Intra-Beam Scattering and Touschek Lifetime Calculations for PEP-X

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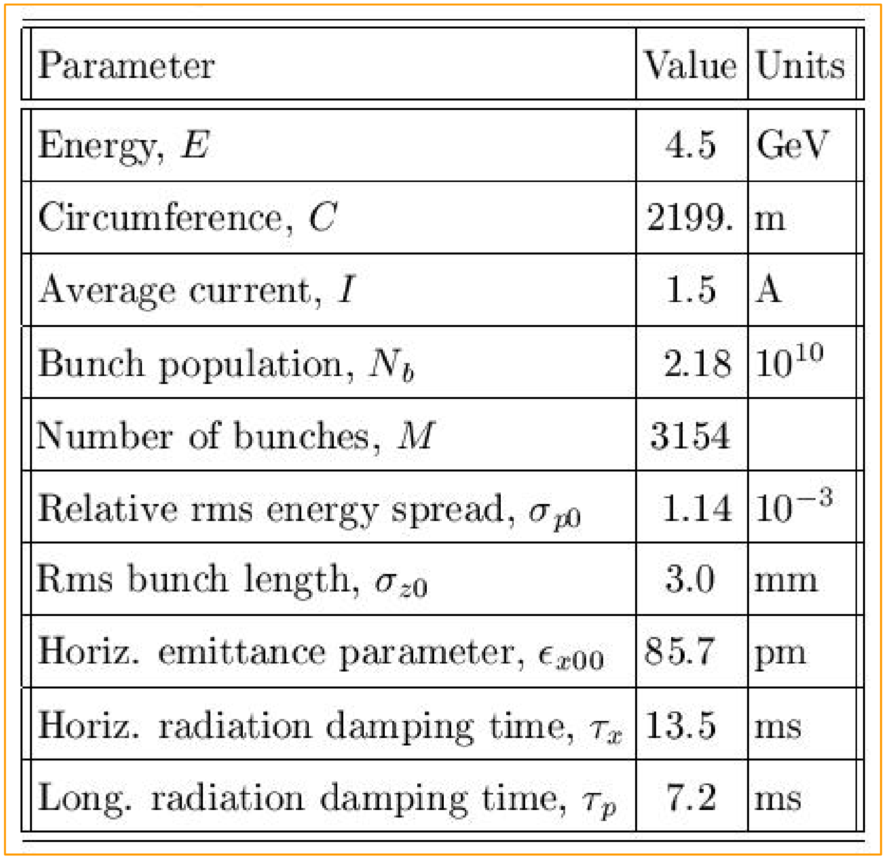
In a low emittance/high current machine such as PEP-X, the collective effects tend to be important

Outline

- PEP-X impedance/instabilities calculations—brief overview
- Intra-beam scattering calculations
- Touschek lifetime calculations
- Conclusions

Selected Parameters for PEP-X



Parameter	Value	Units
Energy, E	4.5	GeV
Circumference, C	2199.	m
Average current, I	1.5	A
Bunch population, N_b	2.18	10^{10}
Number of bunches, M	3154	
Relative rms energy spread, σ_{p0}	1.14	10^{-3}
Rms bunch length, σ_{z0}	3.0	mm
Horiz. emittance parameter, ϵ_{x00}	85.7	pm
Horiz. radiation damping time, τ_x	13.5	ms
Long. radiation damping time, τ_p	7.2	ms

Note that the nominal horizontal emittance $\varepsilon_{x0} = \varepsilon_{x00}/(1+\kappa)$, with κ the x-y coupling parameter

PEP-X Impedance/Instability Calculations—Brief Overview

Simulations and analysis performed by K. Bane, Y. Cai, L.-Q. Lee, C.-K. Ng, G. Stupakov, L. Wang, L. Xiao

Have developed state-of-the-art 3D frequency and time domain finite element programs for simulations

Impedance:

Without an actual vacuum chamber design available, we are developing a straw man design, inspired by objects in other machines, such as PEP-II

Sources include: RF cavities, BPM's, wiggler transitions, undulator transitions, resistive wall, coherent synchrotron radiation (CSR)

For microwave instability, generate (i) a pseudo-Green function wake representing the ring—to be used in simulations (σ_z= .5 mm), and (ii) an impedance budget—to assess relative importance of contributors

Some calculation (e.g. wiggler taper) still ongoing

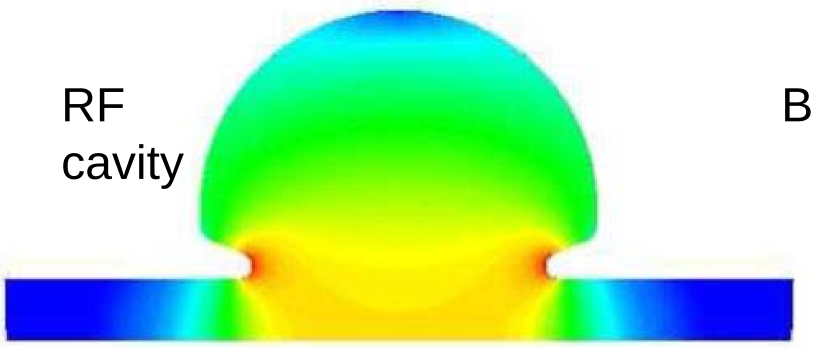
Instability:

Detailed calculations performed for: microwave, transverse mode coupling, transverse multi-bunch, and ion induced instabilities

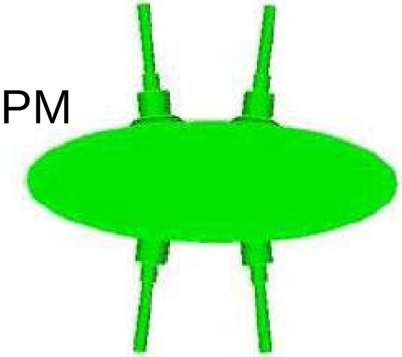
Threshold to microwave instability higher than nominal current, ion instability requires good feedback, vacuum, mitigation methods

Full description of PEP-X work (including on collective effects) will be given in large report to be finished soon

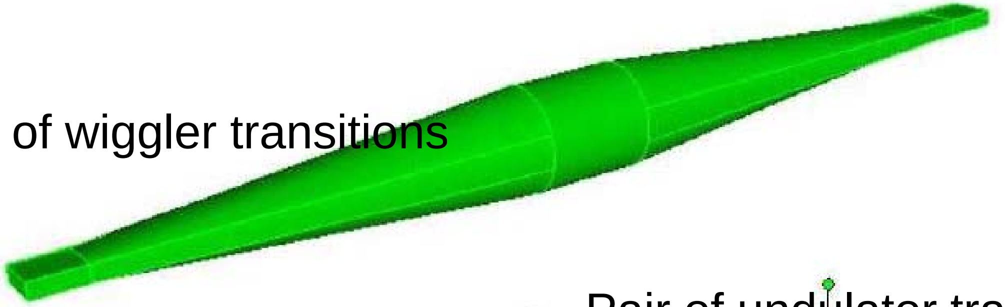
Selected Impedance Sources



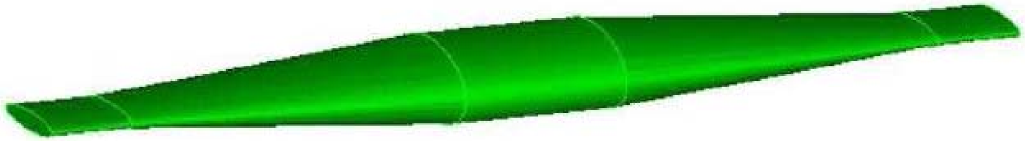
RF
cavity



BPM



Pair of wiggler transitions

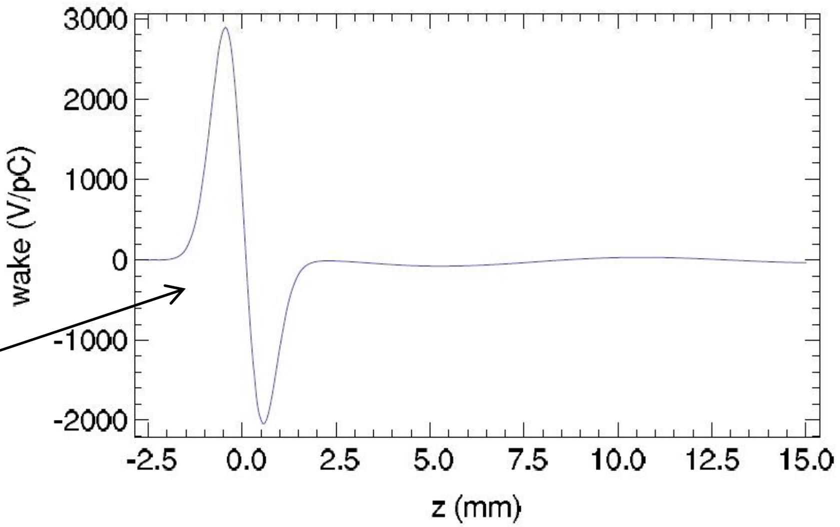


Pair of undulator transitions

Selected impedance objects included in our straw man PEP-X design.
Note: the fundamental mode fields are shown in the RF cavity.

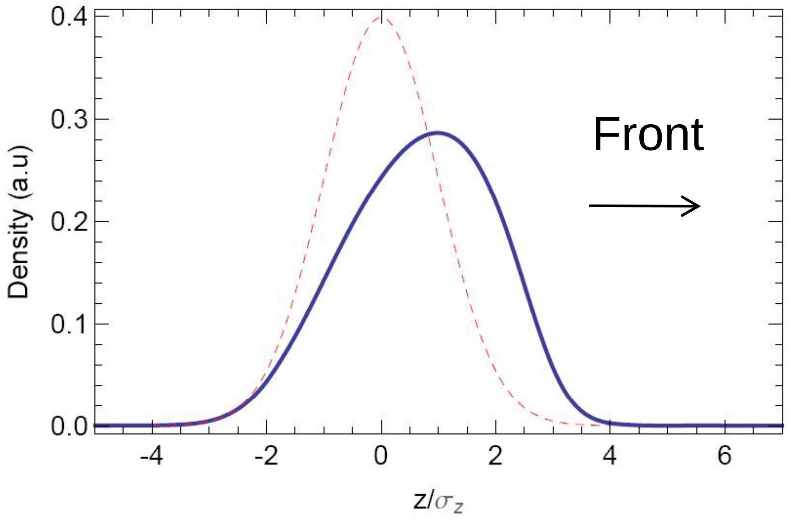
Pseudo-Green Function Wake

Pseudo-Green function wake representing the PEP-X ring: wake of a σ_z= .5 mm bunch (tentative)

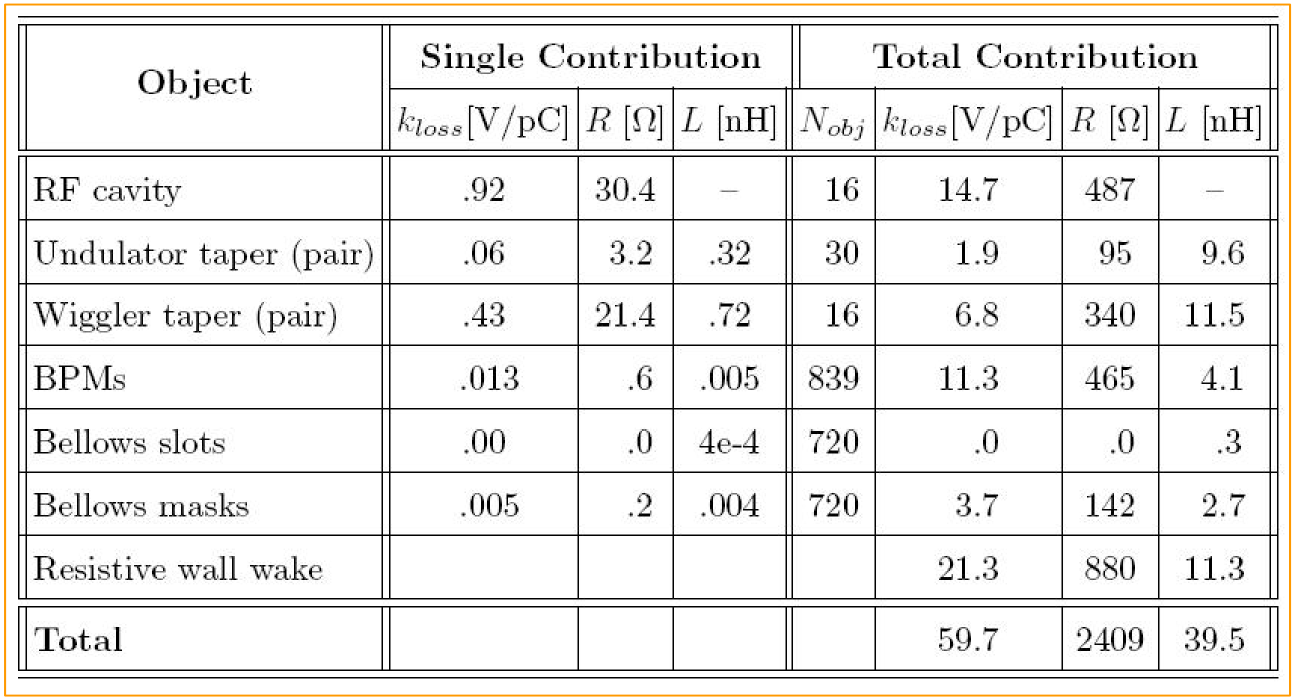


Inductive in character

Haissinski solution, giving the steady-state bunch shape. Bunch length is 25% above nominal length.



Impedance Budget



Object	Single Contribution			Total Contribution			
	k_{loss}[V/pC]	R [Ω]	L [nH]	N_{obj}	k_{loss}[V/pC]	R [Ω]	L [nH]
RF cavity	.92	30.4	–	16	14.7	487	–
Undulator taper (pair)	.06	3.2	.32	30	1.9	95	9.6
Wiggler taper (pair)	.43	21.4	.72	16	6.8	340	11.5
BPMs	.013	.6	.005	839	11.3	465	4.1
Bellows slots	.00	.0	4e-4	720	.0	.0	.3
Bellows masks	.005	.2	.004	720	3.7	142	2.7
Resistive wall wake					21.3	880	11.3
Total					59.7	2409	39.5

Impedance budget for PEP-X, giving the loss factor, and the effective resistance and inductance of the various objects in the ring. The results are at nominal bunch length σ_z*= 3 mm.*

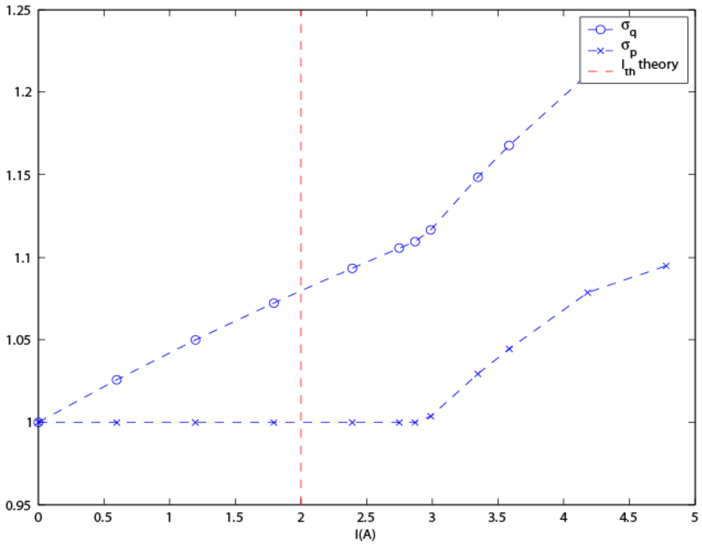
VFP Solver for Microwave Instability

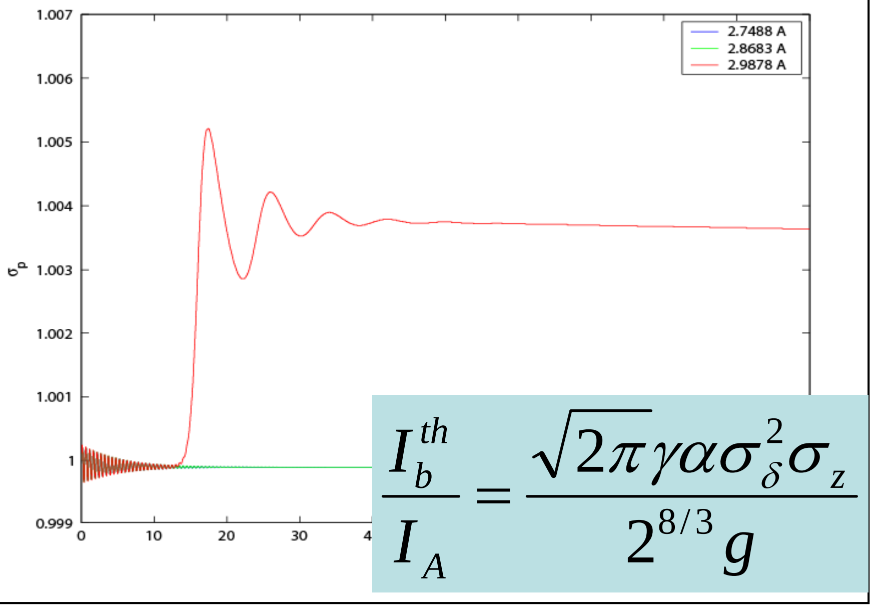
- Vlasov-Fokker-Planck solver based on the work of Warnock and Ellison with an improved interpolation scheme
- Solves Haissinski's integral equation and use its solution as a starting distribution
- Handle singular wakefields, such as CSR wakefield in vacuum or resistive-wall wakefield
- CSR shielding in two parallel plates based on work of Murphy, Krinsky, and Gluckstern
- Many other models: Q=1 resonator, pure inductance or resistance, or any numerical wakefield, for example, wakefield of the SLC damping ring

Coherent Synchrotron Radiation Induced Instability

(Y. Cai)

CSR Instability: For **3 mm** Gaussian bunches, estimated threshold of bunch current for the CSR-driven instability is 0.64 mA, corresponding to a total current of 2.0 A. The simulation, using a VFP solver and parallel plates model, shows a higher threshold near **3.0 A**.





$$\frac{I_b^{th}}{I_A} = \frac{\sqrt{2\pi}\gamma\alpha\sigma_\delta^2\sigma_z}{2^{8/3}g}$$

Intra-Beam Scattering (IBS) Calculations

IBS describes multiple Coulomb scattering that (in electron machines) leads to an increase in all bunch dimensions and in energy spread

Assume coupling dominated: $\varepsilon_y = \kappa\varepsilon_x$

Steady-state IBS emittance and energy spread:

$$\epsilon_x = \frac{\epsilon_{x0}}{1 - \tau_x/T_x} \qquad \text{and} \qquad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p/T_p}$$

Local IBS growth rates $\delta(1/T_x)$, $\delta(1/T_p)$, are functions of beam and lattice parameters; their average around the ring are the growth rates $1/T_x$, $1/T_p$

We follow the Bjorken-Mtingwa (BM) method; solution involves (i) integration at every lattice element, (ii) averaging around the ring, (iii) solving the above two equations simultaneously

Nagaitsev algorithm speeds up the calculation a factor of 25 (in Mathematica)

Raubenheimer pointed out that, due to small impact parameter events, the tails of distributions are not Gaussian; our Coulomb log reflects this (see Kubo and Oide); for PEP-X, (log)≈ 13

Program was spot-checked with SAD, Elegant, MAD—good agreement

Simplified Model of IBS

Longitudinal growth rate:

$$\frac{1}{T_p} \approx \frac{r_e^2 c N_b (\log)}{16\gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_z \sigma_p^3} \left\langle \sigma_H \, g(a/b) \; (\beta_x \beta_y)^{-1/4} \right\rangle$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x} \,, \quad a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}} \,, \quad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}}$$

$$g(\alpha) = \alpha^{(0.021 - 0.044 \ln \alpha)}$$

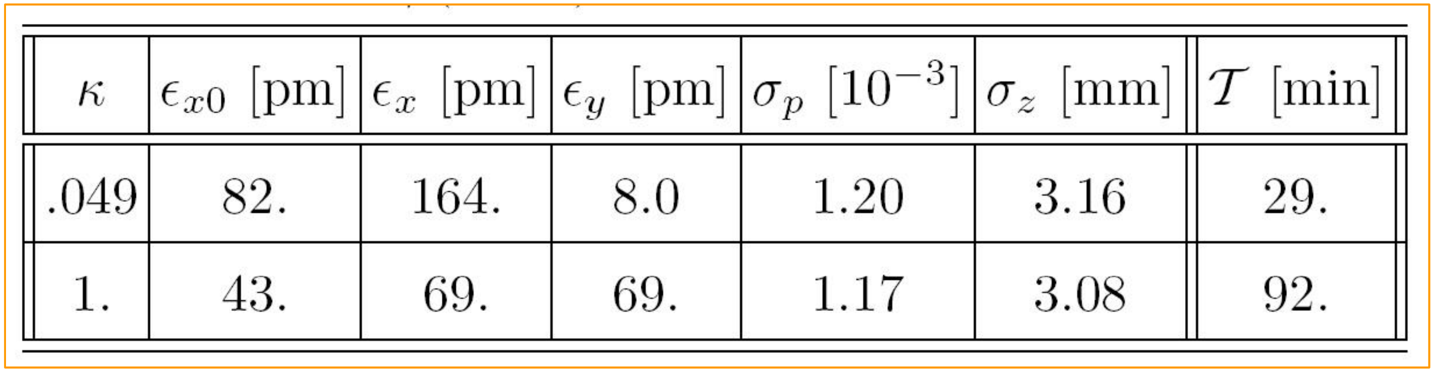
Transverse growth rate:

$$\frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \rangle \frac{1}{T_p} \quad \Longrightarrow \quad \frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \delta(1/T_p) \rangle$$

Valid for a, b<< 1, “high energy approximation”

Solution for PEP-X

- For PEP-X two modes of operation:
  - (1) nominal—adjust κ so that steady-state ε_y= 8 pm (diffraction limited at 1 angstrom)
  - (2) running as FEL in one straight section, so κ= 1



κ	ϵ_{x0} [pm]	ϵ_x [pm]	ϵ_y [pm]	σ_p [10^{-3}]	σ_z [mm]	$\mathcal{T}$ [min]
.049	82.	164.	8.0	1.20	3.16	29.
1.	43.	69.	69.	1.17	3.08	92.

Table. Results for flat and round-beam cases at nominal current I= 1.5 A: coupling parameter, nominal (zero-current) emittance, and steady-state beam properties. The last column gives the Touschek lifetime (discussed below).

- Note: almost no growth in *p* or *z*

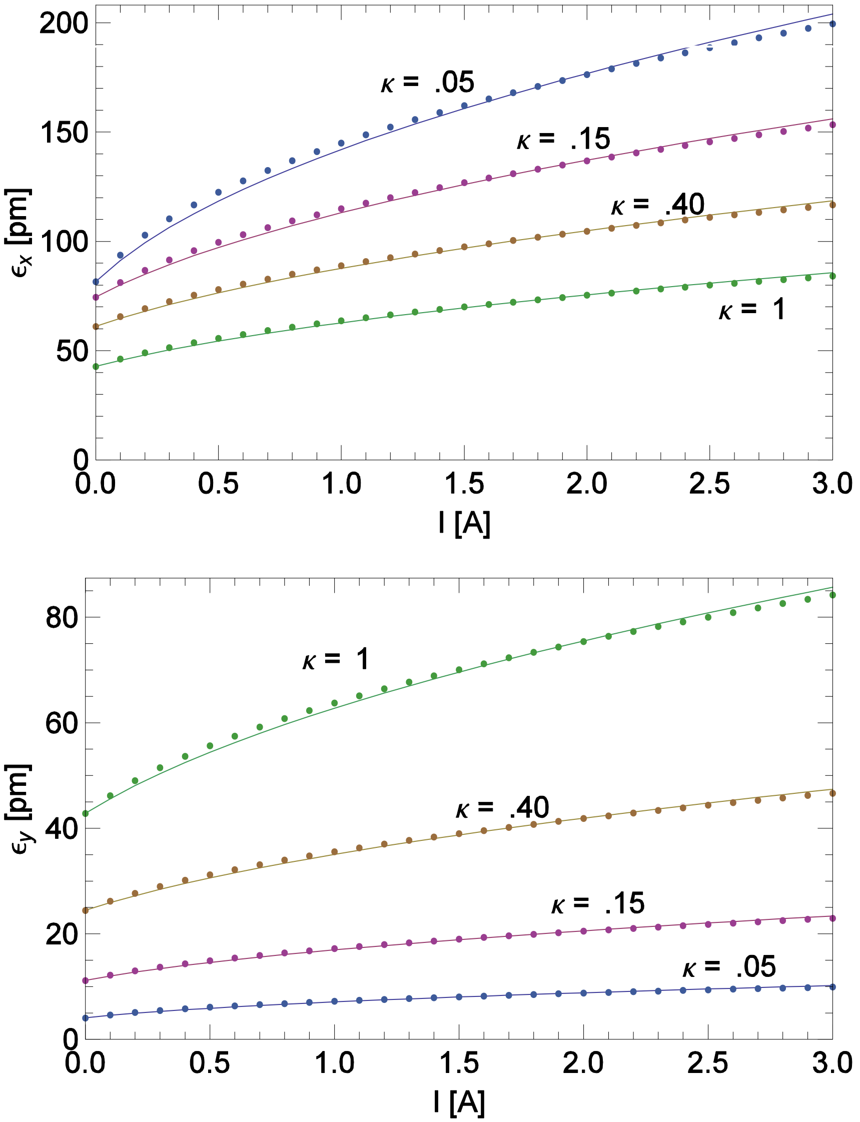
- In nominal configuration T_x^{-1}= 24.7 s^{-1}, T_p^{-1}= 9.5 s^{-1} (simplified model gets T_x^{-1}= 22.9 s^{-1}, T_p^{-1}= 12.0 s^{-1})

- If we assume that σ_p remains unchanged with current , then the simplified model yields a simple equation for emittance as function of current (I_A= 17 kA):

$$\left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{5/2} - \left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{3/2} = \alpha\left(\frac{I}{I_A}\right)$$

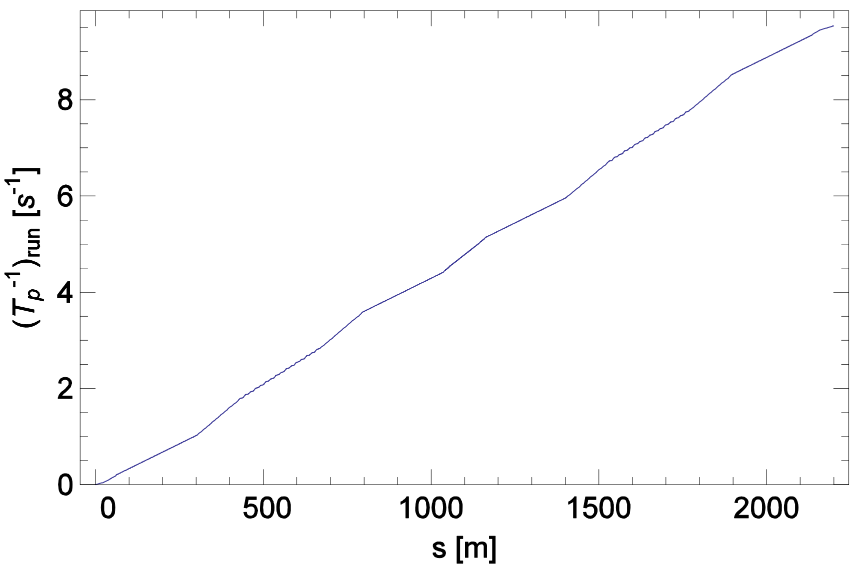
- Fitting to PEP-X BM calculation results, we obtain α= 3570.$(1+\kappa)^{2.08}\kappa^{-.69}$

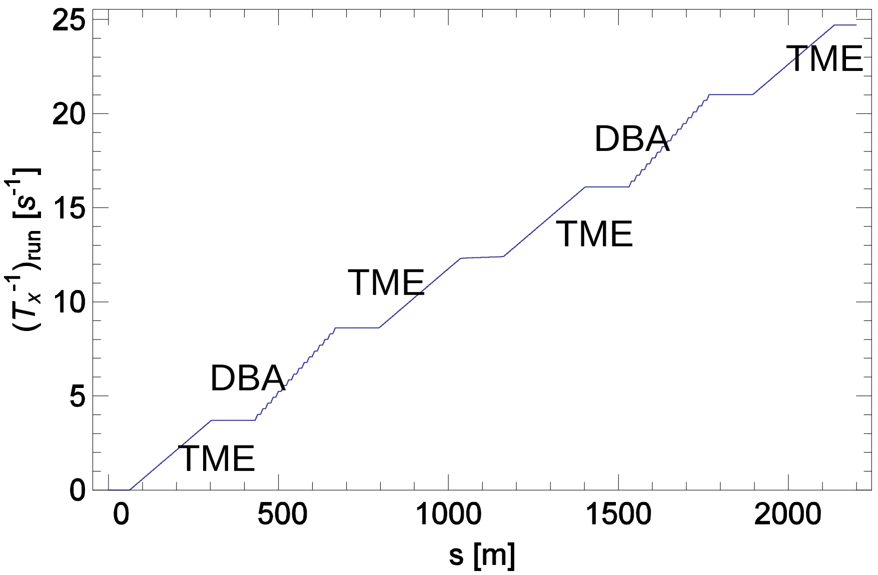
Steady-State Emittance vs Current



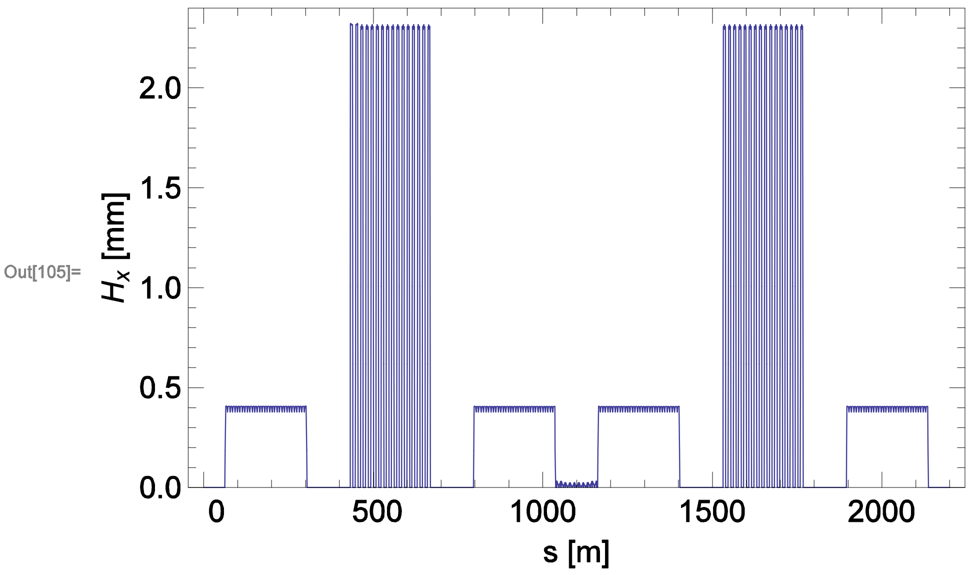
Steady-state emittances as function of current in PEP-X for various couplings. Dots give the solution to the fitted 1d equation.

Accumulated IBS Growth Rates





Accumulated growth rates in p, x;
$\mathcal{H}_x$ optics function



Touschek Lifetime

- Calculation follows method of Brueck, as modified by Piwinski; valid for flat beams

- Number of particles in bunch decays as:

$$N_b = \frac{N_{b0}}{1 + t/\mathcal{T}}$$

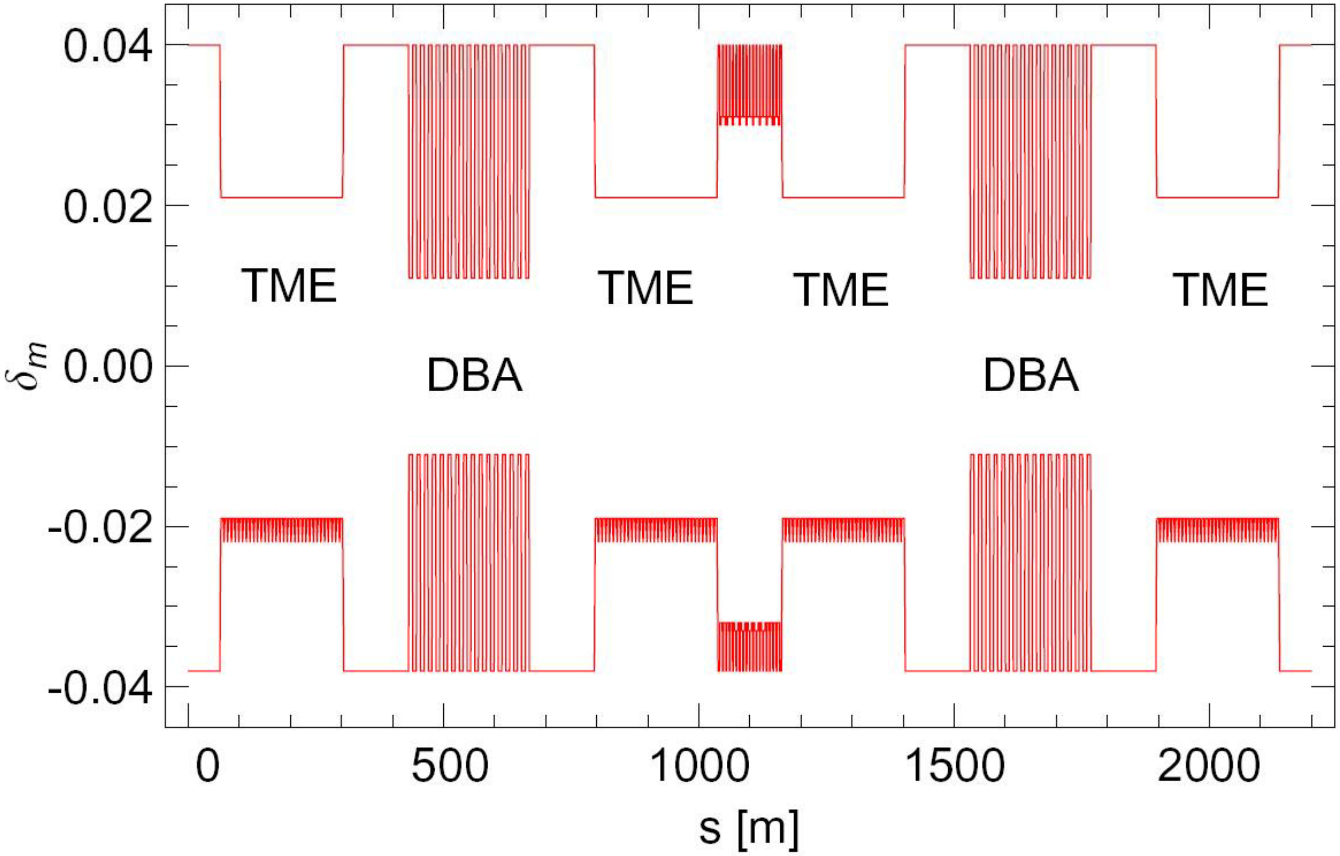
- Inverse of Touschek lifetime (assuming $\varepsilon_y = \kappa\varepsilon_x$) is given by:

$$\frac{1}{\mathcal{T}} = \frac{r_e^2 c N_b}{8\pi\beta^3\gamma^5\sigma_z\kappa^{1/2}\epsilon_x^2}\left\langle \frac{\beta_x^{3/2}\sigma_x^2}{\beta_y^{1/2}\tilde{\sigma}_x^3}\frac{\mathcal{C}(\epsilon_m)}{\epsilon_m}\right\rangle$$

$$\mathcal{C}(\epsilon_m) = -\frac{3}{2}e^{-\epsilon_m} + \int_{\epsilon_m}^{\infty}\left(1 + \frac{3\epsilon_m}{2} + \frac{\epsilon_m}{2}\ln\frac{u}{\epsilon_m}\right)e^{-u}\frac{du}{u}$$

$$\epsilon_m = \frac{\beta_x\sigma_x^2\delta_m}{\gamma^2\epsilon_x\tilde{\sigma}_x^2}$$

(Min-Huey Wang)

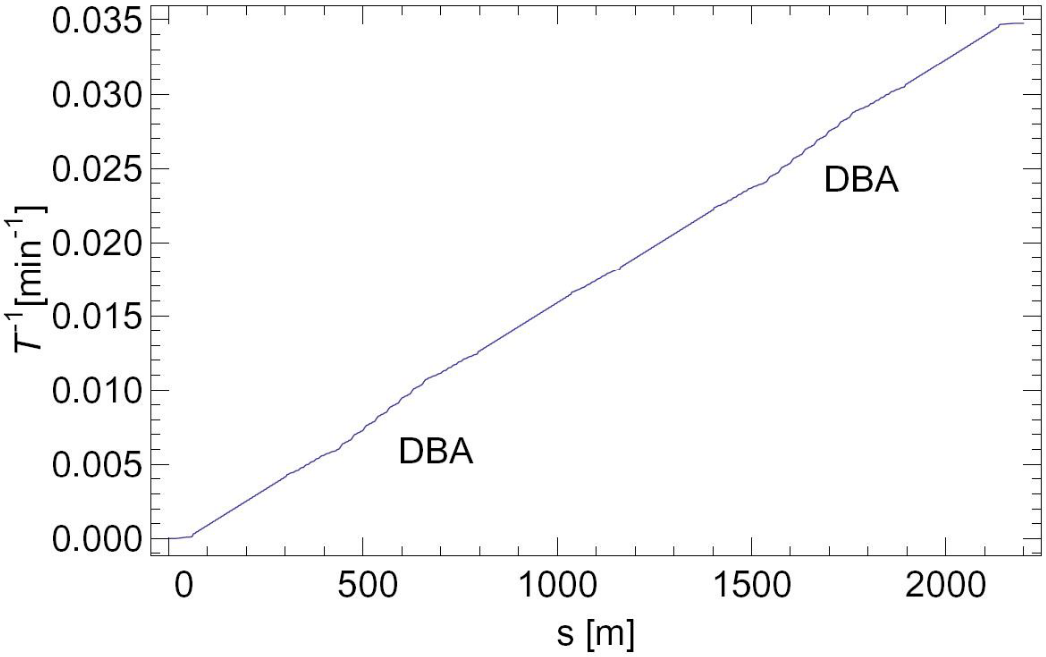


Momentum acceptance due to linear optics for PEP-X. The locations of the TME and DBA arcs are also indicated in the figure.

Touschek Lifetime Results

- Results for the IBS-determined steady-state beam sizes are: T= 29 min (nominal, flat beam), T= 92 min (round beam)

- Results about the same as when taking a fixed aperture $\delta_m = \pm 2\%$

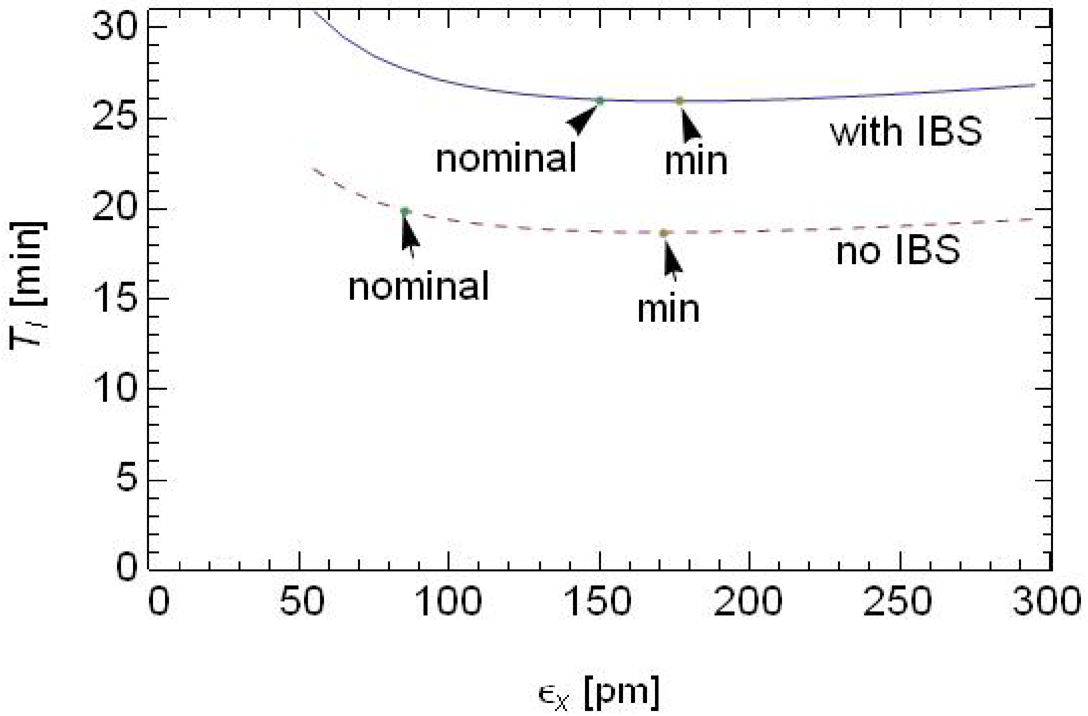


Accumulation around the ring of the Touschek growth rate in the nominal PEP-X configuration.

- Stable top-up injection needed

- If impedance generates potential well bunch lengthening, this will help. Currently the impedance yields a bunch lengthening of 25%, which will increase *T* by about the same amount

- If the emittance becomes small enough, *T* should start increasing again. Can we go to this regime?



Touschek lifetime as function of ε_x, *with* ε_y, σ_p *fixed. This is not a self-consistent calculation. (The lattice is for a slightly earlier version of PEP-X.)*

Conclusions

- Have briefly described impedance/instability effort of our group on PEP-X

- Intra-beam scattering yields ε_x= 165 pm and ε_y= 8 pm in the nominal, flat mode of operation; ε_x= ε_y= 70 pm in the round mode of operation

- The Touschek lifetime is very short, 30 min in the nominal mode; 90 min in the round mode