

Nonlinear dynamics at the SLS storage ring

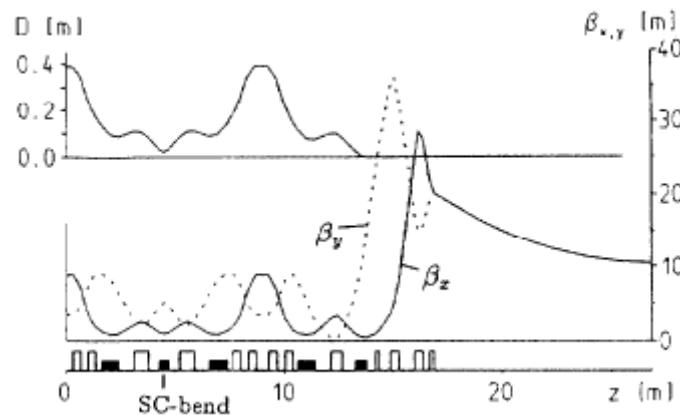
Andreas Streun, Paul Scherrer Institut, Villigen, Switzerland

- ◆ History of SLS design: 7BA vs. TBA
- ◆ The method of sextupole optimization
- ◆ SLS performance: auxiliary sextupoles*
- ◆ The next generation: MAX-IV

* more at NBD-2 WS, DIAMOND, Nov.2-4, 2009
http://www.diamond.ac.uk/Home/Events/Past_events/NBD_workshop.html
coupling issues: → M. Böge, *Reaching ultra-low vertical emittance in the SLS*

1991

SLS History 1



4 × “swiss arc”
(s.c.bend)

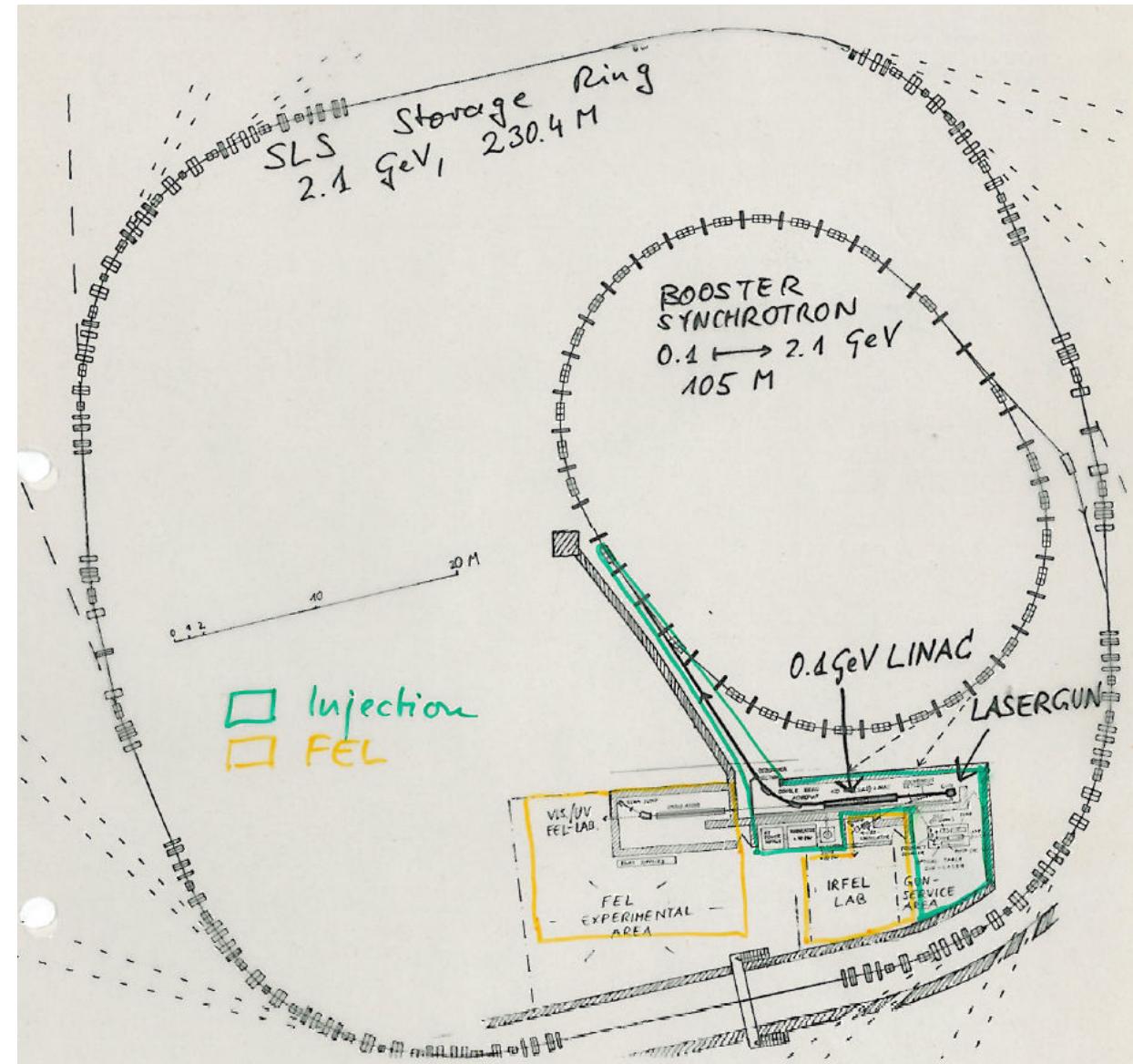
2 × DBA
>20m straights

230.4 m

1.5 / 2.1 GeV

3.6 / 7.0 nm

R.Aabela et al.,
EPAC 1992, p.486



SLS History 2

1993 (CDR)

$6 \times 7\text{BA}$ arc

6 straights:

$4 \times 6\text{ m}$

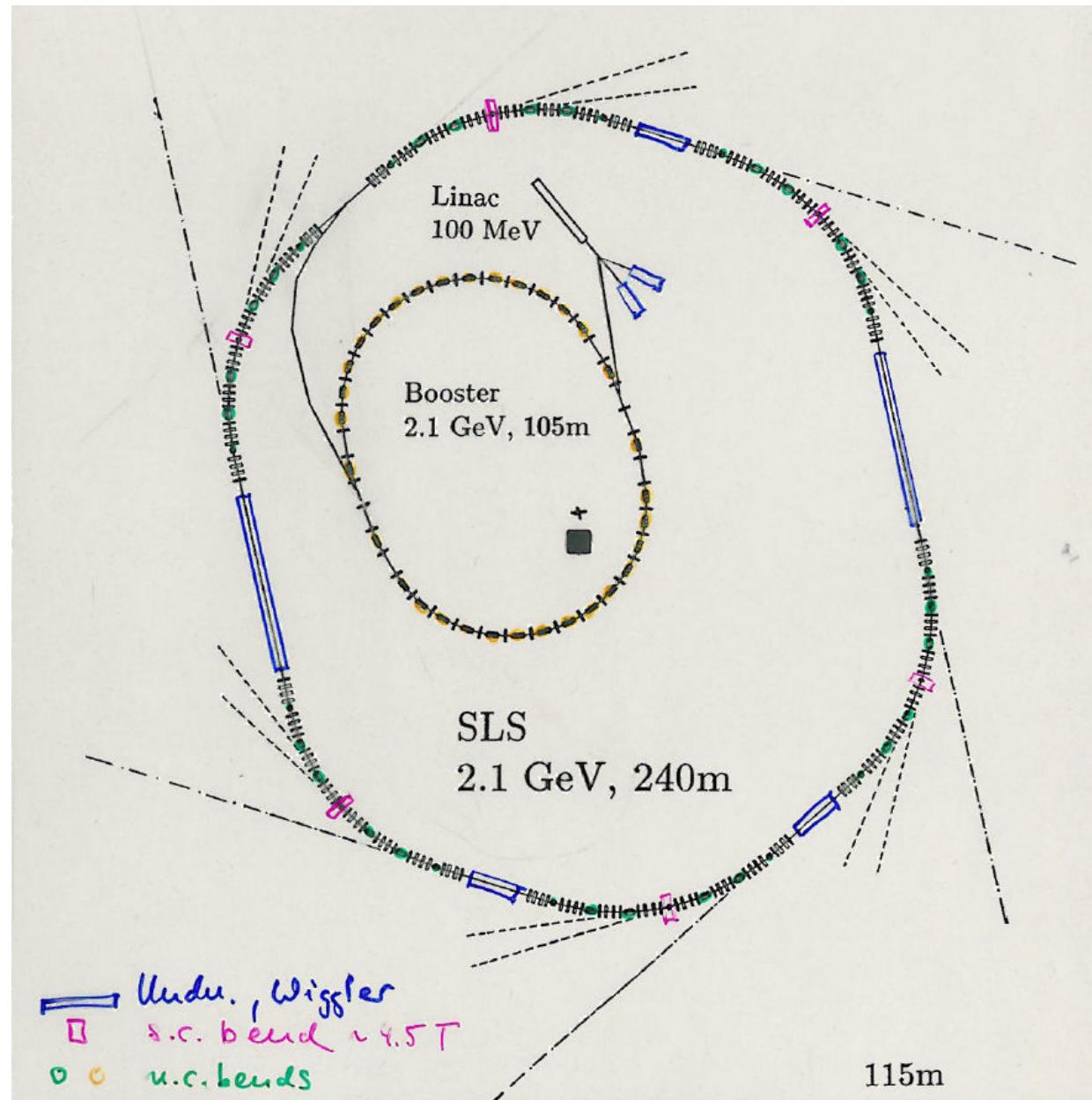
$2 \times 20\text{ m}$

240 m

2.1 GeV

3.2 nm

W. Joho et al.,
EPAC 1994, p.627



SLS History 3

1995

(PSI-TRIUMF collab.)

8 × **5BA** arcs

8 straights:

4 × 6m

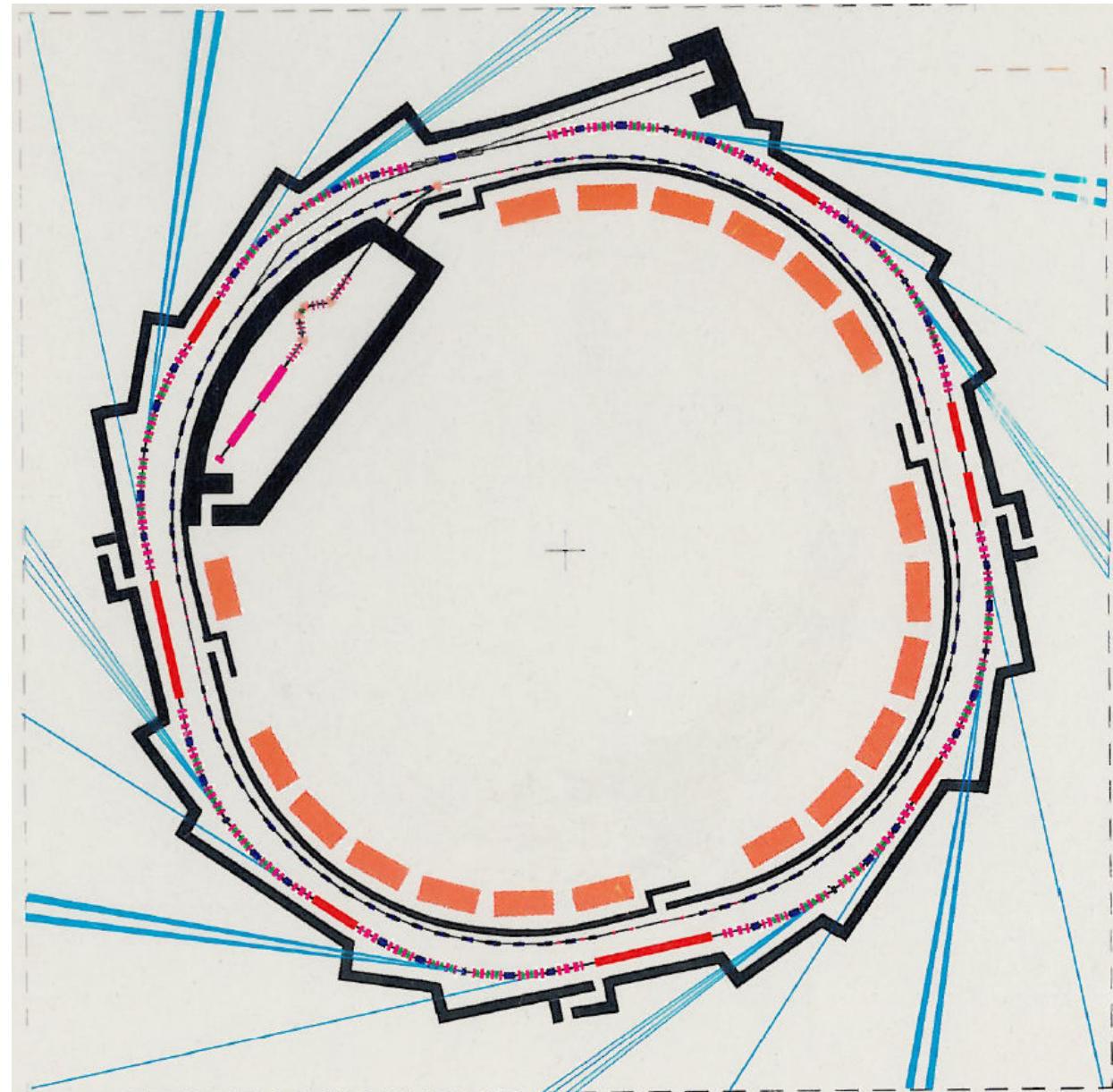
4 × 20m

270 m

2.1 GeV

~ 2 nm

D. Kaltchev et al.,
PAC 1995, p.2823



SLS History 4

1996 → 1997 → 2000✓

(approval 1997, operation 2000)

12 × TBA arc

12 straights

6 × 4 m

3 × 7 m

3 × 11.5 m

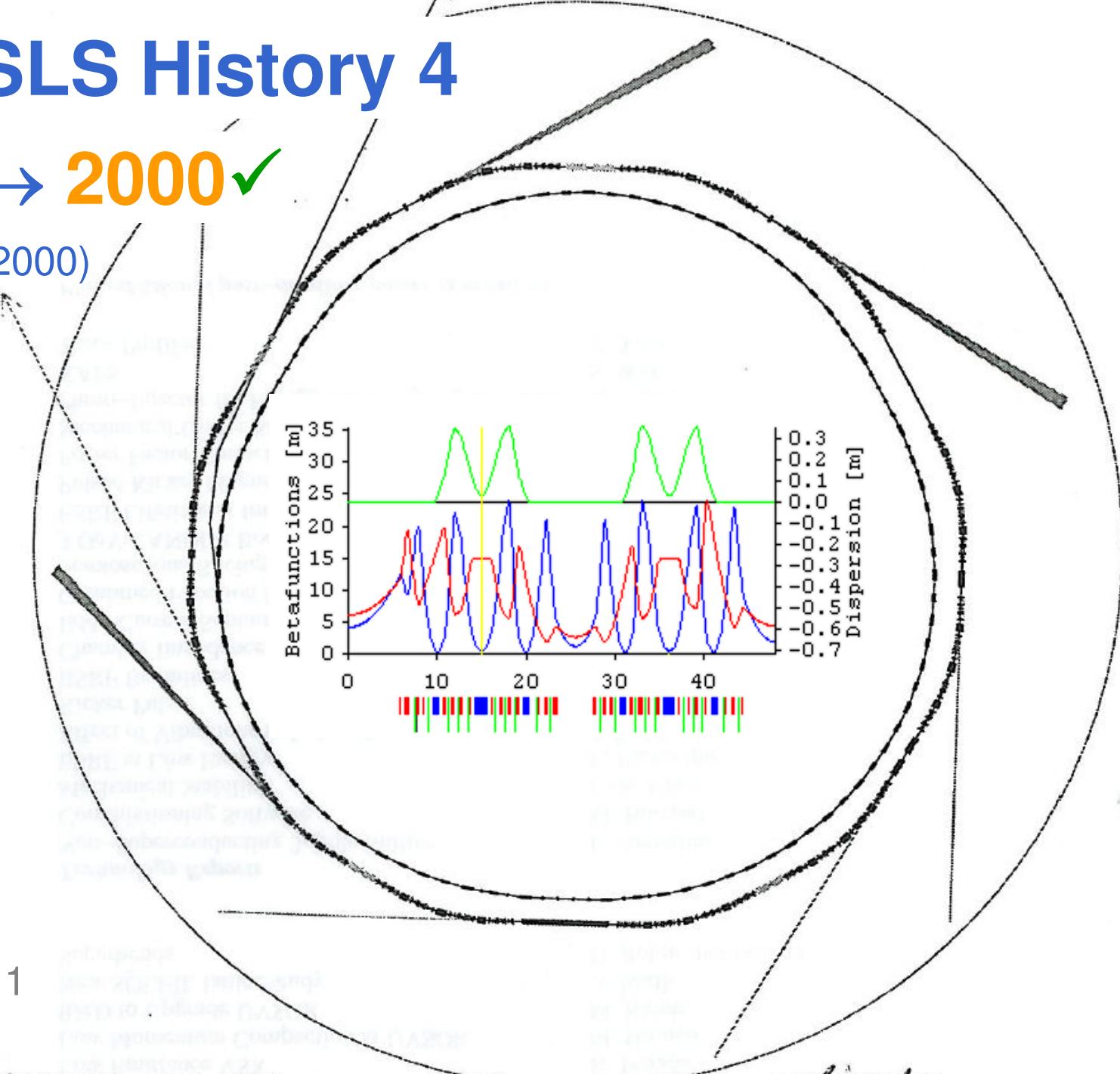
281 → 288 m

2.1 → 2.4 GeV

2.9 → 4.8 nm

A. Streun,
SLS-TME-TA-1996-0011

M. Böge et al.
EPAC 1998, p.623



SLS TBA lattice with booster⁵ in a circular hall of 126 m circumference. Injector linac and transfer lines are not shown. Storage ring circumference is 200 m.

... beyond SLS:

2003 → 2009

12 → 20 × 7BA arc

12 → 20 straights

12 × 4.6 m → 20 × 5 m

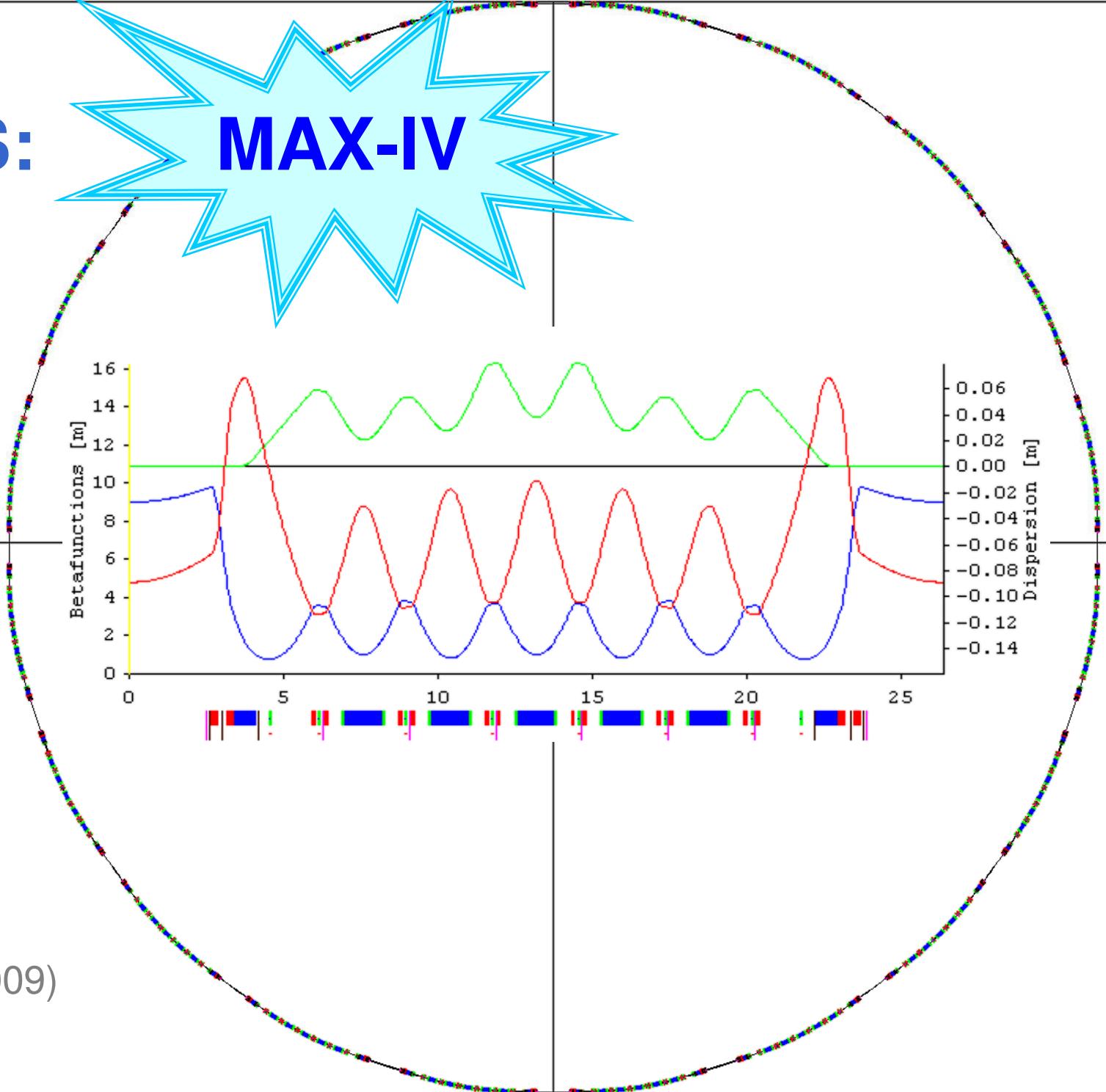
285 → 528 m

3.0 GeV

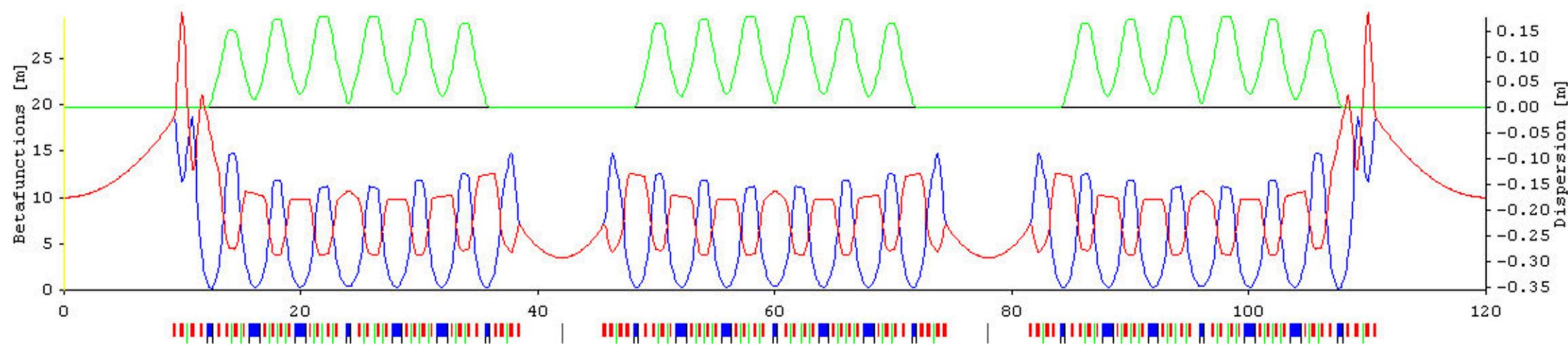
1.2 → 0.33 nm

H. Tarawneh et al.
NIM A 508 (2003) 480

S. C. Leemann et al.
PRST AB 12, 120701 (2009)



The 1993 (CDR) 7BA SLS lattice, *rejected*



- ◆ $6 \times 7\text{BA}$ arcs with centre s.c. bends
- ◆ $Q = 20.2 / 5.4 \quad Q_{\text{cell}} \sim 0.44 / 0.09 \quad \xi = -55 / -18$
- ◆ $E = 2.1 \text{ GeV} \quad C = 240 \text{ m} \quad \varepsilon = 3.2 \text{ nm}$
- ✖ Too few user straights: $2 \times 20 + 4 [2] \times 6 \text{ m}$
- ✖ Problems with energy acceptance
 - ➡ *tool developments...*

The “standard method” for sextupole optimization

J. Bengtsson, *The sextupole scheme for the SLS: an analytic approach*,
Internal report SLS-TME-TA-1997-12

a) get the sextupole [+quadrupole] Hamiltonian:

$$\Rightarrow \int_{\text{cell}} [H_2(s) + H_3(s)] ds = \sum h_{jklmp} \text{ with}$$

$$h_{jklmp} \propto \sum_n^{N_{\text{sext}}} (b_3 L)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} D_n^p e^{i\{(j-k)\phi_{xn} + (l-m)\phi_{yn}\}} - \left[\sum_n^{N_{\text{quad}}} (b_2 L)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} e^{i\{(j-k)\phi_{xn} + (l-m)\phi_{yn}\}} \right]_{p \neq 0}$$

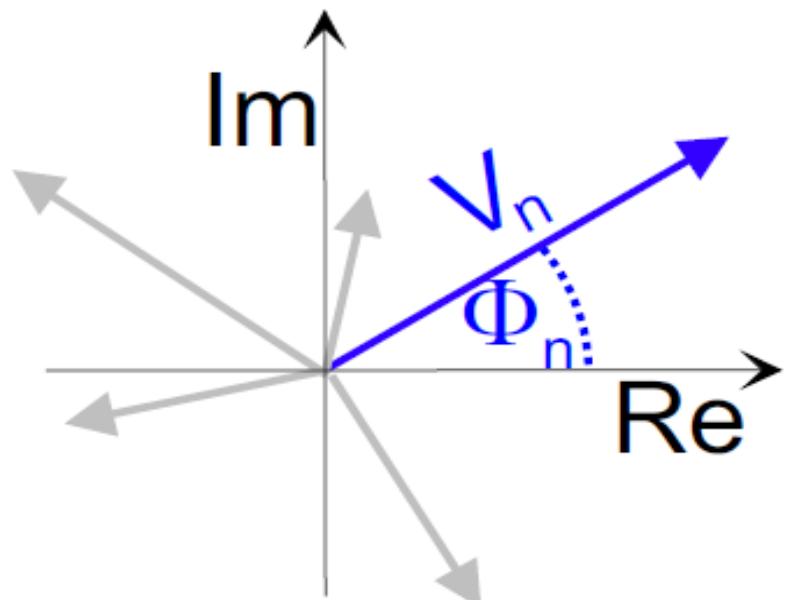
$$h = \sum_n^{N_{\text{sext}}} V_n e^{i\Phi_n} [+ \dots \text{quads for } p \neq 0 \dots]$$

Sextupole_n \leftrightarrow complex vector:

Length $V_n = V_n(b_3, L, \beta_x, \beta_y, D)$

Angle $\Phi_n = \Phi_n(\phi_x + \phi_y)$

- $\Phi_n = 0 \forall n \rightarrow$ tune shifts
- $\Phi_n \neq 0 \rightarrow$ resonances



b) ... 9 first order sextupole terms: adjust 2 real, suppress 7 complex...

First order sextupole [+quadrupole] Hamiltonian

- 2 phase independant terms → chromaticities:

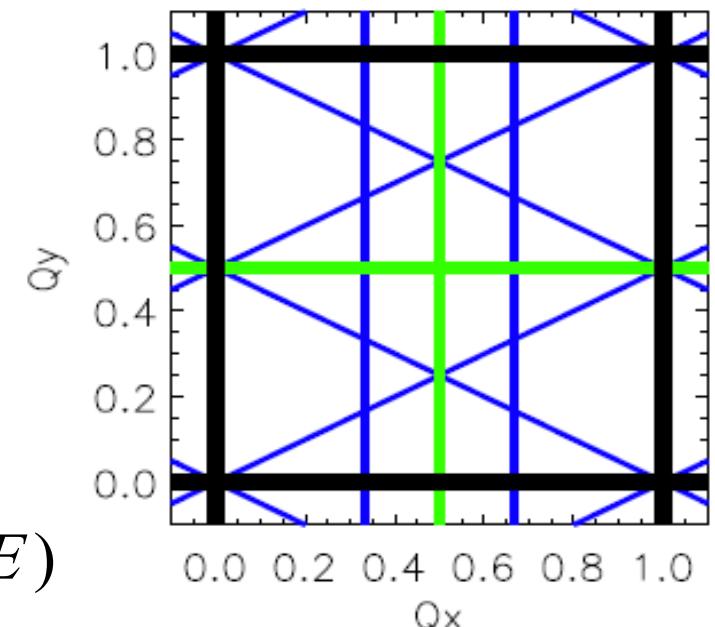
$$h_{11001} = +J_x \delta \left[\sum_n^{N_{sext}} (2b_3 L)_n \beta_{xn} D_n - \sum_n^{N_{quad}} (b_2 L)_n \beta_{xn} \right] \rightarrow \xi_x$$

$$h_{00111} = -J_y \delta \left[\sum_n^{N_{sext}} (2b_3 L)_n \beta_{yn} D_n - \sum_n^{N_{quad}} (b_2 L)_n \beta_{yn} \right] \rightarrow \xi_y$$

- 7 phase dependant terms → resonances: $h^N := h$ for N cells, $N \rightarrow \infty \implies$

$$|h_{jklmp}^\infty| = \frac{|h_{jklmp}|}{2 \sin \pi [a_x Q_x^{\text{cell}} + a_y Q_y^{\text{cell}}]} \quad a_x = (j - k) \quad a_y = (l - m)$$

$$\begin{aligned} h_{21000} &= h_{12000}^* \longrightarrow \mathbf{Q}_x \\ h_{30000} &= h_{03000}^* \longrightarrow 3 Q_x \\ h_{10110} &= h_{01110}^* \longrightarrow \mathbf{Q}_x \\ h_{10200} &= h_{01020}^* \longrightarrow Q_x + 2 Q_y \\ h_{10020} &= h_{01200}^* \longrightarrow Q_x - 2 Q_y \\ h_{20001} &= h_{02001}^* \longrightarrow 2 Q_x \\ h_{00201} &= h_{00021}^* \longrightarrow 2 Q_y \end{aligned} \} \rightarrow d\beta/d\delta \quad (\delta = \Delta E/E)$$



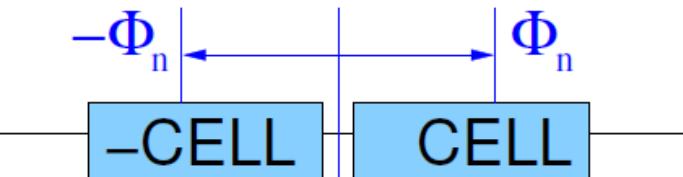
c) a particular problem of multi-bend achromat (MBA) lattices

Systematic first order optimization:

9 terms h_{jklmp} (7 complex, 2 real)

$\rightarrow \mathbf{16}$ sextupole families

\Rightarrow Symmetry:



$Im(h_{jklmp}) = 0 \rightarrow \mathbf{9}$ sextupole families.

Linear system for M families of sextupoles:

$$\left\{ \sum_{n \in \{Sm\}} \beta_n^{(\dots)} D_n^{(\dots)} e^{i\{\dots\}\phi_n} \dots \right\}_{9 \times M} \times \begin{Bmatrix} (b_3 L)_m \end{Bmatrix}_{M \times 1} = \begin{Bmatrix} \sum_{\text{Quad}} (b_2 L) \dots \end{Bmatrix}_{1 \times 9}$$

Light source problem: $\epsilon \downarrow \Rightarrow \Delta\phi_x^{\text{cell}} \rightarrow 180^\circ \Rightarrow e^{i2\phi_x} \approx 1$

SLS 6×7BA:
 $\Phi_x^{\text{cell}} = 159^\circ$

$2Q_x$ resonance driving term h_{20001} proportional to chromaticity $\xi_x \propto h_{11001}$

$$2Q_x \rightarrow \frac{\partial \beta_x}{\partial \delta} \rightarrow \xi_x^{(2)} = \frac{\partial^2 Q_x}{\partial \delta^2} \rightarrow \text{energy acceptance } \downarrow$$

No solution for $\{(b_3 L)_m\} \Rightarrow$ suppression by $\Delta\phi_x^{\text{straight}}$

J. Bengtsson et al., NIM A 404 (1998) 237

d) to optimize the sextupole strengths is not enough....

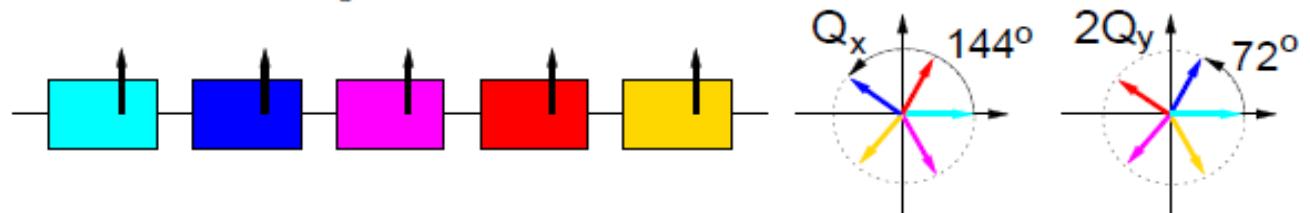
Phase cancellation schemes

Periodicity: N cells

$$N\Delta Q_x^{\text{cell}}, \quad [3N\Delta Q_x^{\text{cell}}, \quad 2N\Delta Q_x^{\text{cell}}], \quad 2N\Delta Q_y^{\text{cell}} \longrightarrow \text{integer!}$$

e.g. $N = 5$, $\Delta Q_x^{\text{cell}} = 0.4$ ($= 144^\circ$), $\Delta Q_y^{\text{cell}} = 0.1$ ($= 36^\circ$)

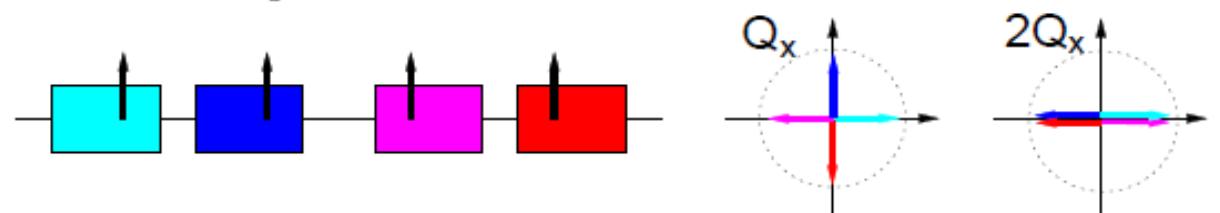
SLS 6×7 BA (i.e. 5 cells)
 $(\varepsilon = 3.2 \text{ nm @ } 2.1 \text{ GeV})$
 $\Rightarrow \Delta Q_x^{\text{cell}} = 0.44$ ☺



Symmetry: Lattice section vs. mirror image

$$\Delta Q_x^{\text{cell}} = \frac{2n_x+1}{4}, \quad \Delta Q_y^{\text{cell}} = \frac{2n_y+1}{4} \quad (n_x, n_y \text{ integers}):$$

SLS $12 \times$ TBA
 $(\varepsilon = 4.9 \text{ nm @ } 2.4 \text{ GeV})$
 $\Delta Q_x^{\text{cell}} \approx 7/4, \Delta Q_y^{\text{cell}} \approx 3/4$ ☺



→ phase advances over straight sections → β_x, β_y in straights.

⇒ Iterate: linear ⇔ nonlinear lattice design

e) ... still not the end: 13 more terms in 2nd order: 5 real, 8 complex
 (Pandora's box has a false bottom!)

Second order sextupole [+first order octupole] Hamiltonian

$$\sum_n \sum_m (b_3 L)_n (b_3 L)_m \times (\beta_n, \phi_n \beta_m, \phi_m \dots) + \left[\sum_q (b_4 L)_q \times (\beta_q, \phi_q \dots) \right]$$

- 3 phase independant terms → amplitude dependant tune shifts:

$$\frac{\partial Q_x}{\partial J_x} = \frac{\partial Q_x}{\partial J_y} = \frac{\partial Q_y}{\partial J_x} = \frac{\partial Q_y}{\partial J_y}$$

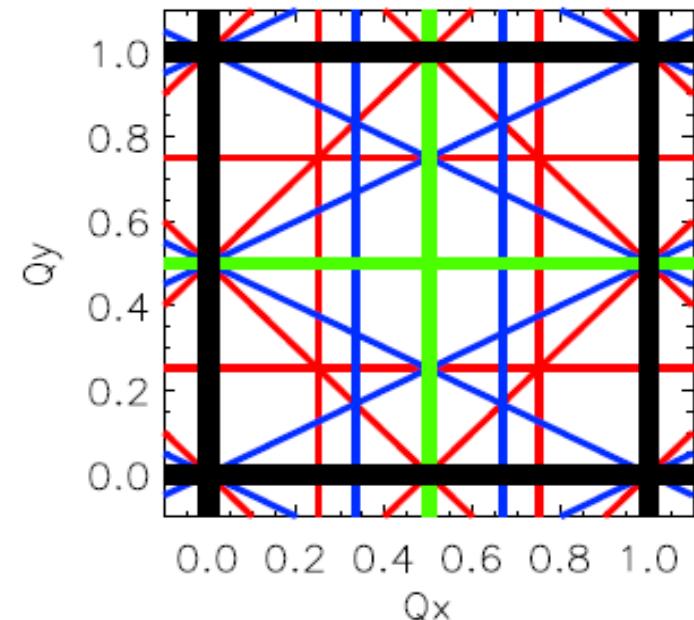
- 2 phase independant off-momentum terms → second order chromaticities:

$$\xi_{x/y}^{(2)} = \frac{\partial^2 Q_{x/y}}{\partial \delta^2}$$

- 8 phase dependant terms

→ octupolar resonances:

$h_{40000} \rightarrow 4Q_x$	$h_{31000} \rightarrow 2Q_x$
$h_{00400} \rightarrow 4Q_y$	$h_{20110} \rightarrow 2Q_x$
$h_{20200} \rightarrow 2Q_x + 2Q_y$	$h_{00310} \rightarrow 2Q_y$
$h_{20020} \rightarrow 2Q_x - 2Q_y$	$h_{01110} \rightarrow 2Q_y$

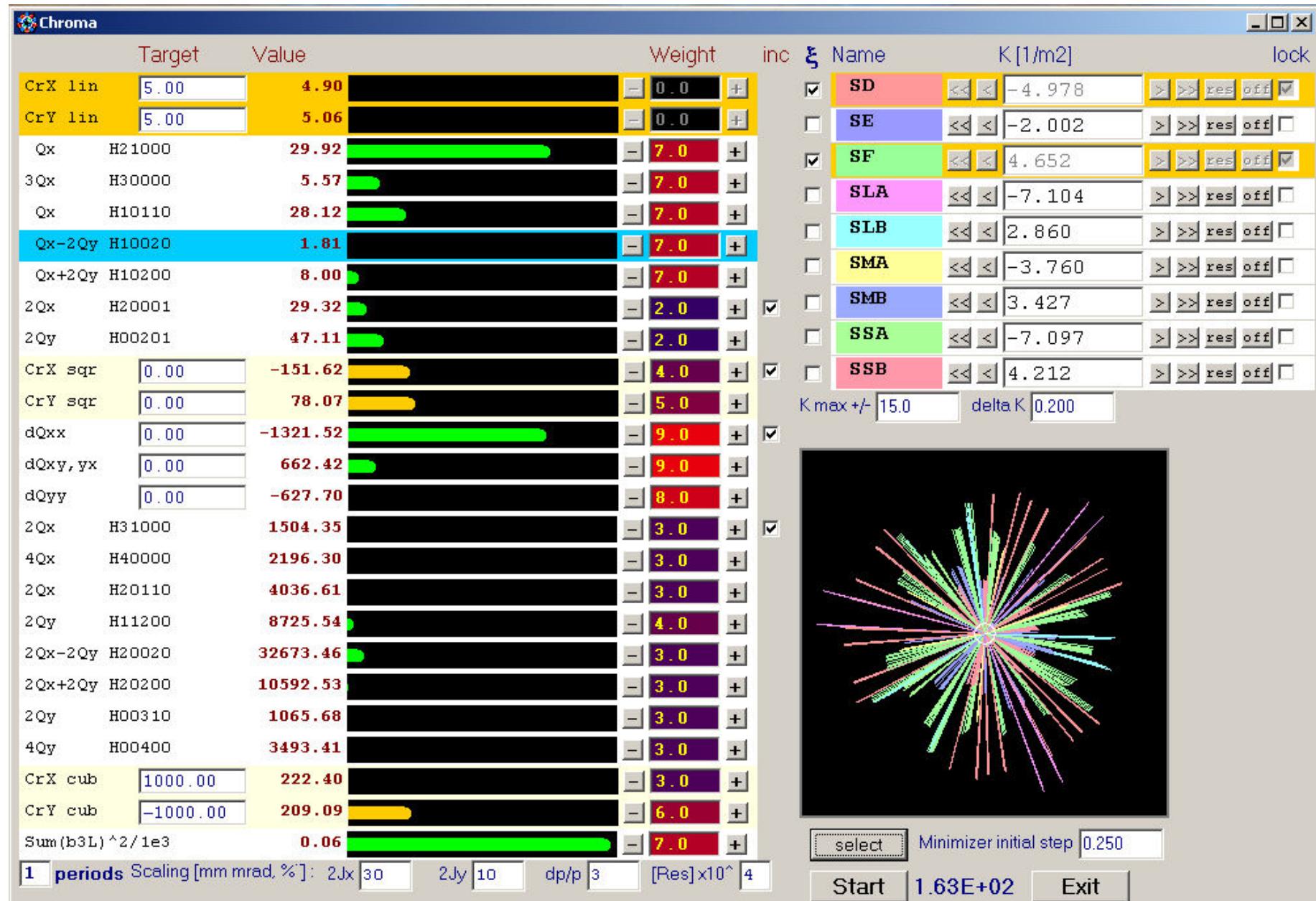


f) Tool for sextupole optimization (OPA)

Analytical
expressions
for 1st and 2nd
Hamiltonian
modes.
(J.Bengtsson)

Numeric
differentiation
for 1st, 2nd, 3rd
chromaticity

$\sum (b_3 l)^2$
included in
minimization



Performance of the SLS 12×TBA lattice

Dynamic apertures

current SLS optics F6CWO

physical aperture

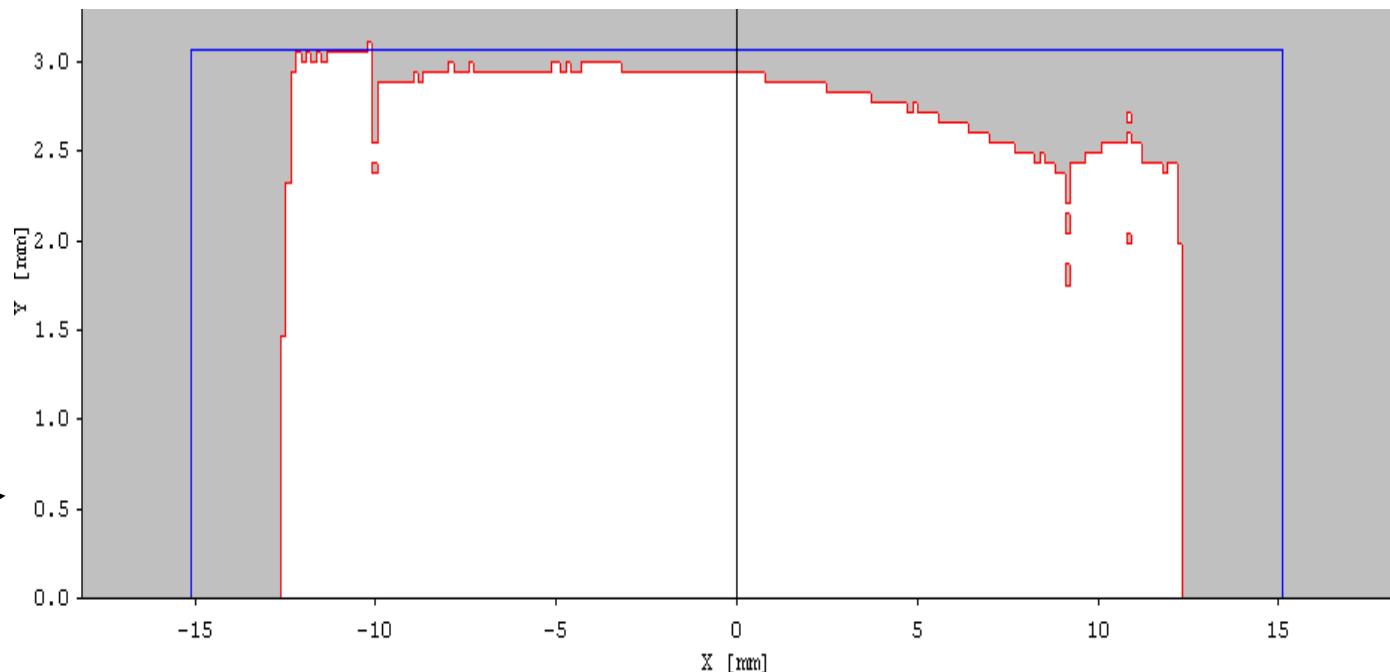
dynamic & phys. aperture

Transverse: x vs. y

corresponding to

$A_x = 34 \text{ mm mrad}$

$A_y = 2 \text{ mm mrad}$



Momentum: x vs. dp/p

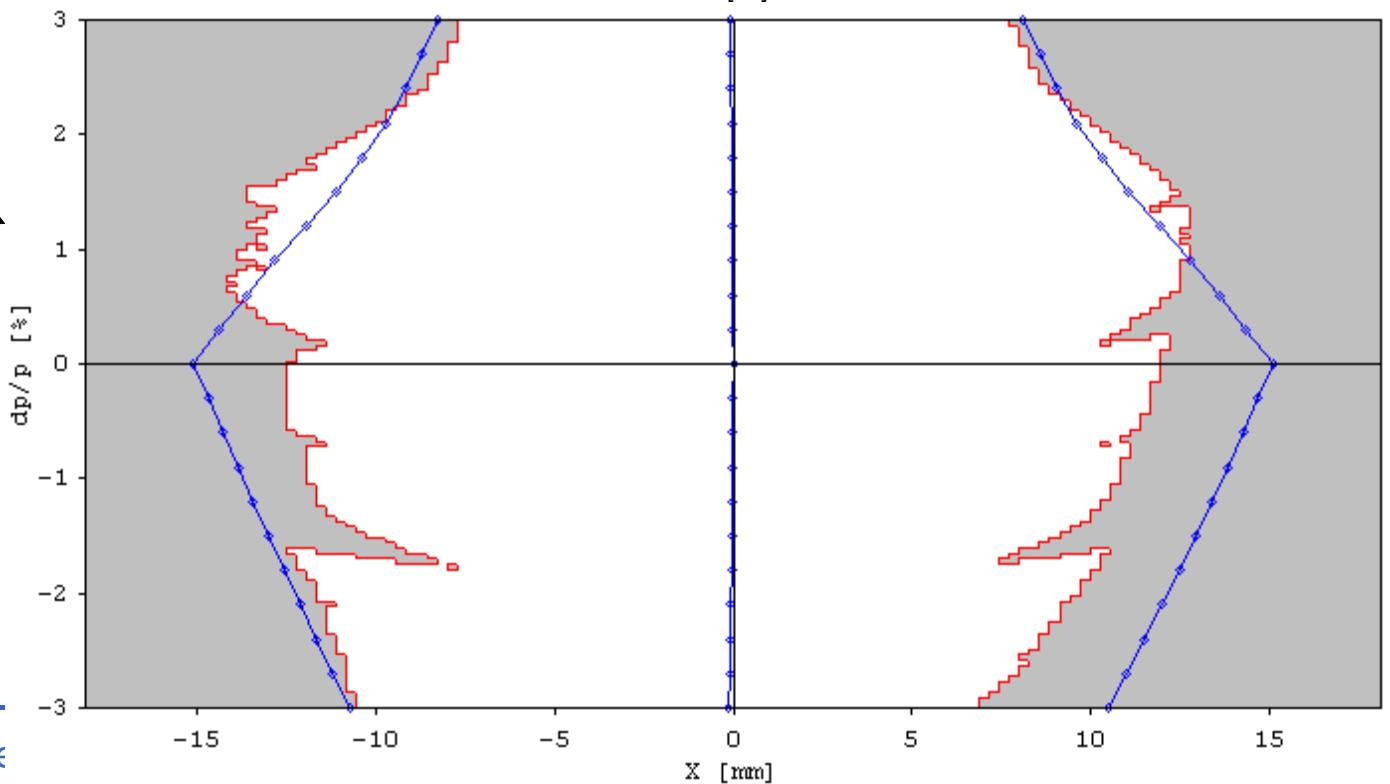
corresponding to

$T_{\text{Touschek}} = 9.7 \text{ hrs}$

in standard user mode.

(6D Tracy tracking)

- 400mA in 390 bunches
- 2.1 MV RF (500 MHz)
- 3rd HC for 2.2 x bunch length
- 0.1% emittance coupling
- $\sigma_y = 9 \mu\text{m}$ at monitor



Realization of performance (2001-2008)

(in terms of lifetime and energy acceptance)

◆ Complications:

- $\xi = +1/+1 \rightarrow +5/+5$ for suppression of instabilities
- Lattice modification for laser slicing: period 3 → 1

◆ Orbit correction

- removal of bumps for users, girder realignments
- beam based BPM calibration (“BB alignment”)

◆ Optics correction: $\Delta\beta/\beta \rightarrow 3\% \text{ rms}$

- 177 quadrupoles with individual power supplies

◆ Coupling suppression

- 12 dispersive & 24 non-disp. skew quadrupoles
- emittance coupling $\rightarrow 5 \cdot 10^{-4}$, $\epsilon_y \rightarrow 2.8 \text{ pm}$

\rightarrow M. Böge, *Reaching ultra-low vertical emittance in the SLS*

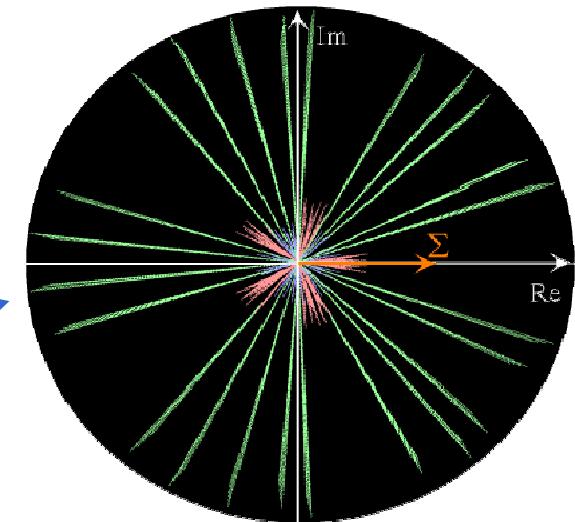
◆ Sextupole symmetrization...

Sextupole symmetrization

$$h_{jklmp} \propto \sum_n^{N_{\text{sext}}} (b_3 L)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} D_n^p e^{i\{(j-k)\phi_{xn} + (l-m)\phi_{yn}\}}$$

Sextupoles in symmetric *families*:

- $\text{Im } (h) = 0$ by lattice symmetry
 - design: optimized for $\text{Re } (h) \rightarrow 0 \forall h$.
- ⇒ Auxiliary sextupoles breaking the symmetry
- compensate parasitic $\text{Re}, \text{Im } (h) \neq 0$.
 - first step: do *empirical* optimization
 - ≥ 9 knobs required for Re and Im of $h_{21000}, h_{30000}, h_{10200}$ and h_{10020} and $\Delta\xi_x = 0$
 - $h_{10110} \propto h_{21000}$ and $\Delta\xi_y \propto \Delta\xi_x$ (SLS: all aux. sext. at same $\beta_x \beta_y \eta$)
 - 12 auxiliary sextupoles installed
- ⇒ energy acceptance 2% → 3%
- auxiliary sextupole strength $\sim 3\%$ of SF strength



Versatile Sextupoles

all 120 sextupoles were delivered with H&V corrector coils
⇒ make skew quadrupoles and auxiliary sextupoles

120 sextupoles in 9 families:

SF(24), SD(24), SE(24) → **chromaticities**

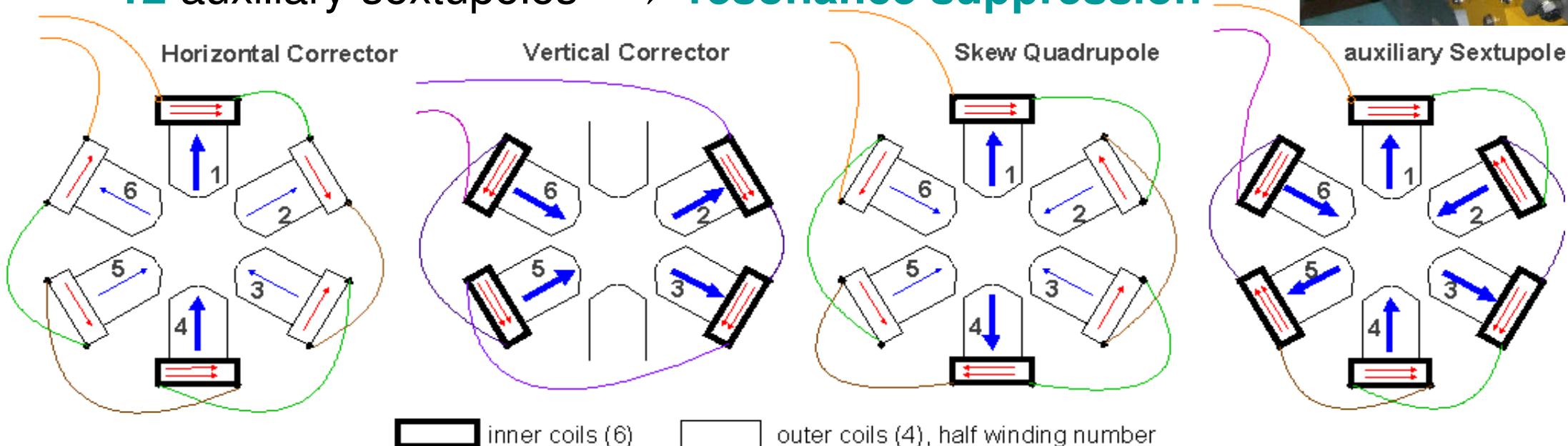
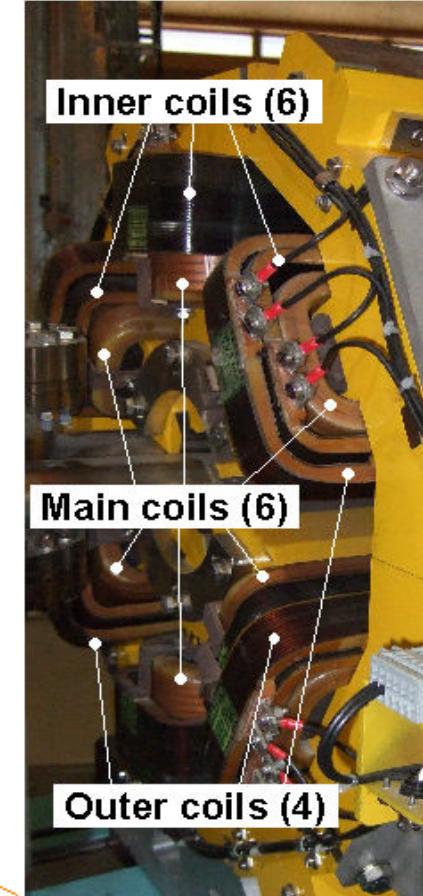
SSA(12), SSB(12), SMA(6), SMB(6), SLA(6), SLB(6) → **D.A.**

SD, SE, S*B: **72** H&V correctors → **orbit correction**

S*A: **24** skew quads ($\eta=0$) → **betatron coupling**

SF: **12** skew quads ($\eta>0$) → **vertical dispersion**

12 auxiliary sextupoles → **resonance suppression**



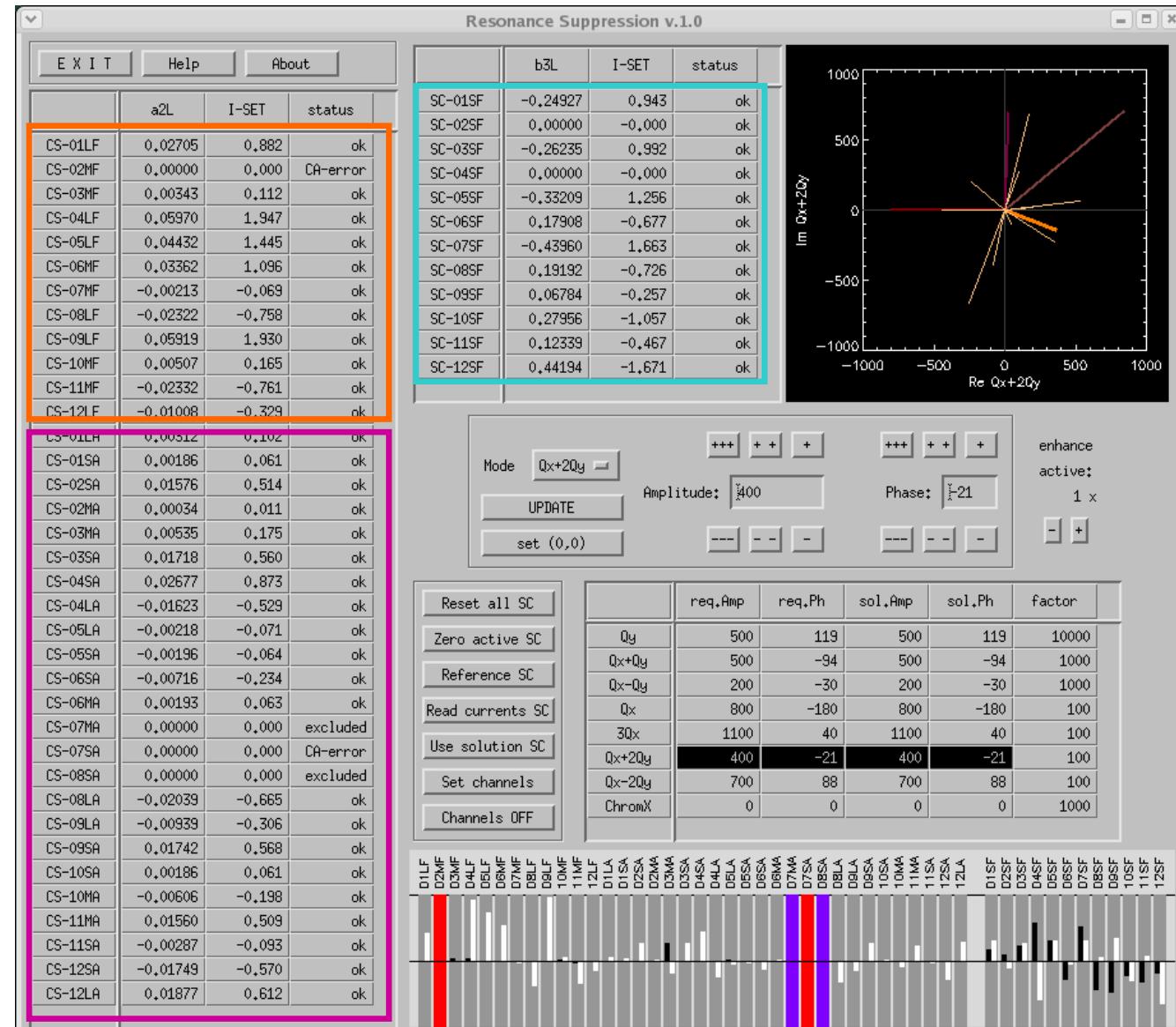
The σ_y/τ operator tool

Dispersive and non-dispersive skew quads

- $h_{00101} \rightarrow Q_y \rightarrow \eta_y$
- $h_{10100} \rightarrow Q_x + Q_y$
- $h_{10010} \rightarrow Q_x - Q_y$

Settings based on vertical dispersion and response matrix measurements

+
some empirical fine tuning



Ratio of vertical beam size to lifetime on stripchart

Auxiliary sextupoles

h_{21000}
 $\Rightarrow Q_x$

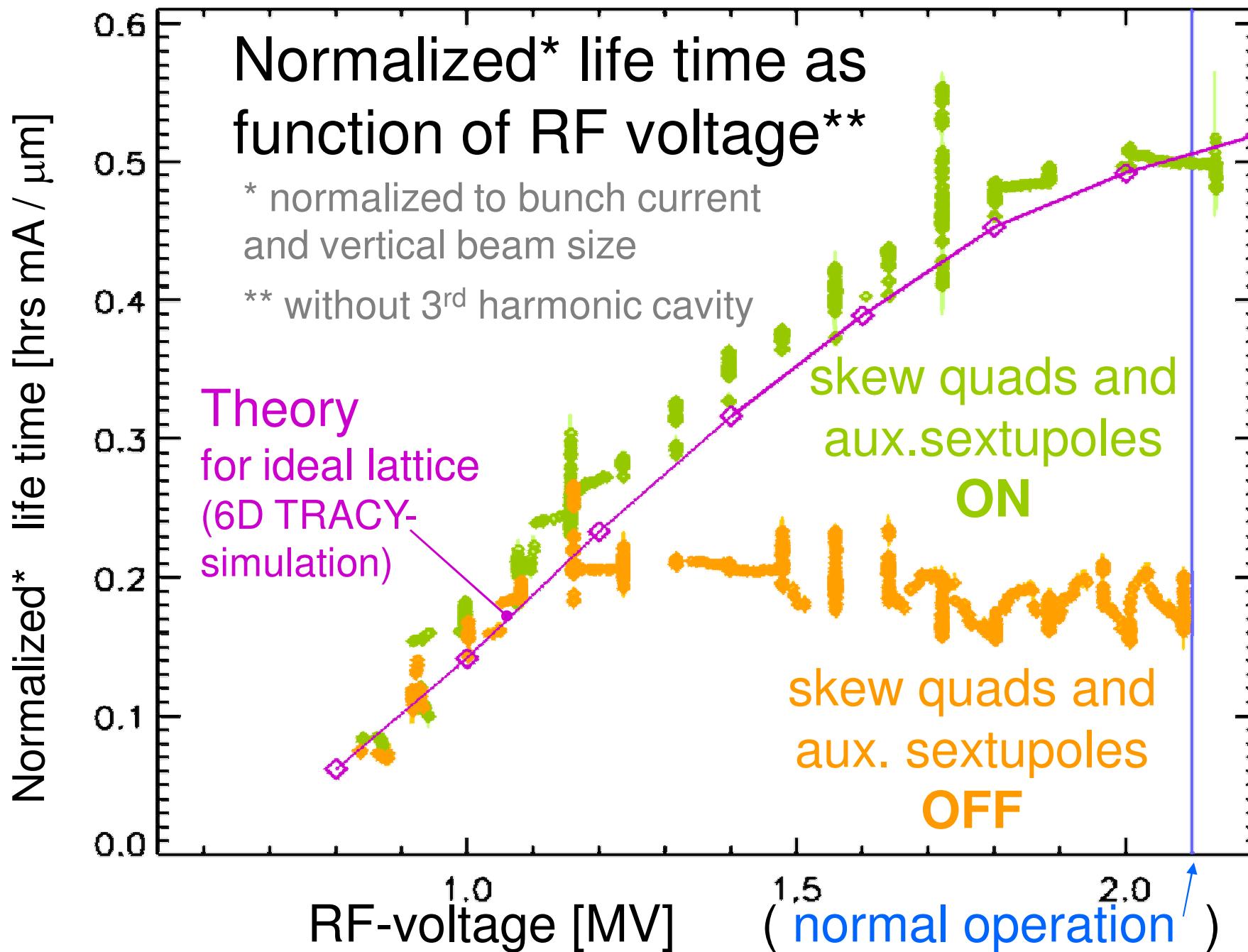
h_{30000}
 $\Rightarrow 3Q_x$

h_{10200}
 $\Rightarrow Q_x + 2Q_y$

h_{10020}
 $\Rightarrow Q_x - 2Q_y$

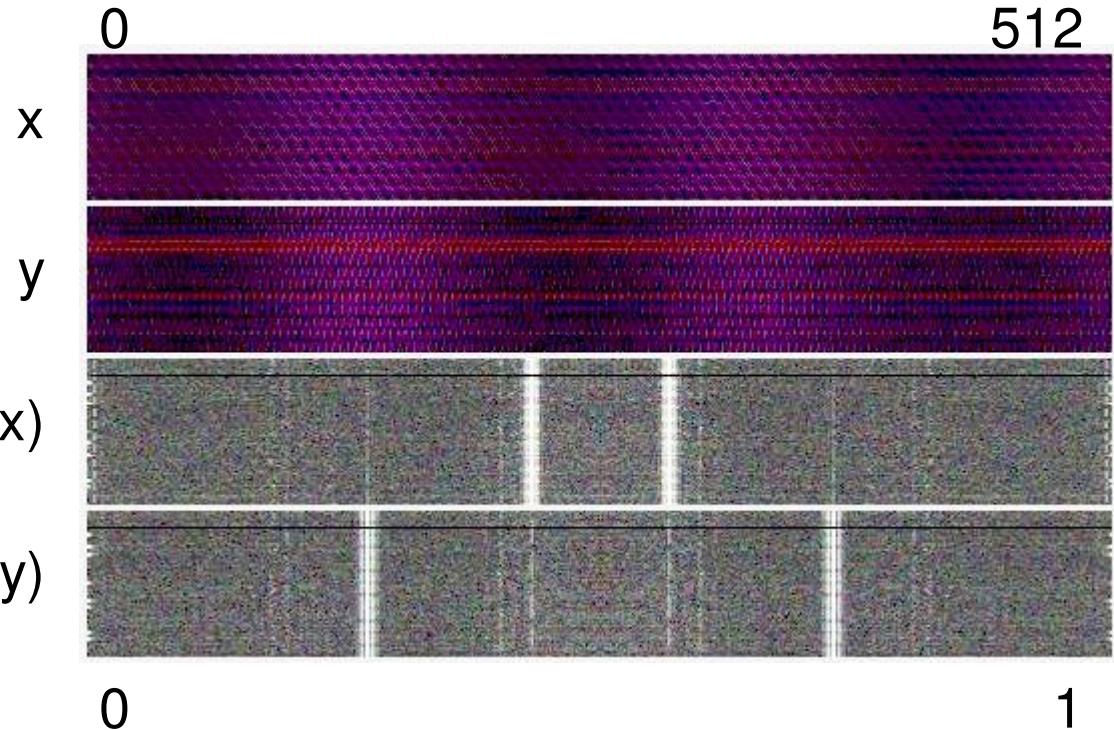
Empirical tuning

Lifetime in agreement with design



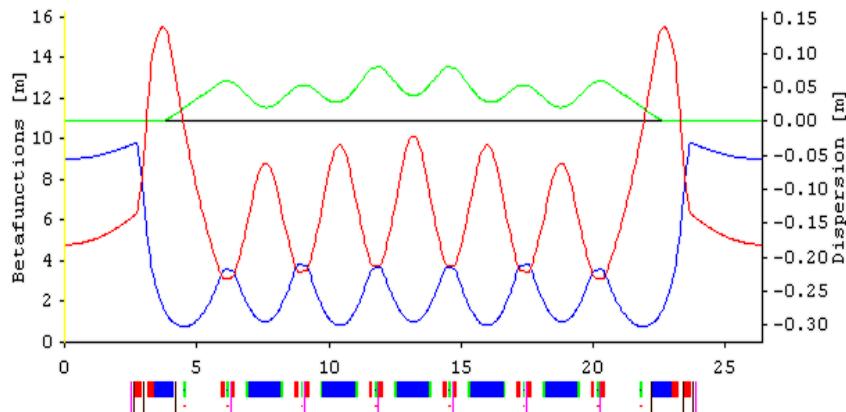
Future work on SLS ...

- ◆ Further suppression of betatron coupling:
 - eliminate steps between girders \Rightarrow girder realignment
 - centering of orbit in sextupoles
 \Rightarrow measurement of misalignments using skew quad coils
- ◆ controlled excitation of vertical dispersion:
 - set (T, ε_y) working point $T \propto \sqrt{\varepsilon_y}$
- ◆ set auxiliary sextupoles based on spectra from all BPMs in turn \times turn mode.
 - \longrightarrow
 - FFT (x)
 - FFT (y)
- ◆ automatize skew quad. & auxiliary sext. tuning.

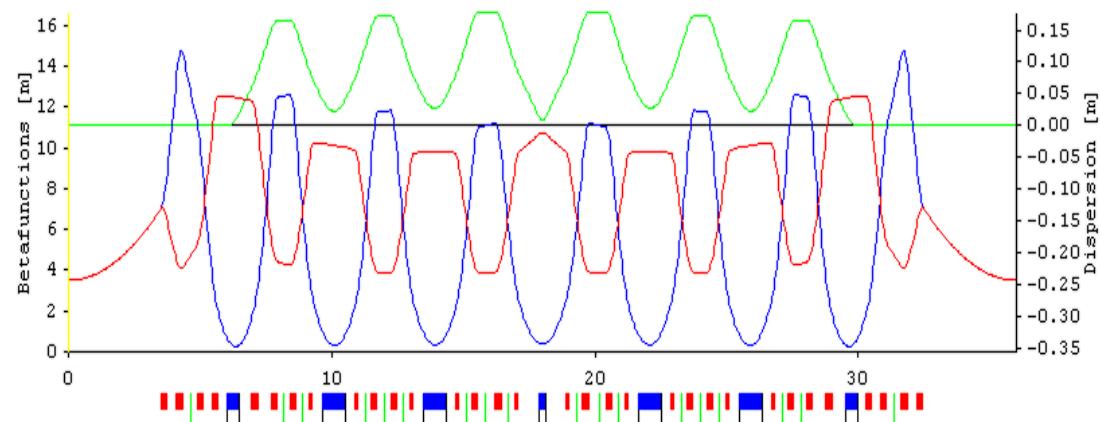


MAX-IV – proper use of the 7BA

1/20 MAX-IV



1/6 SLS'93



	MAX-IV	SLS'93	⇒ ⇒ scaled to
7BA arcs	20	6	20
Energy E	3 GeV	2.1 GeV	3 GeV
Emittance ε	0.33 nm	3.2 nm	0.18 nm
Tune Q	42.2 / 14.3	20.2 / 5.4	67.3 / 18.0
norm. chrom. $-\xi/Q$	1.2 / 3.1	2.7 / 3.4	
Energy acceptance	> 5%	~ 2 %	
Circumference	528 m	240 m	> 800 m

MAX-IV concept

→ S. Leemann, *MAX-4 lattice design*

◆ many (140) small bends → low dispersion

⇒ low emittance with relaxed optics

⇒ small magnet apertures:
high gradients

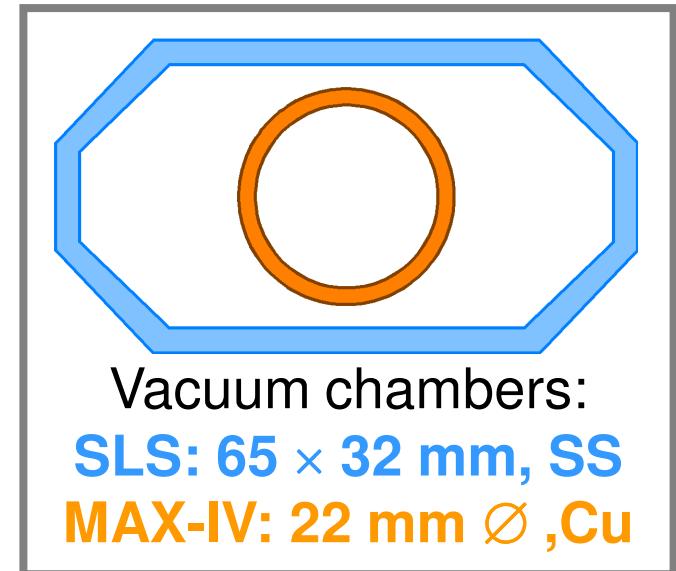
◆ compact lattice

- high gradients: short magnets
- gradients in bending magnets

◆ nonlinear dynamics

- low dispersion: very strong sextupoles
 - sextupoles insufficient to control ADTS $\partial Q/\partial J$
- ⇒ installation of octupoles

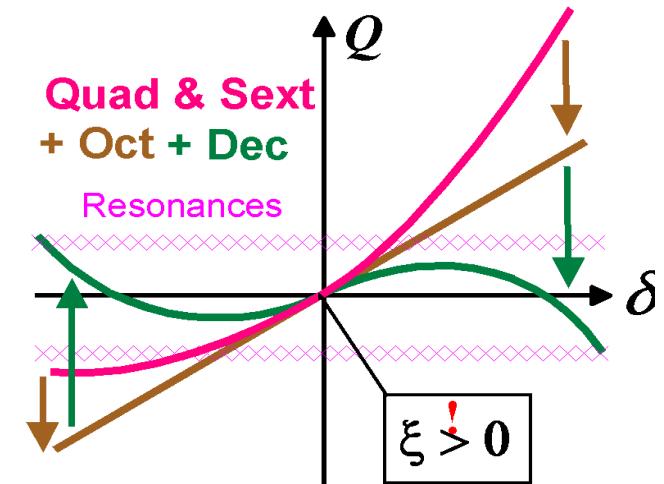
↳ *tool developments ctn'd...*



... g) include octupoles and decapoles

Use higher multipoles to control in 1st order
phase-independent sextupole effects of 2nd and 3rd order

- ◆ Octupoles:
 - linear ADTS: $\partial Q_x / \partial J_x$, $\partial Q_x / \partial J_y = \partial Q_y / \partial J_x$, $\partial Q_y / \partial J_y$
 - quadratic chromaticities: $\partial^2 Q_x / \partial \delta^2$, $\partial^2 Q_y / \partial \delta^2$
- ◆ Decapoles:
 - cubic chromaticities: $\partial^3 Q_x / \partial \delta^3$, $\partial^3 Q_y / \partial \delta^3$
 - quadratic ADTS and off-energy linear ADTS
- ◆ Side-effects (resonances) tolerable [hopefully]
if multipoles are rather weak...
- Taylor (minimize) beam footprint in tune space
 - provide sufficient horizontal dynamic aperture for injection.
 - provide sufficient energy acceptance for Touschek lifetime.
- Provide knobs for control room (linear systems).



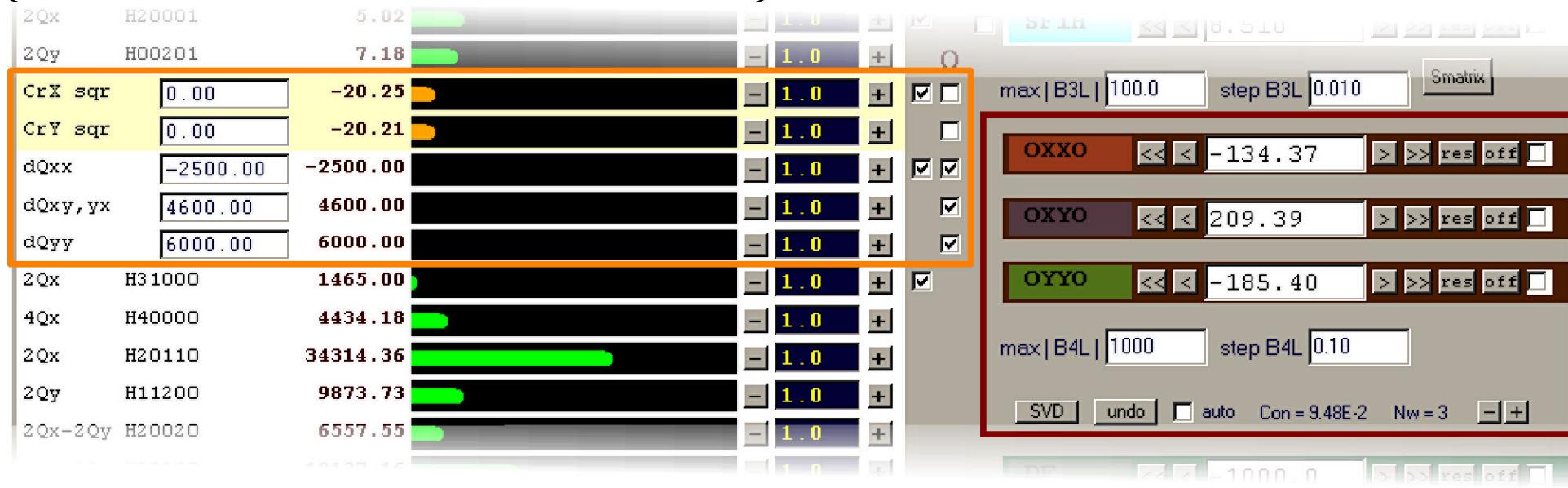
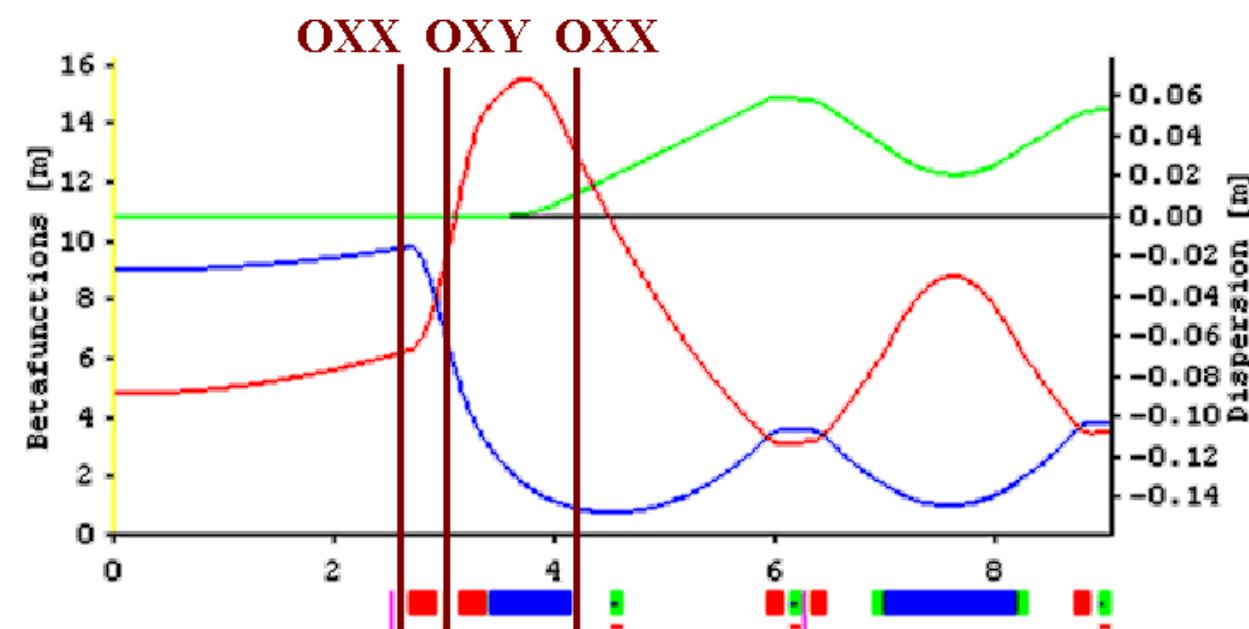
h) ...tool upgrade and application

3 octupole families for **MAX-IV**

S. C. Leemann et al.
PRST AB 12, 120701 (2009)

SVD solver for linear system
for N octupole families:

$$\left\{ \frac{\partial Q_x}{\partial J_x}, \frac{\partial Q_x}{\partial J_y}, \frac{\partial Q_y}{\partial J_y}, \frac{\partial^2 Q_x}{\partial \delta^2}, \frac{\partial^2 Q_x}{\partial \delta^2} \right\}^T = \mathbf{M}_{5 \times N} \cdot \{(b_4 l)_k\}_N^T$$



Conclusions

- ◆ The 12×TBA lattice for the SLS was the right decision
 - compromise: number of straights, emittance, circumference
- ◆ Robust “standard method” of sextupole optimization
 - 1st and 2nd order sextupole Hamiltonian analytical
 - 1st, 2nd and 3rd order chromaticities numerical
- ◆ SLS reached design performance
 - coupling control with 36 skew quadrupoles
 - nonlinear tuning using 12 auxiliary sextupoles
 - there is still room for further improvement
- ◆ Next generation low emittance rings
 - come-back of multi bend achromat lattice: compact & relaxed
 - reduced vacuum chamber and magnet dimensions
 - octupoles [and decapoles] included in lattice design