Robustness considerations for a fast orbit feedback at hadron synchrotrons

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Outline

1. GSI and FAIR project
2. Upgrade of SIS18
   - ❖ Robustness requirements of SIS18 Closed orbit feedback (COFB) system
3. Global orbit correction methods
   - ❖ Harmonic analysis of the perturbed orbit
   - ❖ Singular value decomposition (SVD) of response matrix
   - ❖ Missing connection between two methods
4. Exploitation of Circulant symmetry in response matrix
   - ❖ Simple symmetry
   - ❖ Nearest-Circulant approximation
     - ▪ Application of symmetry exploitation for subtracting the dispersion-induced orbit shift
   - ❖ Block-Circulant symmetry in response matrix
5. Characterizing the effect of model mismatch on orbit correction
   - ❖ Demonstration in COSY synchrotron Julich
6. Summary

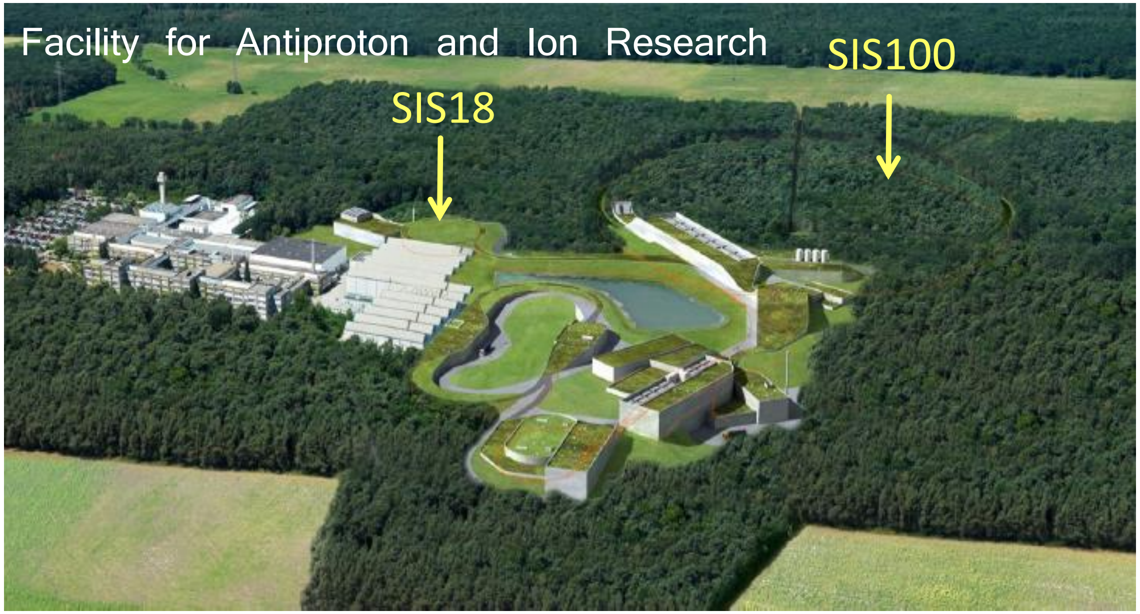
GSI and FAIR: brief introduction

GSI

- UNILAC（2-11.4 MeV/u）
- SIS18（0.010-2 GeV/u）
- Experimental storage ring （ESR）
- Fragment separator FRS

FAIR

- GSI and SIS100
- SIS100 （0.2- 2.7 GeV/u）

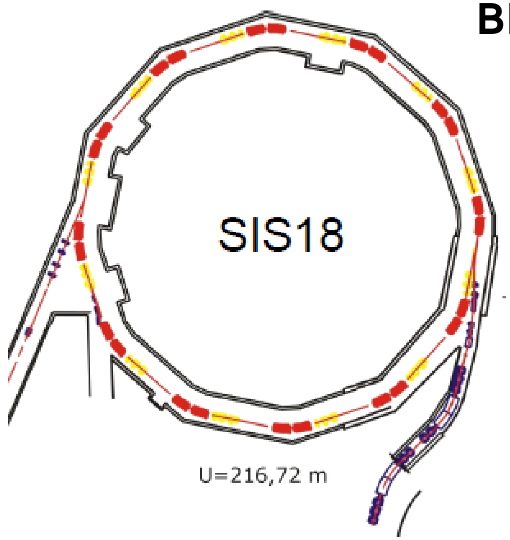


Specialties

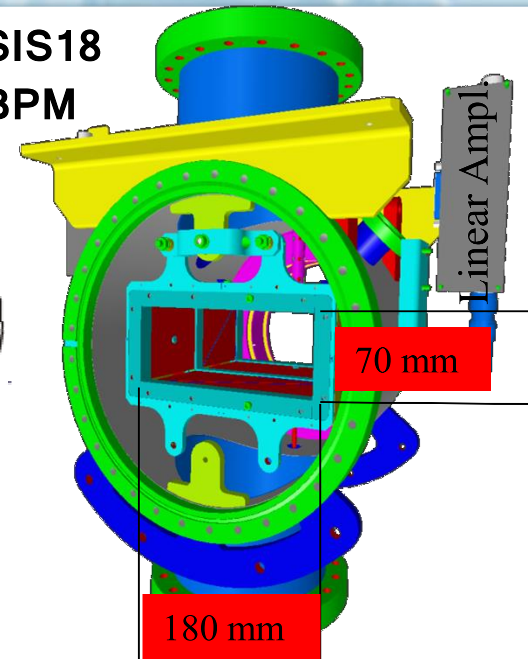
- Large range of ions （p to U）
- Variable charge states
- Ramp rates more than 10 T/s
- Variable ramps （0.1 to 1s）
- Magnet power supplies start at 3% of full power at injection
- Beam size few cm

SIS18 upgrade (as booster ring for SIS100)

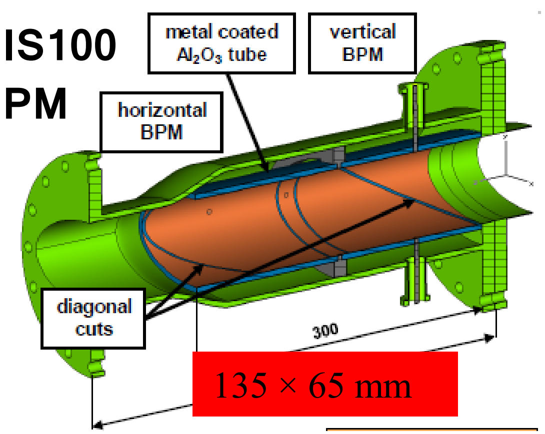
Parameter/Time	Today	FAIR Booster
Reference Ion	U^{73+}	U^{28+}
Maximum Energy GeV/u	1	0.2
Maximum Intensity (Ions per pulse)	4×10^{9}	4×10^{11}
Repetition Rate (Hz)	0.3 – 1	2.7



SIS18 BPM

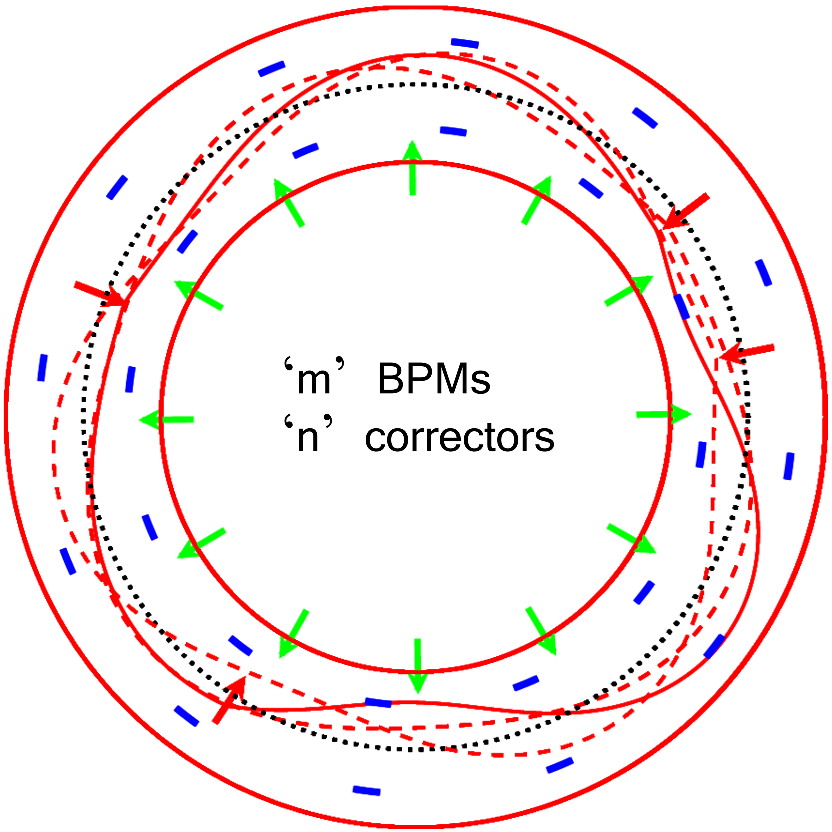


SIS100 BPM



- Higher to lower charge state to avoid space charge effects at high intensities
- Vacuum improvements to avoid beam losses due to residual gas pressures
- More control on beam quality to deliver more intensity to SIS100 (Closed orbit care)

Closed orbit perturbation (distortion)



Perturbed orbit due to single error

$$y(s) = \theta \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{s0}| - \pi Q_y)$$

θ is the kick provided by field error

$\beta(s)$ is the beta function at kick location

$\mu(s)$ is the phase advance

Q is the tune of the machine

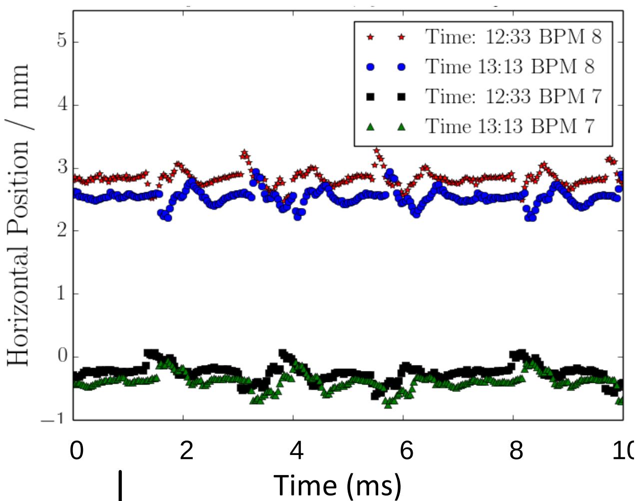
$$y_c(s) = \sum_{i=1}^{N} \theta_i \frac{\sqrt{\beta(s_i)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{si}| - \pi Qy)$$

$$[\mathbf{Y}]_{m\times 1} = [\mathbf{R}]_{m\times n}[\Theta]_{n\times 1}$$

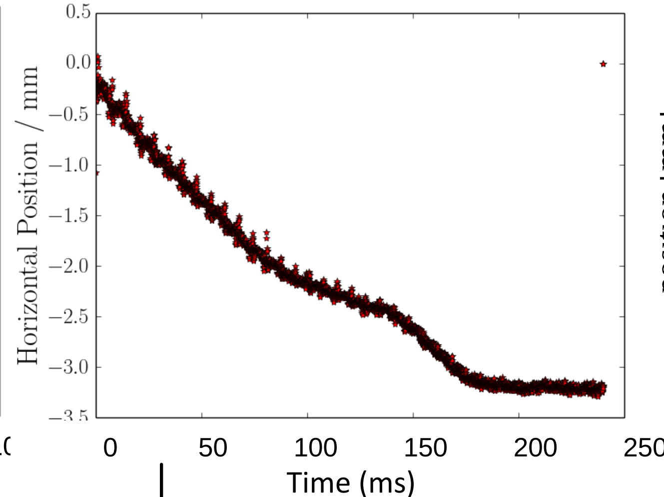
R is called the orbit response matrix

Closed orbit perturbations in SIS18

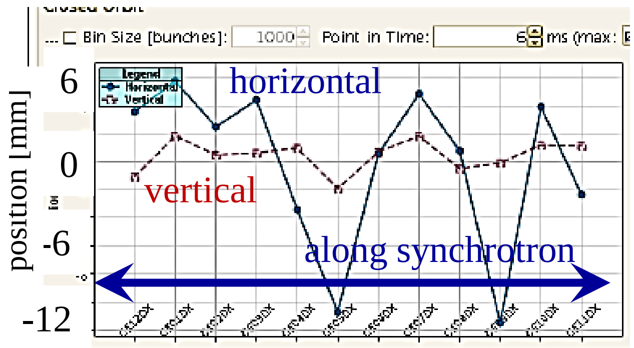
At injection



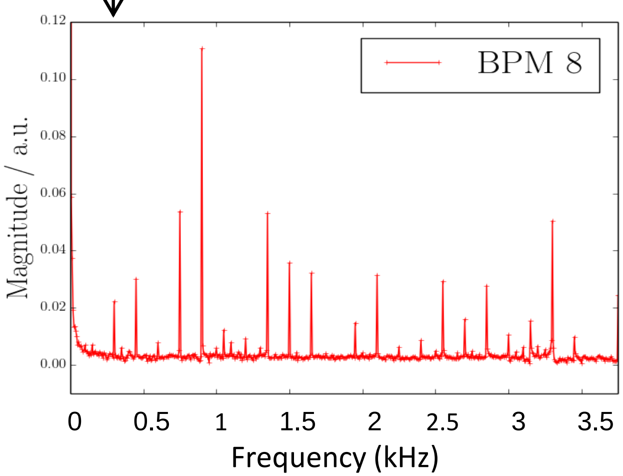
During ramp



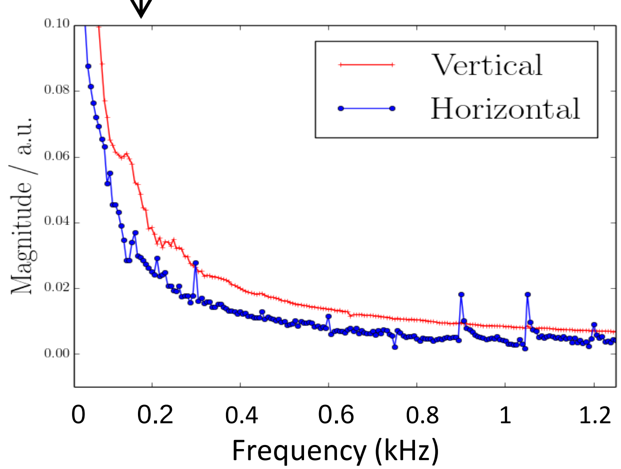
Static perturbation



Fourier Transform



Fourier Transform

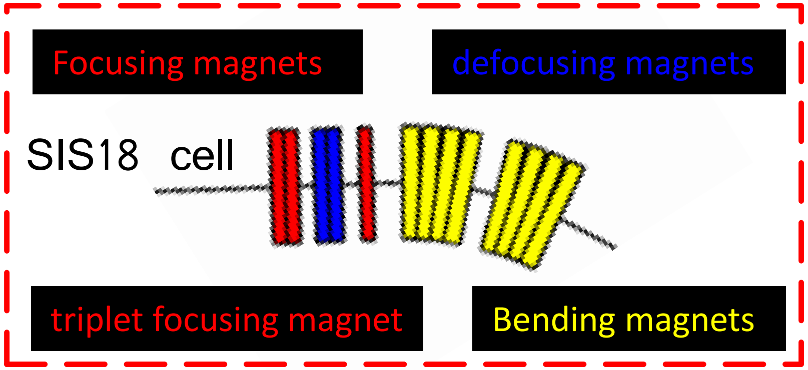


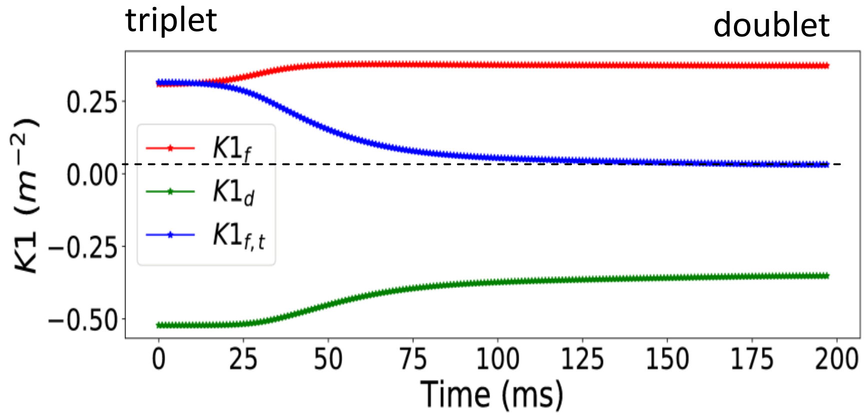
Wall thickness = 300 µm

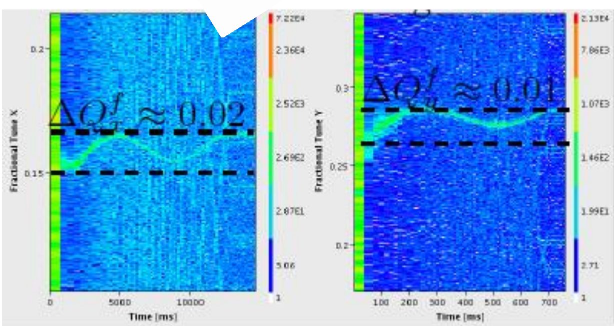
Features of planned SIS18 COFB system

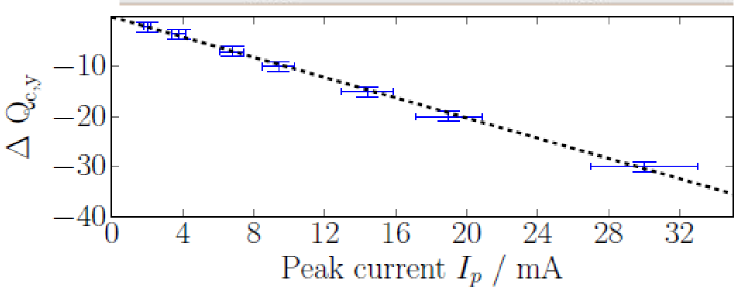
On−ramp orbit correction is planned

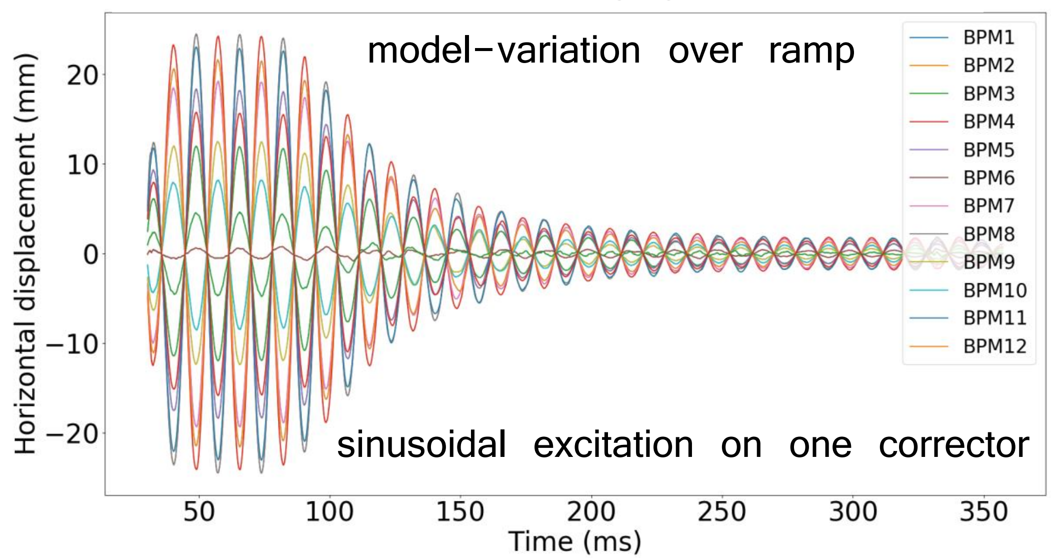
Ions have no synchrotron damping to restore beam quality at high energy









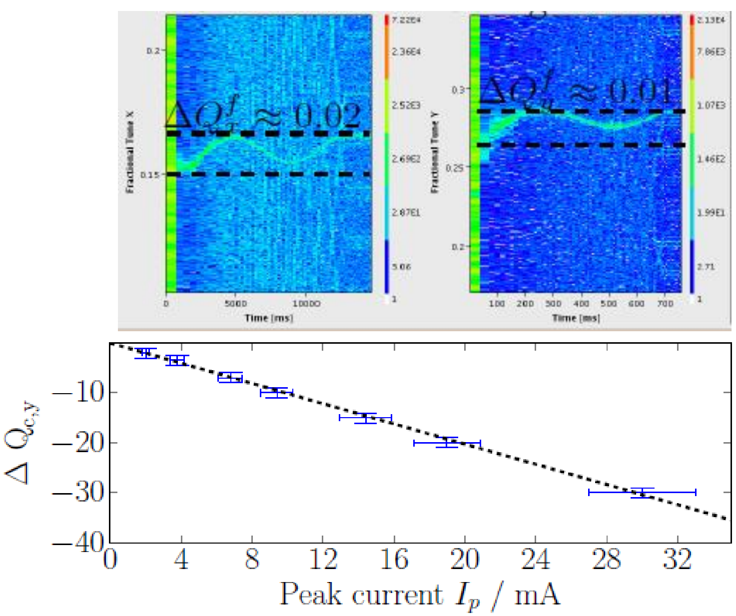


R. Singh, PhD Thesis, TU Darmstadt 2014.

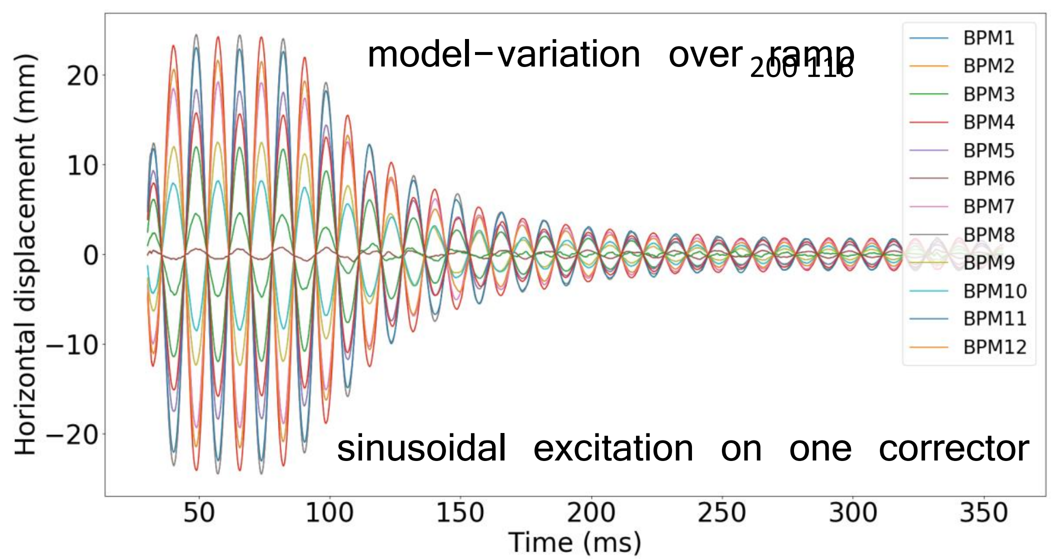
Features of planned SIS18 COFB system

- Online updating of the machine model?
- How many models required for a given ramp?
- Do we really need to update the ORM?
- What if we use wrong ORM for the orbit correction and what are boundaries?

Uncertainty modeling



Orbit correction methods



R. Singh, PhD Thesis, TU Darmstadt 2014.

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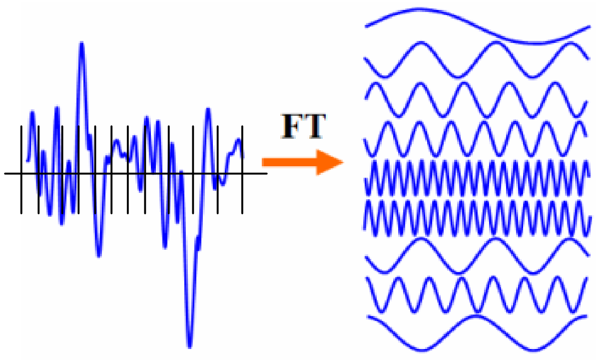
Harmonic analysis (global correction)

$$y(s) = \theta \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{s0}| - \pi Q_y)$$

$$t = \frac{\sqrt{\beta_0}\theta}{2\sin(\pi Q)} \qquad \eta = \frac{y}{\sqrt{\beta}} \qquad \varphi = \frac{\mu}{Q}$$

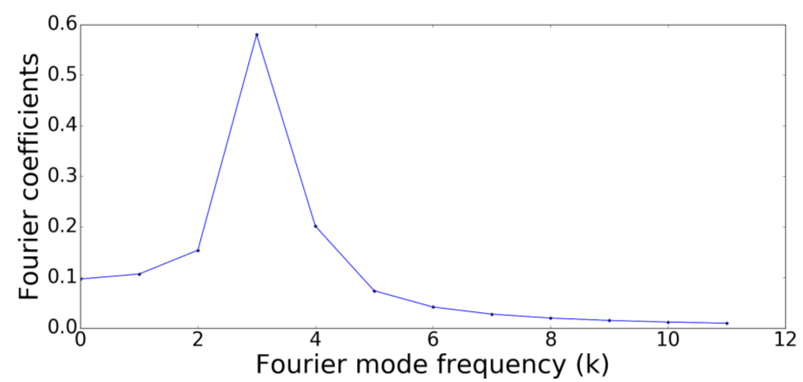
$$\eta = t \cos Q(|\varphi(s) - \varphi_{s0}| - \pi)$$

$$\eta = t \sum_{k=1}^{n} \left(\sigma_{k,r} \cos k\varphi + \sigma_{k,i} \sin k\varphi\right)$$



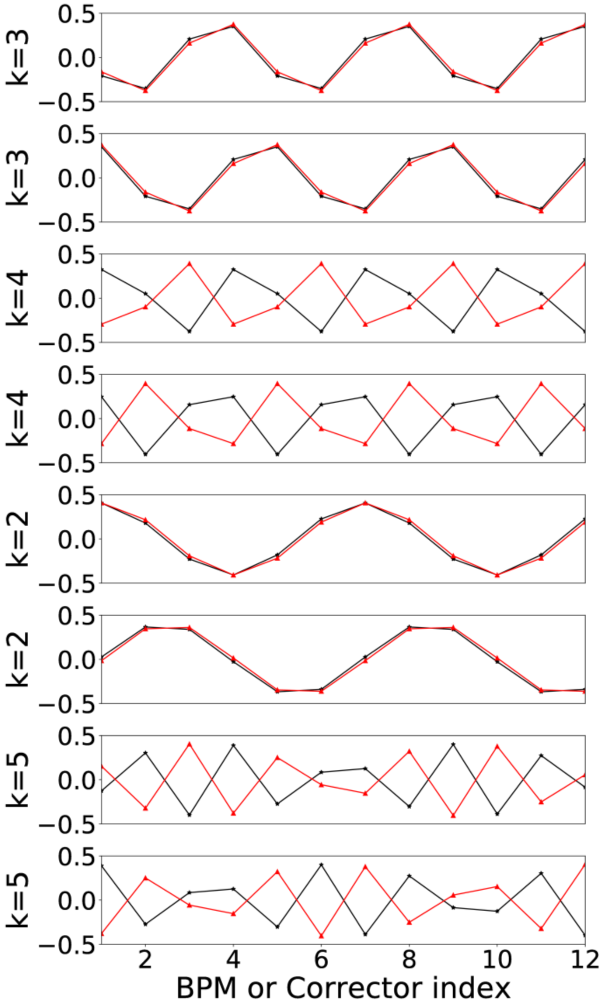
$$\sigma_k = \frac{1}{2\pi} \frac{2Q}{Q^2 - k^2}$$

- Modes to be removed (corrected) are selected before-hand
- measured orbit is fitted over corresponding mode e.g. modes around tune frequency
- <u>Corrector strengths are proportional to the Fourier coefficients</u>
- Mathematically complicated procedure



L.H.Yu et al.“Real time harmonic closed orbit correction”, Nucl. Instr. Meth. A, vol. 284, pp. 268–285, 1989

Singular Value Decomposition (SVD) $\mathbf{R} = \mathbf{USV}^{\mathbf{T}}$



Columns of U:Black
Columns of V: Red

Features of SVD

- **U** and **V** matrices form the BPM and corrector spaces.
- SVD can decompose and invert (or pseudo-invert) "any" matrix
- A robust algorithm for global orbit correction

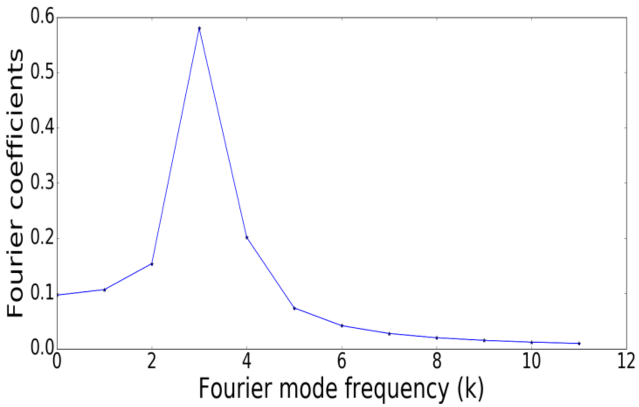
Benefits of SVD over harmonic analysis

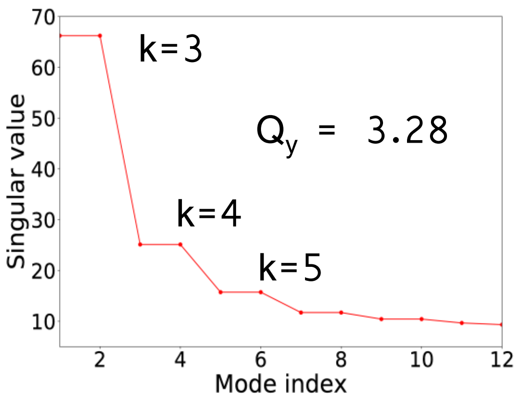
- One needs not to select the modes to be corrected before correction: decompose in all possible modes
- "simple" matrix inversion
- Mode-by-mode correction is still possible through selecting certain eigenvalues

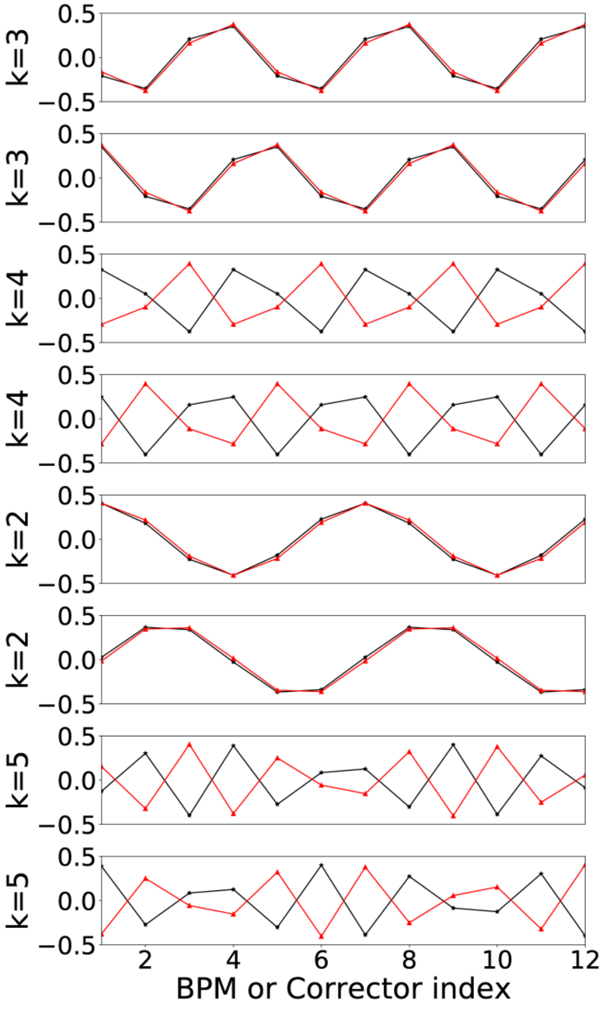
Weaknesses of SVD

$\mathbf{R} = \mathbf{USV^T}$

- **U** and **V** are interconnected through a phase relation
- Over the ramp, updating of all three matrices required
- Uncertainty modeling is required in all three matrices
- Time complexity of the order of n^3, n being dimension of matrix
- Loss of physical meaning of modes (interpolation)
- Fourier coefficients of harmonic analysis have been proposed for uncertainty modeling







Columns of U:Black
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S. Gayadeen, "Synchrotron Electron Beam Control", Ph.D. thesis, St. Hugh's College, University of Oxford, UK, 2014.

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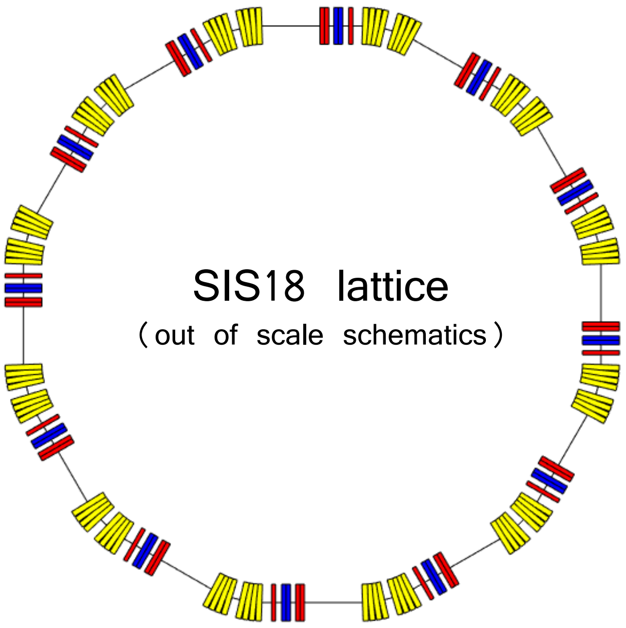
Symmetry exploitation in SIS 18 vertical ORM

$$\beta_{bpm1} = \beta_{bpm2} = \beta_{bpm\mathbf{3}} \cdots \cdots = \beta_{bpm12}$$

$$\beta_{corr1} = \beta_{corr2} = \beta_{corr\mathbf{3}} \cdots \cdots = \beta_{corr12}$$

$$\Delta\mu_{bpm} = constant$$

$$\Delta\mu_{corr} = constant$$



$$R = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & \cdots & R_n \\ R_n & R_1 & R_2 & R_3 & \cdots & R_{n-1} \\ R_{n-1} & R_n & R_1 & R_2 & \cdots & R_{n-2} \\ R_{n-2} & R_{n-1} & R_n & R_1 & \cdots & R_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_2 & R_3 & R_4 & R_5 & \cdots & R_1 \end{bmatrix}$$

Each row is cyclic shift of previous row

All diagonal elements are identical

Such a square matrix is called
Circulant Matrix

Reference: Philips J.Davis, Circulant matrices, (1994), Chelsea

Diagonalization of a Circulant matrix

$$R = \begin{bmatrix} \cancel{R_1} & \cancel{R_2} & \cancel{R_3} & \cancel{R_4} & \cancel{\cdots} & \cancel{R_n} \\ R_n & R_1 & R_2 & R_3 & \cdots & R_{n-1} \\ R_{n-1} & R_n & R_1 & R_2 & \cdots & R_{n-2} \\ R_{n-2} & R_{n-1} & R_n & R_1 & \cdots & R_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_2 & R_3 & R_4 & R_5 & \cdots & R_1 \end{bmatrix}$$

Inverse is straightforward

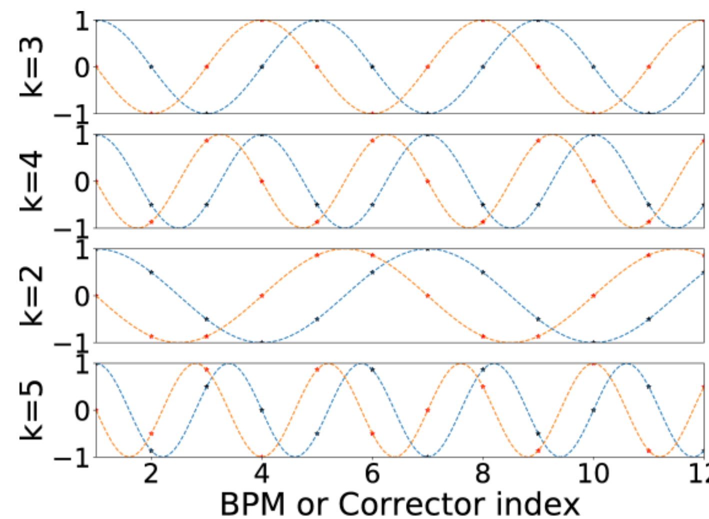
$$R^{-1} = F^* H^{-1} F$$

H^{-1} =diag($\frac{1}{\sigma_k}$) ,k=1…n

$$\sigma_k = \sigma_{rk} + j\,\sigma_{ik} = \sum_i^{n-1} R_n\, e^{-j2\pi ki/n}$$

Standard Fourier matrix

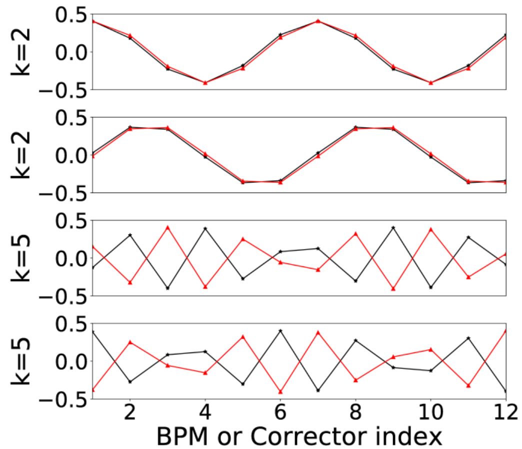
$$\mathrm{R} = \begin{bmatrix} F_{11} & \cdots & F_{1m} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mm} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_2 & \vdots \\ 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{bmatrix}$$



$$F_k = F_{kc} + jF_{ks} \qquad F_{ks} = \sin\left(\frac{2\pi km}{n} + \varphi_k\right)$$

Equivalence of DFT and SVD

DFT: $$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} F_{11} & \cdots & F_{1m} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mm} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_2 & \vdots \\ 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{bmatrix}$$



$$\varphi_{dk} = \text{phase}(\sigma_k)$$

$$s_k = |\sigma_k| = \sqrt{\sigma_{rk}{}^2 + \sigma_{ik}{}^2}$$

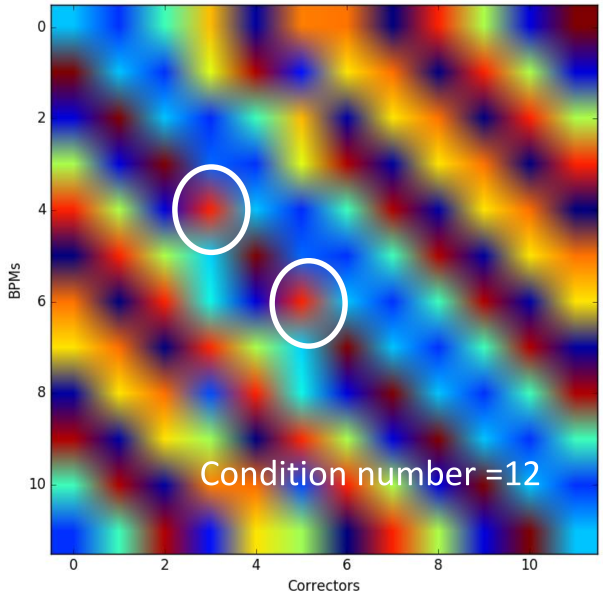
SVD: $$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & \ddots & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \cdots & s_n \end{bmatrix} \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix}$$

Why to do SVD when Circulant symmetry exits?

Herbert Karner et al. Spectral decomposition of real Circulant matrices, Linear Algebra and its Applications, Volume 367, 2003

Nearest-Circulant symmetry

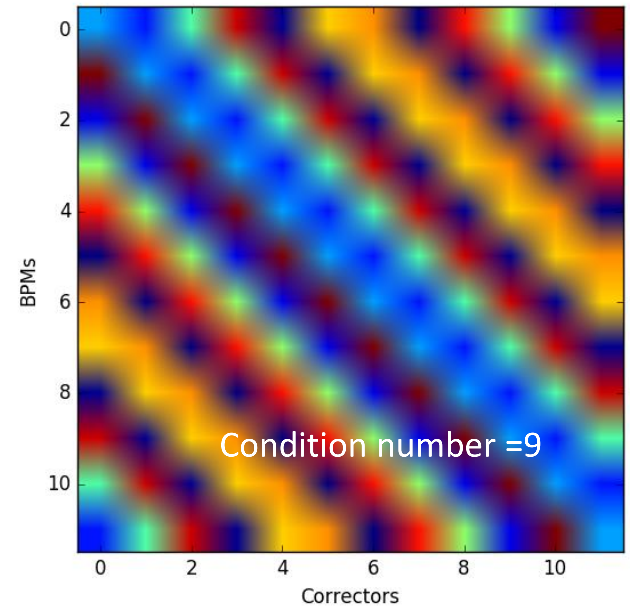
SIS18 horizontal plane



$$c_k = \frac{1}{n} < \mathbf{R}, \pi^k >$$

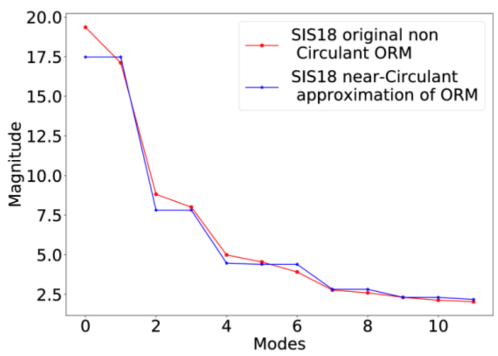
Frobenius product

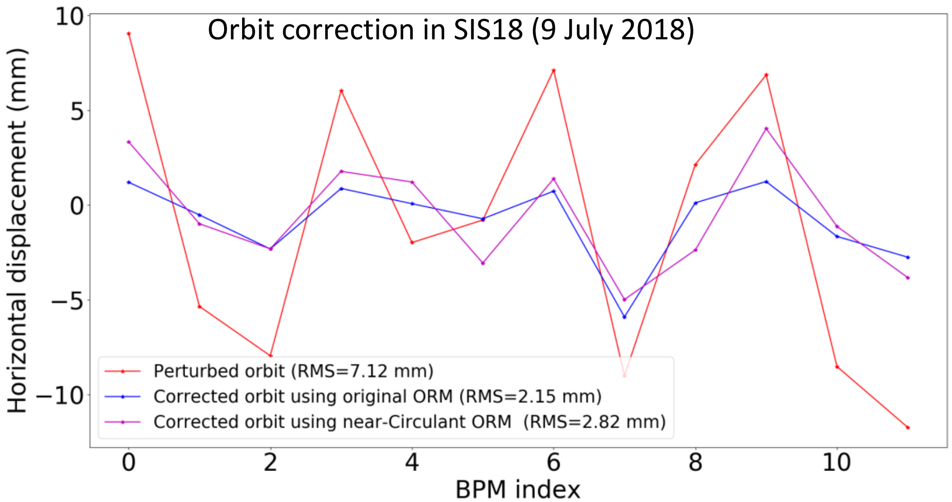
$$\pi^1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ . & . & . & . & . \\ . & . & . & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$



$$c_1 = \frac{1}{n}(R_{12} + R_{23} + \cdots \ldots R_{12,1})$$

$$\mathbf{R}_{nc} = circ(c_0, c_1, c_2, .., c_{11})$$





Dispersion effect in closed orbit

Mismatch between RF frequency and the dipole field

$$\Delta x_D(s) = D(s)\frac{\Delta p}{p}$$

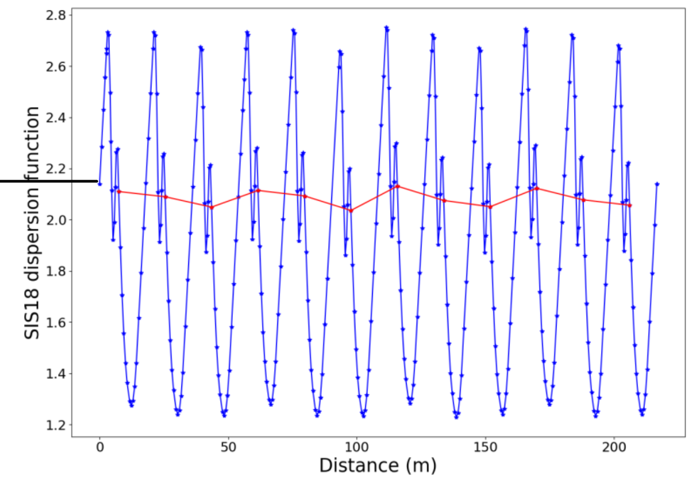
$$\Delta x = \Delta x_{co} + \Delta x_D(s)$$

an attempt to correct it can saturate the corrector magnets.

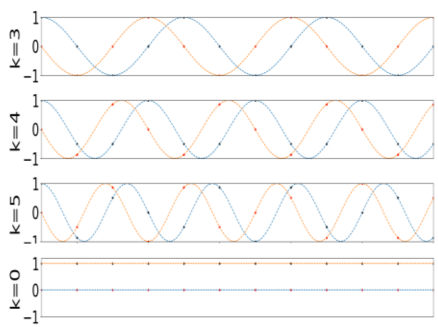
A DC shift in closed orbit

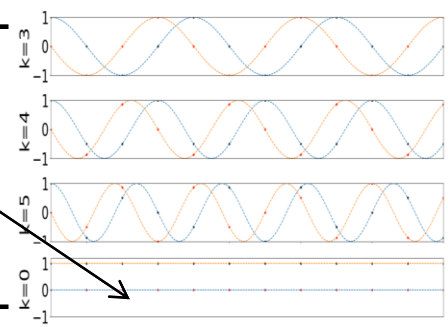
$$\Delta x_D(s) = D(s)\frac{\Delta p}{p}$$

$$\Delta x = \Delta x_{co} + \Delta x_D(s)$$



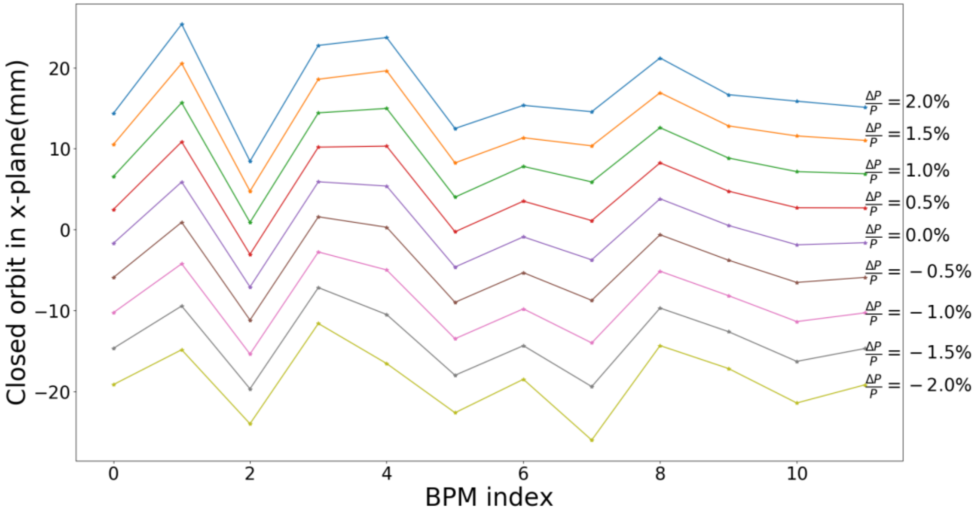
$$\mathbf{R}^{-1} = \left[\text{(plots)} \right] \begin{bmatrix} 1/s_1 & \cdots & 0 \\ \vdots & 1/s_2 & \vdots \\ 0 & \cdots & 1/s_n \end{bmatrix} \left[\text{(plots)} \right] \Delta x$$

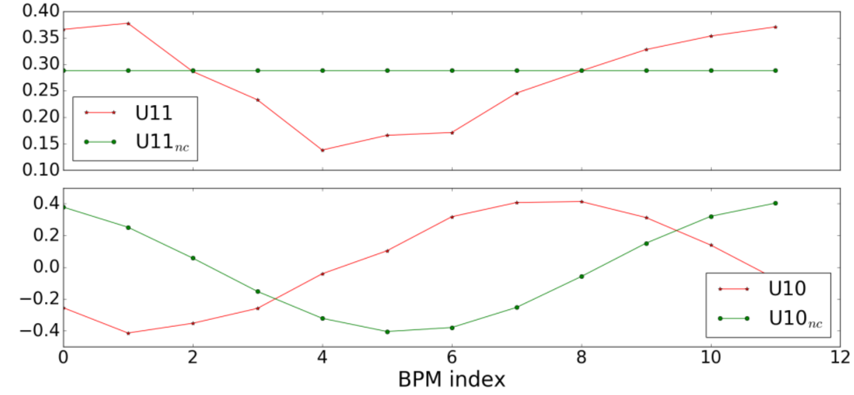




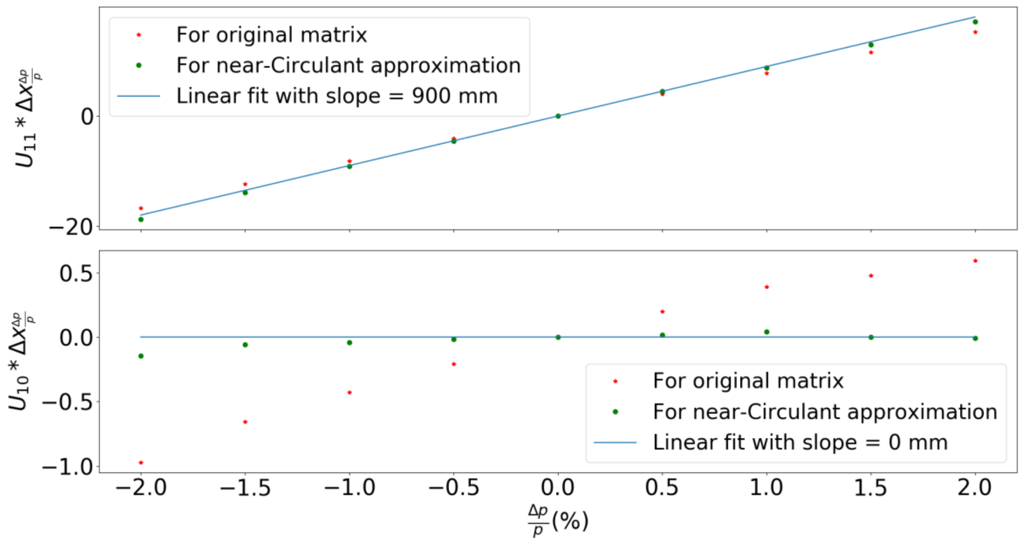
Induced dispersion and closed orbit in x-plane

Experiment at SIS18



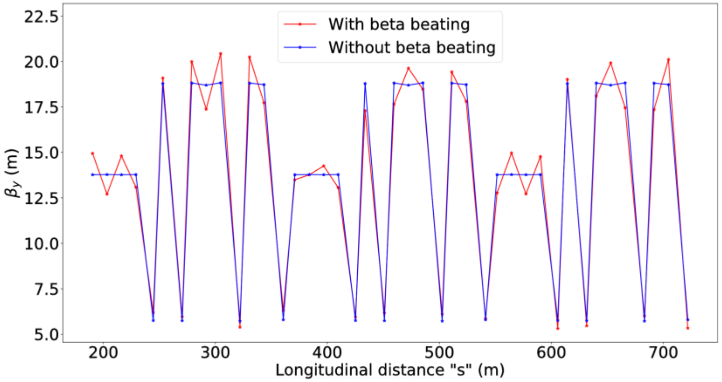


$$\theta = \mathbf{VS^{-1}U} * \Delta x$$



Block Circulant symmetry

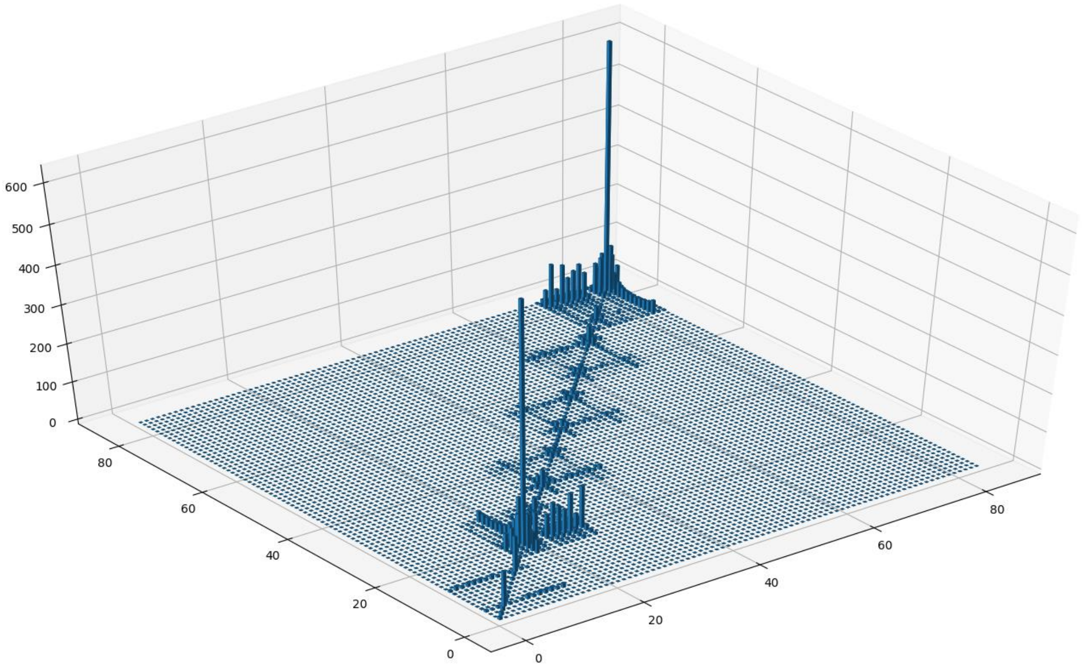
SIS100



$$\mathbf{R}_{BC} = circ(\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots\ldots \mathbf{A}_n)$$

$$\mathbf{R}_{BC} = \sum_{k=0}^{n-1} (\pi_m{}^k \otimes \mathbf{A}_{k+1})$$

$$\mathbf{R}_{BC} = (\mathbf{F}_m \otimes \mathbf{F}_n) diag(\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3, \ldots \mathbf{M}_n)\ (\mathbf{F}_m \otimes \mathbf{F}_n)$$



$$\mathbf{D} = \begin{bmatrix} \mathbf{M}_1 & 0 & 0 & . & . & 0 \\ 0 & \mathbf{M}_2 & 0 & . & . & 0 \\ . & . & . & . & . & 0 \\ . & . & . & . & . & 0 \\ 0 & 0 & 0 & . & . & \mathbf{M}_n \end{bmatrix}$$

$$\mathbf{R}^+{}_{BC} = (\mathbf{F}_m \otimes \mathbf{F}_n)(\mathbf{D}^+)(\mathbf{F}_m \otimes \mathbf{F}_n)$$

Circulant Matrices by P. J. Davis

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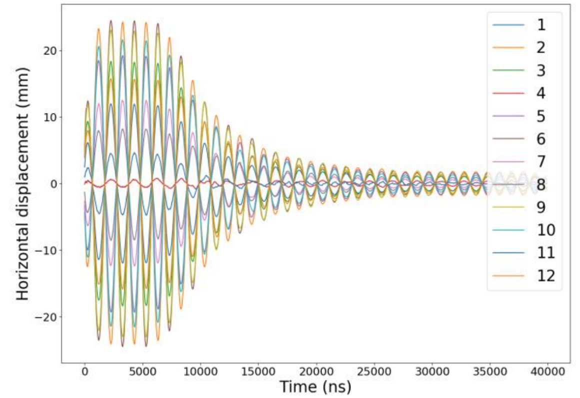
   - ❖ Singular value decomposition （SVD） of response matrix
   - ❖ Missing connection between two methods
4. Exploitation of Circulant symmetry in response matrix
   - ❖ Simple symmetry
   - ❖ Nearest-Circulant approximation
     - Application of symmetry exploitation for subtracting the dispersion-induced orbit shift
   - ❖ Block-Circulant symmetry in response matrix
5. Characterizing the effect of model mismatch on orbit correction
   - ❖ Demonstration in COSY synchrotron Julich
6. Summary

Characterizing the effect of spatial model mismatch

concept of spatial bandwidth

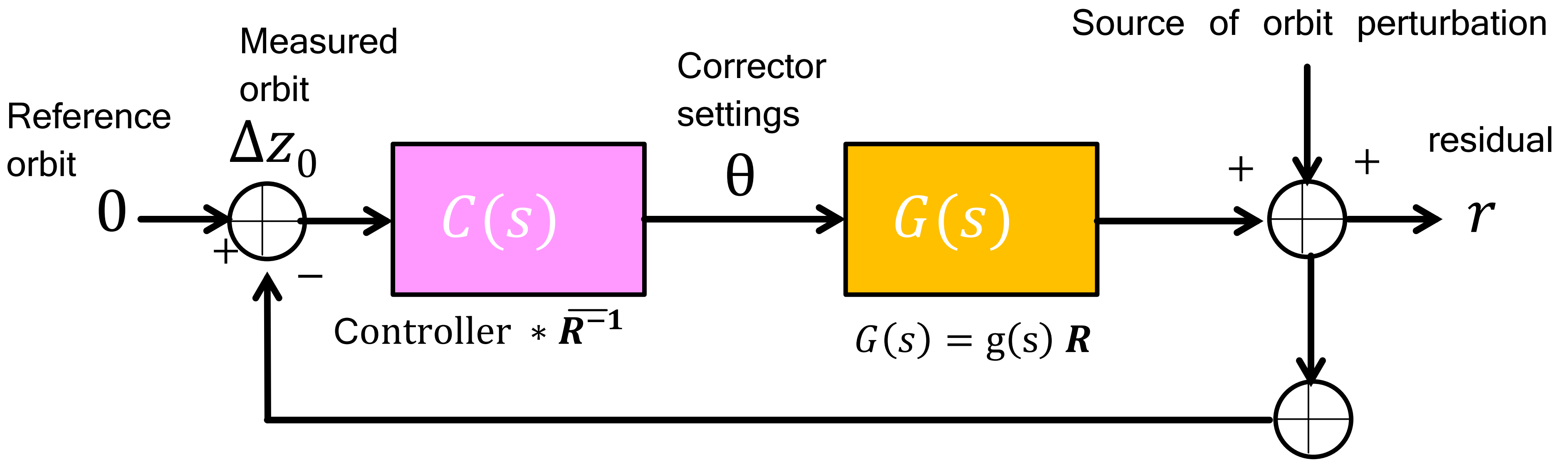
SIS18 is a unique synchrotron in terms of flexibility of operational scenarios.

Variable optics → **Orbit response matrix variations**



❖ How much ORM variation we can live with?

❖ How to quantify the effect of "wrong" model on orbit correction?



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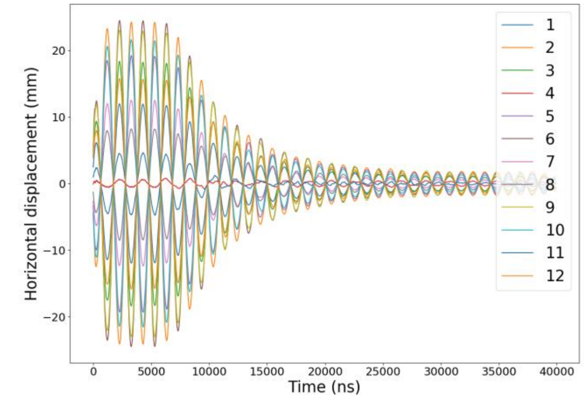

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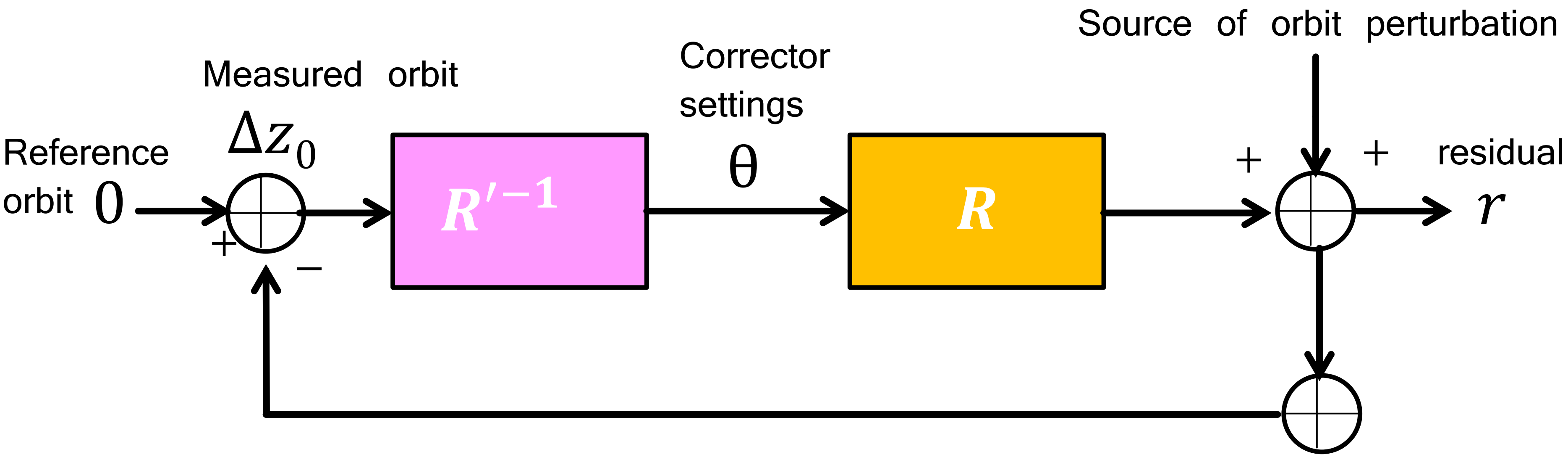


Characterizing the effect of spatial model mismatch

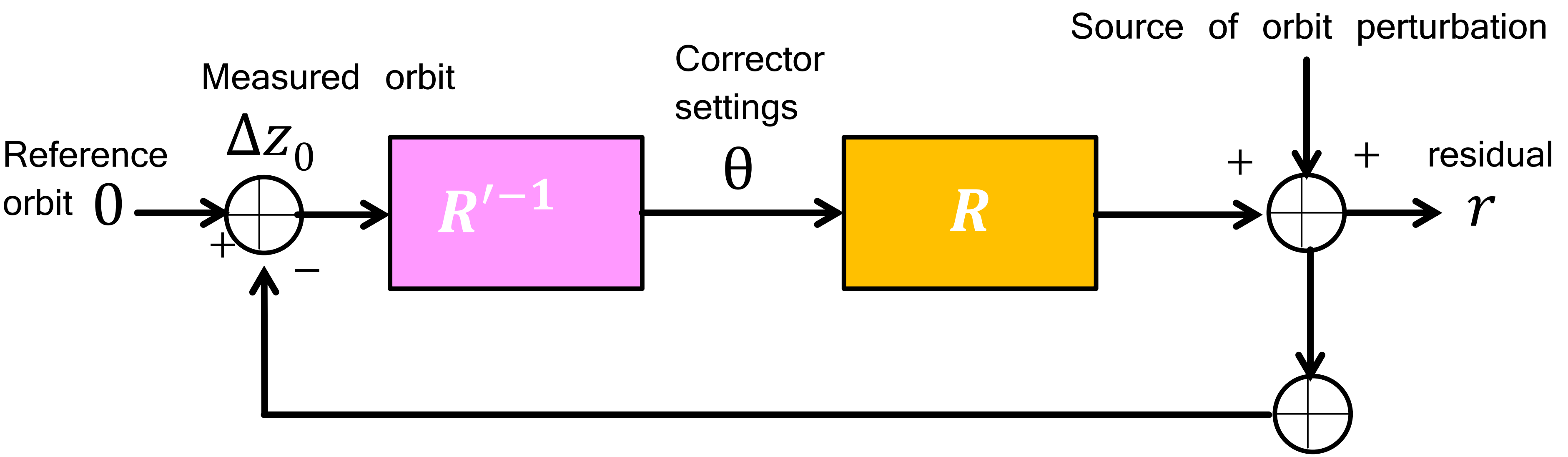
Mathematical treatment

Let θ[k] is the controller output at kth iteration

$$\theta[k] = \theta[k-1] + ae[k] + be[k-1] + ce[k-2]$$

where,

$$a = (k_p + k_i \frac{T_s}{2} + \frac{k_d}{T_s}) \qquad b = (-k_p + k_i \frac{T_s}{2} - \frac{2k_d}{T_s}) \qquad c = \frac{k_d}{T_s}$$



Characterizing the effect of spatial model mismatch

Mathematical treatment

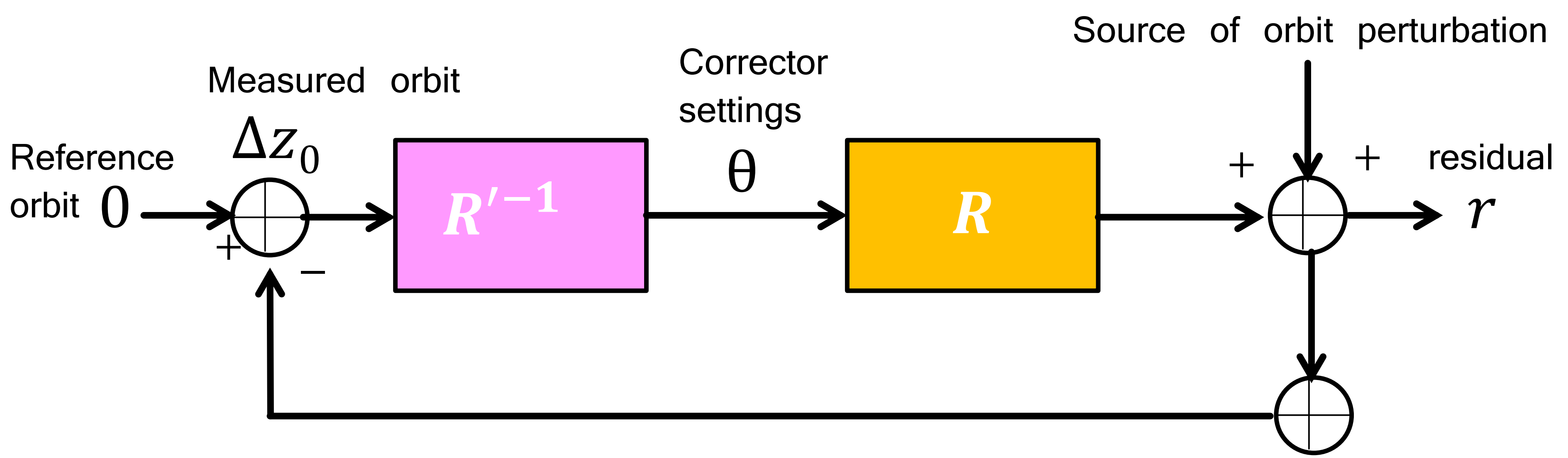
$\theta = \mathbf{R}'^{-1}\Delta z_0$ → Corrector settings

$r_1 = \Delta z_0 - \mathbf{R}\theta$ → First iteration residual

$r_1 = \Delta z_0 - \mathbf{R}\mathbf{R}'^{-1}\Delta z_0$

$r_1 = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})\Delta z_0$

$r_n = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})^n\Delta z_0$ → n^{th} iteration residual (n costs the bandwidth)



Characterizing the effect of spatial model mismatch

Mathematical treatment

$\Theta = \mathbf{R}'^{-1}\Delta z_0$ $\longrightarrow$ Corrector settings

$r_1 = \Delta z_0 - \mathbf{R}\Theta$ $\longrightarrow$ First iteration residual

$r_1 = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})\Delta z_0$

$r_n = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})^n\Delta z_0$ $\longrightarrow$ n^{th} iteration residual （n costs the bandwidth）

$\mathbf{S} = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})$ $\longrightarrow$ Spatial sensitivity function

$\mathbf{S} = \mathbf{P}\boldsymbol{\Lambda}_{\mathbf{S}}\mathbf{P}^{-1}$

$r_n = (\mathbf{P}\boldsymbol{\Lambda}_{\mathbf{S}}\mathbf{P}^{-1})^{n}\Delta z_0$

$r_n = \mathbf{P}(\boldsymbol{\Lambda}_{\mathbf{S}})^{n}\mathbf{P}^{-1}\Delta z_0$

$\boxed{\lambda_k \approx (1 - \lambda_{\boldsymbol{R},\boldsymbol{k}}\lambda^{-1}{}_{\boldsymbol{R}',\boldsymbol{k}})}$

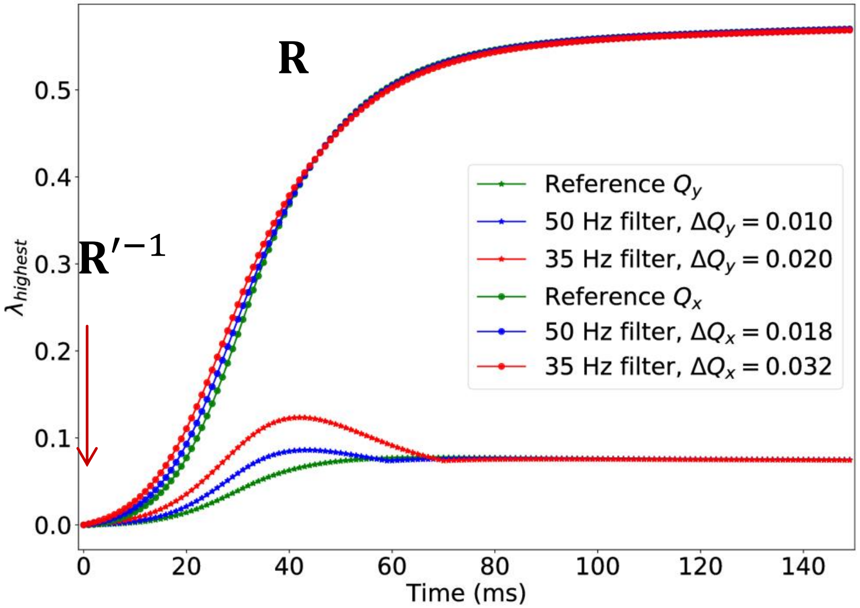
$\lambda_k < 1$	Correctability
$\lambda_k = 1$	No correction
$\lambda_k > 1$	Instability

On-ramp model variation in SIS18

Simulations: Orbit correction using injection ORM

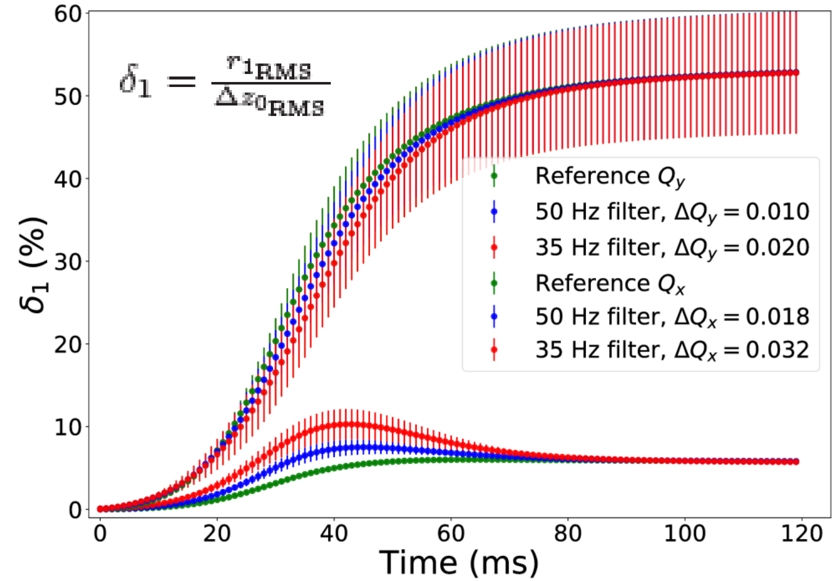
$$\mathbf{S} = (\mathbf{I} - \mathbf{R}\mathbf{R'}^{-1})$$

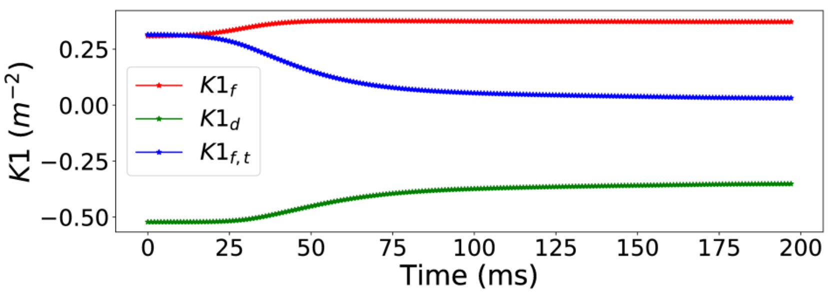
$$\mathbf{S} = \mathbf{P}\boldsymbol{\Lambda}_{\mathbf{S}}\mathbf{P}^{-1}$$



$$\lambda_k \approx (1 - \lambda_{R,k}\lambda^{-1}{}_{R',k})$$

First iteration residual





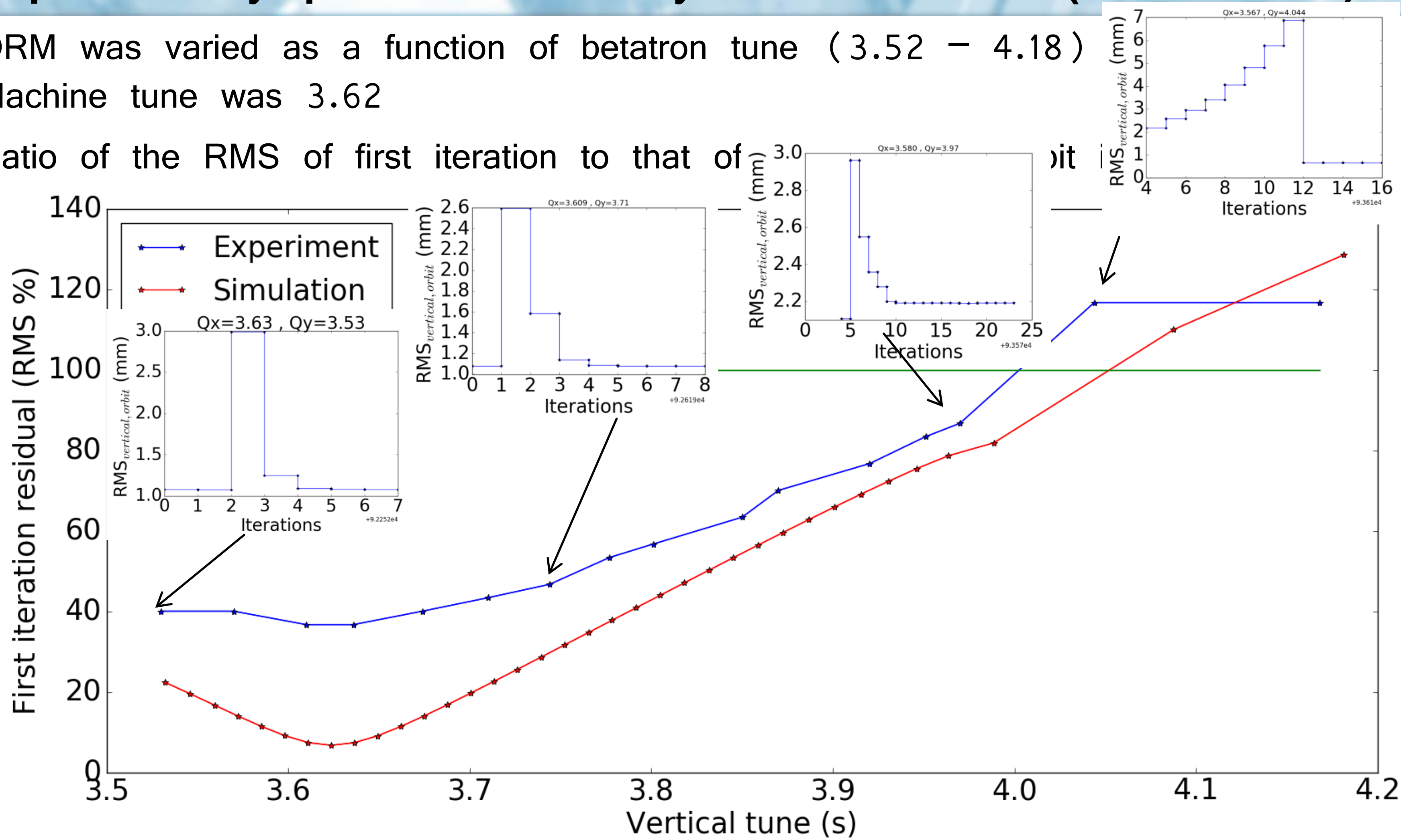
Effect of wrong model on the orbit correction

Experiment: y-plane of COSY synchrotron Julich (31 Oct 2018)

ORM was varied as a function of betatron tune (3.52 – 4.18)

Machine tune was 3.62

Ratio of the RMS of first iteration to that of ...bit i...



Thanks to Lorentz Bernd and Christian Weidemann for helping in the experiment.

Summary

1. On-ramp orbit correction is planned for SIS18 synchrotron of GSI
1. Symmetry exploitation in the ORM can result into faster matrix inversion than SVD
2. DFT based diagonalization also gives physical meaning to SVD modes
3. DFT based diagonalization provides the missing connection between SVD and harmonic analysis
2. Nearest-Circulant approximation can be made for slightly broken symmetries
3. Block-wise Circulant symmetry can also be applied for larger lattices (more than one BPMs per cell)
4. Spatial sensitivity function is defined as tool for the quantification of the effect of model mismatch on the orbit correction
5. 2 to 3 ORM will be required over the ramp orbit correction for SIS18
6. Application of spatial sensitivity function has been demonstrated in COSY synchrotron Julich

I would like to acknowledge Dr. Guenther Rehm for nice discussions during my visit to Diamond Light Source