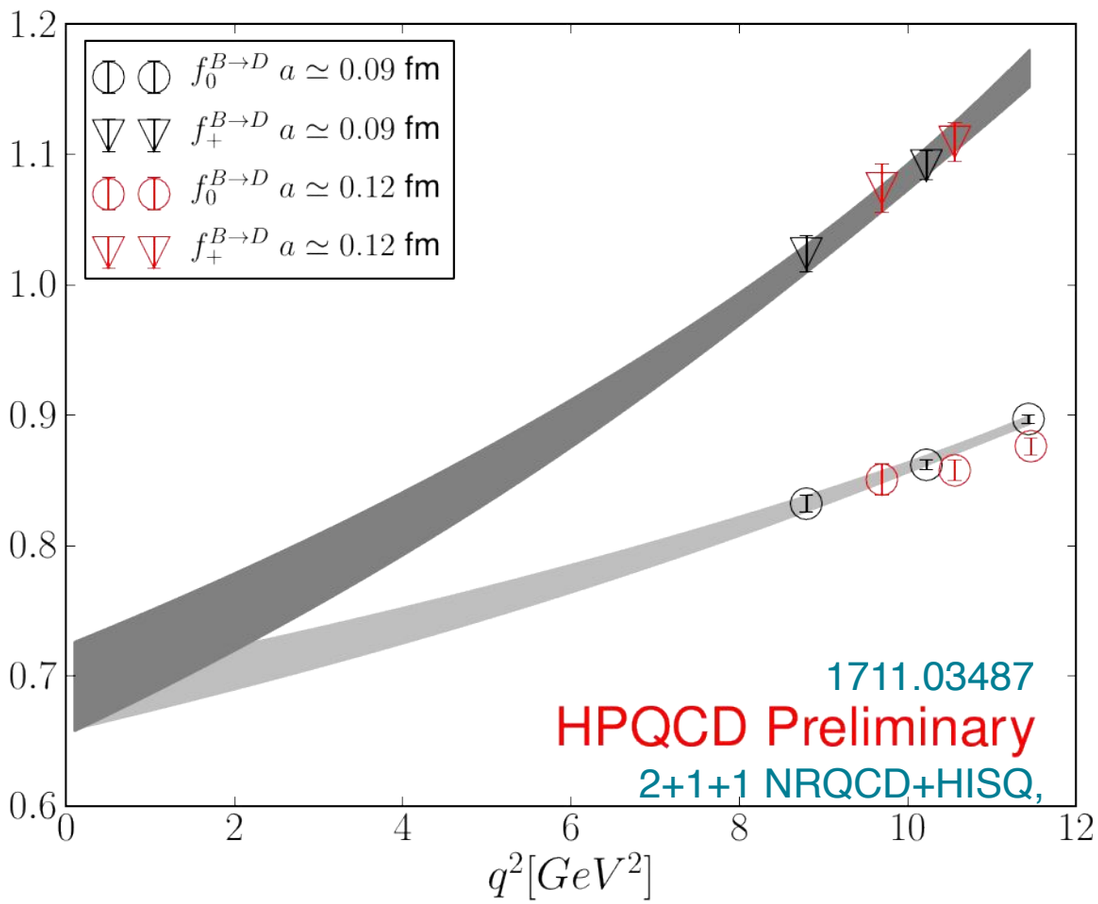


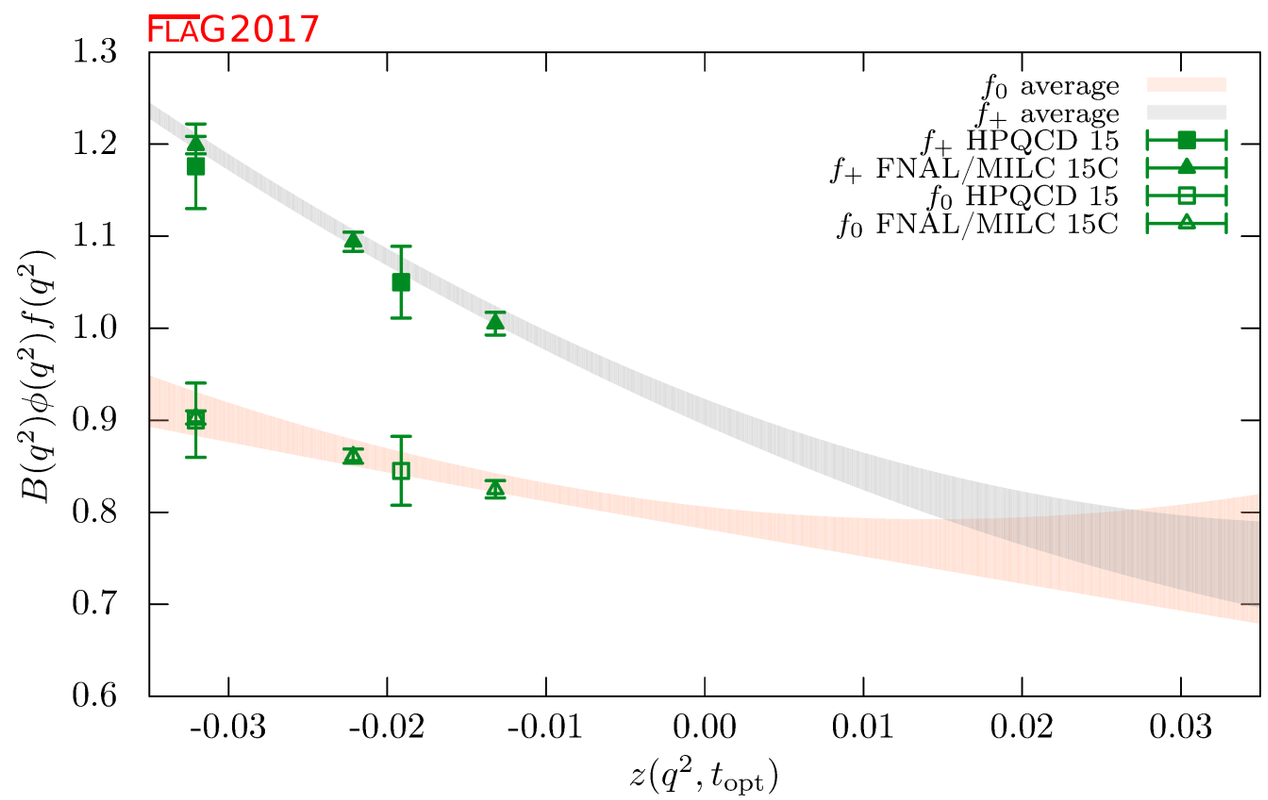




$$b \longrightarrow c$$

B→D



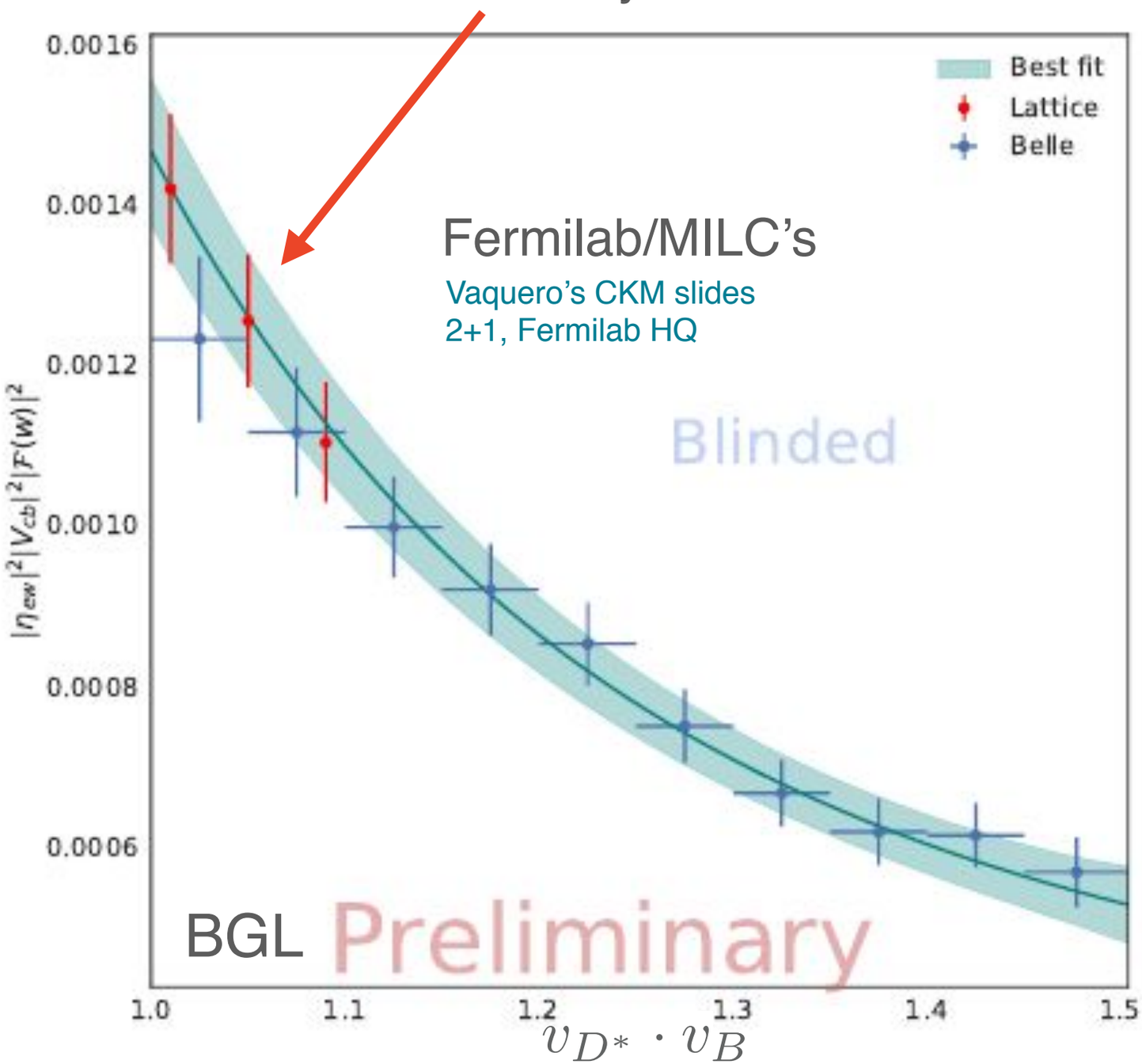


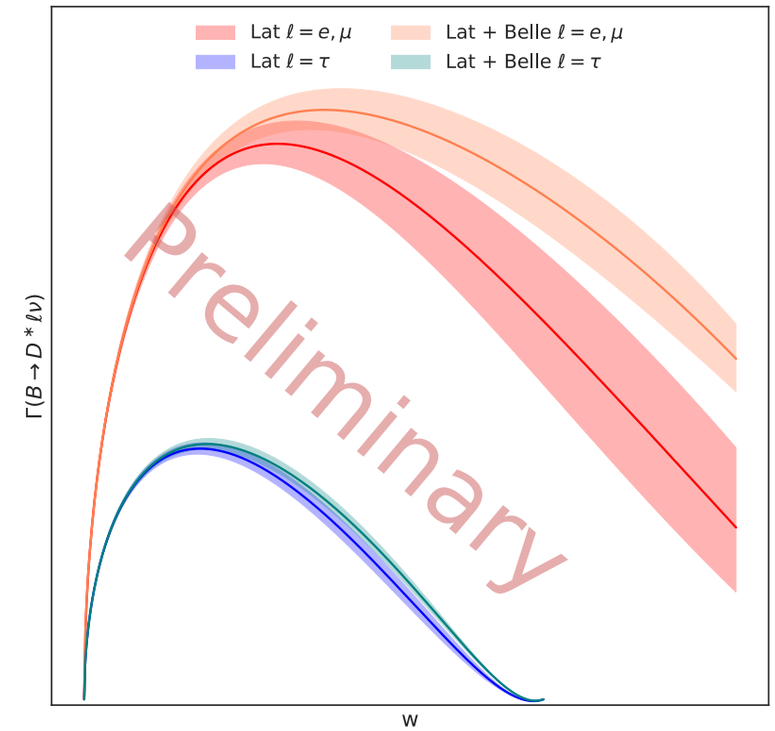
Hard:
b, c and light valence quarks

hopefully more activity here in the near future

$B \to D^*$

lattice data away from zero recoil





"soon results on R(D*)"

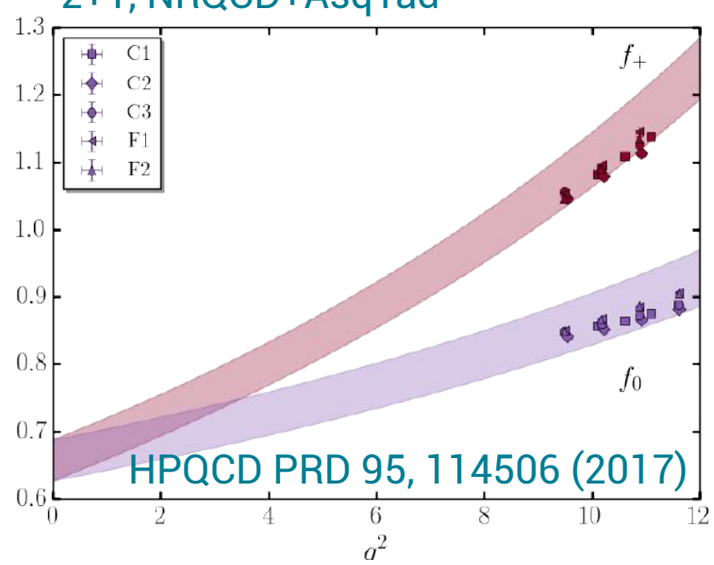
- experimentally precise
- D* assumed stable on in lattice simulations
- issue with parametrisation CLN vs BGL?
  Gambino, Bigi, PRD 94, 094008 (2016)

There is more activity…
(RBC/UKQCD, JLQCD, Bailey et al.…)

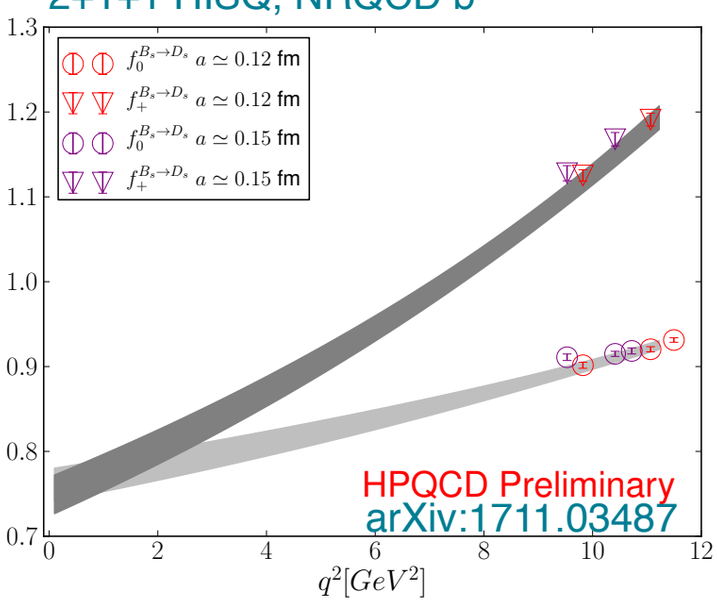
$B_s \to D_s$

Fair bit of activity here mostly preliminary but advanced

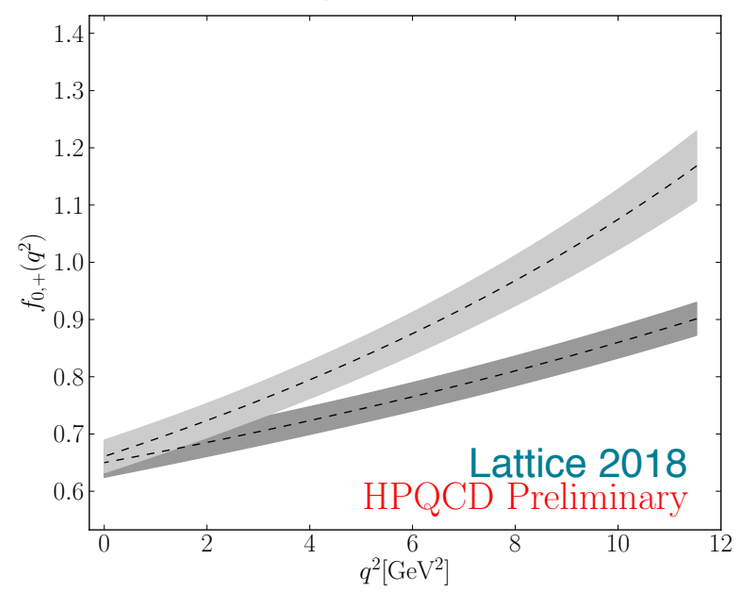
2+1, NRQCD+AsqTad

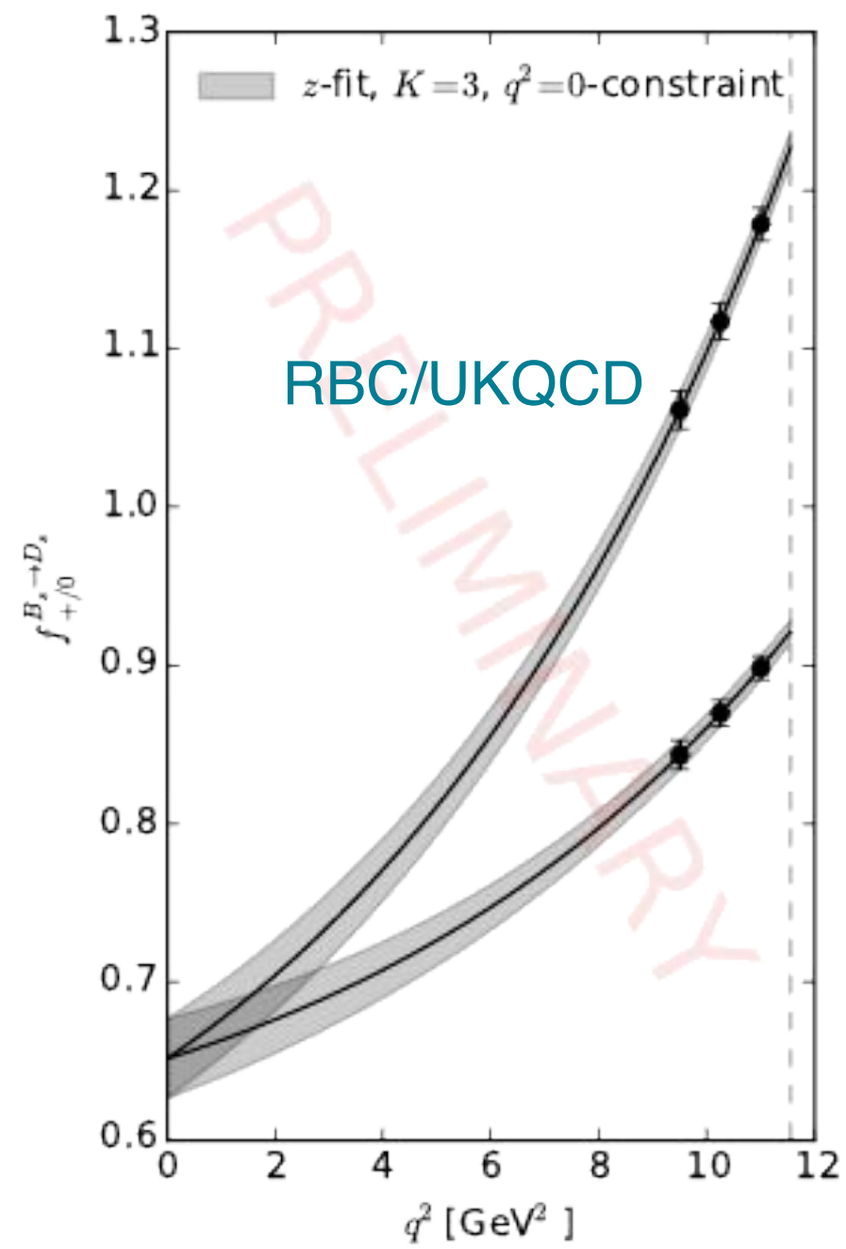


2+1+1 HISQ, NRQCD b

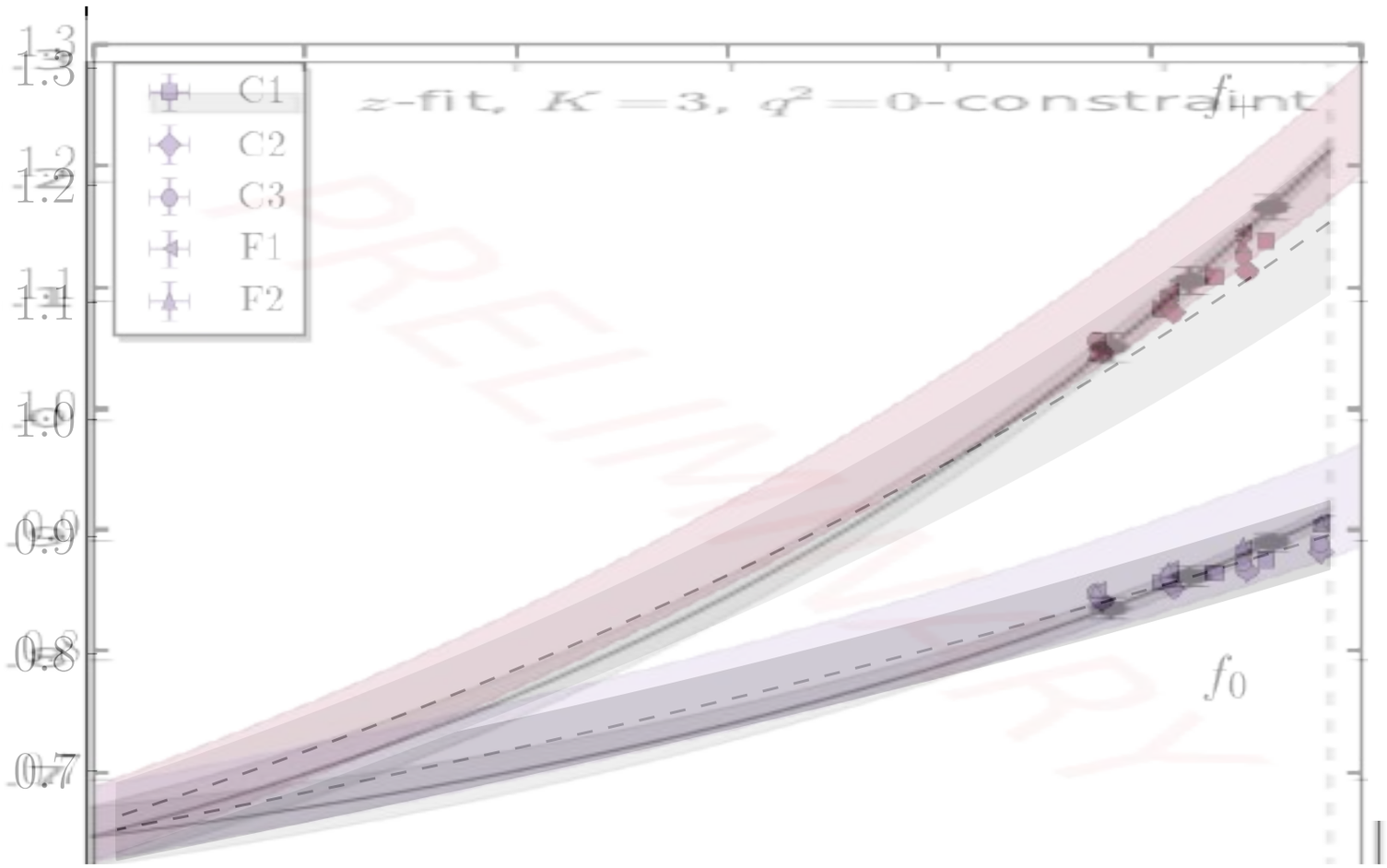


2+1+1 HISQ, HISQ b



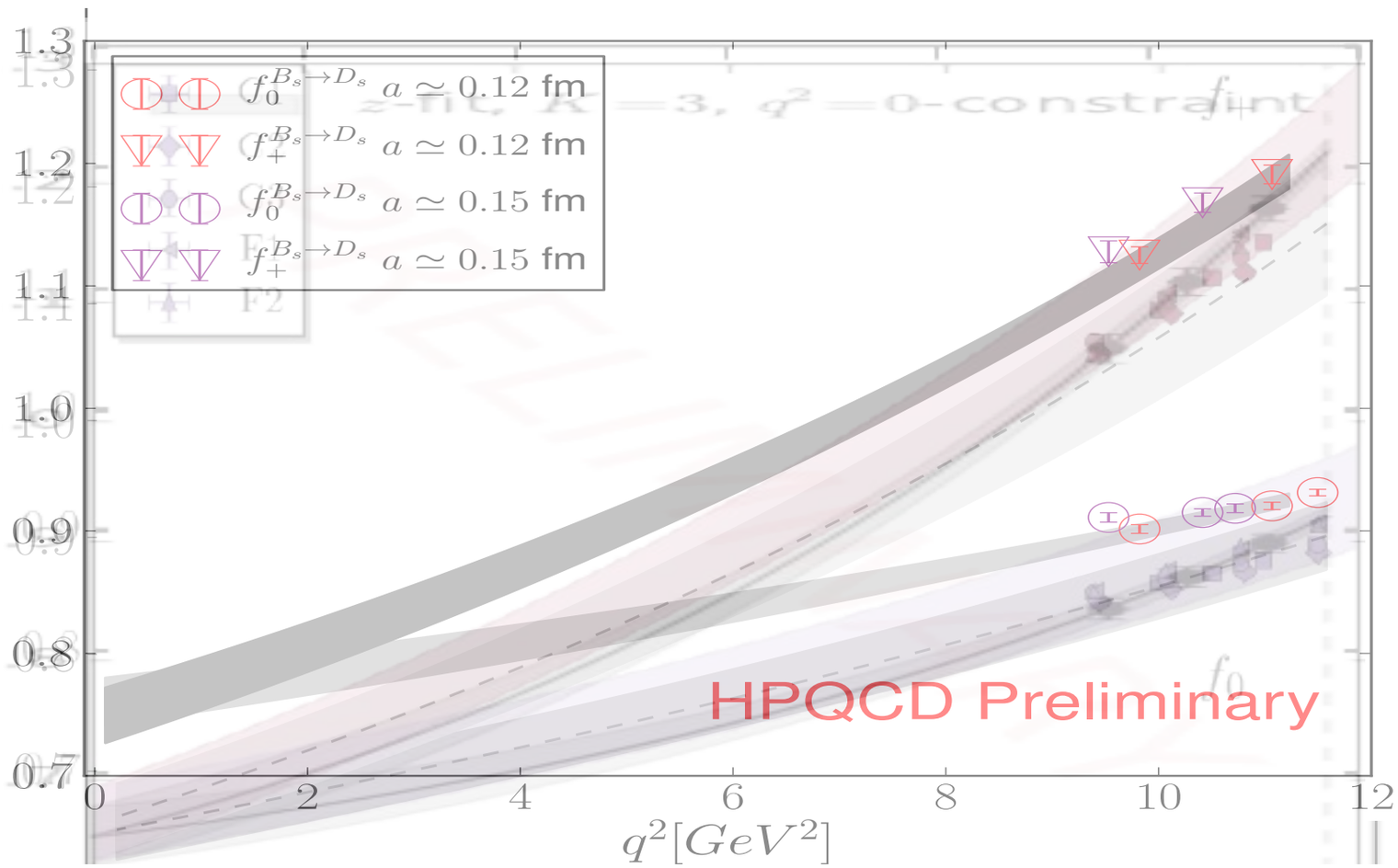


$B_s \to D_s$ comparison



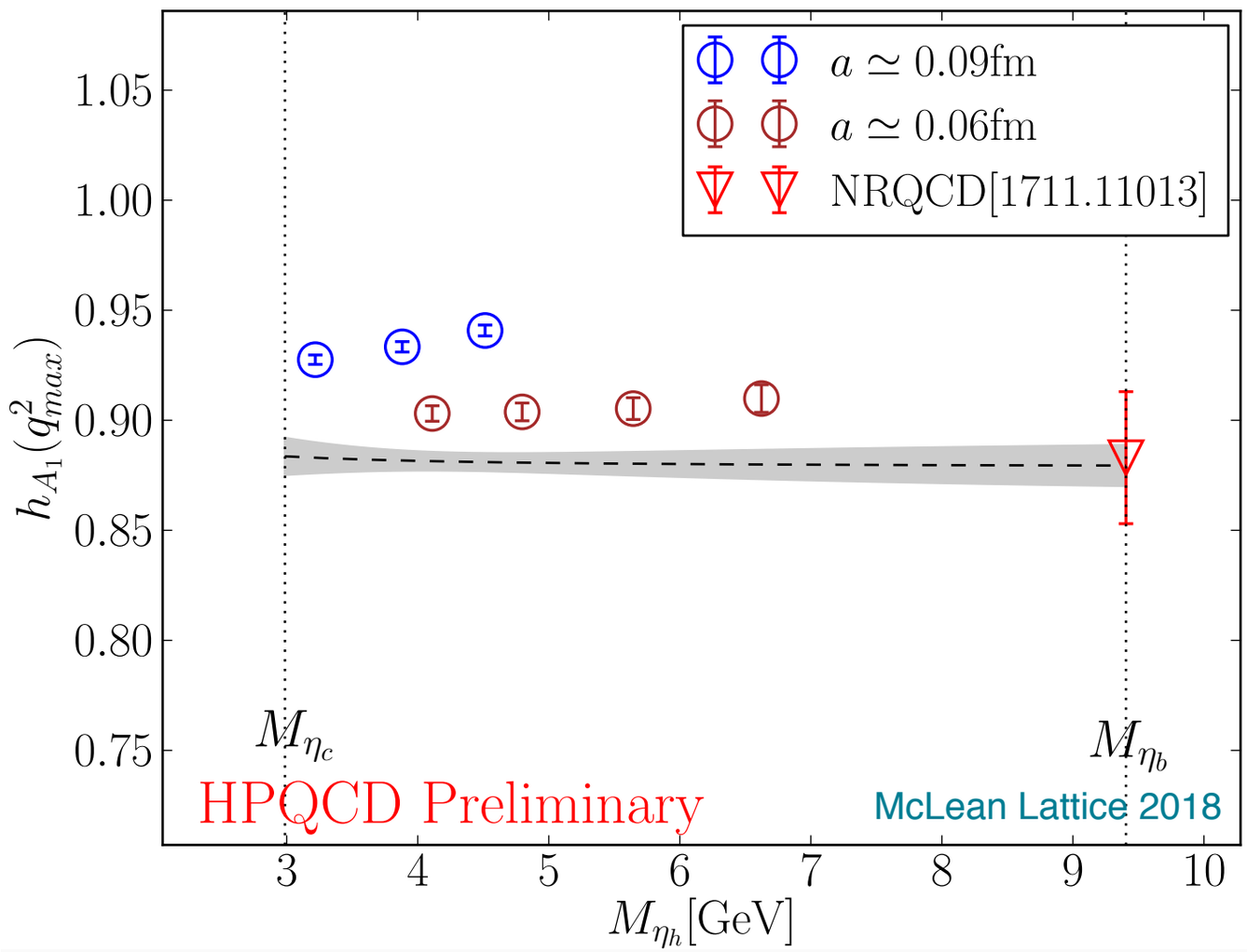
Overlay plot - there is a bit of a spread
let's wait until analyses finalised
will lead to prediction for $R(D_s)$

$B_s \to D_s$ comparison

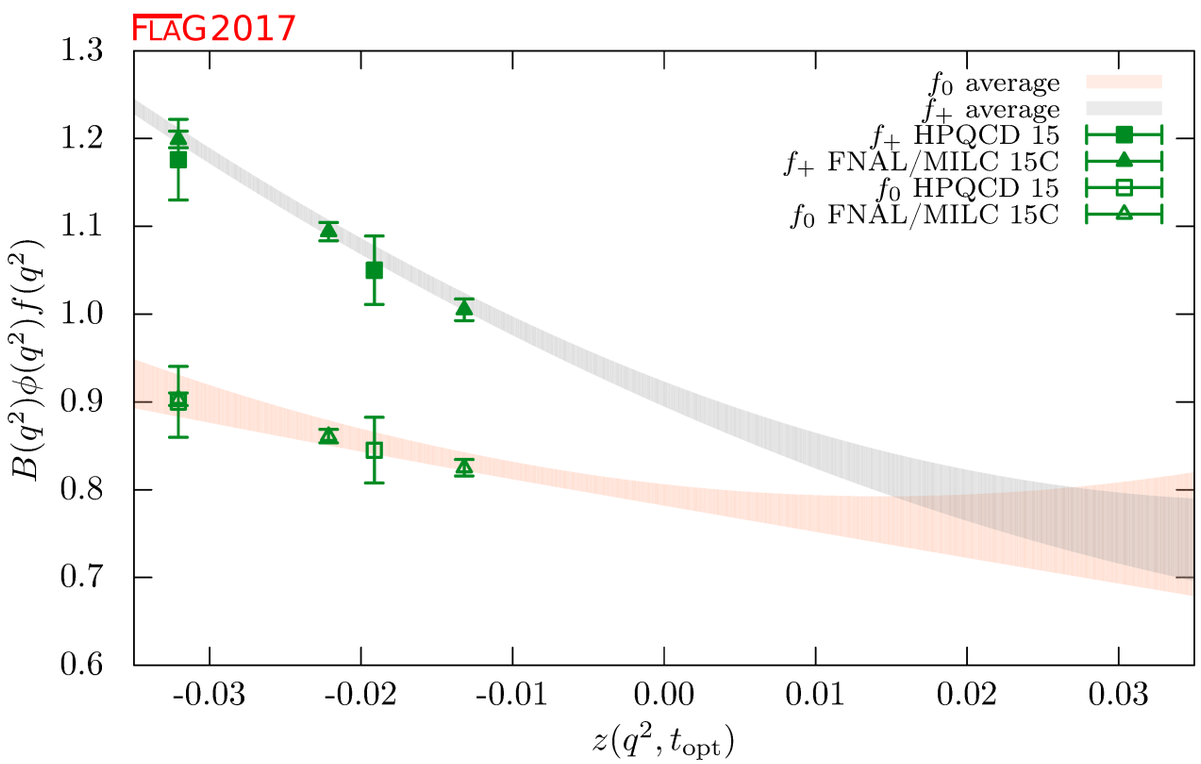


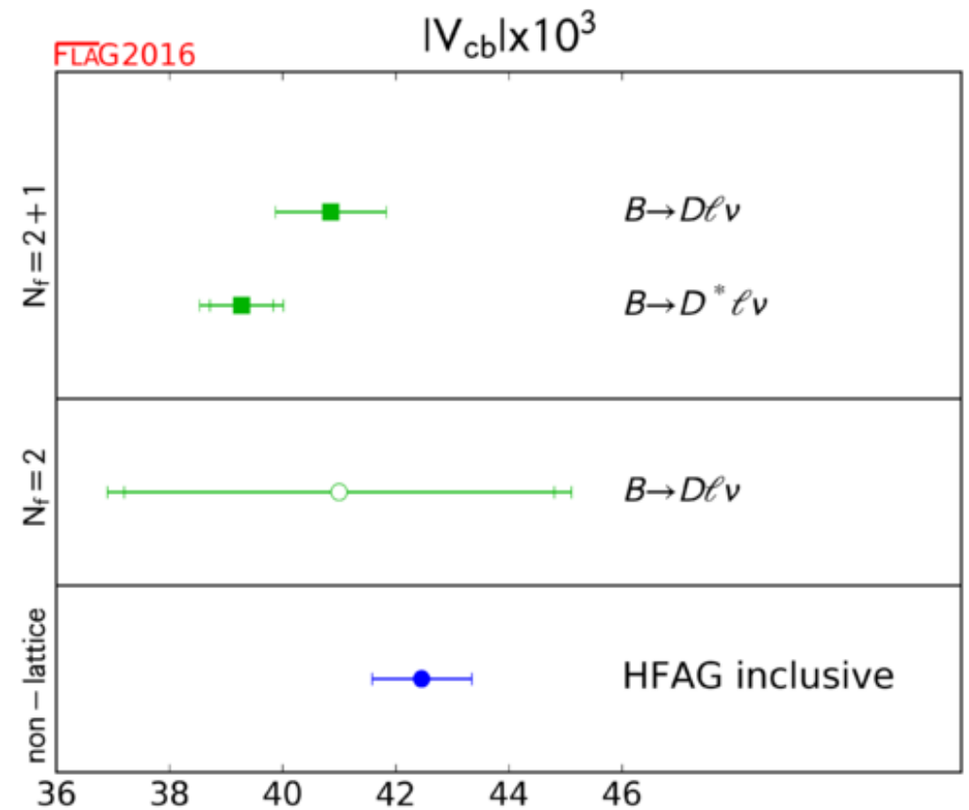
Overlay plot - there is a bit of a spread
let's wait until analyses finalised
will lead to prediction for $R(D_s)$

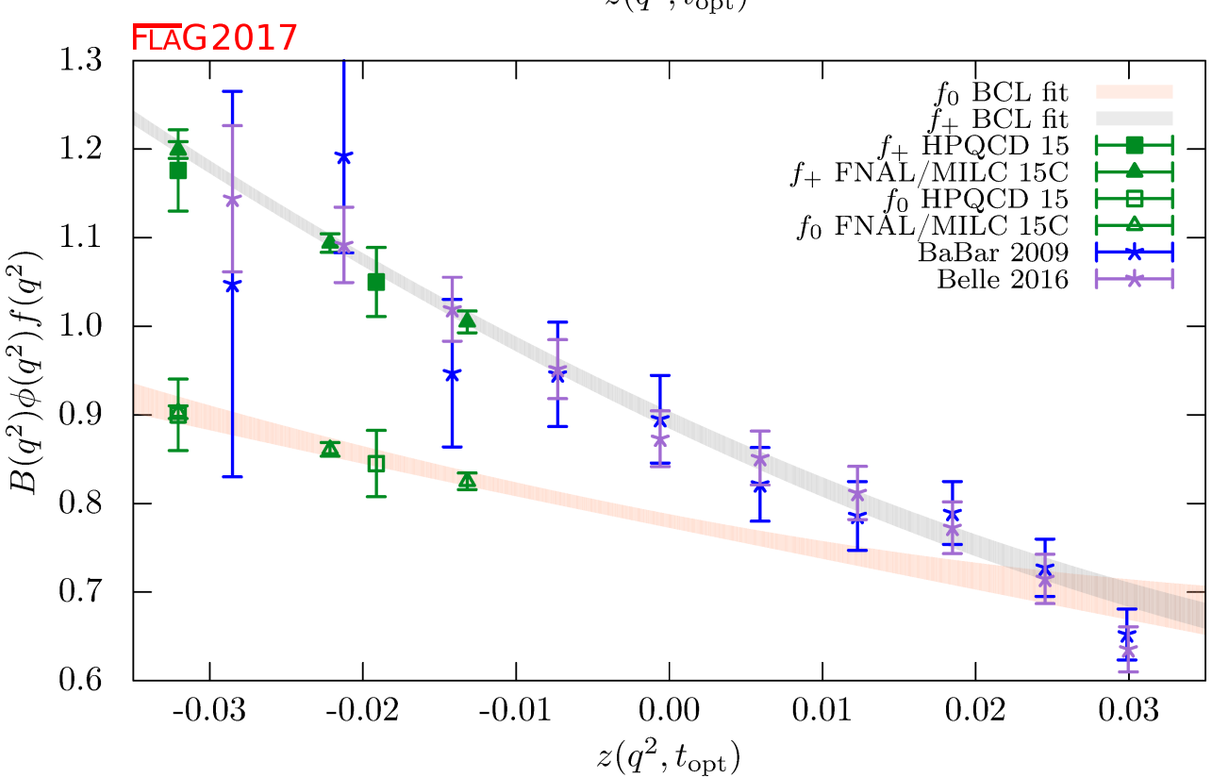
$B_s \to D_s^*$


$a \simeq 0.09$fm
$a \simeq 0.06$fm
NRQCD[1711.11013]
1.05
1.00
0.95
0.90
0.85
0.80
0.75
$h_{A_1}(q^2_{max})$
M_{η_c}
M_{η_b}
HPQCD Preliminary
McLean Lattice 2018
3
4
5
6
7
8
9
10
M_{η_h}[GeV]

V_{cb}







$$B \to D^* l\nu \quad |V_{cb}| \times 10^3 = 39.27(56)(49)$$

$$B \to D l\nu \quad |V_{cb}| \times 10^3 = 40.1(1.0)$$

Ratios

Ratios are great!

Cancellations of

- statistical errors (from Markov Chain Monte Carlo)
- systematic errors
  - renormalisation factors
  - discretisation effects
  - finite volume effects
  - quark-mass extrapolation (in particular heavy quark mass)
  - …

Ratios of branching fractions, differential decay rates, asymmetries…

V_{ub}/V_{cb}

Well known result from ratio of baryonic decay rates (exp.+lattice)

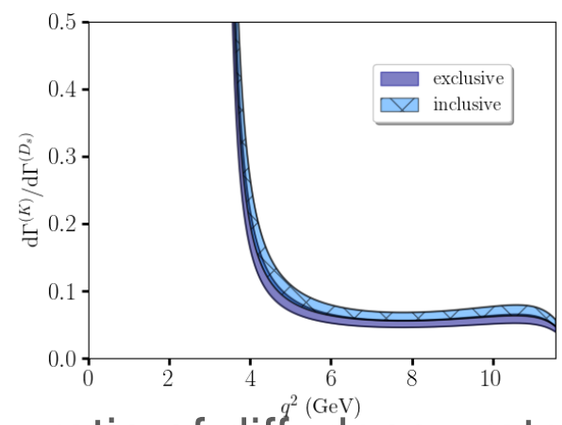
$$\Lambda_b^0 \to \Lambda_c^+ \mu \bar{\nu}$$
$$\Lambda_b^0 \to p \mu \bar{\nu}$$

LHCb, Nature Phys. 11, 743 (2015), 1504.01568.
LHCb PRD96, 112005 (2017), 1709.01920
W. Detmold, C. Lehner, and S. Meinel, PRD92, 034503 (2015), 1503.01421.

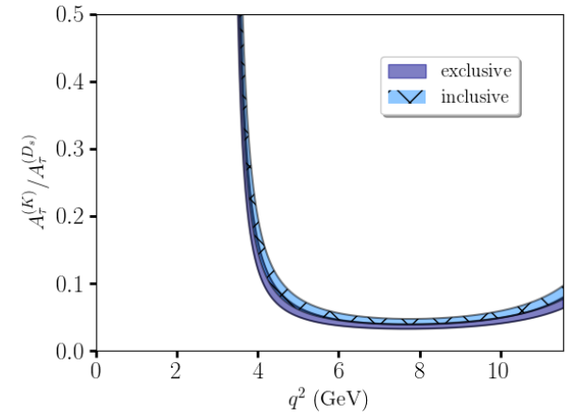
$$|V_{ub}/V_{cb}|^2 = 0.080(6)$$

alternative: ratio of SL B_s decays Monahan et al. arXiv:1808.09285

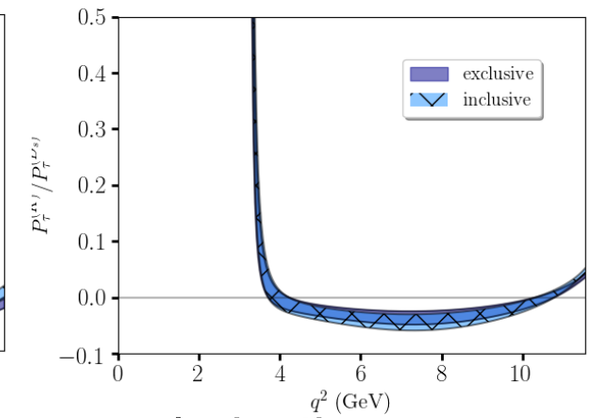
$$B_s \to K l \nu$$
$$B_s \to D_s l \nu$$



ratio of diff. decay rates

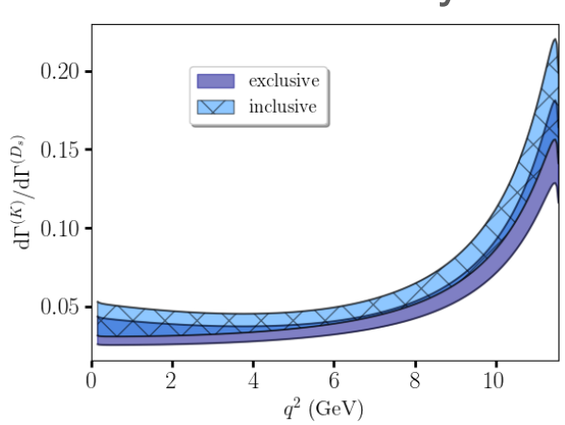


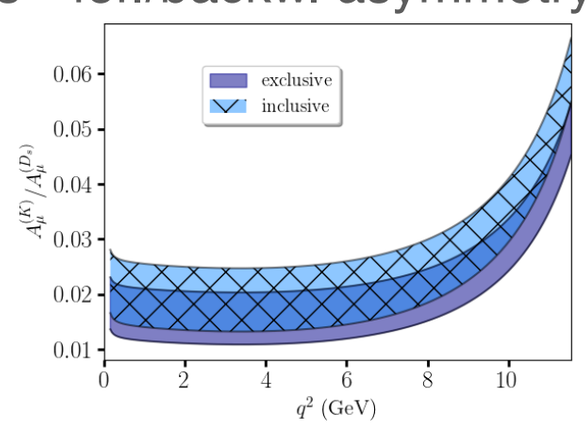
for./backw. asymmetry

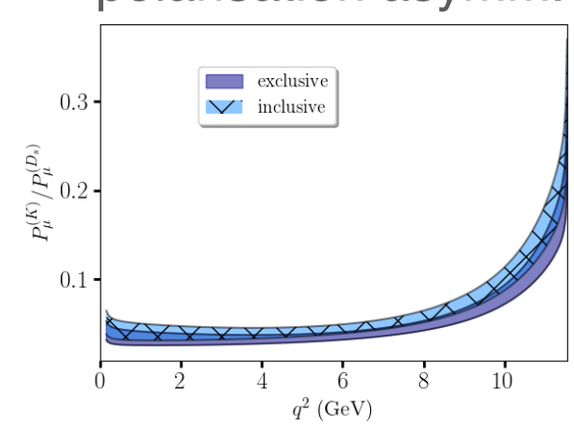


polarisation asymm.

Results assuming above $|V_{ub}/V_{cb}|^2$







RBC/UKQCD is finalising their analysis

What’s next?

Beyond %-level precision

With a sub-percent precision goal we can't ignore isospin breaking effects:

- strong isospin breaking

$$m_u = 2.46(24)\text{MeV} \qquad m_d = 5.03(26)MeV \qquad \overline{\text{MS}}(2\text{GeV})$$

FLAG 3

$$\frac{m_u - m_d}{\Lambda_{\text{QCD}}} \sim O(1\%)$$

- QED

$$\alpha \approx \frac{1}{137} \sim O(1\%)$$

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lattice QCD+QED

Isospin breaking: EM effects

Factorisation Γ = Weak x EM x Strong

lattice will no longer compute decay constants and form factors but aim directly at decay rate

Two major technical difficulties:

- photon is massless and induces large finite-size effects
- IR singularities (Bloch-Nordsiek) need to be dealt with

EM corrections to leptonic decay

Carrasco et al. PRD 91 074506 (2015) **arXiv:1502.00257**

EM corrections to leptonic decay

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- leptonic decay at $O(\alpha^0)$:

$$\Gamma(\pi^+ \to l^+ \nu_l) = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

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- including elm. effects @ $O(\alpha)$:

$$\Gamma(\pi^+ \to l^+ \nu_l(\gamma)) = \Gamma(\pi^+ \to l^+ \nu_l) + \Gamma(\pi^+ \to l^+ \nu_l \gamma)$$

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$$\equiv \quad \Gamma_0 \quad + \quad \Gamma_1$$

IR div. cancel between terms on r.h.s.
between virtual and real photons
(Bloch Nordsieck)

Summary

- Increased activity in $B_{(s)}$ SL decay calculations, mostly preliminary but anomaly-hype has created some momentum
- more results — more independent systematics
- most results based on effective theory for *b*-quark — this will continue for a while but increasingly extrapolations towards *b*
- we are learning how to include QED effects which become relevant at %-level. First applications leptonic decay under way.
  This will change the way analyses are done…

Backup - Baryon SL decays

Early work on $\Lambda_b \to \Lambda_c$ (quenched, focused on Isgur-Wise function):

K. C. Boweler *et al.* (UKQCD Collaboration), arXiv:hep-lat/9709028/PRD 1998
S. Gottlieb and S. Tamhankar, arXiv:hep-lat/0301022/Lattice 2002

Our work, using RBC/UKQCD $2+1$ flavor ensembles:

Transition	m_b	a [fm]	m_π [MeV]	Reference
$\Lambda_b \to \Lambda$	∞	0.11, 0.08	230–360	WD, DL, SM, MW, arXiv:1212.4827/PRD 2013
$\Lambda_b \to p$	∞	0.11, 0.08	230–360	WD, DL, SM, MW, arXiv:1306.0446/PRD 2013
$\Lambda_b \to p$	phys.	0.11, 0.08	230–360	WD, CL, SM, arXiv:1503.01421/PRD 2015
$\Lambda_b \to \Lambda_c$	phys.	0.11, 0.08	230–360	WD, CL, SM, arXiv:1503.01421/PRD 2015; AD, SK, SM, AR, arXiv:1702.02243/JHEP 2017
$\Lambda_b \to \Lambda$	phys.	0.11, 0.08	230–360	WD, SM, arXiv:1602.01399/PRD 2016
$\Lambda_b \to \Lambda^*$	phys.	0.11	340	SM, GR, arXiv:1608.08110/Lattice 2016
$\Lambda_b \to \Lambda_c^*$	phys.	0.11, 0.08	300–430	SM, GR, Later in this talk
$\Lambda_c \to \Lambda$		0.11, 0.08	**140**–360	SM, arXiv:1611.09696/PRL 2017
$\Lambda_c \to p$		0.11, 0.08	230–360	SM, arXiv:1712.05783/PRD 2018

WD = William Detmold
DL = C.-J. David Lin
SM = Stefan Meinel
MW = Matthew Wingate
CL = Christoph Lehner
AD = Alakabha Datta
SK = Saeed Kamali
AR = Ahmed Rashed
GR = Gumaro Rendon (graduate student at U of A)

Meinel's slide, CKM 2018

EM corrections to leptonic decay

- cut on small photon momentum < ΔE → γ *sees* point-like π
  $\Delta E \approx 20$MeV experimentally accessible and π point like

point approximation

$$\Gamma(\Delta E) = \lim_{V\to\infty} \boxed{(\Gamma_0 - \Gamma_0^{\mathrm{pt}})} + \lim_{V\to\infty} \boxed{(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E))}$$

$\Gamma(\pi^+ \to l^+ \nu_l)$

lattice and analytical
finite V

$\Gamma(\pi^+ \to l^+ \nu_l \gamma(\Delta E))$

analytically in $V \to \infty$

both terms separately IR finite, gauge invariant on its own

Sub precent precision requires inclusion of IB breaking effects for which new techniques in QFT are being developed