From R_{D^*} to $R_{D\pi}$: The role of longitudinal corrections

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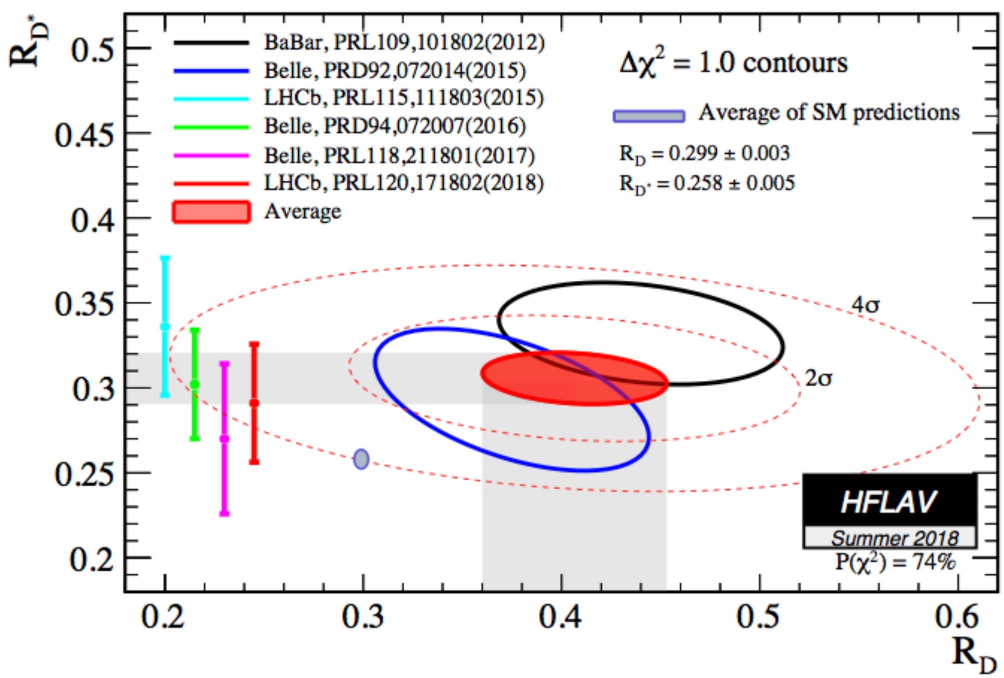
Evidence of LFU violation?

$$R_{D^{(*)}} \equiv \frac{\mathrm{Br}(B \to \tau\nu_\tau D^{(*)})}{\mathrm{Br}(B \to l\nu_l D^{(*)})}$$



$R_{D^*}^{SM} = 0.252 \pm 0.003$
$R_{D^*}^{expWA} = 0.304 \pm 0.015$ } 3.7 σ

$R_D^{SM} = 0.299 \pm 0.011$
$R_D^{exp} = 0.407 \pm 0.046$ } 2.3 σ

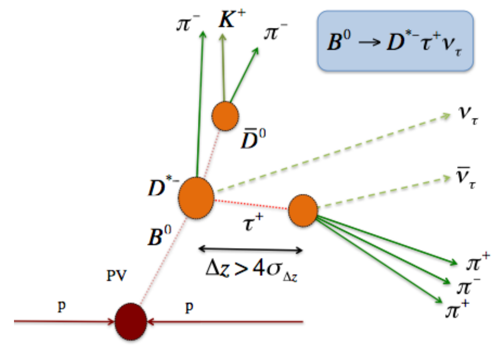
Y. Amhis et al. (HFLAV), Eur. Phys. J. C77, 895 (2017), 1612.07233.
See Befani, S. et al. arXiv:1809.06229v1 for a recent review

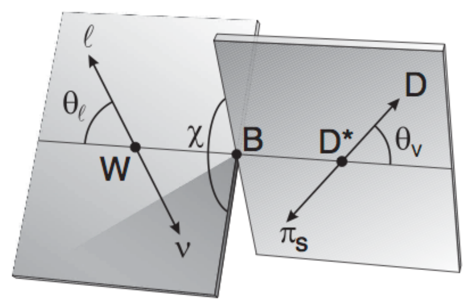
$B \to D^{*} l \nu$ reconstruction

- LHCb
  - Uses Neutral B's (charged D*)
  - D* -> Dπ , $\Delta M = M_{D^*} - M_D < 2$ MeV/c²
  - D -> Kπ,
  - τ, leptonic and hadronic modes (π)

- Belle
  - Uses Neutral and Charged B's
  - D* -> Dπ, Dγ, $\Delta M = M_{D^*} - M_D < 3\,\sigma$
  - D -> Kπ, Kππ
  - τ, leptonic and hadronic modes (π, ρ)

- BaBar
  - Uses Neutral and Charged B's
  - D* -> Dπ, Dγ, $\Delta M = M_{D^*} - M_D < 4\,\sigma$
  - D -> Kπ, Kππ, Kπππ, KKπ…
  - τ, leptonic modes





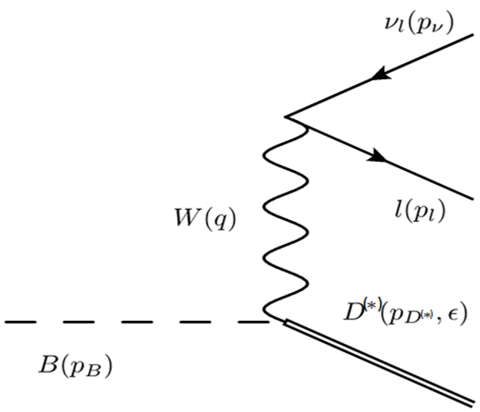
D* is detected through its daughter particles

Br(D*+ -> Dπ)= 98 %
Br(D*0 -> Dπ)= 64.7 %
Br(D*0 -> Dγ)= 35.3 %

M. Tanabashi et al. (PDG) PRD 98, 030001 (2018).

Standard model calculation

- 3-body decay



$$M_3 = \frac{G_F}{\sqrt{2}} V_{cb} J^\lambda l_\lambda$$

Leptonic current

$$l_\lambda \equiv \bar{u}_l \gamma_\lambda (1-\gamma^5) v_\nu$$

Parametrization of the hadronic vertex

WSB ZPC 29 637(1985). Form factors param. *Falk A and Neubert, M. PRD 47 2965 (93) Caprini I. et al Nucl. Phys. B 530 (1998) 15*

$$\begin{aligned}
&\langle D^*(p_{D^*}, \epsilon_\mu)|J^\lambda|B(p_B)\rangle \\
&= \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon^{\lambda\mu\alpha\beta} \epsilon^*_\mu (p_B)_\alpha (p_{D^*})_\beta - 2m_{D^*} A_0(q^2) \frac{q^\lambda q \cdot \epsilon^*}{q^2} \\
&\quad - (m_B + m_{D^*}) A_1(q^2) \left(\epsilon^{*\lambda} - \frac{q^\lambda q \cdot \epsilon^*}{q^2} \right) \\
&\quad + \frac{A_2(q^2) q \cdot \epsilon^*}{m_B + m_{D^*}} \left((p_B + p_{D^*})^\lambda - \frac{m_B^2 - m_{D^*}^2}{q^2} q^\lambda \right),
\end{aligned}$$

- V, A0, A1, A2 have been reconstructed by Belle using HQEFT analysis, but only for e and μ measurements – not τ! W. *Dungel et al. (Belle Collaboration), Phys. Rev. D 82, 112007 (2010).*
- A0 is heavily suppressed for e and μ since it represents a longitudinal state of the lepton-neutrino system
- Instead, A0 is derived from A2 using HQEFT approximation. A direct measurement from τ data could test it!

Form factors measurement

$$\langle D^*(p_{D^*},\epsilon_\mu)|J^\lambda|B(p_B)\rangle$$
$$=\frac{2iV(q^2)}{m_B+m_{D^*}}\epsilon^{\lambda\mu\alpha\beta}\epsilon^*_\mu(p_B)_\alpha(p_{D^*})_\beta - 2m_{D^*}A_0(q^2)\frac{q^\lambda q\cdot\epsilon^*}{q^2}$$
$$-(m_B+m_{D^*})A_1(q^2)\left(\epsilon^{*\lambda}-\frac{q^\lambda q\cdot\epsilon^*}{q^2}\right)$$
$$+\frac{A_2(q^2)q\cdot\epsilon^*}{m_B+m_{D^*}}\left((p_B+p_{D^*})^\lambda-\frac{m_B^2-m_{D^*}^2}{q^2}q^\lambda\right),$$

	Belle	Average
$h_{A_1}(1)\|V_{cb}\|$	$(34.6\pm0.2\pm1.0)\times10^{-3}$	$(35.61\pm0.43)\times10^{-3}$
ρ^2	$1.214\pm0.034\pm0.009$	1.205 ± 0.026
$R_1(1)$	$1.401\pm0.034\pm0.018$	1.404 ± 0.032
$R_2(1)$	$0.864\pm0.024\pm0.008$	0.854 ± 0.020

$$w\equiv1-\frac{1}{2}\left(\frac{p_B}{m_B}-\frac{p_{D^*}}{m_{D^*}}\right)^2=\frac{p_B\cdot p_{D^*}}{m_Bm_{D^*}}=\frac{m_B^2+m_{D^*}^2-q^2}{2m_Bm_{D^*}}$$

$$h_{A_1}(w)=\frac{A_1(q^2)}{R_{D^*}}\frac{2}{w+1}\qquad R_{D^*}\equiv2\sqrt{m_Bm_{D^*}}/(m_B+m_{D^*})$$

$$V(q^2)=R_1(w)\frac{h_{A_1}}{R_{D^*}},$$

$$A_0(q^2)=R_0(w)\frac{h_{A_1}}{R_{D^*}},$$

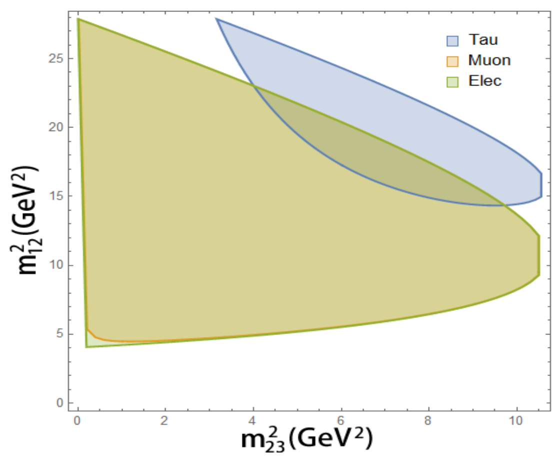
$$A_2(q^2)=R_2(w)\frac{h_{A_1}}{R_{D^*}}.$$

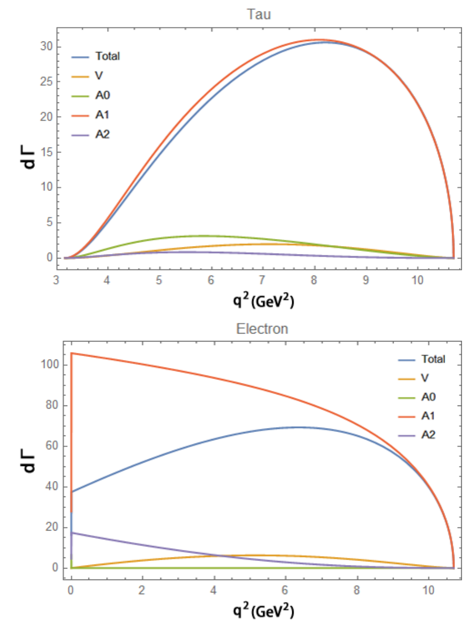
$$R_1(w)=R_1(1)-0.12(w-1)+0.05(w-1)^2,$$
$$R_0(w)=R_0(1)-0.11(w-1)+0.01(w-1)^2,$$
$$R_2(w)=R_2(1)+0.11(w-1)-0.06(w-1)^2,\ \text{and}$$
$$h_{A_1}(w)=h_{A_1}(1)\left[1-8\rho^2z+(53\rho^2-15)z^2-(231\rho^2-91)z^3\right],$$
$$\text{with } z\equiv\frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}},$$

$$\frac{R_2(1)(1-m_{D^*}/m_B)+(m_{D^*}/m_B)\left[R_0(1)(1+m_{D^*}/m_B)-2\right]}{(1-m_{D^*}/m_B)^2}=0.97$$

Form factor measurements

- Phase space for different leptons. Form factors are evaluated in different regions.





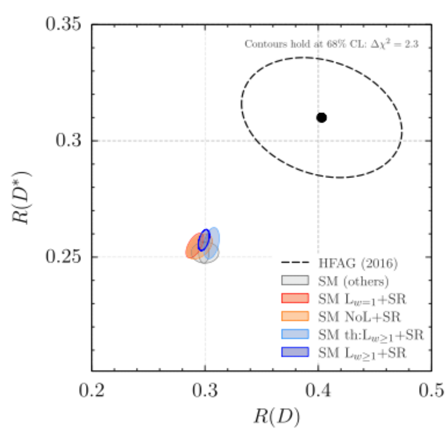
form factors contributions to the differential decay width

- Different parametrizations, NLO corrections and theoretical restrictions for the form factors have been studied

Boyd, C. et al. PRD 56 (1997) 6895

Bigi,D. et al. JHEP11(2017)061

Berlochner,F. et al. PRD 95, 115008 (2017)



R_{D^*} SM prediction

R_{D^*}	Reference
0.252	Chuan-Hung, C. et al. JHEP 10(2006) 053 (LFQM)
0.252 ±0.003	S. Fajfer, S. et al. PRD 85, 094025 (2012) (CLN)
0.260 ± 0.008	Bigi, D. et al. JHEP 11(2017)06 (LCSR)
0.257 ± 0.003	Berlochner, F. et al. PRD 95, 115008 (2017)
0.259 ± 0.006	Jaiswal, S. et al. JHEP 12 (2017)060 CLN
0.257 ± 0.005	Jaiswal, S. et al. JHEP 12 (2017)060 BGL

What is the correction from considering the D* as intermediate state?

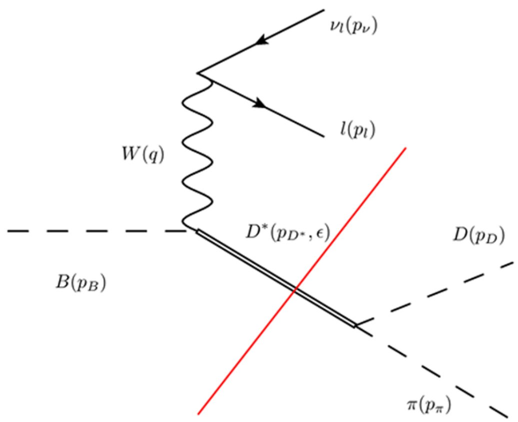
Longitudinal dof, finite width,…

From R_{D^*} to $R_{D\pi}$

$$R_{D\pi} \equiv \frac{BR(B \to \tau\nu_\tau \pi D)}{BR(B \to l\nu_\pi D)}$$

Standard model calculation contd.

- 4-body decay



$$M = M_{3\mu} D^{\mu\nu} M_{2\nu},$$

$$M_3^\mu = \frac{G_F}{\sqrt{2}} V_{cb} J^{\lambda\mu} l_\lambda$$

$$J^\lambda = J^{\lambda\mu} \epsilon_\mu$$

Consider hadronic vertex as in the 3-body decay additional structures might be important.

$$M_{2\nu} = -ig(p_D - p_\pi)_\nu$$

2-body decay vertex

$D^{\mu\nu}$ is the D^* propagator

Proper description of unstable states is important

Standard model calculation contd.

Unstable states description

For gauge bosons (ex.W) The tree level propagator $D_0^{\mu\nu}(q) = -i(g^{\mu\nu} - \frac{q^\mu q^\nu}{M^2})/(q^2 - M^2 + i\epsilon).$

With the replacement $M^2 \to M^2 - iM\Gamma$

The tree level electromagnetic vertex W(q1)W(q2)γ(k) $\Gamma_0^{\mu\nu\lambda} = g^{\nu\lambda}(q_1+q_2)^\mu - g^{\mu\nu}(q_1+k)^\lambda - g^{\mu\lambda}(q_2-k)^\nu$

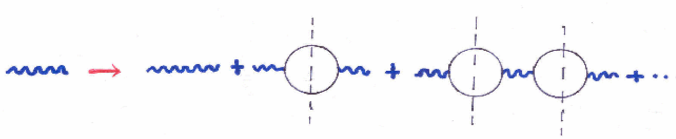
- Ward identity is not fullfiled !!! $k_\mu \Gamma_0^{\mu\nu\lambda} = iD_0^{\nu\lambda}(q_1)^{-1} - iD_0^{\nu\lambda}(q_2)^{-1} - iM\Gamma(\frac{q_1^\nu q_1^\lambda}{q_1^2 - M^2} + \frac{q_2^\nu q_2^\lambda}{q_2^2 - M^2})$

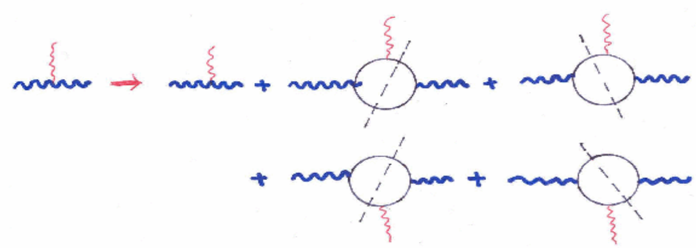
Fermion-loop scheme *U. Baur and D. Zeppenfeld PRL 75(1995)1002; E.N. Argyres et al PLB 358(1995)339; M. Beuthe et al. NPB 498 55(1997)*

In QFT, widths arise naturally from the imaginary parts of higher-order diagrams describing boson self-energies, resummed to all orders

Linearity of the Ward identity is fullfiled order by order in PT

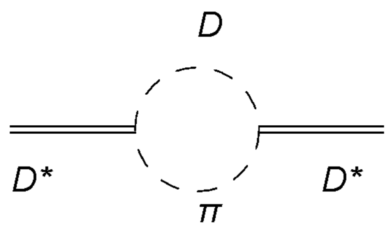
- Absorptive part resummation of the fermion loops in propagators AND electromagnetic vertices.





Absorptive corrections to the propagator: Boson loop scheme

- In a similar procedure, for vector mesons we computed the absorptive contribution due to the bosons in the loop of the propagator and vertex. *G.Lopez Castro and GT PRD 61 (00)033007; L.A. Jimenez Pérez and GT JPG 44 125003 (2017)*



The corrected propagator, split in Transversal (T) and Longitudinal (L) is:

$$D^{\mu\nu} = \frac{-iT^{\mu\nu}}{p_{D^*}^2 - m_{D^*}^2 + i\mathrm{Im}\Pi_T} + \frac{iL^{\mu\nu}}{m_{D^*}^2 - i\mathrm{Im}\Pi_L},$$

$$T^{\mu\nu} \equiv g^{\mu\nu} - \frac{p_{D^*}^\mu p_{D^*}^\nu}{p_{D^*}^2} \qquad L^{\mu\nu} \equiv \frac{p_{D^*}^\mu p_{D^*}^\nu}{p_{D^*}^2}$$

where the absorptive functions are:

$$\mathrm{Im}\Pi_T = \sqrt{p_{D^*}^2}\,\Gamma_{D^*}(p_{D^*}^2)$$

introduces the finite decay width

$$Im\Pi_L = -\frac{g^2\lambda^{1/2}(p_{D^*}^2, m_D^2, m_\pi^2)}{16\pi}\left(\frac{m_D^2 - m_\pi^2}{p_{D^*}^2}\right)^2$$

the longitudinal correction is proportional to $\sim \frac{m_D^2 - m_\pi^2}{m_{D^*}^2} = \Delta^2 = 0.86$

- The interference between T and L may be relevant. Earlier calculations of $R_{D\pi}$ considered (compared to our case) Im Π_L= 0 and drop off $im_{D^*}\Gamma_{D^*}$ in the longitudinal part. *C. S. Kim, G. Lopez-Castro, S. L. Tostado, and A. Vicente, Phys. Rev. D 95, 013003 (2017).*

$$\left[\frac{N_{D^*}^{\nu\beta}(p_1 + p_2)}{(p_1 + p_2)^2 - m_{D^*}^2 + im_{D^*}\Gamma_{D^*}}\right] \qquad N_{V^*}^{\nu\beta}(q) = T_{\nu\beta}(q) + L_{\nu\beta}(q)(m_{V^*}^2 - q^2)/m_{V^*}^2$$

For other approaches see P. Roudeau talk

Standard model calculation contd.

- 4-body decay. Narrow width approximation $|p_{D^*}^2 - m_{D^*}^2 + i m_{D^*}\Gamma(p_{D^*}^2)|^{-2} \approx \frac{\pi}{m_{D^*}\Gamma_{D^*}}\delta(p_{D^*}^2 - m_{D^*}^2)$

  - Transverse $$|M_T|^2 = M_{3\mu}M_{3\alpha}^* T^{\mu\nu}T^{\alpha\beta}M_{2\nu}M_{2\beta}^* \frac{\pi\delta(p_{D^*}^2 - m_{D^*}^2)}{m_{D^*}\Gamma_{D^*}}$$ $$\Gamma_{D^*} \equiv \Gamma_{D^*}(m_{D^*}^2)$$

  - Longitudinal $$M_L = igM_{3\mu}\frac{p_{D^*}^\mu}{p_{D^*}^2}\frac{m_D^2 - m_\pi^2}{m_{D^*}^2 - i\mathrm{Im}\Pi_L}$$ $$|M|_L^2 = |M_3^\mu L_{\mu\nu} M_2^\nu|^2 \frac{1}{m_{D^*}^4 + (Im\Pi_L)^2}.$$

  - Interference $$-iM_3^{\mu_1}M_3^{*\mu_2}\left(\frac{T_{\mu_1\nu_1}L_{\mu_2\nu_2}}{m_{D^*}^2 + iIm\Pi_L} + \frac{L_{\mu_1\nu_1}T_{\mu_2\nu_2}}{m_{D^*}^2 - iIm\Pi_L}\right)M_2^{\nu_1}M_2^{*\nu_2}\pi\delta(p_{D^*}^2 - m_{D^*}^2)$$

$$= l^{\lambda_1\lambda_2}\left(H_{\lambda_1}^T H_{\lambda_2}^{L*} + H_{\lambda_1}^L H_{\lambda_2}^{T*}\right)$$
$$= 2l_S^{\lambda_1\lambda_2} Re\left(H_{\lambda_1}^T H_{\lambda_2}^{L*}\right) + 2l_A^{\lambda_1\lambda_2} Im\left(H_{\lambda_1}^T H_{\lambda_2}^{L*}\right)$$

Relevant contribution

Levi-Civita from leptonic part

Metric tensor in T and BD*W vertex prop. to A1 only

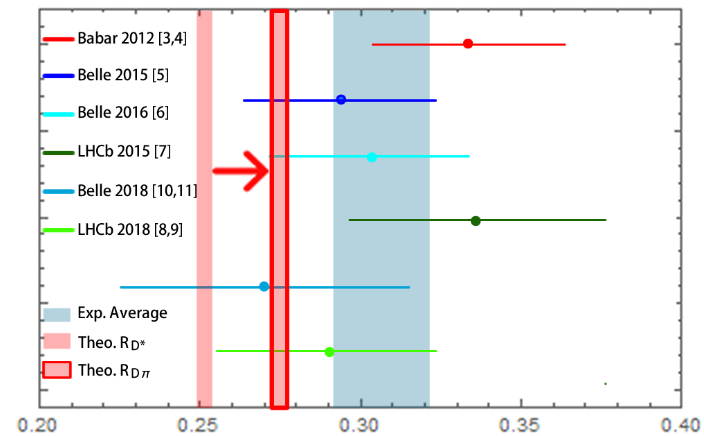
P_D from D*Dπ vertex

we explored the longitudinal contribution around $(p_D + p_\pi)^2 = (m_{D^*} \pm \delta)^2$ δ is in the range $\Gamma_{D^*}/2$ to 1 MeV.

Results for $R_{D\pi}$

Br (in %)	Transversal	Longitudinal $(\delta = \Gamma_{D^*})$,	Interference
Electron	4.6(3)	$5.0(3) \times 10^{-6}$	$7.6(6) \times 10^{-8}$
Muon	4.6(3)	$5.0(3) \times 10^{-6}$	$1.6(1) \times 10^{-3}$
Tau	1.16(8)	$1.1(6) \times 10^{-6}$	$1.02(7) \times 10^{-1}$
$R^e_{D\pi}$	0.252	0.252	0.274 (3)
$R^\mu_{D\pi}$	0.252	0.252	0.275 (3)

$R_{D\pi}$ values as each contribution is added from left to right.
Our final result is highlighted in the red square



LHCb 1.1σ to 0.48 σ

Belle 0.42σ to 0.10 σ

WA 3.7σ to 2.10 σ

Independent of the value of $\delta < 1$ MeV at the current precision.
Integrating out in the full phase space, the longitudinal contribution becomes:

$(R_{D\pi})_L = 0.111(3)$ This work

$(R_{D^*})_L = 0.115(2)$ *S. Fajfer, S. et al. PRD 85, 094025 (2012)*

Outlook

- By considering $R_{D\pi}$, the SM model prediction is brought into better agreement with the latest LHCb and Belle results for B $\rightarrow D^* l\,\nu$ ($\rightarrow D\pi\, l\,\nu$) measurements.

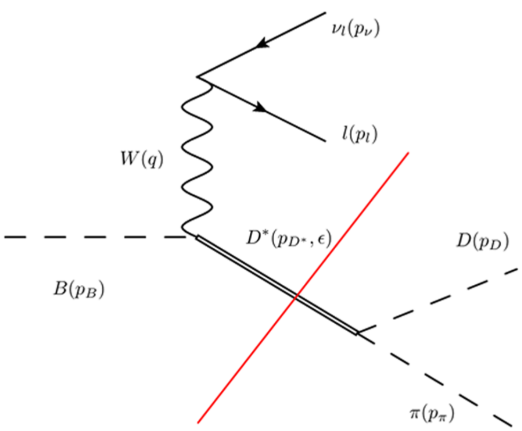
- Reconstruction of D* using other decay modes are not affected by the corrections here considered

- The SM prediction could be refined further with a more careful analysis of the BD*W vertex.
  - τ data could provide a better approximation of Ao
  - form factors parametrizations and HQET assumptions.

- Radiative corrections may be important at the few percent level
  - see S. de Boer talk.

- Correlations with other observables and implications on scenarios for new physics may provided hints on remaining discrepancies.
  - see M. Fedele and M. Blanke talks

4-body kinematics



Decay width

$$\Gamma_4 = \int \frac{|M|^2 ds_2 ds_1}{2m_B(2\pi)^8} \frac{du_1 du_2}{\lambda^{1/2}(m_B^2, s_2, s_2')\lambda^{1/2}(m_B^2, m_\pi^2, u_2)} \times \frac{dt}{\lambda^{1/2}(m_B^2, u_1, m_l^2)[(1-\xi^2)(1-\eta^2)(1-\zeta^2)]^{1/2}},$$

phase space factors

5 independent variables

R. Kumar, Phys. Rev. 185, 1865 (1969).

$$s_2 \equiv (p_B - p_l - p_\nu) = p_{D^*}^2 \qquad s_1 \equiv (p_B - p_l)^2$$

$$u_1 \equiv (p_B - p_\nu)^2 = (p_l + p_\pi + p_D)^2 \qquad u_2 \equiv (p_B - p_\pi)^2 = (p_l + p_\nu + p_D)^2$$

$$t \equiv (p_B - p_\nu - p_\pi)^2 = (p_l + p_D)^2$$

integration limits, in general

$$s_2^- = (m_\pi + m_D)^2, \qquad s_2^+ = (m_B - m_l - m_\nu)^2,$$

$$s_1^- = (\sqrt{s_2} + m_\nu)^2, \qquad s_1^+ = (m_B - m_l)^2,$$

$$u_1^\pm = m_B + m_\nu^2 - \frac{(s_1 + m_\nu^2 - s_2)(m_B + s_1 - m_\nu^2)}{2s_1} \pm \frac{\lambda^{1/2}(s_1, m_\nu^2, s_2)\lambda^{1/2}(m_B^2, s_1, m_\nu^2)}{2s_1},$$

$$u_2^\pm = m_B + m_\pi^2 - \frac{(s_2 + m_\pi^2 - m_D^2)(m_B - s_2 - s_2')}{2s_2} \pm \frac{\lambda^{1/2}(s_2, m_\pi^2, m_D^2)\lambda^{1/2}(m_B, s_2, s_2')}{2s_2},$$

$$t^\pm = u_1 + m_\pi^2 - \frac{(m_B^2 + m_\pi^2 - u_2)(m_B^2 + u_1 - m_l^2)}{2m_B^2} + \frac{\lambda^{1/2}(m_B^2, m_\pi^2, u_2)\lambda^{1/2}(m_B^2, u_1, m_l^2)}{2m_B^2}\left(-\xi\eta \pm \sqrt{(1-\xi^2)(1-\eta^2)}\right)$$

The B-D-pi-W vertex

The most general Lorentz structure of the hadronic matrix element in the 4-body decay allows terms proportional to $p_{D^*}{}^{\mu}$ which are null when taking the D* on-shell, in the 3-body decay.