Lattice Predictions for Bound Heavy Tetraquarks

Anthony Francis*
Renwick James Hudspith
Randy Lewis
Kim Maltman

Implications of LHCb measurements and future prospects

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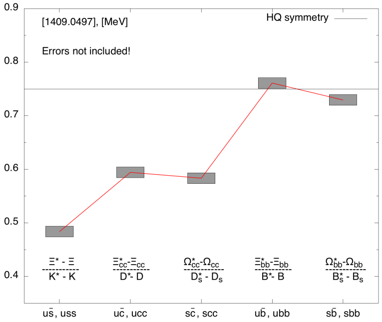
Away from the X, Y, Z states, the observed heavy hadron spectrum suggests a phenomenological binding mechanism from "good" diquark configurations for ground state tetraquarks containing heavy quarks, e.g. $qq'\bar{b}\bar{b}$ and $qq'\bar{c}\bar{b}$.

Assumptions and observations:

- HQS $\rightarrow$ heavy quark spin decouples and $(\bar{h}\bar{h})_3 \leftrightarrow h$
- Good approx. in $(B^* - B)/(\Xi^*_{bb} - \Xi_{bb})$ and $(B^*_s - B_s)/(\Omega^*_{bb} - \Omega_{bb})$



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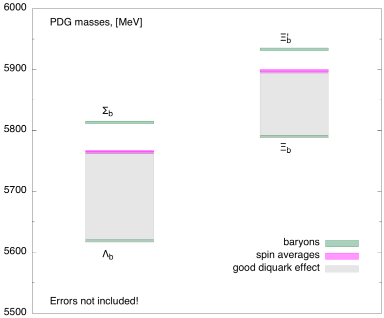
Naive binding using the spin average $B_{sp} = \frac{1}{4}[3(\mathrm{spin0}) + (\mathrm{spin1})]$:

- $\Sigma_b - \Lambda_b \approx 194\mathrm{MeV}$ vs. $B_{sp} - \Sigma_b \sim -145\mathrm{MeV}$
- $\Xi'_b - \Xi_b \approx 162\mathrm{MeV}$ vs. $B_{sp} - \Xi'_b \sim -106\mathrm{MeV}$



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Consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- *Bound ground state* tetraquark below meson-meson threshold
- Deeper binding with heavier $\bar{Q}'\bar{Q}$ diquarks, $\sim 1/m_Q$
- Binding set by the reduced mass of $\bar{Q}$ and $\bar{Q}'$ in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark

$\Rightarrow$ Great opportunity for lattice theory prediction.

Direct calculation of $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$

Goals:

- Determine $\Delta E = E_{\text{tetra}} - E_{\text{meson}-\text{meson}}$ for $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$
  $\Rightarrow$ we use binding correlator $\sim e^{-\Delta E\, t}$
- Study quark mass dependence $\Rightarrow$ Verify, quantify predictions

Sea-quark sector:

- $N_f = 2 + 1$ Wilson-Clover fermions with Iwasaki gauge action

Valence-quark sector:

- Wilson-Clover quarks for $u = d, s$
- Fermilab/Tsukuba relativistic effective HQ action for c
- NRQCD for b and non-relativstic, unphysical $Q' = b'$

PACS-CS,'09	$32^3 \times 64$	$a^{-1} = 2.194$[GeV]	$m_{s,lat} = m_{s,phys}$
Label	E_H	E_M	E_L
m_π[MeV]	415	299	163
$m_\pi L$	6.1	4.4	2.4

Diquark-Diquark operator:

$$D(x) = \Big((q_a^\alpha(x))^T (C\gamma_5)^{\alpha\beta} q'^\beta_b(x) \Big) \times \\ \big[(\bar{Q}_a^\kappa(x)(C\gamma_i)^{\kappa\rho}(\bar{Q}'^\rho_b(x))^T - (\bar{Q}_b^\kappa(x)(C\gamma_i)^{\kappa\rho}(\bar{Q}'^\rho_a(x))^T \big] .$$

Dimeson-Dimeson operator:

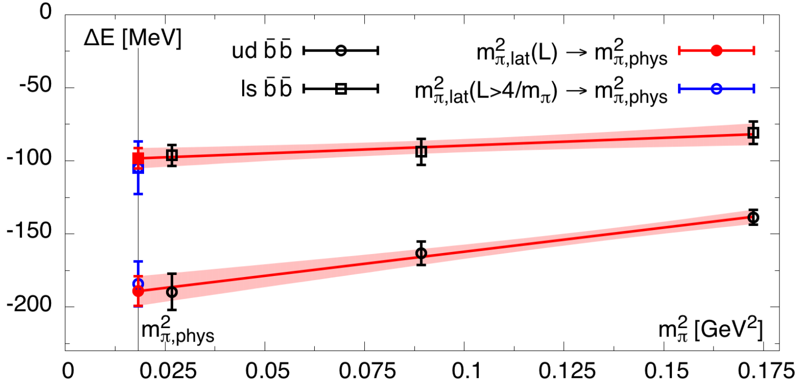
$$M(x) = \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta} u_a^\beta(x)\ \bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho} d_b^\rho(x) - \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta} d_a^\beta(x)\ \bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho} u_b^\rho(x) .$$

Compute the energies from a 2×2 GEVP

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu ,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t) C_{VV}(t)} , \ \lambda(t) = A e^{-\Delta E (t-t_0)} .$$

Tetraquarks with $\bar{Q}'\bar{Q}$: 3×3 GEVP via second two-meson threshold.



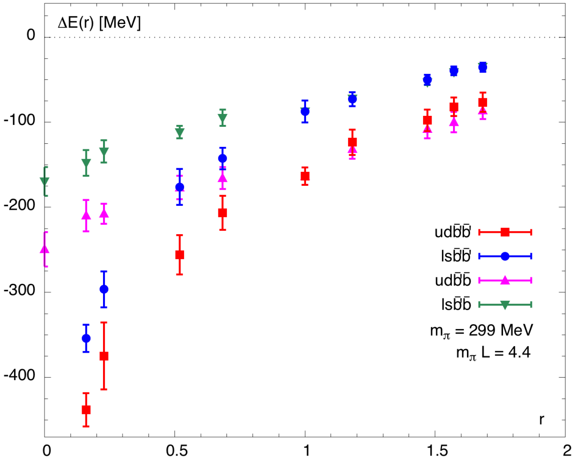
Physical point: $\Delta E_{ud\bar{b}\bar{b}} = 189(10)(3)$ MeV and $\Delta E_{ls\bar{b}\bar{b}} = 98(7)(3)$ MeV

Heavy quark mass dependence

- *Bound ground state* tetraquark below meson-meson threshold ✓
- Deeper binding with heavier $\bar{Q}'\bar{Q}$ diquarks, $\sim 1/m_Q$
- Binding set by the reduced mass of $\bar{Q}$ and $\bar{Q}'$ in the $\bar{Q}'\bar{Q}$ diquark
- Deeper binding for lighter quarks in the qq' diquark ✓

The direct calculation of $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ validates the prediction of a state below the meson-meson threshold and reveals a deeper binding with lighter qq' diquarks.

$\Rightarrow$ Further insight into by varying heavy quark mass $\bar{Q}' = \bar{b}'$



- Channels investigated are: $ud\bar{b}'\bar{b}'$, $ls\bar{b}'\bar{b}'$, $ud\bar{b}'\bar{b}$ and $ls\bar{b}'\bar{b}$
- $m^b/m^{b'} = r$, i.e $r = 0$ means $m'_Q = \infty$
- $r^{-1} = 0.594, ..., 6.287, \infty(=\text{static})$

- *Bound ground state* tetraquark below meson-meson threshold ✓
- Deeper binding with heavier $\bar{Q}'\bar{Q}$ diquarks, $\sim 1/m_Q$ ✓
- Binding set by the reduced mass in the $\bar{Q}'\bar{Q}$ diquark ✓
- Deeper binding for lighter quarks in the qq' diquark ✓

- Is it possible to further quantify these findings in a model?
- Can we gain insight away from the HQS validity regime, e.g. in the charm quark region?

Direct lattice calculation at the charm quark mass is not possible with this set-up due to the loss of control of discretization effects ($aM < 1$ in $1/(aM)$-expansion).

- Requires more resource intensive treatment of the c-quark
- Pool resources and invest on the most probable tetraquark candidate.

Towards a phenomenological model description

Modeled, expected terms contributing to the tetraquark binding:

1. Coulomb attraction between two heavy antiquarks $\rightarrow \mathcal{O}(\mu)$
   1 parameter: C_0
2. Mass independent term from the good light diquark
   $\rightarrow \mathcal{O}(const.)$
   2 parameters for ud, ℓs: C_1^{ud}, $C_1^{\ell s}$
3. Residual heavy-light interaction (tetraquark state)
   $\rightarrow \mathcal{O}(1/m_{Q_1} + 1/m_{Q_2})$
   2 parameters: C_2^{ud}, $C_2^{\ell s}$
4. Residual heavy-light interaction (two-meson threshold states)
   $\rightarrow \mathcal{O}(1/m_{Q_1} + 1/m_{Q_2})$
   Fixed by PDG

A model with in total 5 independent parameters can be formed.

$\bar{b}'\bar{b}'$:

$$\Delta E_{ud\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud}\,(2r) + (23\text{ MeV})\,r,$$

$$\Delta E_{\ell s\bar{b}'\bar{b}'} = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s}\,(2r) + (24\text{ MeV})\,r$$

$\bar{b}'\bar{b}$, $r < 1$:

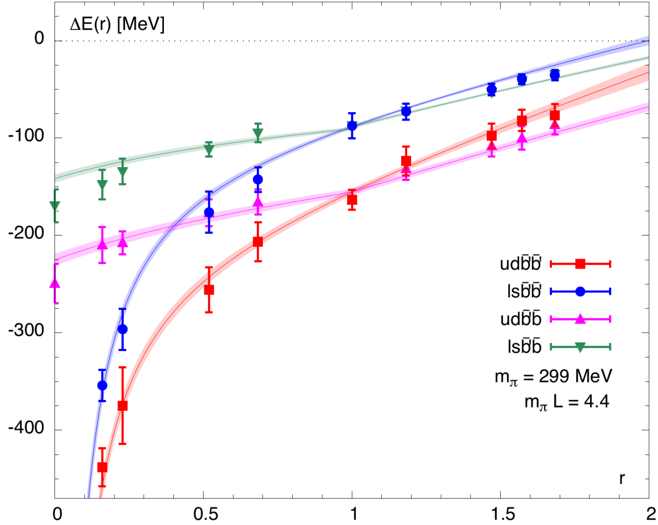
$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34\text{ MeV} - 11\text{ MeV}\,r)\,,$$

$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (34\text{ MeV} - 12\text{ MeV}\,r)$$

$\bar{b}'\bar{b}$, $r > 1$:

$$\Delta E_{ud\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34\text{ MeV}\,r - 11\text{ MeV})\,,$$

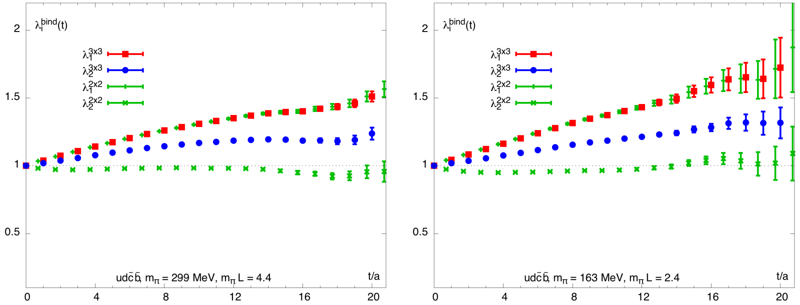
$$\Delta E_{\ell s\bar{b}'\bar{b}} = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (36\text{ MeV}\,r - 11\text{ MeV})$$



- Most likely additional bound tetraquark in charm quark region: $ud\bar{c}\bar{b}$

Direct calculation of $ud\bar{c}\bar{b}$ tetraquarks

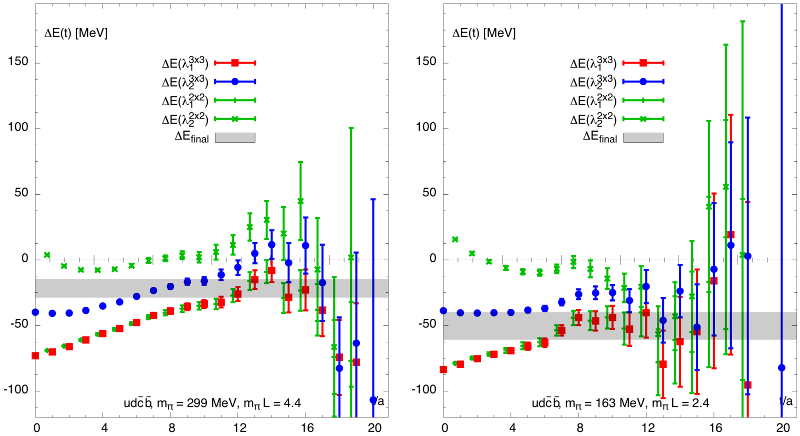
Most likely bound state: $ud\bar{c}\bar{b} \Leftarrow$ resource focus.



At $m_\pi = 299, 163$ MeV

- λ-correlator results show increasing exp. for the ground state
- indication that $\Delta E < 0$

In our calculation: RHQ *c*- and NRQCD *b*-quark. Only a single physical volume.



Direct lattice calculation reveals evidence of a $ud\bar{c}\bar{b}$ tetraquark state with a binding at the level of 15 – 61 MeV.

Synopsis and future work

- Previous study revealed bound $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquarks with $\Delta E_{ud\bar{b}\bar{b}} = 189(10)(3)$ MeV and $\Delta E_{ls\bar{b}\bar{b}} = 98(7)(3)$ MeV
- Studying $qq'\bar{Q}\bar{Q}'$ with unphysically heavy quarks we find broad agreement with phenomenological considerations.
- Model study suggests the $ud\bar{c}\bar{b}$ is likely a bound tetraquark of this type in the charm quark region.
- At $m_\pi = 299, 163$ MeV there is evidence of binding in the $ud\bar{c}\bar{b}$ channel:

$$\Delta E_{ud\bar{c}\bar{b}} = 15 - 61 \text{ MeV}$$

$\Rightarrow$ broadly across e/m-stability threshold, possibly strong stable

BUT: Possible FV scattering $\Delta E_L^{V,\text{scatt}} \approx (0.4)\Delta E^{bind}$ in $ud\bar{c}\bar{b}$
$\Rightarrow$ Need in depth study of volume effects!

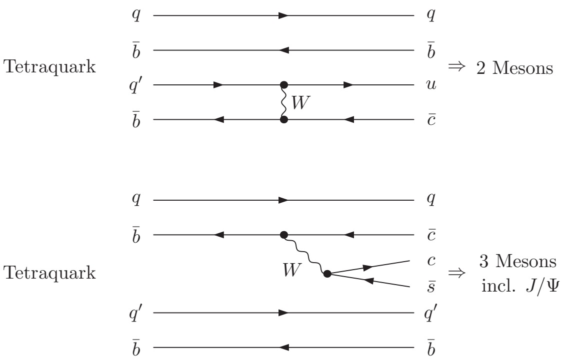
*Positive outlook for experimental detection (1806.09288)

Thank you for your attention!

Backup

Detection possibilities in experiment: $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$

With such deep ΔE, both $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ tetraquarks decay only weakly



$ud\bar{b}\bar{b} \rightarrow B^+ D^0$

$\rightarrow J/\psi B^+ K^0$

$us\bar{b}\bar{b} \rightarrow B^+ D_s^0$

$\rightarrow B_s \bar{D}^+$

$\rightarrow J/\psi B^+ \phi$

$\rightarrow J/\psi B_s K^+$

$ds\bar{b}\bar{b} \rightarrow B^+ D_s^-$

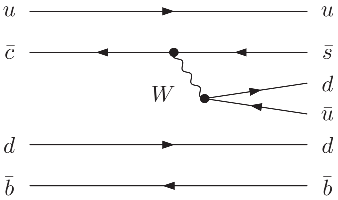
$\rightarrow B_s \bar{D}^0$

$\rightarrow J/\psi B^0 \phi$

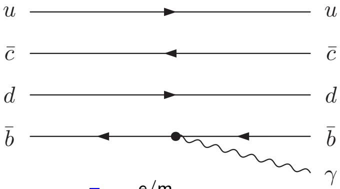
$\rightarrow J/\psi B_s K^0$

Detection possibilities in experiment: $ud\bar{c}\bar{b}$

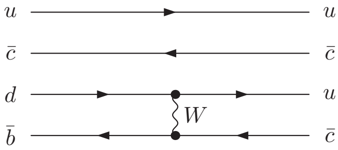
At this point $ud\bar{c}\bar{b}$ could decay only weakly or also electromagnetically



$us\bar{c}\bar{b} \overset{\text{weak}}{\Longrightarrow} (\pi^- K^+ B^0)$

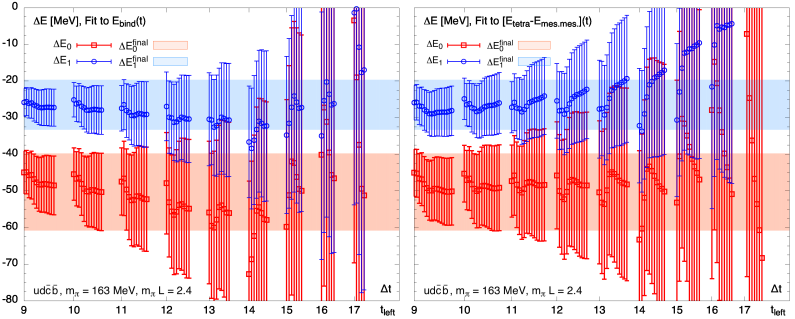


$ds\bar{c}\bar{b} \overset{\text{e/m}}{\Longrightarrow} (D^- B^+ \gamma)$



$ud\bar{c}\bar{b} \overset{\text{weak}}{\Longrightarrow} (\bar{D}^0 \bar{D}^0)$

Backup: Energy of $ud\bar{c}\bar{b}$ on E_L



Backup: Ensembles in preparation

κ_l	L	T	m_π[MeV]	$m_\pi L$	L[fm]
0.13781	32	64	163	2.4	2.88
	48	64		3.6	4.32
	64	64		4.8	5.76
0.13779	32	64	$\sim$185	2.7	2.88
	48	64		4.1	4.32
0.13777	32	64	$\sim$205	3.0	2.88
	48	64		4.5	4.32
	64	64		6.0	5.76
0.13770	32	64	299	4.4	2.88
0.13754	32	64	415	6.1	2.88

Table: Throughout $a^{-1} = 2.194\text{GeV}^{-1}$ and $\kappa_{s,sea} = 0.13640$